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# STANDARD MODEL STABILITY BOUNDS FOR NEW PHYSICS WITHIN LHC REACH *

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**Abstract**

We analyse the stability lower bounds on the Standard Model Higgs mass by carefully controlling the scale independence of the effective potential. We include resummed leading and next-to-leading-log corrections, and physical pole masses for the Higgs boson, $M_H$, and the top-quark, $M_t$. Particular attention is devoted to the cases where the scale of new physics $\Lambda$ is within LHC reach, i.e. $\Lambda \leq 10$ TeV, which have been the object of recent controversial results. We clarify the origin of discrepancies and confirm our earlier results within the error of our previous estimate. In particular for $\Lambda = 1$ TeV we find that

$$M_H[GeV] > 52 + 0.64\ (M_t[GeV] - 175) - 0.50\ \frac{\alpha_s(M_Z) - 0.118}{0.006}.$$

For fixed values of $M_t$ and $\alpha_s(M_Z)$, the error from higher effects, as the lack of exact scale invariance of the effective potential and higher-order radiative corrections, is conservatively estimated to be $\lesssim 5$ GeV.

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# 1 Introduction

The requirement of vacuum stability in the Standard Model (SM) imposes a severe lower bound on the Higgs mass, $M_H$, which depends on the precise value of the top-quark mass, $M_t$, and on the scale $\Lambda$ beyond which the SM is no longer valid [1]. The relationship between the scale of new physics $M$ [the mass of new particles or resonances] which can stabilize the effective potential, and the instability scale $\Lambda$ has recently been studied in Ref. [2]. The conclusion is that $M$ can be a few times $\Lambda$ provided that new physics is strongly coupled. However, if new physics is weakly coupled, consistently with the idea of Grand [or String] Unification, then we expect that [1] $\Lambda \sim M$. In this case it is of the utmost interest to study scenarios with $\Lambda$ scales below roughly 10 TeV since they correspond to cases where new (weakly interacting) physics should be produced at LHC. This is stressed by the fact that the typical lower bounds on the Higgs mass in these scenarios lie precisely around the range accessible to LEP2.

Updated stability bounds have been presented in Refs. [3, 4], AI and CEQ bounds, respectively. Both papers agree for large values of $\Lambda$ [i.e. $\Lambda = M_P$ or even several orders of magnitude smaller, depending on the cases], while they differ substantially for low values of $\Lambda$. In particular, for $\alpha_s(M_Z) = 0.118$, $M_t = 175$ GeV and $\Lambda = 10^{19}$ GeV, AI quote $M_H > 137$ GeV and CEQ $M_H > 133$ GeV, a difference well within the theoretical errors. However, for the same value of $M_t$ and $\Lambda = 1$ TeV, AI quote $M_H > 73$ GeV while CEQ give a much lower bound $M_H > 55$ GeV, a substantial difference which cannot be absorbed in the quoted errors of the two calculations. Notice that the region of discrepancy lies precisely in the region of interest for LHC prospects as mentioned above.

Very recently, the lower stability bounds on the SM Higgs mass have been reconsidered [5, 6]. The discrepancy between AI and CEQ results has been claimed [6] to be explained by the non inclusion in Ref. [4] of the Higgs-Yukawa sector contributions (more precisely, tadpole contributions) in the relation between the top-quark pole and $\overline{\rm MS}$ running masses.

In this letter we will show that the claim in Ref. [6] is incorrect because finite electroweak tadpoles are automatically included in the treatment of Ref. [4]. As we will see, the discrepancy between AI and CEQ bounds for low values of $\Lambda$ can be traced back to the more accurate description of the effective potential in Ref. [4], in particular with the inclusion of one-loop effects. For large values of $\Lambda$, which correspond to the region of maximum concern in Refs. [3, 4], these effects are in fact negligible and both calculations are in agreement. However, these one-loop effects become relatively important for low values of the instability scale and, by removing them, we will precisely recover the AI bounds. Moreover, a detailed analysis of the scale independence of the effective potential provides a bound somewhat lower than in our previous analysis. This makes, for the same value of $M_t$ and $\Lambda$ as above, the lower bound to decrease to $\sim 52$

[1] A good example is the case of the Minimal Supersymmetric Standard Model where the scalar particles that can stabilize the effective potential are the third generation squarks, with a multiplicity N=12 and a coupling [in the notation of Ref. [2]] $\delta = h_t^2/2$. For $m_t \lesssim 190$ GeV we obtain $\delta \lesssim 0.6$, which corresponds from Fig. 2 of [2] to $M \lesssim \Lambda$.

GeV. This procedure will also allow a reliable estimate of the various theoretical errors (coming from the lack of scale invariance of the effective potential, the non-considered two-loop corrections, the gauge and renormalization scheme dependence of the result, etc.) involved in the calculation. All these precise estimates will be essential if a Higgs boson with Standard Model properties is found at LEP2 (with a mass $\lesssim 90$ GeV) and will concern the need of new physics at LHC.

## 2 Higgs and top masses in the standard vacuum

It has been shown [7] that the one-loop effective potential improved by two-loop renormalization group equations resums the next-to-leading-log contributions. To this order of approximation, the SM effective potential can be written in the 't Hooft-Landau gauge and the $\overline{\rm MS}$ renormalization scheme as

$$V_{\rm eff} = V_0 + V_1, \tag{1}$$

where the tree-level, $V_0$, and one-loop, $V_1$, terms are given by:

$$V_0 = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{8}\lambda(t)\phi^4(t), \tag{2}$$

$$V_1 = \sum_i \frac{n_i}{64\pi^2} M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2(t)} - C_i\right] \; + \; \Omega(t), \tag{3}$$

with [2] $i = W, Z, t$, $M_i^2 = \kappa_i\phi^2(t)$, and

$$C_W = C_Z = \frac{5}{6}, \quad C_t = \frac{3}{2},$$

$$n_W = 6, \quad n_Z = 3, \quad n_t = -12,$$

$$\kappa_W = \frac{1}{4}g^2(t), \quad \kappa_Z = \frac{1}{4}[g^2(t) + g'^2(t)], \quad \kappa_t = \frac{1}{2}h^2(t).$$

In the previous expressions the parameters $\lambda(t)$ and $m(t)$ are the SM quartic coupling and mass, and $g(t)$, $g'(t)$ and $h(t)$ are the $SU(2)$, $U(1)$ and top Yukawa couplings, respectively. The running of the Higgs field is $\phi(t) = \xi(t)\phi_c$, $\phi_c$ being the classical field, and $\xi(t) = \exp\{-\int_0^t \gamma(t')dt'\}$ where $\gamma(t)$ is the Higgs anomalous dimension. The scale $\mu(t)$ is related to the running parameter $t$ by $\mu(t) = M_Z\exp(t)$. Finally, $\Omega(t)$ is the (field-independent) one-loop contribution to the cosmological constant [in particular we set it to $\Omega(t=0) = 0$], which as we will see is irrelevant for the results of the present work.

From here on the procedure to fix the standard electroweak minimum and the pole masses for the top-quark and the Higgs boson is that specified in Refs. [4, 8]. For the sake of the discussion we will summarize the main points here. The scale

[2] The contribution from Higgs and Goldstone bosons can be easily incorporated, though it is numerically irrelevant as we have checked.

independence of the effective potential allows fixing the renormalization scale $\mu(t)$ at will for different values of the field [7]. Actually, the scale-invariance properties of $V$ permit to perform the substitution either before or after taking the derivative $\partial^n V/\partial\phi^n$, with equivalent results [8], which in turn allows to ignore $\Omega$ for this task. On the other hand, although the whole effective potential is scale-invariant, the one-loop approximation is not. Therefore, one needs a criterion to choose the appropriate renormalization scale in the previous equations. As was shown in [8] a sensible criterion is to choose as the optimal scale the value $\mu^* = \mu(t^*)$ where the potential is more scale-independent. This issue was carefully examined in Ref. [8], where $\mu(t^*)$ was shown to be close to the top mass [its detailed value, however, is not very important because of the high degree of scale independence of the one-loop potential around $\mu(t^*)$]. Then, we minimize the potential (1) at the scale $\mu(t^*)$. For the sake of the discussion of the tadpole contribution, we next consider two (equivalent) ways to do this. The first one is to define the tree-level vacuum expectation value (VEV) by means of the condition $\partial V_0(\phi, t^*)/\partial\phi|_{\phi=\langle\phi(t^*)\rangle_0} = 0$, i.e.

$$\langle\phi(t^*)\rangle_0^2 = \frac{2m^2(t^*)}{\lambda(t^*)}. \tag{4}$$

In this case one-loop corrections from (3) shift the VEV (4) as

$$\langle\phi(t^*)\rangle^2 = \langle\phi(t^*)\rangle_0^2 + \delta\phi^2, \tag{5}$$

where (leaving apart gauge corrections)

$$\delta\phi^2 = -\frac{2}{\lambda\langle\phi\rangle}\frac{\partial V_1}{\partial\phi} \sim \frac{h_t^4}{\lambda}\langle\phi\rangle^2, \tag{6}$$

which amounts diagrammatically to the contribution of the one-loop tadpoles. This correction is large for low Higgs masses and has to be taken into account when relating $\langle\phi(t^*)\rangle_0^2$ to physical observables like $G_\mu$.

Another possibility [4] is to define the one-loop VEV by means of the condition $\partial V_{\rm eff}(\phi, t^*)/\partial\phi|_{\phi=\langle\phi(t^*)\rangle} = 0$, which can be expressed as

$$m^2(t^*) = \frac{1}{2}\lambda(t^*)\langle\phi(t^*)\rangle^2 - \frac{3}{16\pi^2}h_t^4(t^*)\langle\phi(t^*)\rangle^2\left[\log\frac{h_t^2(t^*)\langle\phi(t^*)\rangle^2}{2\mu^2(t^*)} - 1\right] + \cdots \tag{7}$$

where the ellipsis refers to gauge corrections. Diagrammatically this procedure amounts [9, 10] to a cancellation between the bare one-point vertex and the one-loop tadpole contribution. In other words, tadpoles are absorbed in the one-loop VEV and will never appear (except if we desired to compute quantities from $V_0$). This is the procedure followed in Ref. [4] and the procedure we will adopt here. We now impose the physical condition on the VEV at $t = t^*$:

$$\langle\phi(t^*)\rangle = \xi(t^*)\ v, \tag{8}$$

where $v = (\sqrt{2}G_\mu)^{-1/2} = 246.22$ GeV.

Then the running of $m_H^2 \equiv \partial^2 V_{\rm eff}/\partial\phi^2$ and the $\overline{\rm MS}$ renormalized top-quark mass, $m_t$, are determined by [8]

$$\begin{aligned} m_H^2[\mu(t)] = & \left.\frac{\xi^2[\mu(t^*)]}{\xi^2[\mu(t)]}\,\frac{\partial^2 V_{\rm eff}}{\partial\phi^2[\mu(t^*)]}\right|_{\phi[\mu(t^*)]=\langle\phi[\mu(t^*)]\rangle}, \\ m_t[\mu(t)] = & \frac{1}{\sqrt{2}} h[\mu(t)]\xi[\mu(t)]v\ , \end{aligned} \tag{9}$$

and the pole masses $M_H$ and $M_t$ by [4, 11]

$$\begin{aligned} M_H^2 = & \ m_H^2[\mu(t)] + {\rm Re}\left[\Pi_{HH}(p^2 = M_H^2) - \Pi_{HH}(p^2 = 0)\right], \\ M_t = & \left[1 + \frac{4}{3}\frac{\alpha_s(M_t)}{\pi}\right] m_t[M_t]. \end{aligned} \tag{10}$$

It is clear that the effect of tadpole corrections is automatically included in both formulae.

We can now comment on the errors associated with the estimates of the pole masses (10). As for $M_t$, one-loop electroweak corrections (besides the tadpole ones) amount [9] to $\sim +1\%$, and the unconsidered two-loop QCD corrections [11] amount to $\sim -1\%$, so they almost cancel. In this way a conservative estimate of the total error is $\sim 1\%$, i.e. $\Delta M_t \lesssim 2$ GeV. As for $M_H$, the lack of scale invariance is a measure of the error associated with the unconsidered corrections. This was studied in Ref. [4], where an uncertainty of $\Delta M_H \lesssim 2$ GeV was conservatively estimated.

It is clear from the above discussion that the claim in Ref. [6], where it is argued that the discrepancy between AI and CEQ results comes from the unconsidered large tadpole corrections in Ref. [4], is incorrect. In fact, as we have explained in this section, the large tadpole corrections are absorbed in the one-loop VEV and should be nowhere in the calculation relating the pole and $\overline{\rm MS}$ running masses, either for the top-quark or the Higgs boson. The reason for the discrepancy should be traced back to the different approximations used in the effective potential to compute the bounds in Refs. [3] and [4], and the large uncertainty due to the lack of scale invariance in the approximation of Ref. [3], as we will explain in the next section.

# 3 The lower bound on $M_H$ and the origin of the discrepancy

It is well known that for certain values of the top-quark and Higgs boson masses the effective potential (1) develops an instability, i.e. the potential becomes deeply negative, for large values of the field [1]. Eventually, the potential raises again yielding a (deep) non-standard minimum, although this may happen for values of the field beyond the Planck scale. To ensure that the electroweak minimum is the deepest one, the SM should be cutoff at a value $\Lambda$ of the field such that the depth of the potential equals the depth of the standard electroweak minimum. For a fixed value of $\Lambda$ and $M_t$ this provides a lower bound on $M_H$ such that the latter condition is barely fulfilled. This

procedure was recently refined in Refs. [3, 4] where, as mentioned in the introduction, agreement was found, within the quoted errors, for large values of $\Lambda$, while a large discrepancy, impossible to reconcile within the quoted errors, remains at low values of $\Lambda$. In this section we will explain the origin of the discrepancy, reproduce the results of Ref. [3] and refine our previous analysis by controlling the scale invariance of the result.

The effective potential (1) improved by two-loop renormalization group equations is highly scale independent [8]. This allows fixing the renormalization scale as $\mu(t) \sim \phi(t)$ in order to tame potentially dangerous logarithms at large values of the field [7] (where the instability is expected to appear). In particular, fixing

$$\mu(t) = \alpha\phi(t) \ , \tag{11}$$

allows to translate the scale-independence of the (whole) effective potential into the $\alpha$ independence. Now, similarly to the procedure followed in Ref. [8] to determine $t^*$ for the standard electroweak minimum, we can find out the optimum value of $\alpha$ ($\alpha^*$ in what follows) to study the instability region using the one-loop approximation to the potential [Eq. (1)] and thus evaluating a more precise lower bound on $M_H$. The value of $\alpha^*$ will be that for which the results are more scale-invariant [3].

Using (11) we can write the potential (1) as

$$V_{\rm eff} = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{8}\lambda_{\rm eff}\phi^4(t) + \Omega(t) \tag{12}$$

where

$$\lambda_{\rm eff}(t) = \lambda(t) + \sum_i \frac{n_i}{8\pi^2}\kappa_i^2\left[\log\frac{\kappa_i}{\alpha^2} - C_i\right] . \tag{13}$$

and $t = \log[\mu(t)/M_Z]$ is fixed by Eq. (11). The value of the scale $\Lambda$ where new physics has to stabilize the SM potential is given by the value of the field $\phi$ where the depth of the potential equals the depth of the potential at the standard electroweak minimum. In practice, due to the steepness of the potential around that point, we can identify $\Lambda$ with the value of the field where the potential vanishes, i.e.

$$V_{\rm eff}(\phi)|_{\phi=\Lambda} = 0 \ \ , \tag{14}$$

which can be written, using (12) as

$$\left[\lambda_{\rm eff} - 4\frac{m^2}{\Lambda^2} + 8\frac{\Omega}{\Lambda^4}\right]\Bigg|_{\mu=\alpha\Lambda} = 0 \ \ , \tag{15}$$

Since $\Omega$ obeys the one-loop RG equation [7] $8\pi^2 d\Omega/dt = m^4(t)$, it is clear that for $\Lambda \gtrsim 1$ TeV its contribution to (15) is $\ll 1$ [4], and thus can be compensated by a shift

[3] Again, the precise value of $\alpha^*$ is not very important due to the high degree of scale independence of the one-loop potential around $\alpha^*$, by the very definition of the latter. In Ref. [4] the reasonable choice $\alpha = 1$ was made from the very beginning.

[4] The order of magnitude of $m(0)$ is provided by the tree-level result $m^4 \sim M_H^4/4$. Then, using the boundary condition $\Omega(0) = 0$ we obtain $\Omega(\Lambda) \sim M_H^4/(32\pi^2)\log(\Lambda/M_Z)$.

on $\lambda$ of the same magnitude, with negligible consequences for the value of $M_H$. Hence, the presence of $\Omega$ can be safely ignored. The $-4m^2/\Lambda^2$ term in (15) can have a small effect for low cut-offs being also negligible for larger ones. We include its effect in the numerical calculations although it is a very good approximation to write (15) as $\lambda_{\rm eff} \simeq 0$ for all values of the cut-off. In particular this will be assumed below when deriving some helpful analytical formulae.

In Fig. 1 we plot [thick solid line] the bound on $M_H$ as a function of $\alpha$, for $\Lambda = 1$ TeV, $\alpha_s(M_Z) = 0.118$ and $M_t = 175$ GeV, from the condition (15). Note that (15) depends on $\alpha$ through Eq. (11). We see the mild dependence of $M_H$ on $\alpha$, while the bound is flatter for the value $\alpha^* \sim 0.4$. This behaviour was qualitatively expected from the analysis of Ref. [8] where the role of $\alpha^*$ here was played there by the scale $t^*$ at the standard electroweak minimum. In the same way that $t^*$ was an average of the masses appearing in the one-loop corrections to the potential such that the effect of these corrections was cancelled, we have checked that $\alpha^*$ here can be estimated analytically with high precision as the value of $\alpha$ that cancels the one-loop corrections in Eq. (13), which is conceptually quite satisfactory. We recover, for $\alpha = 1$, the bound $M_H \gtrsim 55$ GeV, in agreement with our previous result in Ref. [4]. Nevertheless, in the region where the bound is more scale independent [i.e. around $\alpha = \alpha^*$] we find $M_H \gtrsim 52$ GeV, which we consider as the most reliable value. In particular, for $\alpha^*/2 \leq \alpha \leq 2\alpha^*$ we find $\Delta M_H \lesssim 2$ GeV, which is a conservative estimate of the error [5].

We are now ready to discuss the origin of the discrepancy between the AI and CEQ results. As we will see, the AI results are essentially equivalent to a calculation based on the tree level potential improved by RG evolution at two-loops and thus resums all-loop leading and part of next-to-leading-logs. Our results are obtained using the full one-loop potential with parameters running also at two-loops. This approximation for the potential resums all-loop leading and next-to-leading-logs and exhibits a high degree of scale invariance, as was discussed in Ref. [8]. In fact, as shown there, the tree-level (leading-log) approximation exhibits a strong scale dependence so that only provides a good approximation after a judicious choice of the scale. Actually, we will explicitly show that the results in Ref. [3] are highly scale dependent at low $\Lambda$ and can be made to coincide with the present results for a particular value of the scale. Let us see this in somewhat more detail.

The scale $\Lambda$ is defined as the value of the $\phi$-field at which the potential $V$ becomes negative. In principle, to study the value of $V$ at $\phi = \Lambda$ any value of the renormalization scale, $\mu$, could be used since the complete scalar potential is exactly scale invariant. But, when a perturbative approximation for the potential is used, exact scale invariance is lost and a convenient scale must be carefully chosen. Although the authors of Ref. [3] write in principle the complete one-loop potential, their criterion, in practice, to identify $\Lambda$ as the value of the scale at which $\lambda$ crosses zero, namely $\lambda(\Lambda) = 0$, is essentially equivalent to use $\mu = \Lambda$ [i.e. $\alpha = 1$ in Eq. (11)] and require $V_0(\phi = \Lambda) = 0$; or, in other words, to ignore [6] the one-loop corrections to $\lambda_{\rm eff}$ given by (13). However, although it

[5] Of course, going to too large or small values of $\alpha$ increases the size of logarithms in the perturbative expansion, and makes perturbation theory unreliable. This phenomenon, expected from the point of view of perturbation theory, was already observed with respect to the $t^*$ dependence in Ref. [8].

[6] As we will see, this is in fact a good approximation for large values of $\Lambda$, but not for the low $\Lambda$

is clear that an appropriate scale to evaluate the potential should be of the same order as the value of the field in which we are interested, there is no reason to expect them to be exactly equal. This consideration is particularly relevant for the AI results since the (RG improved) potential $V_0$ is strongly dependent on the renormalization scale, and so are the corresponding results on $M_H$. This can be seen from Fig. 1 where we have plotted the corresponding bound on $M_H$ as a function of $\alpha$, i.e. the renormalization scale [thin solid line]. As it is clear from the figure, in this approximation the bound is, as expected, strongly dependent on $\alpha$. For $\alpha = 1$ we obtain the bound $M_H \gtrsim 72$ GeV, which is the AI bound quoted in Ref. [3]. For $\alpha \sim \alpha^*$, the two approximations coincide since, as mentioned above, for that scale the radiative corrections to $V_0$ are essentially cancelled [7]. Therefore, for a given value of $M_H$ the value for $\Lambda$ obtained by CEQ and the one obtained by AI (say $\Lambda_0$) are related by

$$\Lambda_0 \equiv \alpha^* \Lambda = \exp\left\{\Delta\lambda(\Lambda_0)/\beta_\lambda(\Lambda_0)\right\} \Lambda \ , \tag{16}$$

where we have used the scale invariance of the effective potential to obtain the second equality, and [8]

$$\begin{aligned} \Delta\lambda(\Lambda_0) = & \sum_i \frac{n_i}{8\pi^2}\kappa_i^2 \left[\log \kappa_i - C_i\right] \\ \beta_\lambda(\Lambda_0) = & \frac{1}{16\pi^2}\left[-12h_t^4 + \frac{9}{4}\left(g^4 + \frac{2}{3}g^2 g'^2 + \frac{1}{3}g'^4\right)\right] . \end{aligned} \tag{17}$$

For illustrative purposes we have plotted in Fig. 2 the effective potential (12) [thick solid line], for $\alpha = \alpha^*$, $\Lambda = 1$ TeV, $\alpha_s(M_Z) = 0.118$, $M_t = 175$ GeV and $M_H = 52$ GeV. We see that the value of the field $\phi$ where the potential satisfies condition (14) and where the value of the potential equals the depth of the standard electroweak minimum are almost indistinguishable, as anticipated. We also plot $\lambda_{\rm eff}(\alpha^*\phi)$ [thick dashed line] as a function of the field and subject to the boundary condition $\lambda_{\rm eff}(\alpha^*\Lambda) = 0$. As expected, $\lambda_{\rm eff}$ crosses zero exactly at the value of the field $\phi$ where the potential itself crosses zero so that it keeps track perfectly of the potential destabilization. The plot of $\lambda(\phi)$ [thin dashed line] crosses zero at the scale $\Lambda_0 \sim 370$ GeV, in good agreement with Eq. (16). In other words, had we used the tree-level condition $\lambda(\Lambda) = 0$, we would have obtained the same value for the Higgs mass lower bound, i.e. 52 GeV, but for the scale $\sim 370$ GeV.

Let us now discuss why the AI results are in good agreement with the CEQ ones for large values of $\Lambda$. This can be intuitively understood from the very small dependence of the $M_H$ bound on $\Lambda$ for large $\Lambda$ (see either Refs. [3] or [4]). Then, it is clear that the uncertainty derived from the choice of the scale in the AI approximation becomes very small and hence AI and CEQ results get agreement. A more precise way to understand the agreement is to note that $\Delta\lambda(\Lambda_0)$ gets reduced for large scales because the top Yukawa coupling runs to smaller values at high scales and, besides, there is a

regime, in which we are concerned here.

[7] A similar *coincidence* was also observed in Ref. [8] concerning the scale $t^*$.

[8] Notice that $\Delta\lambda(\Lambda_0)$ has a clear interpretation. If we fix the stability scale $\Lambda$ and define two different boundary conditions (Higgs masses) for $\lambda$: the AI boundary condition [i.e. $\lambda_{\rm AI}(\Lambda) = 0$] and the CEQ boundary condition [i.e. eq.(15), or equivalently $\lambda_{\rm CEQ}(\Lambda_0) \simeq 0$]; then, $\Delta\lambda(\Lambda_0) \simeq \lambda_{\rm AI}(\Lambda_0) - \lambda_{\rm CEQ}(\Lambda_0)$.

cancellation between the top Yukawa and gauge effects [9] in $\Delta\lambda(\Lambda_0)$. On top of that, the fact that $M_H$ is larger for higher scales contributes to the coincidence between AI and CEQ results. Therefore, for a given discrepancy $\delta M_H^2$, the effect in $\delta M_H \sim \delta M_H^2/2M_H$ is suppressed as $M_H$ grows.

To illustrate these effects we present in Table 1 the values of $\alpha^*$ [i.e. $\Lambda_0$], the bounds on $M_H$, and the couplings $g$, $g'$ and $h_t$ evaluated at the scale $\Lambda_0$ for two different sets of parameters: the case discussed in Fig. 2, $\Lambda = 1$ TeV, and the case of a high value of $\Lambda$, in particular $\Lambda = 10^{19}$ GeV.

| $\Lambda$[GeV] | $\Lambda_0$[GeV] | $M_H$[GeV] | $g'$ | $g$ | $h_t$ |
|---|---|---|---|---|---|
| $10^3$ | 370 | 52 | 0.358 | 0.643 | 0.912 |
| $10^{19}$ | $3.6 \times 10^{17}$ | 134 | 0.457 | 0.514 | 0.414 |

Table 1

The squared mass difference between AI and CEQ calculations can be approximated by (see footnote 7)

$$\delta M_H^2 = \Delta\lambda(M_t)v^2 = \Delta\lambda(\Lambda_0)v^2 + \cdots \tag{18}$$

where $\Delta\lambda(\Lambda_0)$ is defined in Eq. (17) and the ellipsis stands for the renormalization of $\Delta\lambda$ from $\Lambda_0$ to $M_t$, which is a small effect in all cases. Then, using Table 1 and Eq. (17) one can obtain, disregarding radiative corrections as in (18), for $\Lambda = 1$ TeV, $\delta M_H^2 \sim (60\ \text{GeV})^2$, and for $\Lambda = 10^{19}$ GeV, $\delta M_H^2 \sim (10\ \text{GeV})^2$, which explains qualitatively the agreement (disagreement) between AI and CEQ for large (small) values of $\Lambda$.

To close the discussion on the bound for large cut-offs we would also like to comment on the results of Ref. [5]. Although it is clear that the recipe given in that paper to compute the bound is exactly the same used by AI, somewhat larger bounds are found for the case of $\Lambda = M_P$. Part of this effect can be explained by the fact that [5] integrates one-loop RGEs while AI is using two-loop equations instead. As is well known [3], this will result in an overestimated bound, the effect being more important the longer the running is.

# 4 Detailed results and estimate of errors

In Fig. 3 we have plotted the lower bounds on $M_H$ based on condition (15) [solid lines], for $\Lambda = 1$ TeV and $\alpha_s(M_Z) = 0.118 \pm 0.006$, as functions of $M_t$. A very accurate fit (with an error below 1 GeV), is given by

$$M_H[GeV] > 52 + 0.64\ (M_t[GeV] - 175) - 0.50\ \frac{\alpha_s(M_Z) - 0.118}{0.006}. \tag{19}$$

For the sake of comparison we also plot [dashed line] the corresponding bound based on the condition $\lambda(\Lambda) = 0$, for $\alpha_s(M_Z) = 0.118$. We can see that its prediction agrees well with AI bounds in Ref. [3]. In Fig. 4 we plot the lower bounds based on our condition

[9] The value of $\beta_\lambda$ is also much smaller for the same reasons, producing the above mentioned insensitivity of $M_H$ to the value of $\Lambda$ for high $\Lambda$.

(15) as functions of $\Lambda$ for values $M_t = 150$, 175 and 200 GeV, and $\alpha_s(M_Z) = 0.118$. For a fixed value of $M_t$ the corresponding curve provides an upper bound on the scale of new physics necessary to stabilize the SM potential. This bound can be read from Fig. 4 as a function of $M_H$. A very good fit is provided by

$$\log \Lambda[TeV] < a_0 + a_1 x + a_2 x^2, \tag{20}$$

where

$$x = \frac{M_H[GeV] - 75}{10}, \tag{21}$$

and $a_i$ (i=0,1,2) are given in Table 2.

| $M_t$[GeV] | $a_0$ | $a_1$ | $a_2$ |
|---|---|---|---|
| 150 | 4.62 | 1.84 | 0.17 |
| 175 | 1.39 | 0.76 | 0.08 |
| 200 | 0.24 | 0.36 | 0.04 |

Table 2

We will conclude by making an evaluation of the errors affecting our analysis. First of all, those from the determination of the pole masses for the top-quark and Higgs boson (10) have already been mentioned in section 2. We have seen that unconsidered two-loop QCD and one-loop electroweak corrections to $M_t$ lead to an uncertainty in $M_t$ of $\sim 1\%$, i.e. to $\Delta M_t \lesssim 2$ GeV, which translates from (19) to $\Delta M_H \sim 1$ GeV. On the other hand, the scale dependence of $M_H$, which measures the unconsidered higher-order corrections, was evaluated in Ref. [8] to $\Delta M_H \sim 2$ GeV. The other source of theoretical uncertainty comes from the lack of exact renormalization-scale invariance of the one-loop effective potential at the instability region (which has been encoded into the parameter $\alpha$). This (mild) scale dependence reflects all the unconsidered higher-order effects, in particular higher-loop corrections, thus being a good measure of the theoretical uncertainty of the calculation. As it was stated in the previous section, a variation of $\alpha$ in the range $[\alpha^*/2, 2\alpha^*]$ yields $\Delta M_H \lesssim 2$ GeV, which we consider to be a conservative estimate of the theoretical uncertainty [values of $\alpha$ far from the 'optimum' value, $\alpha^*$, lead to large logarithms in the perturbative expansion, thus becoming unreliable, see footnote 4]. Note that the value of $\alpha^*$ we are using corresponds to the choice that cancels one-loop corrections (including finite contributions). This way of fixing $\alpha^*$ has sometimes the drawback that higher order logarithms $[\log(\kappa_i/\alpha^2)]^n$ are not automatically zero and can be enventually important. It would be nice to confirm this by the explicit computation of the next higher order corrections to our calculation. This in fact can be done by adding the leading $\mathcal{O}(g_s^2 h_t^4)$ and $\mathcal{O}(h_t^6)$ two-loop effective potential corrections (the full two-loop potential can be found in Ref. [12]), which provide a two-loop correction to the effective coupling $\lambda_{\rm eff}$ given by

$$\begin{aligned}\Delta_{2-\rm loop}\lambda_{\rm eff} &= \frac{2h_t^4}{(16\pi^2)^2}\left\{g_s^2\left[24\left(\log\frac{h_t^2}{2\alpha^2}\right)^2 - 64\log\frac{h_t^2}{2\alpha^2} + 72\right]\right. \\ &\left. -\frac{3}{2}h_t^2\left[3\left(\log\frac{h_t^2}{2\alpha^2}\right)^2 - 16\log\frac{h_t^2}{2\alpha^2} + 23 + \frac{\pi^2}{3}\right]\right\}. \end{aligned} \tag{22}$$

This correction (which is positive for any value of $\alpha$) raises the value of $\lambda_{\rm eff}$ if the boundary condition of $\lambda$ is maintained. Therefore, condition (15) indicates that for a given $\Lambda$, the boundary condition of $\lambda$ must actually be slightly lowered, and thus the $M_H$ bound. For the typical case $\Lambda = 1$ TeV, $M_t = 175$ GeV, we find $\Delta M_H \sim -1$ GeV, well within the previous conservative estimate $\Delta M_H \lesssim 2$ GeV.

Let us now discuss the dependence of our bounds on the renormalization scheme. In principle one could expect small changes associated with the choice of the scheme. However, since an exact calculation would remove all the ambiguities as well as the scale dependence for any physical quantity, assigning an additional error to this effect would be redundant. It is illustrative to comment how this works when one compares the results obtained in the MS and $\overline{\rm MS}$ schemes. For them, the change in the definition of $\lambda_{\rm eff}$ in Eq. (13), and the subsequent modification in condition (15), is exactly compensated at one loop by the modification in the relation to extract the value of $M_H$, Eqs. (9), (10). This is not surprising since the relation between MS and $\overline{\rm MS}$ schemes can be viewed as a redefinition of the renormalization scale, and our calculation is scale invariant up to higher order corrections. However, the use of the tree-level condition $\lambda(\Lambda) = 0$ does produce different results in the MS and $\overline{\rm MS}$ schemes due to the lack of scale invariance of the approach.

Concerning the gauge dependence of the bounds, it is expected to be well below the previously estimated errors. Note that the main effect of loop corrections in the effective potential can be assigned to the Yukawa sector where no gauge dependence arises. In fact, including the one-loop gauge corrections (as we have done numerically) amounts to shifting the bound by $\Delta M_H \sim 0.5$ GeV, a negligible quantity as compared to our previously estimated errors

Altogether we can conclude that a realistic evaluation on the error associated with our calculational method yields an uncertainty of

$$\Delta_{\rm tot} M_H \lesssim 5\ GeV,$$

which can be taken as a conservative estimate. This error does not take into account those coming from the measured values of $M_t$ and $\alpha_s(M_Z)$. For $\Lambda = 1$ TeV, the former can be readily read off from Eq. (19) to be given by

$$\Delta M_H = 0.64\ \Delta M_t$$

while the latter (much smaller) is estimated to be

$$\Delta M_H \sim -0.50\ \frac{\Delta\alpha_s(M_Z)}{0.006}.$$

# 5 Conclusions

In this letter we have clarified some recent controversy concerning stability bounds on the SM Higgs mass for low values of the SM cutoff, $\Lambda$. We have reanalyzed our previous results by making a refined analysis of the scale dependence of the effective potential, which these bounds depend upon, reobtained other results existing in the literature

and shown that the latter can be explained as a consequence of the corresponding approximations made in evaluating the effective potential. We have restricted ourselves to scales ($\Lambda \lesssim 10$ TeV) which are within the reach of LHC. For that reason our detailed analysis is relevant for the phenomenology of the planned colliders. In particular, from Fig. 4 we see that for $M_t \gtrsim 175$ GeV, if the Higgs is found at LEP2 with a mass $M_H \lesssim 85$ GeV, then $\Lambda \lesssim 10$ TeV, which implies that new physics (if weakly coupled), necessary to stabilize the SM effective potential, should be produced at LHC. Even if we admit living in a metastable minimum, with a lifetime to the non-standard minimum longer than the present age of the universe, the detailed bounds, obtained from the calculation of the decay to the non-standard minimum [13], as a function of $\Lambda$ do depend on the very existence and location of the latter, and thus a detailed knowledge of it is still relevant.

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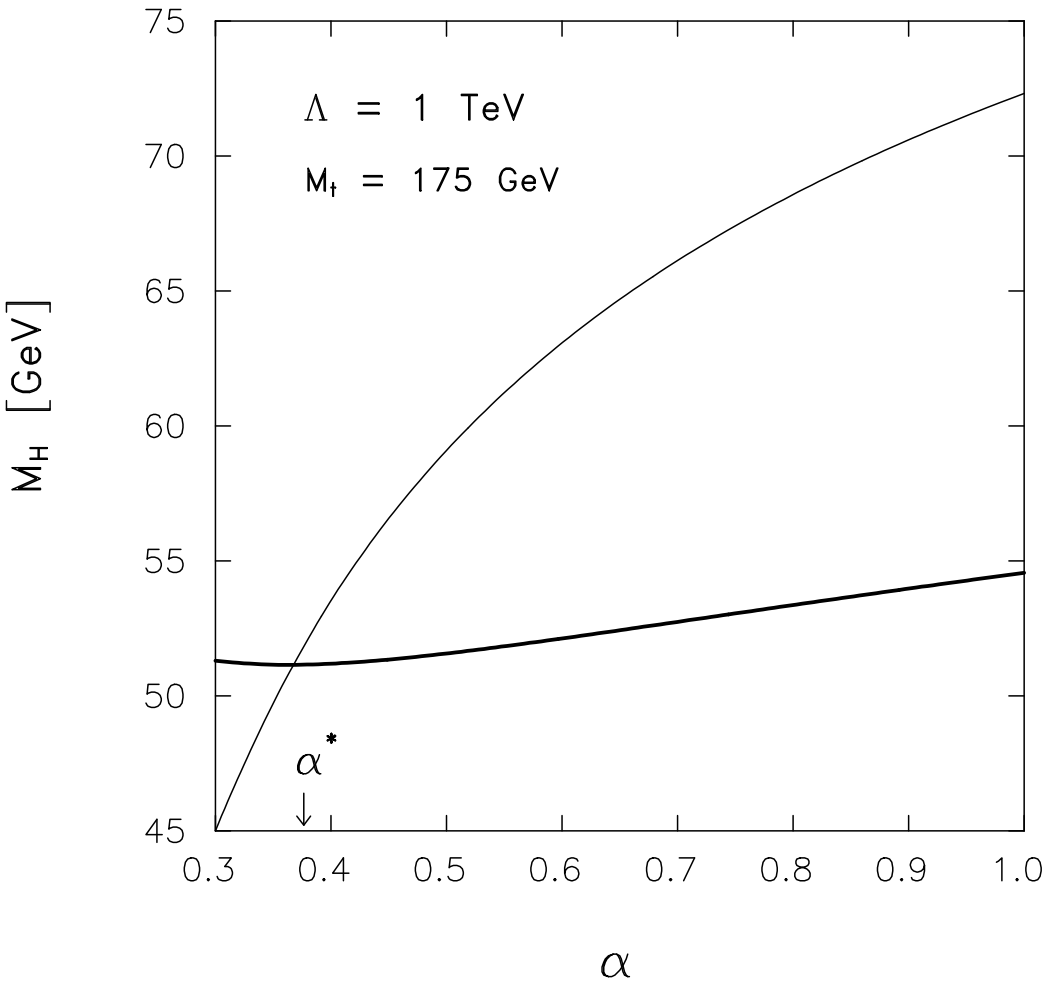


Figure 1: Plots of the lower bound on the Higgs mass based on condition (15) [thick solid line], and on the condition $\lambda(\alpha\Lambda) = 0$ [thin solid line], as functions of the parameter $\alpha$, defined by $\mu(t) = \alpha\phi(t)$, for $\Lambda = 1$ TeV, $\alpha_s(M_Z) = 0.118$ and $M_t = 175$ GeV.

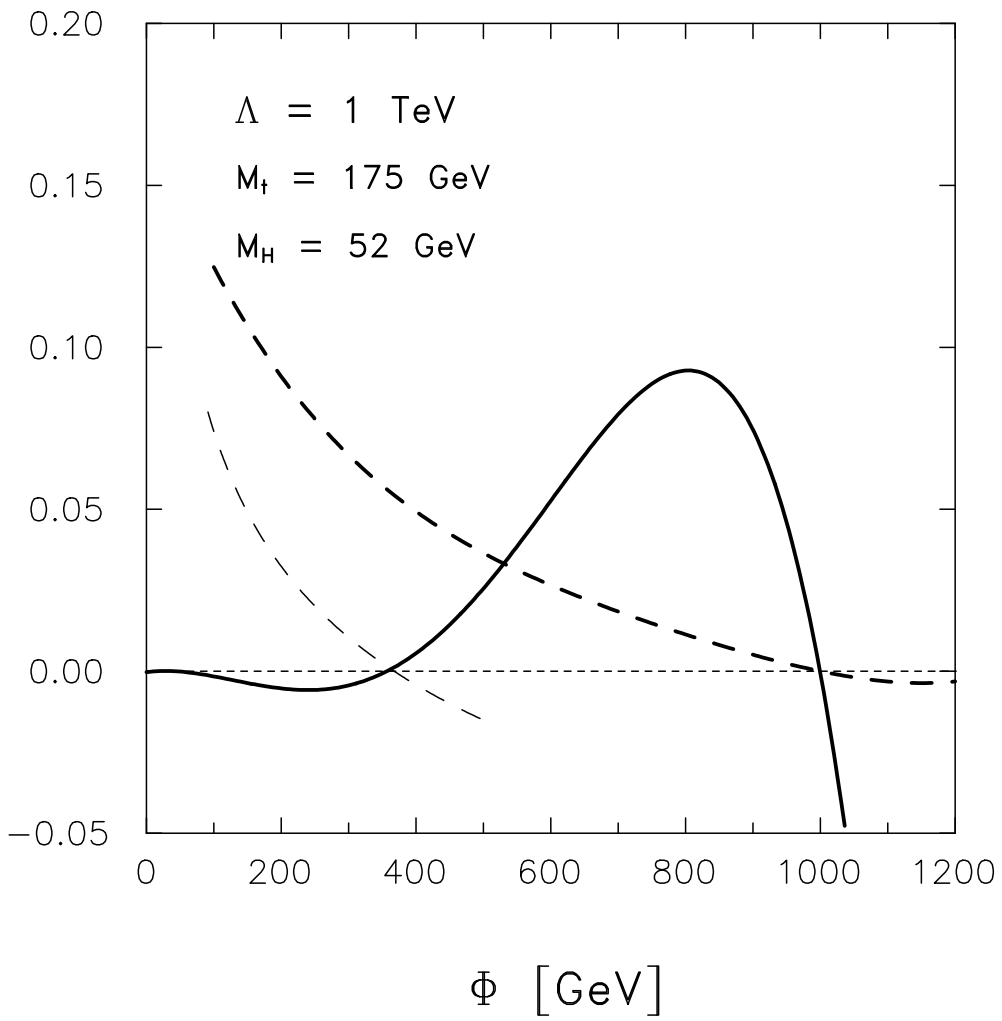


Figure 2: Plot of the effective potential [thick solid line] for $\Lambda$, $\alpha_s(M_Z)$ and $M_t$ as in Fig. 1, and $M_H = 52$ GeV. Dashed lines are plots of $\lambda_{\rm eff}(\mu = \alpha^*\phi)$ [thick one] and $\lambda(\mu = \phi)$ [thin one].

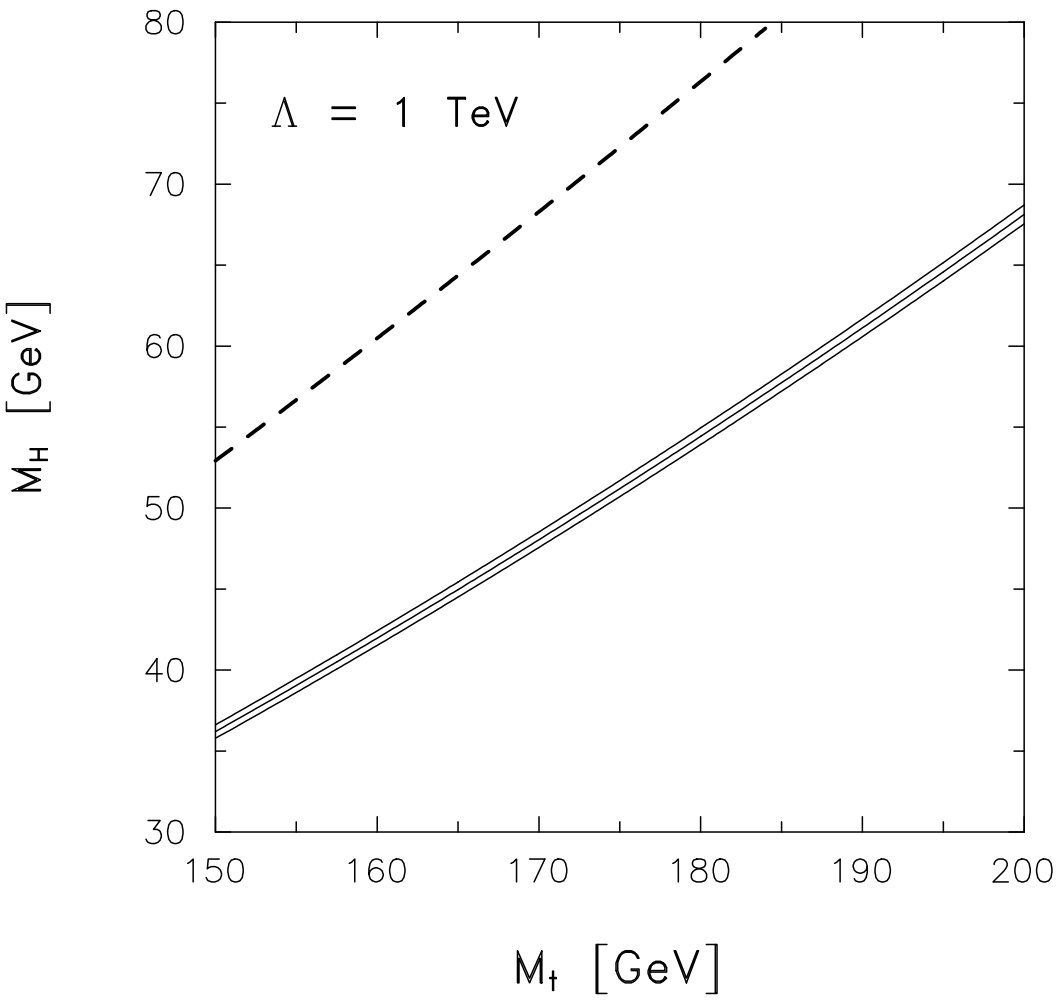


Figure 3: Plots of the lower bound on $M_H$ as a function of $M_t$ from condition $\lambda_{\rm eff} = 0$ [solid lines] as in Fig. 1, for $\alpha_s(M_Z) = 0.118$ [central line], $\alpha_s(M_Z) = 0.124$ [lower line] and $\alpha_s(M_Z) = 0.112$ [upper line]. The bound based on the condition $\lambda(\Lambda) = 0$ and $\alpha_s(M_Z) = 0.118$ is also plotted for the sake of comparison [dashed line].

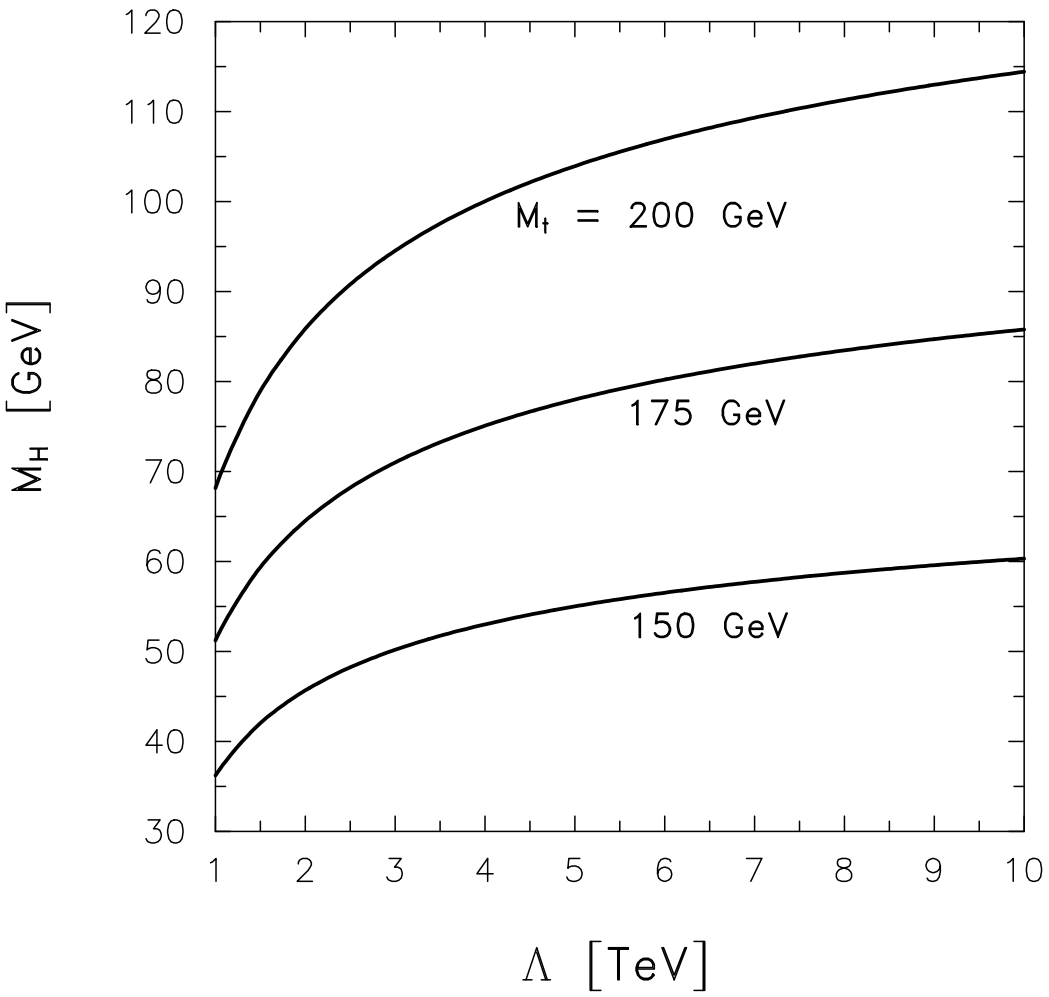


Figure 4: Plots of the lower bound on $M_H$ as a function of $\Lambda$ for different values of $M_t$ and $\alpha_s(M_Z) = 0.118$.