DESY 95-188

# Charged Current Couplings in $\tau$ Decay

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October 19, 1995

**Abstract**

We discuss the impact of the recent measurements of the Michel parameter in $\tau$ decay. These measurements are used to extract limits on the charged current couplings beyond the Standard Model, first in a model independent way and then with some additional but very general assumptions. The measurements substantiate the Standard Model V-A hypothesis and restrict right handed couplings of the $\tau$. On the other hand, left handed scalar and tensor couplings of the $\tau$ cannot be bounded in the model independent framework with presently available data.

## 1 Introduction

One hundred years ago Becquerel discovered the phenomenon of radioactivity. One of its components, the nuclear $\beta$ decay, led to the assumption and later discovery of the neutrino. A theory for this 'weak interaction' was established by Fermi with the famous coupling constant $G_F$. Later, many particles have been found with similar decay characteristics, and $G_F$ was assumed to be a universal constant. Today, the Standard Model of electroweak interactions is thought to provide a complete description of the weak interaction. To a large extent, the predictive power of the Standard Model is rooted in the very precise value of the Fermi constant, $G_F$, obtained from the measurement of the muon lifetime. Moreover, the detailed study of $\mu$ decay has placed bounds on charged current interactions beyond the Standard Model [1, 2].

It has been pointed out that leptonic decays of the $\tau$ are well suited for further investigation of the charged current structure [3]. First, the two leptonic decay modes $\tau \to \mu\nu_\mu\nu_\tau$ and $\tau \to e\nu_e\nu_\tau$ can be used to test universality. Second, the larger mass of the $\tau$ expands on the $Q^2$ region previously examined in $\mu$ decay, and can be a probe for models with couplings proportional to the lepton mass.

In this paper we interpret the currently available measurements of the Michel parameters [4] in $\tau$ decay in terms of the fundamental parameters of the theory, the coupling constants. We adopt the following philosophy: rather than proving V-A, we ask how much room is left for interactions

outside the Standard Model. We attempt to do this first in a model independent way as outlined in [3], and then with the additional assumptions of universality and factorisation [5].

## 2 Production and Decay of $\tau$ Leptons

The production of $\tau$ lepton pairs in $e^+e^-$ annihilations proceeds via an intermediate photon at low energies and a Z° at LEP energies. At energies above $\approx$ 10 GeV, the decay of the vector bosons forces the helicities of the $\tau^-$ and $\tau^+$ to be almost anti-correlated. Couplings other than V or A in the production, which would lead to a reduction of the anti-correlation, are neglected.[1]

Furthermore, we assume that lepton flavour is conserved, so that the $\tau$'s are produced pairwise and that they decay in association with a $\tau$ neutrino. The neutrinos are treated as massless Dirac particles with two helicities. Given the current upper limits on the neutrino masses [7] any corrections to the Michel parameters from neutrino mass terms are much less than the current experimental accuracy.

## 3 Leptonic Decays

The leptonic decays $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ can be described phenomenologically by the most general, derivative-free, four-fermion contact interaction Hamiltonian. In order to minimize the number of interference terms and to reduce the Standard Model V-A interaction to only one non-vanishing coupling the Matrix element is written in the helicity projection form [3, 8]:

$$\mathcal{M}_\ell = \frac{4G_\ell}{\sqrt{2}} \sum_{\substack{\gamma = S,V,T \\ i,j = L,R}} g^\gamma_{ij} \langle \bar{\ell}_i | \Gamma^\gamma | (\nu_\ell)_m \rangle \langle (\bar{\nu}_\tau)_n | \Gamma_\gamma | \tau_j \rangle, \tag{1}$$

where $G_\ell$ is a constant equivalent to the Fermi constant in $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. This matrix element contains ten complex coupling constants, $g^\gamma_{ij}$, where the type of interaction – $\Gamma^S = 1$ for scalar, $\Gamma^V = \gamma^\mu$ for vector, $\Gamma^T = \frac{1}{\sqrt{2}}\sigma^{\mu\nu}$ for tensor – is labelled by $\gamma$ and the chiral projections of the leptons – left, right – by $i, j$. The helicities $n, m$ of the neutrinos are fixed by $\gamma$ and $i, j$. In the Standard Model V-A interaction the only non-zero coupling constant is $g^V_{LL} = 1$.

Current experiments measure the energy of the final state lepton. Without radiative corrections and neglecting terms of $\mathcal{O}(\frac{m_\ell^2}{m_\tau^2})$ the decay spectrum in the $\tau$ rest frame is given by

$$\begin{aligned} \frac{d\Gamma(\tau^\mp)}{dx\,d\Omega} &= \frac{G_\ell^2 m_\tau^5}{192\pi^4}\, x^2 \Big[ 3(1-x) + \tfrac{2}{3}\rho(4x-3) + 6\eta \tfrac{m_\ell}{m_\tau}\tfrac{1-x}{x} \\ &\quad \mp P_\tau \cos\theta \Big( \xi(1-x) + \tfrac{2}{3}\Delta(4x-3) \Big) \Big], \end{aligned} \tag{2}$$

where $x = \frac{2E_\ell}{m_\tau}$, $P_\tau$ the $\tau$ polarisation, and $\theta$ the angle between the lepton momentum and the $\tau$ spin. The Michel parameters $\rho, \eta, \xi$ and $\Delta^2$ are bilinear combinations of the coupling constants $g^\gamma_{ij}$. A detailed discussion of these parameters is given in the appendix. The number of experimentally accessible Michel parameters in $\tau$ decay is at present still smaller than in $\mu$ decay. This is because the polarisation of the final lepton is not measured and the inverse process $\nu_\tau e \to \tau \nu_e$ has yet to be observed [9].

[1] S,P,T type couplings in the production would in addition change the cross section and the distribution of the production angle. A preliminary investigation of the LEP data shows that these couplings contribute less than 1% to the cross section [6].

[2] We use $\Delta \equiv \delta\xi$.

# 4 Measurements and Interpretation

The first studies of the Lorentz structure in $\tau$ decays were limited to the measurement of the Michel parameter $\rho$. Only recently several experiments succeeded to measure Michel parameters besides $\rho$. Table 1 summarises the current status. In obtaining the average values, the results from the independent experiments are treated, when available, with the full error matrices, also taking

| Experiment | Ref. | $\rho$ | $\rho_e$ | $\rho_\mu$ |
|---|---|---|---|---|
| DELCO (79) | [10] | 0.72 ± 0.15 | 0.72 ± 0.15 | |
| CLEO (85) | [11] | 0.71 ± 0.10 | 0.60 ± 0.13 | 0.81 ± 0.13 |
| MAC (87) | [12] | 0.79 ± 0.14 | 0.62 ± 0.22 | 0.89 ± 0.16 |
| CBAL (89) | [13] | 0.64 ± 0.09 | 0.64 ± 0.09 | |
| ARGUS (95) | [15, 17] | 0.732 ± 0.040 | 0.735 ± 0.041 | 0.734 ± 0.061 |
| ARGUS (95) | [14, 16] | 0.738 ± 0.038 | 0.79 ± 0.10 | 0.76 ± 0.11 |
| ALEPH (95) | [18] | 0.751 ± 0.045 | 0.793 ± 0.056 | 0.693 ± 0.064 |
| L3 (95) | [19] | 0.810 ± 0.049 | | |
| average | | 0.757 ± 0.018 | 0.736 ± 0.028 | 0.735 ± 0.038 |
| | | $\Delta$ | $\Delta_e$ | $\Delta_\mu$ |
| ARGUS (95) | [16] | 0.65 ± 0.12 | | |
| ALEPH (95) | [18] | 0.88 ± 0.13 | 1.11 ± 0.18 | 0.71 ± 0.15 |
| L3 (95) | [19] | 0.73 ± 0.19 | | |
| average | | 0.785 ± 0.072 | 1.11 ± 0.18 | 0.71 ± 0.15 |
| | | $\xi$ | $\xi_e$ | $\xi_\mu$ |
| ARGUS (95) | [16] | 0.97 ± 0.14 | | |
| ALEPH (95) | [18] | 1.18 ± 0.16 | 1.03 ± 0.25 | 1.23 ± 0.24 |
| L3 (95) | [19] | 0.95 ± 0.25 | | |
| average | | 1.07 ± 0.09 | 1.03 ± 0.25 | 1.23 ± 0.24 |
| | | $\eta$ | | $\eta_\mu$ |
| ARGUS (95) | [17] | 0.03 ± 0.22 | | 0.01 ± 0.34 |
| ALEPH (95) | [18] | -0.04 ± 0.19 | | -0.24 ± 0.29 |
| L3 (95) | [19] | 0.10 ± 0.23 | | |
| $\Gamma_\mu/\Gamma_e$ | [20] | 0.01 ± 0.03 | | |
| average | | 0.01 ± 0.03 | | -0.14 ± 0.22 |

error matrix

| | $\rho$ | $\eta$ | $\xi$ | $\Delta$ |
|---|---|---|---|---|
| $\rho$ | .33E-03 | .77E-04 | -.22E-03 | -.17E-03 |
| $\eta$ | .77E-04 | .83E-03 | .14E-03 | .65E-04 |
| $\xi$ | -.22E-03 | .14E-03 | .84E-02 | .27E-03 |
| $\Delta$ | -.17E-03 | .65E-04 | .27E-03 | .52E-02 |

Table 1: Measurements of the Michel parameters in $\tau$ decay. The non-subscripted parameters are obtained by assuming $e-\mu$ universality (which amounts to averaging $\rho_e$ and $\rho_\mu$ for the experiments before 1995. In ARGUS publications from 1995 $\rho$ is determined from a multidimensional fit including the correlations with other Michel parameters while $\rho_e$ and $\rho_\mu$ are obtained from single parameter fits).

into account the hidden parameters, the hadronic chirality $\xi_h \equiv h(\nu_\tau)$ and $P_\tau$[3]. Since [16] and [17] have an approximately 50% overlapping data sample an additional correlation between their $\rho$ values of 0.5 is included.

Our procedure is to determine the 90% CL contours in the multi-dimensional parameter space $(\rho, \eta, \xi, \Delta)$. The probability density distribution is obtained with the full correlation matrix and is always normalised to unity over the physically allowed ranges. From the contours limits on the positive quantities $a^\pm, b^\pm, c^\pm$ (see appendix) are determined, which are then reinterpreted in terms of the coupling constants $g^\gamma_{ij}$. As input we use the average values of the Michel parameters from Table 1 with $e-\mu$ universality. It turns out that the limits show only little influence from the correlations between the parameters.

Despite the fact that the 90%CL contour of the measurements reduces the a priori allowed volume by more than 99%, only weak limits can be set on the coupling constants. The model independent analysis leads to the limits shown in row a) of Table 2. These limits are correlated in the sense that not all couplings can simultaneously achieve their maximum value. The additional assumption of universality of the couplings, i.e., $g^\gamma_{LR} = (g^\gamma_{RL})^*$, leads to row b). Adding the plausible assumption that the interaction is mediated by a superposition of three intermediate bosons, scalar, vector and tensor [5], then the limits of c) apply. Finally, we have considered only a vector boson interaction, i.e., a mixture of V and A, for which we derive $|g^V_{LL}| > 0.90$ and $|g^V_{RR}| < 0.10$.

Assuming that infinitely precise measurements of $(\rho, \eta, \xi, \Delta)$ yield the Standard Model expectations of $(\frac{3}{4}, 0, 1, \frac{3}{4})$ then the model independent limits which can be set are listed in row d).

| | $g^S_{RR}$ | $g^S_{RL}$ | $g^S_{LR}$ | $g^S_{LL}$ | $g^V_{RR}$ | $g^V_{RL}$ | $g^V_{LR}$ | $g^V_{LL}$ | $g^T_{RL}$ | $g^T_{LR}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) | 0.27 | 1. | 0.36 | 1. | 0.28 | 0.52 | 0.28 | 1. | 0.90 | 0.36 |
| b) | 0.20 | 0.21 | 0.21 | 1. | 0.20 | 0.17 | 0.17 | 1. | 0.17 | 0.17 |
| c) | 0.20 | 0.16 | 0.16 | 1. | 0.20 | 0.17 | 0.17 | 1. | 0.12 | 0.12 |
| d) | 0. | 1. | 0. | 1. | 0. | 0.5 | 0. | 1. | 0.86 | 0. |

Table 2: 90%CL upper limits on the absolute values of the normalised coupling constants $g^\gamma_{ij}/\max(|g^\gamma_{ij}|)$: a) model independent b) universality c) universality and boson exchange d) model independent with infinite precise Michel parameters.

## 5 Summary and Outlook

Present measurements of the Michel parameters in $\tau$ decay impose some limits on the Lorentz structure of the charged current. The strongest restrictions apply to right-handed $\tau$ couplings. The limits are shown graphically in Fig. 1 and can be compared with those obtained in $\mu$ decay, Fig. 2. The current type of experiments leave room for non-standard couplings in $\tau$ decay. Even when we assume infinite precision for the measurements of the Michel parameters we do not improve the limits on the left-handed $\tau$ couplings. Thus, the measurements of the polarisation of the final state lepton, or of neutrino correlations, or of the cross section for $\nu_\tau e \to \tau \nu_e$ are needed to prove V-A in the same sense as has been done in $\mu$ decay.

[3]The correlation matrix in [18] needs to be modified[21].

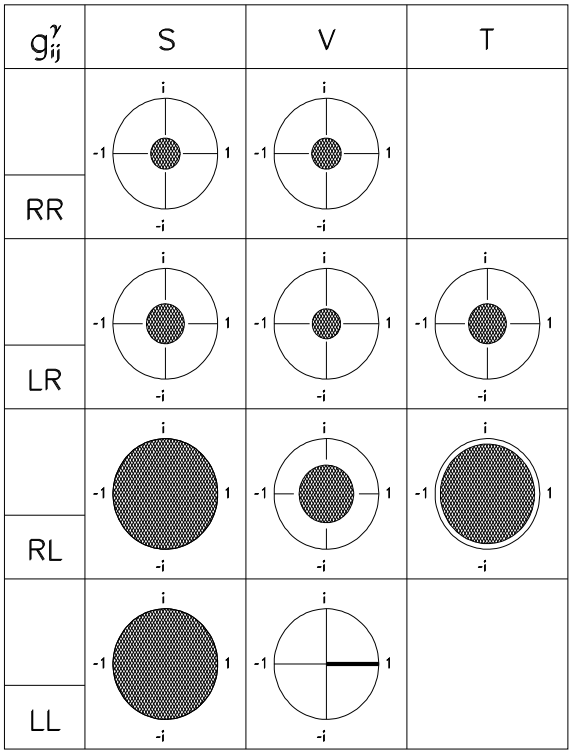


Figure 1: Model independent limits at 90%CL on the normalised coupling constants $g^{\gamma}_{ij}/\max(|g^{\gamma}_{ij}|)$ obtained from the measurements of the Michel parameters in $\tau$ decay.

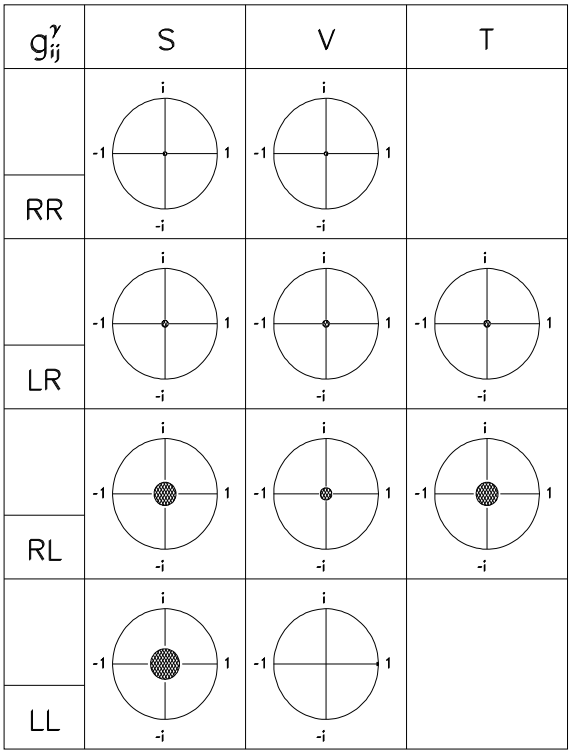


Figure 2: Model independent limits at 90%CL on the normalised coupling constants $g^{\gamma}_{ij}/\max(|g^{\gamma}_{ij}|)$ obtained from the study of $\mu$ decay [22].

# 6 Appendix

Using the notation of [23] we define

$$\begin{aligned} a^+ &= |g^V_{RL}|^2 + \tfrac{1}{16}|g^S_{RL} + 6g^T_{RL}|^2 \\ b^+ &= |g^V_{RR}|^2 + \tfrac{1}{4}|g^S_{RR}|^2 \\ c^+ &= \tfrac{3}{16}|g^S_{RL} - 2g^T_{RL}|^2 \end{aligned} \qquad \begin{aligned} a^- &= |g^V_{LR}|^2 + \tfrac{1}{16}|g^S_{LR} + 6g^T_{LR}|^2 \\ b^- &= |g^V_{LL}|^2 + \tfrac{1}{4}|g^S_{LL}|^2 \\ c^- &= \tfrac{3}{16}|g^S_{LR} - 2g^T_{LR}|^2 \end{aligned} \tag{3}$$

The normalisation condition

$$4(|g^S_{LL}|^2+|g^S_{LR}|^2+|g^S_{RL}|^2+|g^S_{RR}|^2)+16(|g^V_{LL}|^2+|g^V_{LR}|^2+|g^V_{RL}|^2+|g^V_{RR}|^2)+48(|g^T_{LR}|^2+|g^T_{RL}|^2) \equiv 16$$

is equivalent to

$$a^+ + a^- + b^+ + b^- + c^+ + c^- = 1 \tag{4}$$

and shows that $0 \leq a^{\pm}, b^{\pm}, c^{\pm} \leq 1$.

The Michel parameters are then

$$\rho = \tfrac{3}{4}(b^+ + b^-) + (c^+ + c^-) \tag{5}$$

$$\xi = 3(a^- - a^+) + (b^- - b^+) + \tfrac{7}{3}(c^+ - c^-) \tag{6}$$

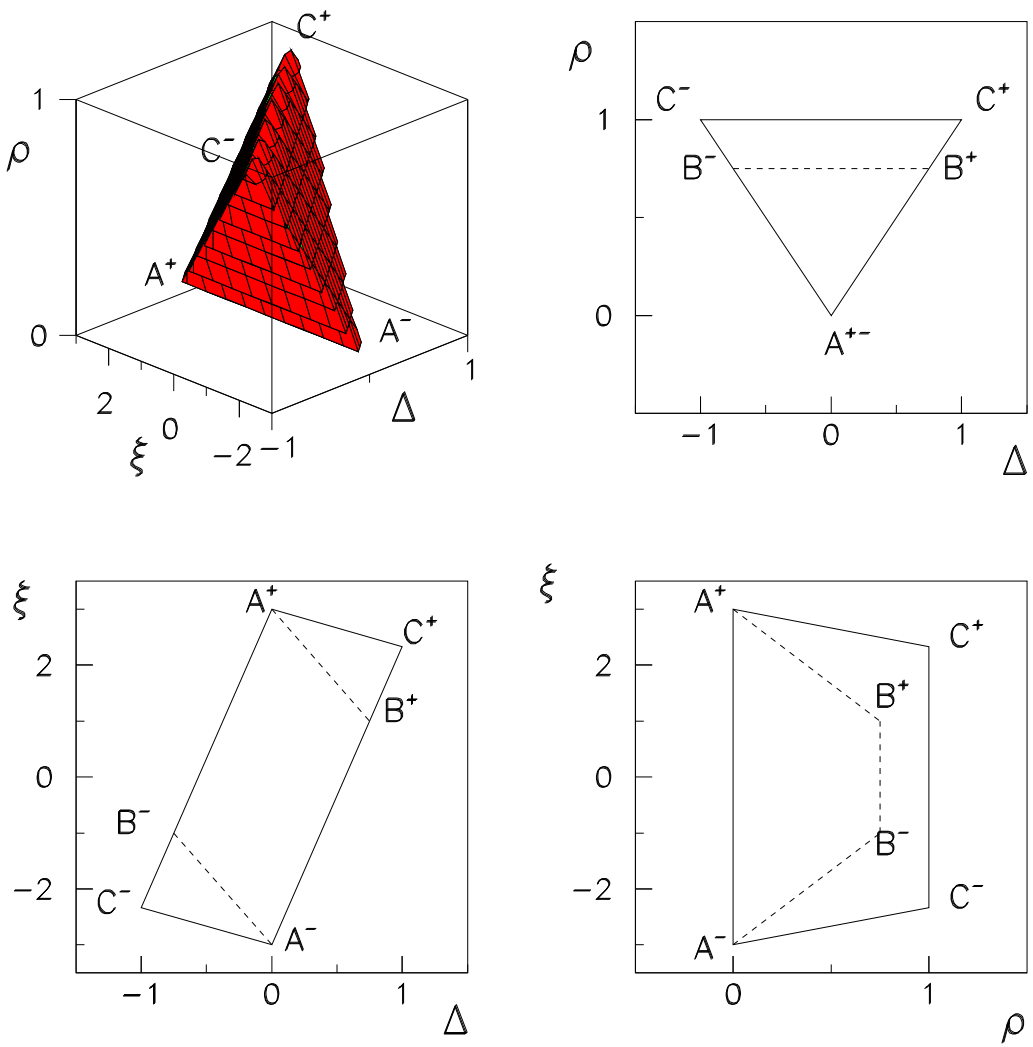


Figure 3: The physically allowed parameter space in $\Delta, \xi, \rho$ is a tetrahedron. The projections in the different planes are also shown.

$$\Delta = \tfrac{3}{4}(b^- - b^+) + (c^+ - c^-) \tag{7}$$

$$\eta = \tfrac{1}{2}\mathcal{R}e\left[g^V_{RL}(g^S_{LR} + g^T_{LR})^* + g^V_{LR}(g^S_{RL} + g^T_{RL})^* + g^V_{RR}g^{S\,*}_{LL} + g^V_{LL}g^{S\,*}_{RR}\right] \tag{8}$$

The physically allowed region can be determined by setting $a, b, c$ to their extreme values, 0 or 1. In the 3-d space $\Delta, \xi, \rho$ one finds the tetrahedron in Fig. 3 with the corners given by $A^\pm = (0, \pm 3, 0)$ and $C^\pm = (\pm 1, \pm\frac{7}{3}, 1)$.

Additional assumptions restrict the parameter space. For example, universality between the $\tau$ and the decay lepton implies $g^\gamma_{LR} = g^{\gamma\,*}_{RL}$, so that $a^+ = a^-, c^+ = c^-$. Therefore the parameters are limited to the intersection of the plane $\Delta = \frac{3}{4}\xi$ with the tetrahedron. It is interesting to note that the combinations V, S+V and V+T have identical parameter ranges which are marked by the points $A^\pm, B^\pm$ in Fig. 3. The Standard Model V-A interaction corresponds to the point $B^+ = (\frac{3}{4}, 1, \frac{3}{4})$.

The range of $\eta$ is $\pm 1$ and $\eta$ is non-zero only when right-handed lepton couplings exist. It is

possible to show analytically that

$$\begin{aligned} |\eta| &\leq 1 - \tfrac{2}{3}\rho \qquad \rho \leq \tfrac{3}{4} \\ &\leq 2(1-\rho) \qquad \rho \geq \tfrac{3}{4} \end{aligned} \tag{9}$$

In determining limits on the couplings useful combinations are

$$\begin{aligned} P_\tau^R &= \tfrac{1}{2}(1 + \tfrac{1}{3}\xi - \tfrac{16}{9}\Delta) \\ &= a^- + b^+ + c^- \end{aligned} \tag{10}$$

$$\begin{aligned} \kappa &= \rho - \Delta \\ &= \tfrac{3}{2}b^+ + 2c^-, \end{aligned} \tag{11}$$

where $P_\tau^R$ is the probability for right-handed $\tau$ couplings. Both of these combinations are greater than or equal to zero, and give upper bounds on $a^-, b^+, c^-$. In addition, $\rho$ sets a weak upper bound on $c^+$.

**Acknowledgements**

One of us (JR) thanks the staff and scientists at DESY-IfH for their support and for providing a very friendly working environment during his brief visits. We thank R. Waldi, S. Theisen and H. Kolanoski for very useful discussions and comments.

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| | $\rho_\ell$ | $(\delta\xi)_\ell$ | $\eta_\ell$ | $\xi_\ell$ | $\xi_h$ |
|---|---|---|---|---|---|
| $p_\tau$ | 0.39 | −0.09 | 0.04 | −0.09 | −0.30 |
| $\rho_\ell$ | 1 | −0.10 | 0.56 | −0.06 | −0.21 |
| $(\delta\xi)_\ell$ | | 1 | 0.10 | 0.05 | 0.26 |
| $\eta_\ell$ | | | 1 | 0.24 | −0.01 |
| $\xi_\ell$ | | | | 1 | 0.17 |

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