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Thermodynamic potential with condensate fields in an $SU(2)$ model of QCD

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Abstract

We calculate the thermodynamic potential of the quark-gluon plasma in an $SU(2)$ model of QCD, taking into account the gluon condensate configuration with a constant A_4-potential and a uniform chromomagnetic field H. Within this scheme the interplay of condensate fields, as well as the role of quarks in the possible dynamical stabilization of the system is investigated.

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1 Introduction

Statistical properties of systems with non-Abelian gauge fields have recently attracted much attention in connection with the description of the early hot Universe and investigations of the behaviour of the hadronic matter in heavy-ion collisions. Hadronic matter is believed to exist in two different phases: a low-temperature confined phase, and a high-temperature deconfined phase referred to as the quark-gluon plasma. Many papers devoted to the study of kinetic properties of the quark-gluon plasma have appeared during the last years. First, since the effective gauge coupling constant has to be small due to the property of asymptotic freedom, perturbative methods were extensively used to explore the high temperature dynamics [?]. Thereby it turned out that perturbative calculations have serious problems. Thus, for example, one-loop results for the damping rate of the plasma are unstable with respect to higher order corrections [?]. Moreover, perturbative expressions for the free energy density suffer from infrared divergences. After an early unjustified optimism that the generation of a chromoelectric Debye screening mass $m_D \sim gT$ [?] would eliminate these infrared divergences of perturbative QCD, Linde has shown this not to be true [?]. For example, in the case of the free energy such divergence appears in order g^6. It is expected that these divergences, related to zero modes of spatial components of the gauge field A_i, are effectively cut off at momenta of order g^2T due to the so-called magnetic mass.

It is generally believed that the non-linear nature of non-Abelian gauge fields may lead to the formation of condensates which under certain conditions can serve as infrared regulators. Quark and gluon condensates were first considered in the QCD sum rule approach [?]. Also a simple analytical model of the gluon condensate with a uniform chromomagnetic field $H = \text{const}$ (the so called "colour ferromagnetic state") [?] has been proposed which, however, suffered from an instability [?]. Various attempts have been made to improve this with assumptions of a certain domain-like [?] or non-Abelian structure [?] of the chromomagnetic field. Moreover, the consideration of finite temperature effects did not cure this instability [?]. On the other hand, the possibility of the formation at $T \neq 0$ of a new condensate of the 4-th component of the gluon potential $A_4 = \text{const}$ has been discussed in refs.[?]-[?]. This condensate minimizes the effective potential at high temperatures due to the behaviour of the infrared leading perturbative

two-loop and higher-loop terms. It has the order of $gA_4 \sim g^2T$. In particular, as the problem of the homogeneous chromomagnetic field instability stayed unsolved, an attempt was made to use the A_4-potential as a stabilizing term [**?**].

The purpose of the present paper is to study the combined effect of external field configurations with $H = \text{const}$ and $A_4 = \text{const}$ on the quark-gluon plasma in a simplified $SU(2)$ gauge model of QCD. To this end, we consider the thermodynamic potential as a function of these fields and study its minimum corresponding to the stable thermodynamic state of the system. This determines the values of the condensate fields H_{min} and $(A_4)_{min}$ as functions of the temperature. In particular, we are interested in the influence of the chromomagnetic field on the stability of the A_4-condensate taking into account the quark degrees of freedom. Thereby, the interplay of different condensate field components in various cases of their relative strengths is studied. Finally, we estimate the condensate contributions to the Debye mass which serves as a regulator of infrared singularities at $T \neq 0$.

The paper is organized as follows. Section 2 contains the general formulas for the effective potential in the one-loop approximation with condensate fields. In section 3 we investigate the gluon sector, and sections 4 and 5 contain the discussion of the quark sector and the conclusions, respectively.

2 Effective potential in the one-loop approximation

In the following we shall consider an $SU(2)$ gauge model of QCD in an external background field $\overline{A}$, whose generating functional in Euclidean space-time ($it = x_4$) can be written as follows

$$Z\left[\overline{A}, j, \eta, \overline{\eta}\right] = \int da^a_\mu d\overline{\chi} d\chi d\overline{\psi} d\psi \exp\left[-\int d^4x (\mathcal{L} + j^a_\mu a^a_\mu + \overline{\psi}\eta + \overline{\eta}\psi)\right], \quad (1)$$

where the QCD Lagrangian in the background gauge has the form

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2\alpha}(\overline{D^{ab}}_\mu a^b_\mu)^2 + \overline{\chi}_a(\overline{D}^2)_{ab}\chi_b + \sum_{i=1}^{N_f} \overline{\psi}_i[\gamma_\mu(\partial_\mu - ig\frac{1}{2}\lambda_a A^a_\mu) + m_i]\psi_i. \quad (2)$$

Here $A^a_\mu = \overline{A^a_\mu} + a^a_\mu$, $\overline{A^a_\mu}$ is the background field, a^a_μ are quantum fluctuations of the gluon field, $\overline{D}^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}\overline{A}^c_\mu$ is the covariant derivative in the background field, $\chi, \overline{\chi}$ are ghost fields and $(\overline{D}^2)_{ab} = \overline{D}^{ac}_\mu \overline{D}^{cb}_\mu$ (the Lorentz gauge $\alpha = 1$ will be adopted in the following).
For the one-loop calculations to be considered here, the gluon Lagrangian is expanded as

$$\mathcal{L}_g = \mathcal{L}^{(0)}_g(\overline{A}) + \mathcal{L}^{(1)}_g(\overline{A}, a), \tag{3}$$

where $\mathcal{L}^{(0)}(\overline{A})$ is the Lagrangian of the background field $\overline{A_\mu}$, and $\mathcal{L}^{(1)}(\overline{A}, a)$, the Lagrangian quadratic in gluon fluctuations, is given by

$$\mathcal{L}^{(1)}_g(\overline{A}, a) = -\frac{1}{2} a^a_\mu [(\overline{D}^2)_{ab}\delta_{\mu\nu} + 2g\overline{F}^c_{\mu\nu} f_{acb}] a^b_\nu. \tag{4}$$

The quark Lagrangian used in the one-loop approximation takes the form

$$\mathcal{L}_q = \sum_{i=1}^{N_f} \overline{\psi}_i (\gamma_\mu \overline{\nabla}_\mu + m_i)\psi_i, \tag{5}$$

where $\overline{\nabla}_\mu = \partial_\mu - ig\frac{\lambda_a}{2}\overline{A}^a_\mu$ is the covariant derivative of quarks in the external field.
Gaussian path integrations in (**??**) then lead to the result

$$\begin{aligned} Z\left[\overline{A}\right] &= \exp\left[-\frac{1}{4}\int d^4x \left(\overline{F^a}_{\mu\nu}\right)^2\right] \left[\det\left(-\overline{D}^2\delta_{\mu\nu} - 2g\overline{F}_{\mu\nu} f\right)\right]^{-\frac{1}{2}} \\ &\quad \times \det\left(-\overline{D}^2\right) \prod_{i=1}^{N_f} \det(\gamma_\mu \overline{\nabla}_\mu + m_i). \end{aligned} \tag{6}$$

By writing $Z = \exp(W_E)$, we obtain for the Euclidean effective action

$$W^{(1)}_E = -\frac{1}{2}\int \frac{dq_4}{2\pi} \sum_r \ln(q_4^2 + \varepsilon_r^2(G)) + \sum_{i=1}^{N_f} \int \frac{dp_4}{2\pi} \sum_k \ln(p_4^2 + \varepsilon_k^2(Q_i)). \tag{7}$$

Here $\varepsilon_r(G)$ and $\varepsilon_k(Q_i)$ are the energy spectra of gluons G and quarks Q_i with flavour i in a constant external field with quantum numbers r and k, respectively. Note that unphysical degrees of freedom of gluons are cancelled by ghost contributions, and in (**??**) only physical degrees of freedom are counted by the boson quantum numbers r.

The effective potential is defined as

$$V = -\ln Z/(\ L^4) = -\frac{1}{L^4} W_E, \tag{8}$$

where L^4 is the 4-volume. In the one-loop approximation we obtain

$$V = V^{(0)} + v, \tag{9}$$

where $V^{(0)} = (\overline{F}^a_{\mu\nu})^2/4$ is the energy density of the classical field, and v is the one-loop effective potential.
For finite temperature T the fields are (anti)periodic

$$A^a_\mu(x_4+\beta) = A^a_\mu(x_4), \quad \psi(x_4+\beta) = -\psi(x_4), \tag{10}$$

where $\beta = 1/T$, and we substitute

$$\begin{aligned} &\int_{-\infty}^{+\infty} dx_4 \to \int_0^\beta dx_4, \\ &q_4 \to \frac{2\pi l}{\beta} - i\mu_1\kappa, \quad \int \frac{dq_4}{2\pi} \to \frac{1}{\beta}\sum_{l=-\infty}^{+\infty}, \quad \text{(bosons)}, \\ &p_4 \to \frac{2\pi}{\beta}(l+\frac{1}{2}) - i\mu_2\kappa, \quad \text{(fermions)}. \end{aligned} \tag{11}$$

Here $\mu_{1,2}\kappa$ ($\kappa = \pm 1$) are the chemical potentials corresponding to opposite charges of particles (gluons, quarks).

In the case of finite T and μ the partition function of the grand canonical ensemble is given by

$$Z = \mathrm{Tr}\exp[-\beta(H - \mu_1\widehat{Q}_1 - \mu_2\widehat{Q}_2)], \tag{12}$$

where H is the Hamiltonian of the system, $\widehat{Q}_{1,2}$ are charge matrices, and the trace is taken over physical states only. In this case the thermodynamic potential $\Omega = -T\ln Z$ can be calculated, and for the one-loop effective potential depending on $T$and μ we obtain

$$\begin{aligned} v &= \frac{\Omega^{(1)}}{L^3} = \frac{1}{2\beta L^3}\sum_{l=-\infty}^{+\infty}\sum_{r(\kappa=\pm1)} \ln[(\frac{2\pi l}{\beta} - i\kappa\mu_1)^2 + \varepsilon_r^2(G)] \\ &- \frac{1}{\beta L^3}\sum_{i=1}^{N_f}\sum_{l=-\infty}^{+\infty}\sum_{k(\kappa=\pm1)} \ln[(\frac{2\pi(l+1/2)}{\beta} - i\kappa\mu_2)^2 + \varepsilon_k^2(Q_i)]. \end{aligned} \tag{13}$$

If we use the formal integral representation (valid up to a counter term)

$$\ln A = -\int_0^\infty \frac{ds}{s}\exp(-sA)$$

where A has a positive definite real part, the expression (**??**) reads

$$\begin{aligned} v &= -\frac{1}{\beta L^3}\sum_{l=-\infty}^{+\infty}\int_0^\infty \frac{ds}{s}\{\frac{1}{2}\sum_{r(\kappa=\pm 1)}\exp(-s[(\frac{2\pi l}{\beta}-i\kappa\mu_1)^2+\varepsilon_r^2(G)]) \\ &\quad -\sum_{i=1}^{N_f}\sum_{k,\kappa}\exp(-s[(\frac{2\pi(l+1/2)}{\beta}-i\kappa\mu_2)^2+\varepsilon_k^2(Q_i)])\} \qquad (14) \\ &= v^G + v^Q, \end{aligned}$$

where v^G and v^Q are the gluon and quark contributions to the one-loop effective potential.

Let us assume that the constant external fields A_4 and H modelling the gluon condensate are described by the following potential

$$\overline{A}^a_\mu = \delta_{\mu 2}\delta_{a3}Hx_1 + \delta_{\mu 4}\delta_{a3}A_4 = \delta_{a3}\overline{A}_\mu. \qquad (15)$$

The energy spectrum of charged gluons in this field

$$\varepsilon^2_{n,\sigma,q_3}(G) = 2gH(n+1/2-\sigma)+q_3^2, \qquad (16)$$

depends on the following quantum numbers: $n = 0,1,2,...$ is the Landau quantum number, σ is the spin orientation parallel ($\sigma = 1$) and antiparallel ($\sigma = -1$) to the external field direction (the $\sigma = 0$ mode is cancelled by the ghost contribution), and q_3 is the longitudinal momentum. For quarks of two colours we have

$$\varepsilon^2_{n,\sigma,p_3}(Q_i) = gH(n+1/2+\sigma/2)+p_3^2+m_i^2, \qquad (17)$$

where for the ground state $n = 0$ we have only one value of the spin orientation $\sigma = -1$. Note that the naive ground state of gluons ($n = 0, \sigma = +1, q_3 = 0$) is tachyonic , $\varepsilon_0^2(G) = -gH < 0$. Hence, since an unstable mode is associated with it, it should be considered separately (see below, (**??**)). As for the true ground state ($n = 1, \sigma = +1, q_3 = 0$), its energy is real, $\varepsilon_{true} = \sqrt{gH}$, and plays the role of a "mass" of stable excitation modes of gluons.

The contribution of the A_4 -potential is easily accounted for if we substitute $i\mu_1 \to gA_4$, $i\mu_2 \to g\frac{A_4}{2}$ for gluons or quarks, respectively, in expression (??). We remark that the role of the A_4-potential in some way resembles that of the mass term, since for the gluon Lagrangian (??) we may write

$$\mathcal{L}_g^{(1)} = \frac{1}{2}(\partial_\mu^2 a_\mu^a)^2 + igA_4\partial_4 a_\mu^a f^{ab3} a_\mu^b + \frac{1}{2}g^2A_4^2(\delta_{cd} - \delta_{c3}\delta_{d3})(a_c^\mu a_d^\mu) + \mathcal{L}_{gH}^{(1)}. \quad (18)$$

Here $\mathcal{L}_{gH}^{(1)}$ stands for the H field contribution, and for charged gluons ($c = 1, 2$; $d = 1, 2$) $g^2A_4^2$ plays the role of a finite real mass term. This effective mass, together with the "field mass" squared gH (see also sect.3 below), provide for the regularization of the infrared divergencies arising in finite T perturbative calculations. Notice that a non-vanishing constant external potential A_4 associated to mass generation leads to a spontaneous breakdown of local SU(2) gauge symmetry.

3 Gluon sector

We study here the boson contribution to the one-loop effective potential and discuss in more detail the case of a strong A_4 background as compared to the H background. After integration over q_3 in (??) (as understood in the summation over r, cf.(??)) and separating the temperature independent contribution with the help of the formula

$$\sum_{l=-\infty}^{+\infty} \exp[-s(2\pi l/\beta + gA_4)^2] = \frac{\beta}{2\sqrt{\pi s}}[1 + 2\sum_{l=1}^{\infty} \exp(-\frac{\beta^2 l^2}{4s})\cos(gA_4\beta l)], \quad (19)$$

we obtain for the gluon effective potential

$$\begin{aligned} v^G = v_{T=0}^G + v_T^G &= -\frac{gH}{8\pi^2}\sum_{n=0(\sigma=\pm 1)}^{\infty}\int_0^\infty \frac{ds}{s^2}\exp[-2gHs(n + 1/2 - \sigma)] \\ &\times[1 + 2\sum_{l=1}^{\infty}\exp(-\beta^2 l^2/4s)\cos(gA_4\beta l)]. \end{aligned} \quad (20)$$

The first term, corresponding to zero temperature, gives after performing regularization and renormalization procedures the well known result [?, ?]

$$v_{T=0}^G = \frac{11}{48\pi^2}(gH)^2(\ln\frac{gH}{M^2} + c) - i\frac{(gH)^2}{8\pi}, \quad (21)$$

where M is the renormalization point, and c is a corresponding numerical constant which we do not specify here. This expression contains an imaginary part indicating the instability of this ferromagnetic state of the gauge field at zero temperature. The second term in (**??**) determines the temperature dependent part of the effective potential. Performing the summations over n and σ we obtain

$$v_T^G = v_{T,H=0}^G + v_{T,H}^G, \tag{22}$$

where we separated the $H = 0$ effective potential $v_{T,H=0}^G$ from the finite H part $v_{T,H}^G$

$$v_{T,H=0}^G = -\frac{1}{(2\pi)^2}\int_0^\infty \frac{ds}{s^3}\sum_{l=1}^\infty \exp(-\beta^2 l^2/4s)\cos(gA_4\beta l), \tag{23}$$

$$v_{T,H}^G = -\frac{gHT^2}{(2\pi)^2}(A+B+C), \tag{24}$$

and

$$\begin{aligned} A &= \int_0^\infty \frac{dy}{y^2}\left(\frac{2\exp(-3\lambda y)}{1-\exp(-2\lambda y)} - \frac{1}{\lambda y}\right)\sum_{l=1}^\infty \exp(-l^2/4y)\cos(gA_4\beta l), \\ B &= \int_0^\infty \frac{dy}{y^2}\exp(-\lambda y)\sum_{l=1}^\infty \exp(-l^2/4y)\cos(gA_4\beta l), \\ C &= \int_0^\infty \frac{dy}{y^2}\exp(\lambda y)\sum_{l=1}^\infty \exp(-l^2/4y)\cos(gA_4\beta l). \end{aligned} \tag{25}$$

Here we have introduced a new integration variable $y = s/\beta^2$ and a dimensionless parameter $\lambda = gH\beta^2$. The exponential $\exp(\lambda y)$ in C (**??**) leads to a divergence of the integral as $y \to \infty$. This divergence originates from the incorrect procedure of integrating over the proper time variable s for the $n = 0$, $\sigma = +1$ mode. The gluon state with $n = 0, \sigma = +1$ which gives this divergent exponent in C, is a tachyonic state with imaginary energy, $\varepsilon_0^2 < 0$. Note that this unstable mode is responsible for the imaginary part of the effective potential v^G.

The $H = 0$ term (**??**) is easily computed. After integrating over s the series over l reduces to

$$\sum_{l=1}^\infty \frac{\cos(l\xi\beta)}{l^4} = -\frac{(2\pi)^4}{48}B_4(\xi\beta/2\pi), \tag{26}$$

where $B_4(x) = x^4 - 2|x|^3 + x^2 - 1/30$ is the Bernoulli polynomial with argument defined modulo 1, and $\xi = gA_4$. Adding the contribution $(-\frac{\pi^2 T^4}{45})$ from uncharged gluons, we obtain

$$v^G_{T,H=0} = -\frac{\pi^2}{15}T^4 + \frac{4}{3}\pi^2 T^4(\xi\beta/2\pi)^2(1-\xi\beta/2\pi)^2 \tag{27}$$

(for convenience, here and in the following we use also the dimensionless variable $\xi\beta/2\pi$ being taken modulo 1).

Now let us consider the limit of high temperatures, $T \gg \sqrt{gH}$, $T \gg gA_4$ as compared to the background fields and proceed to compute $v^G_{T,H}$. First we suppose that the potential A_4 is smaller than the chromomagnetic field H, i.e.,

$$\xi = gA_4 \ll \sqrt{gH}. \tag{28}$$

Then in the high temperature limit the expressions A, B, C in (**??**) with the help of analytical continuation $C(\lambda) = B(-\lambda - i\varepsilon)$ can be asymptotically evaluated and for the real part of (**??**) plus (**??**) we obtain

$$\begin{aligned} V^{(0)} + \mathrm{Re}(v^G_{T=0} + v^G_{T,H}) &= \frac{H^2}{2} + \frac{11}{48\pi^2}(gH)^2[\ln(\frac{4\pi T}{M})^2 - \gamma] - \frac{1}{3}aT(gH)^{3/2} \\ &\quad -b\xi^2\sqrt{gH}T - c\xi^4(gH)^{-1/2}T, \end{aligned} \tag{29}$$

where γ is the Euler constant, and a, b, c are numerical constants given by

$$\begin{aligned} a &= \frac{3}{2\pi}[1 - \zeta(3/2)\frac{\sqrt{2}-1}{2\pi}], \\ b &= \frac{1}{4\pi}[(\sqrt{2}-2)\zeta(1/2) + 1], \\ c &= \frac{1}{16\pi}[(2 - \frac{1}{\sqrt{2}})\zeta(3/2) - 1], \end{aligned} \tag{30}$$

with $\zeta(x)$ being the Riemann zeta function. This approximate expression was obtained for the first time for the case of finite chemical potential $\mu = -igA_4$ in ref.[**?**]. For $\xi = 0$ it agrees with the result of ref.[**?**]. It should be noted that the logarithmic dependence on H present in $v^G_{T=0}$ is cancelled by a corresponding term in $v^G_{T,H}$.

We are now able to find the minimum of the real part of the effective potential determined by

$$\partial \mathrm{Re} v^G / \partial(\sqrt{gH}) = 0. \tag{31}$$

The solution of this equation $gH = (gH)_{\min}$ can be presented in the form (see also ref.[?])

$$\sqrt{gH_{\min}} = 2\pi a \alpha_s(T) T + (\frac{\xi}{T})^2 \frac{bT}{2\pi a^2 \alpha_s(T)}, \tag{32}$$

where the positive numerical constants a and b are defined in (??), and we have omitted a smaller term proportional to $c\xi^4$. Moreover

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{g^2}{4\pi(1 + \frac{11}{24\pi^2} g^2 \ln(T^2/\widetilde{M}^2))} \tag{33}$$

is the effective coupling constant of the pure gluon sector at the scale determined by T, and $\widetilde{M}^2 = M^2 e^{\gamma} (4\pi)^{-2}$. The main term in (??) is proportional to the infrared regulating parameter $\alpha_s(T)T$ [?], which has to be expected since the "ferromagnetic" background itself is an infrared, i.e. long distance phenomenon. The second term in (??), proportional to $(gA_4)^2$, enlarges the value of $gH_{\min}$, indicating that there is no tendency of a transition to the perturbative vacuum with zero magnetic field at temperatures where $\xi \ll \alpha_s(T)T$.

The expressions (??), (??) can be used to find the Debye mass of a temporal gluon influenced by the condensate fields

$$\begin{aligned} m_D^2 &= \partial^2 v^G / \partial A_4^2 = \frac{2}{3} g^2 T^2 - 4 g^2 T^2 \frac{\xi\beta}{2\pi}(1 - \frac{\xi\beta}{2\pi}) - \\ &\quad - 2bg^2 T \sqrt{gH} - 12 c g^4 A_4^2 \frac{T}{\sqrt{gH}}. \end{aligned} \tag{34}$$

This expression was derived for the case of small gA_4, satisfying (??). Extrapolating it to the condensate value $\xi_{min} \sim g^2 T$ (cf.(??) below), and using $\sqrt{gH_{\min}} \sim g^2(T)T$ from (??) , we obtain

$$m_D^2 = \frac{2}{3} g^2 T^2 + C_1 g^4 T^2, \tag{35}$$

where C_1 is a certain numerical coefficient. From (??) we see that keeping field dependent terms in m_D^2 and determining the condensates by the infrared behaviour of the theory, is in some sense equivalent to going beyond the one-loop approximation.

Let us now consider the imaginary part of v^G, originating from the contribution of the unstable tachyonic mode $n = 0$, $\sigma = +1$. Then $\mathrm{Im}v^G$, which does not diverge, can easily be computed taking the term $n = 0$, $\sigma = +1$ in the sum over r in (??)

$$\mathrm{Im}v^G = \frac{gH}{2\pi\beta}\mathrm{Im}\sum_{l=-\infty}^{+\infty}\int\frac{dq_3}{2\pi}\ln[(\frac{2\pi l}{\beta} - gA_4)^2 - gH + q_3^2]. \qquad (36)$$

At high temperatures $T \gg \sqrt{gH}$ only the $l = 0$ term can give a non-zero contribution to the imaginary part and we obtain

$$\mathrm{Im}v^G = -\frac{gH}{2\pi\beta}\sqrt{gH - (gA_4)^2}\,\theta(gH - (gA_4)^2), \qquad (37)$$

where $\theta(x)$ is the usual step function. In fact, (??) admits a simple physical interpretation in the sense of a tachyonic instability. For large H-fields the tachyonic contribution $-gH$ to the effective mass squared of charged gluons dominates over the contribution $(gA_4)^2$ from the A_4-condensate.

However, when the gA_4 condensate is larger in magnitude than the magnetic one, i.e., $gA_4 > \sqrt{gH}$, the imaginary part vanishes

$$\mathrm{Im}v^G = 0, \quad gH < (gA_4)^2 \qquad (38)$$

and the non-perturbative field configuration may become thermodynamically stable. Thus, the system stabilizes itself as a result of the formation of condensates $\sqrt{gH_{\min}}$ and $(gA_4)_{\min}$ (see the discussion below). It is just the interaction of gluons with the constant field A_4 which cures the "ferromagnetic" unstable mode $n = 0$, $\sigma = +1$ so that the imaginary part of v disappears.

In the region $gA_4 > \sqrt{gH}$, we can write

$$\begin{aligned} v^G &= -\frac{\pi^2T^4}{15} + \frac{4}{3}\pi^2T^4(\frac{\xi\beta}{2\pi})^2(1 - \frac{\xi\beta}{2\pi})^2 \\ &\quad + \frac{\Phi^4}{2g^2(T)} - \frac{2F(\nu)}{\pi}\Phi^3T, \quad \mathrm{Im}v^G = 0, \end{aligned} \qquad (39)$$

where $\Phi = \sqrt{gH}$, and $F(\nu)$ is the function of $\nu = (1/2)(1 + \xi^2/\Phi^2)$, introduced in ref.[?]

$$F(\nu) = \frac{\sqrt{2}}{3}\nu^{3/2} + \int_0^\infty \frac{xdx}{4(x^2+\nu^2)^{1/4}(\exp(2\pi x) - 1)}. \tag{40}$$

Unfortunately, the one-loop effective potential (??) has no nontrivial minimum in ξ. At the same time, it possesses the well-known $Z(2)$-symmetry, as is seen from the symmetry properties of the Bernoulli polynomials (??). The authors of refs.[?]-[?] claim that higher loop contributions to v^G lead to the formation of a A_4 condensate. In fact, let the one-loop result (??) be supplemented by the two-loop contribution [?]

$$v^{G(2)} = -\frac{1}{3}g^2T^4(\frac{\xi\beta}{2\pi}) + g^2\frac{T^4}{24} \tag{41}$$

calculated in the Feynman gauge $\alpha = 1$ for the case $H = 0$. Then a nontrivial minimum arises

$$\xi_{\min} = (gA_4)_{\min} = (g^2/4\pi)T + ..., \tag{42}$$

where the omitted terms are of higher orders in g^2 and $\Phi^2/(gA_4)^2$. Generally speaking, one may expect that the condensate value of A_4 will be of the same order as (??), i.e. $(gA_4)_{\min} = g^2(T)Tf(H^2, g^2(T))$, where f is a certain unspecified function of order unity. Then, for $\Phi < \xi$ the "ferromagnetic" background estimated from the result (??), (??) instead of (??) becomes

$$\Phi^2_{\min} \simeq \frac{1}{2\pi}g^2(T)T\xi_{\min}. \tag{43}$$

Substituting the extreme values of ξ and Φ from (??) and (??) into (??), (??), one arrives at

$$v^G_{\min} + v^{G\ (2)}_{\min} = -\frac{\pi^2T^4}{15} + g^2\frac{T^4}{24} - \frac{g^4T^4}{48\pi^2} - \frac{g^6(T)T^4}{24\pi^4}(\frac{\xi_{\min}}{\Phi_{\min}})^6. \tag{44}$$

Thus, one can state that, as a result of gluon self-interactions, at high temperatures specific condensates $\sqrt{gH_{\min}}$ and $(gA_4)_{\min}$ are formed. The former is determined in the one-loop approximation, whereas the latter is a higher-loop effect. Though both condensate values are of the same order g^2, their

contributions to the thermodynamic potential differ in the order of magnitudes: the "ferromagnetic" one is of the order g^6, and the A_4 contribution is of the order g^4.

Let us mention that the above results have been obtained in the Feynman gauge $\alpha = 1$. In ref.[?] it was shown that the effective potential containing an A_4 field does not depend on the gauge parameter at the two-loop level. It should be expected that this conclusion remains valid for the general background configuration (??).

Concluding this section, let us compare our results with those of ref.[?], where the case of an inhomogeneous chromomagnetic field was considered. The approach, advocated in that paper differs from ours in using the operator method which gives only the terms of the perturbative expansion in powers of the chromomagnetic field. Nonperturbative nonanalytical terms in H, such as those in (??) are not described by such an expansion. The analogous situation of an insufficiency of the operator expansion for the full description of effects influenced by the external field was mentioned in ref.[?]. The exact description of the external uniform field, advocated in the present paper, accounts for the most interesting infrared region and thus cures the infrared divergences present in the resulting expression of ref.[?]. Indeed, the infrared divergent term (see eq.(6) of that paper) is reproduced from our expressions (??), (??) by expansion in powers of gH and subsequent integration in the proper time representation (??) with an infrared cutoff on the upper limit. Without expansion, the proper time integral for the real part of v^G in (??), (??) converges, the parameter gH being the natural infrared regulator.

4 Quark sector

We now examine the role of quarks concerning the stability of the condensate fields A_4, H. In the chiral limit, when masses of quarks are zero, the one-loop effective potential of quarks is described according to (??) and (??) by the formula

$$N_f^{-1} v^Q = -\frac{1}{\beta} \sum_{l=-\infty}^{+\infty} \frac{gH}{4\pi} \int \frac{dp_3}{2\pi} \times \sum_{n=0,1,\ldots;\sigma=\pm 1,\kappa=\pm 1} \ln[(\frac{2\pi(l+1/2)}{\beta} - \kappa g A_4/2)^2 + \varepsilon^2_{n,\sigma,p_3}]. \quad (45)$$

Eq.(??) can be considered in the same way as the boson case. Using (??) to separate the zero temperature term and introducing the proper time representation (??) we finally arrive at

$$v^Q = v^Q_{T=0} + v^Q_{T\neq 0}, \tag{46}$$

where

$$N_f^{-1} v^Q_{T=0} = \frac{gH}{2(2\pi)^2} \int_0^\infty \frac{ds}{s^2} \coth \frac{1}{2} gHs \tag{47}$$

is the quark contribution to the effective potential at zero temperature and

$$\begin{aligned} N_f^{-1} v^Q_{T\neq 0} &= \frac{gH}{2(2\pi)^2} \int_0^\infty \frac{ds}{s^2} \coth(gHs/2) \sum_{l=1}^\infty \exp(-\frac{\beta^2 l^2}{4s}) \\ &\times [\cos(\frac{\beta\xi}{2} + \pi)l + \cos(\frac{\beta\xi}{2} - \pi)l] \end{aligned} \tag{48}$$

is the corresponding finite temperature part of v^Q. Here three terms may be separated

$$v^Q_{T\neq 0} = v^Q_{T,H} + v^Q_{T,\xi} + v^Q_{T,\xi,H}, \tag{49}$$

where the first one

$$\begin{aligned} N_f^{-1} v^Q_{T,H} &= \frac{gH}{(2\pi)^2} \int_0^\infty \frac{ds}{s^2} (\coth(gHs/2) - \frac{2}{gHs}) \\ &\times \sum_{l=1}^\infty (-1)^l \exp(-\frac{\beta^2 l^2}{4s}), \end{aligned} \tag{50}$$

describes the chromomagnetic field contribution, the second one

$$\begin{aligned} N_f^{-1} v^Q_{T,\xi} &= \frac{1}{(2\pi)^2} \int_0^\infty \frac{ds}{s^3} \sum_{l=1}^\infty \exp(-\frac{\beta^2 l^2}{4s}) \\ &\times [\cos(\frac{\beta\xi}{2} + \pi)l + \cos(\frac{\beta\xi}{2} - \pi)l], \end{aligned} \tag{51}$$

is the contribution for the fermionic gas in the A_4 potential, and

$$\begin{aligned} N_f^{-1} v^Q_{T,\xi,H} &= \frac{gH}{2\,(2\pi)^2} \int_0^\infty \frac{ds}{s^2} (\coth \frac{gHs}{2} - \frac{2}{gHs}) \\ &\times \sum_{l=1}^\infty \exp(-\frac{\beta^2 l^2}{4s}) [\cos(\frac{\beta\xi}{2} + \pi)l + \\ &+ \cos(\frac{\beta\xi}{2} - \pi)l - 2(-1)^l], \end{aligned} \tag{52}$$

describes the combined effect of A_4 and H on quarks.
The ξ contribution is most easily calculated. After performing the s- integration and summing over l according to (??) we obtain

$$N_f^{-1} v^Q_{T,\xi} = -\frac{(2\pi)^2}{3\beta^4}[B_4(\frac{x+1}{2}) + B_4(\frac{1-x}{2})], \tag{53}$$

where $x = \frac{\beta\xi}{2\pi}$. Since

$$B_4(\frac{x+1}{2}) = B_4(\frac{1-x}{2}),$$

we finally have

$$N_f^{-1} v^Q_{T,\xi} = -\frac{7\pi^2}{90}T^4 - \frac{2}{3}(2\pi)^2 T^4 (\frac{x}{2})^2((\frac{x}{2})^2 - \frac{1}{2}). \tag{54}$$

Here the first term corresponds to the free fermion gas with two colour degrees of freedom. Moreover, due to the modified periodicity properties in x of the Bernoulli polynomials in (??) as compared to (??), this expression breaks the $Z(2)$-symmetry of the gluon contribution (??).

Next, let us introduce as in the gluon case a new variable $y = \frac{s}{\beta^2}$ and the parameter $\lambda = \frac{g}{2}H\beta^2$ into the chromomagnetic term (??) which yields

$$\begin{aligned} N_f^{-1} v^Q_{T,H} &= \frac{gH}{(2\pi)^2\beta^2}\int_0^\infty \frac{dy}{y^2}(\coth\lambda y - \frac{1}{\lambda y}) \\ &\quad \times \sum_{l=1}^{\infty} \exp(-\frac{l^2}{4y})(-1)^l. \end{aligned} \tag{55}$$

Here again the identity (??) is useful, which we put into the form

$$\begin{aligned} \sum_{l=1}^{\infty} \exp(-\frac{l^2}{4y})\cos(\pi l) &= -\frac{1}{2} + \exp(-\pi^2 y)\sqrt{\pi y} \\ &\quad \times[1 + 2\sum_{l=1}^{\infty}\exp(-4\pi^2 l^2 y)\cosh(4\pi^2 l y)]. \end{aligned} \tag{56}$$

In the high temperature limit $T \gg \sqrt{gH}$ ($\lambda \ll 1$) the following terms can be separately calculated

$$J_1 = \int_0^\infty \frac{dy}{y^2}(\coth\lambda y - \frac{1}{\lambda y})e^{-\pi^2 y}\sqrt{\pi y} = \frac{\sqrt{\pi}}{3}\lambda + O(\lambda^3), \tag{57}$$

$$J_2 = -\frac{1}{2}\int_0^\infty \frac{dy}{y^2}(\coth\lambda y - \frac{1}{\lambda y}) + \frac{2}{3}\lambda\sqrt{\pi}\int_0^\infty dy\, y^{\frac{3}{2}} e^{-\pi^2 y}$$
$$\times \sum_{l=1}^{\infty} \exp(-4\pi^2 l^2 y)\cosh(4\pi^2 l y) = C\lambda, \tag{58}$$

where C is a constant, whose value is evidently expressed by the left-hand side of this expression with $\lambda = 1$. The third term

$$J_3 = 2\sqrt{\pi}\int_0^\infty \frac{dy}{y^{\frac{3}{2}}}(\coth\lambda y - \frac{1}{\lambda y} - \frac{1}{3}\lambda y)e^{-\pi^2 y}\times$$
$$\times \sum_{l=1}^{\infty} \exp(-4\pi^2 l^2 y)\cosh(4\pi^2 l y)$$

is approximately calculated with the observation that the term -$\lambda y/3$ provides the main contribution. In the approximation with $\ln\lambda \gg 1$ we obtain

$$J_3 = \frac{\lambda}{6}\ln(\lambda C_1), \tag{59}$$

where C_1 is a numerical constant, which we do not specify here. Combining (??), (??) and (??) we find

$$N_f^{-1} v_{T,H}^Q = \frac{gH}{(2\pi)^2\beta^2}(\frac{\sqrt{\pi}}{3}\lambda + C\lambda + \frac{\lambda}{6}\ln(\lambda C_1)). \tag{60}$$

The final result looks as follows

$$N_f^{-1} v_{T,H}^Q = \frac{(gH)^2}{48\pi^2}[\ln\frac{(gH/2)}{T^2} + C' + O(\frac{gH}{T^2})^2]. \tag{61}$$

As it is seen from this formula, there is no linear temperature term $(gH)^{3/2}T$ in the fermionic effective potential. The zero temperature effective potential (??) after performing regularization and renormalization procedures can be written in the form

$$N_f^{-1} v_{T=0}^Q = \frac{(gH)^2}{48\pi^2}\ln\frac{2M^2}{gH}, \tag{62}$$

where M is the renormalization point. Therefore, the $\ln\frac{gH}{2}$ contribution is cancelled between $v^Q_{T,H}$ and $v^Q_{T=0}$, and we arrive at

$$N_f^{-1}(v^Q_{T=0}+v^Q_{T,H})=\frac{(gH)^2}{48\pi^2}\ln[\frac{M^2}{T^2}+C']. \tag{63}$$

Note that in the case of weak chromomagnetic fields $\sqrt{gH}\ll gA_4$ the combined effect of A_4 and H described by (??) is small and can be neglected.

If we consider the quark contribution (??) where there is no linear in T term, we see that the value of the "ferromagnetic" field strength (??) is not affected, though the effective strong coupling constant (??) is replaced by

$$g^2(T)=g^2/(1+(11-N_f)\frac{g^2}{24\pi^2}\ln\frac{T^2}{\widetilde{M^2}}). \tag{64}$$

Clearly, this has to be expected since asymptotic freedom holds at high temperature. This value of $g^2(T)$ has to be substituted in (??)-(??) as well.

Finally, we can find the quark contribution to the Debye mass

$$m_D^2=\frac{\partial^2}{\partial A_4^2}v^Q=\frac{N_f}{6}g^2T^2(1-3\frac{g^2\beta^2A_4^2}{4\pi^2}). \tag{65}$$

For the condensate estimate $A_4\sim gT$ the second term in (??) is again of the order of the higher perturbative terms and hence must be omitted.

The case of finite current quark masses is somewhat more complicated. After some transformations we can write the effective potential in the form

$$\begin{aligned} v^Q &= \sum_{i=1}^{N_f}\frac{(gH/2)^{3/2}}{4\pi^2\beta}\sum_{\kappa=\pm1}\frac{1}{2\pi i}\int_C[\frac{2\sqrt{2}}{\beta\sqrt{gH}}]^sF(-e^{ix\kappa},s+1) \\ &\quad\times\frac{[\Gamma(\frac{s+1}{2})]^2}{s-1}\{2\zeta(\frac{s-1}{2},\frac{m_i^2}{gH})-(\frac{gH}{m_i^2})^{\frac{s-1}{2}}-(\frac{gH}{m_i^2+gH})^{\frac{s-1}{2}}\}ds, \end{aligned} \tag{66}$$

where $x=gA_4\beta/2$, $F(a,s)=\sum_{k=1}^{\infty}\sum a^k/k^s$, $\zeta(n,b)$ is the generalized Riemann zeta function, and the integration contour is an infinite circle closed in the left half-plane, which is bounded on the right-hand side by the straight line parallel to the ordinate axis and situated to the right of the point $s=3$ (i.e. $s=3+\varepsilon$, $0<\varepsilon<1$). As one can see from (??), $s=3,1,...$are the

first order poles leading to the free fermion gas contributions. The general expression

$$v^Q = v^Q_{mon} + v^Q_{osc}, \tag{67}$$

has an oscillating part v^Q_{osc} and a monotonic part v^Q_{mon}. In the high temperature limit $m\beta \ll 1$ the oscillating part is negligible (for the corresponding expression in QED describing Landau oscillations, see [**?**]). The monotonic part takes the form

$$\begin{aligned} v^Q_{mon} &= -\frac{2}{\pi^2}\sum_{i=1}^{N_f}\sum_{n=1}^{\infty}\sum_{\kappa=\pm 1}(e^{-ix\kappa})^n \\ &\times\{\frac{K_2(nm_i\beta)}{n^2\beta^2} + \sum_{k=1}^{\infty}\frac{\mid B_{2k}\mid}{(2k)!}(\frac{gH}{2m_i})^{2k}(\beta n)^{2(k-1)}K_{2(k-1)}(nm_i\beta)\}, \end{aligned} \tag{68}$$

where $K_s(x)$ is the Macdonald function and B_{2k} are the Bernoulli numbers. In the high temperature limit ($m_i\beta \ll 1$) after performing summation of the Macdonald functions series [**?**] we obtain

$$v^Q_{mon} = \sum_{i=1}^{N_f}[-\frac{7\pi^2}{90\beta^4} + \frac{m_i^2}{6\beta^2} + \frac{m_i^4}{4\pi^2}(\ln\frac{m_i\beta}{\pi} - \frac{3}{4} + \gamma +$$

$$+\sum_{k=1}^{\infty}\frac{\mid B_{2k}\mid}{k(k-1/2)}(\frac{gH}{m_i^2})^{2k}) + f(gA_4, gH, m_i\beta)],$$

where γ is the Euler constant, and the dependence on A_4 is included in the function f. Taking into account quarks as well as gluons, the expression for the Debye mass is replaced by

$$\begin{aligned} m_D^2 &= \sum_{i=1}^{N_f}\frac{1}{6}g^2T^2[1 - \frac{3}{2\pi^2}(m_i/T)^2 + \\ &+\frac{7\zeta(3)}{8\pi^3}(\frac{gH}{2m_i^2})^2(m_i/T)^4 + ...] + \frac{2}{3}g^2T^2, \end{aligned} \tag{69}$$

where smaller contributions of terms associated to A_4 have been omitted.

5 Conclusions and outlook

In this paper we have studied the quark-gluon plasma in the framework of a non-perturbative approach using background gauge fields as variational parameters. Our main result is the calculation of the thermodynamic potential in a simplified $SU(2)$ model of QCD and the investigation of the formation of the condensate fields $(gA_4)_{min}$ and $(gH)_{min}$ as functions of the temperature. These condensates determine the effective dynamical masses of charged gluons. They appear as a result of the interaction of thermal gluons in the infrared region, and on their part improve the infrared properties of perturbation theory at finite temperatures. The neutral spatial components of the gauge field remain massless at this level, whereas the temporal component acquires a Debye mass which is affected by the background fields.

The analysis of the interplay of these condensate fields as well as of contributions of both quarks and gluons shows that the H-condensate does not destroy the state with a A_4-condensate up to its critical value $gH_{cr} = g^2A_4^2$. Larger H-fields lead to the appearance of a tachyonic instability. In this region the tachyonic contribution $-gH$ to the effective mass squared of charged gluons dominates over the contribution $(gA_4)^2$ from the A_4-condensate. On the other hand, in the region of stability $H < H_{cr}$ the chromomagnetic field lowers the value of the thermodynamic potential (cf.(**??**)). It minimizes this potential already in the one-loop approximation, while a non-vanishing value of the A_4-condensate is expected to arise only at the two-loop level. Moreover, a detailed analysis of the thermodynamic potential in the comparatively large A_4-condensate field shows that in this region the dynamically determined condensate value $H_{\min}$ is proportional to the value of the A_4-condensate (cf.(**??**)).

It is worth emphasizing that there is a distinction between the usually studied contribution of gluon (quark) condensates to hadron masses, as considered in the strong coupling region, and our consideration. In the present paper we have studied the temperature-dependent condensates in a region far above the critical temperature of the hadronization phase transition, and it was shown that condensates together with the Debye mass turn out to be important for the infrared stabilization of the quark-gluon plasma.

Concerning quarks, their influence does not alter the principle conclusions made from considerations of the gluon sector, though their quantitative contributions to the Debye mass and effective coupling constant at high T are

substantial. In particular, we note that quarks in the chromomagnetic field region $H < H_{cr}$ do not change the conclusion about the possibility of the formation of the A_4 -condensate.

Let us, finally, add some remarks about possible extensions of the results obtained here. The most apparent step is to generalize them to the realistic case of $SU(3)$ and to calculate also higher-loop contributions in the presence of the external fields. The method of introducing background fields as non-perturbative variational parameters, used in this paper, can further be employed for investigations of the chiral phase transition from hadron bound states to the quark-gluon matter. Here it could promote a deeper understanding of the question how strong the gluon condensate affects the quark condensate and the meson masses at finite temperature (see e.g. ref.[**?**]). Further results concerning the role of external fields in the Standard Model, in QED and in calculations of the photon polarization operator can be found in refs.[**?**, **?**, **?**].

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