This is a problem for all theories of nuclear forces, not just for the chiral Lagrangian approach. One possible way of resolving the problem within the present formalism is to note that the nucleon mass is in a sense an independent parameter, since it is proportional to the number N_c of colors* while other QCD mass scales like m_ρ are not. According to this view, the factors $1/m_N$ in energy denominators like that in (16) enhance the effect of pure-nucleon intermediate states, leading to a divergence of the perturbation series at energies much less than the QCD scale. Be that as it may, because m_N is large the problem of accounting for the large nucleon-nucleon scattering lengths and small deuteron binding energy is not as severe as one might think. Eq. (7) indicates that each loop contributes a factor of Q^2 to the effective potential, but it also contributes a numerical factor of order $1/2\pi^2$, so in our power counting we are really assuming that a coupling of dimensionality $mass^{-d}$ is of the order** of $(2\pi^2)^{d/2} m_{QCD}^{-d}$. Hence it is $C_R/2\pi^2$ rather than

*It was shown by Witten[8] that in the limit $N_c \to \infty$ of quantum chromodynamics the nucleon is a skyrmion, so if the nucleon mass is to be regarded as of a different order of magnitude than the QCD scale then we ought to treat nucleons as skyrmions. Unfortunately the results obtained in this way depend on the choice of terms introduced into the Lagrangian to stabilize the skyrmion. With the simplest choice of such terms, the skyrmion-skyrmion interaction potential in the spin triplet state turns out[9] to be attractive, with a depth at its deepest point of about 25 MeV. (It would be interesting to repeat these calculations for the spin singlet state.) Of course for $N_c \to \infty$ the nucleon kinetic energy is negligible, so this would be the deuteron binding energy. From this point of view, as pointed out to me by Witten, the very small binding energy of the deuteron (and the even smaller energy of the singlet virtual state) would have to be regarded as due to a fortuitous cancellation of the potential and kinetic energies for the physical value $N_c = 3$.

**This is the same sort of counting of powers of 2 and π as used in the 'naive dimensional analysis' of Georgi et. al.[10] .

C_R itself that must be compared with m_{QCD}^{-2}. In fact, Eq. (21) indicates that in order to give a deuteron binding energy of 2.2 MeV, $C_R/2\pi^2$ in the triplet state must have the large value (260 MeV) $^{-2}$. The problem is worse in the singlet state; the singlet neutron-proton scattering length $a_s = -24.3 \times 10^{-13}$ cm requires that $-C_R/2\pi^2$ in the singlet state must have a very large value, (110 MeV) $^{-2}$.

The same difficulty can be seen in a different way if we define the renormalized value of C as the value of T at a variable renormalization point, $\vec{k}^2 = -\mu^2$. The solution of the renormalization group equation here is obtained by simply substituting $-\mu^2$ for $\vec{k}^2$ in Eq (19):

$$C(\mu) = \frac{C_R}{1 + m_N C_R \mu/4\pi}. \tag{22}$$

Note that for $\mu \gg 4\pi/m_N C_R$, the dimensionless quantity $C(\mu)\mu m_N$ approaches the fixed point 4π. It is not surprising that the renormalized value of C scales as μ^{-1}. At the renormalization scale μ the delta-function potential (14) is effectively smeared over a radius $b \simeq \mu^{-1}$, so in order to have a volume integral $C(\mu)$, the depth V_0 of the potential must be of order $C(\mu)\mu^3$; the familiar condition $V_0 \sim b^{-2}$ for a short-ranged potential to have a finite effect thus requires that $C(\mu) \propto \mu^{-1}$. What *is* surprising is that for $\mu \gg 4\pi/m_N C_R$, the parameter $C(\mu)\mu m_N$ takes the specific value 4π. Quantum chromodynamics does not seem to provide any reason why this should be the case. (It is like having a square well potential of range b and depth V_0 with $m_N V_0 b^2$ very near the value $\pi^2/4$ that yields a single bound state of

zero binding energy.) Of course, the problem only arises if $4\pi/m_N C_R$ is small compared to QCD scales like m_ρ, because otherwise Eq. (22) loses its validity before the fixed point is approached. In fact, $4\pi/m_N C_R$ is approximately 46 MeV in the triplet state and 8 MeV in the singlet state, so $C(\mu)\mu m_N$ comes near its fixed point long before (22) becomes invalid.

In any case, we shall from now on adopt the rule that a pure-nucleon intermediate state counts as if it contributed *two* more factors of $1/Q$ than other intermediate states, so that each pure-nucleon intermediate state compensates for one additional loop in the diagrams for the T-matrix.

4. Three Nucleons

The leading terms in the three-nucleon effective potential may be identified by the power counting rules described in Section 2. They are given by the graphs of Figure 2, which yield the result quoted in I:

$$\begin{aligned} V_{123} \;=\; & -\frac{1}{2}\left(\frac{2g_A}{F_\pi}\right)^4 (\vec{\sigma}_1\cdot\vec{\nabla}_1)(\vec{\sigma}_2\cdot\vec{\nabla}_1)(\vec{\sigma}_2\cdot\vec{\nabla}_3)(\vec{\sigma}_3\cdot\vec{\nabla}_3)(\mathbf{t}_1\cdot\mathbf{t}_2)(\mathbf{t}_2\cdot\mathbf{t}_3) \\ & \times\left[\tilde{Y}(|\vec{x}_1-\vec{x}_2|)Y(|\vec{x}_2-\vec{x}_3|)+\tilde{Y}(|\vec{x}_2-\vec{x}_3|)Y(|\vec{x}_1-\vec{x}_2|)\right] \\ & -\delta^3(\vec{x}_1-\vec{x}_2)\left(\frac{2g_A}{F_\pi}\right)^2[C_S+C_T(\vec{\sigma}_1\cdot\vec{\sigma}_2)] \\ & \times(\vec{\sigma}_2\cdot\vec{\nabla}_2)(\vec{\sigma}_3\cdot\vec{\nabla}_3)Y(|\vec{x}_2-\vec{x}_3|) \\ & \pm \text{ permutations}, \end{aligned} \tag{23}$$

where

$$\tilde{Y}(r)\equiv(2\pi)^{-3}\int\frac{e^{i\vec{q}\cdot\vec{r}}d^3q}{(q^2+m_\pi^2)^{3/2}}, \tag{24}$$

and '$\pm$ permutations' indicates that we must add or subtract terms involving

even or odd permutations of initial as well as final nucleons. Nothing new needs to be said here about the three-nucleon effective potential itself, but there is an important complication in the use of the effective potential for three or more nucleons that was not mentioned in I, and needs to be explained here.

It is not true that the complete S-matrix element for interactions among $N \geq 3$ nucleons is given by summing ladder graphs with $n = 1, 2, \cdots$ connected N-nucleon-irreducible subgraphs (i.e., effective potentials), connected by $n-1$ N-nucleon links. In order to calculate the S-matrix as a series of this sort, shown in Figure 3, we would have to include in the sum of three-nucleon-irreducible subgraphs not only the effective potential graphs of Figure 2, but also terms that include disconnected parts* , such as those shown in Figure 4. These disconnected parts in fact make a larger contribution to the three-nucleon-irreducible kernel than the three-nucleon effective potential itself. According to Eq. (7), the leading tree graphs for the effective two-nucleon and three-nucleon potentials make contributions respectively of order Q^0 and Q^{-1}, but the extra momentum-conservation delta function in the disconnected graphs makes a contribution effectively of order Q^{-3}, so the disconnected graphs are more important by a factor Q^{-2}. For instance, Figure 5 shows two different graphs for the connected three-nucleon S-matrix, each with a *single* three-nucleon intermediate state connecting a

*I thank S. Coleman for this important remark.

pair of three-nucleon-irreducible subgraphs. Figure 5a, in which these two three-nucleon-irreducible subgraphs are connected terms appearing in the effective three-nucleon potential, contains two extra loops, and hence makes a contribution to the connected three-nucleon S-matrix which is smaller by a factor Q^4 than Figure 5b, where the two three-nucleon-irreducible subgraphs are both disconnected. We conclude that if we sum the series in Figure 3 by re-writing it as a Lippmann-Schwinger integral equation, then the kernel must be calculated using a potential

$$V = V_{12} + V_{13} + V_{23} \tag{25}$$

with the three-nucleon effective potential (23) entering only in corrections of higher order in Q. This of course is just what has usually been assumed in dealing with the three-nucleon system.

The disconnected terms in the three-nucleon- irreducible kernel contain two delta-functions, and hence are of order Q^{-6}. Each three-nucleon intermediate state contributes a factor Q^9 from the three momentum space integrals, so if as discussed above we take the energy denominator for such a state as being of order Q^3, then each such intermediate state contributes a net factor Q^6, just compensating for one of the three-nucleon-irreducible kernels. Thus the leading graphs for the three-nucleon S-matrix does indeed consist of a series of terms like those in Figure 3. It is only the presence of the disconnected three-nucleon irreducible terms that makes it necessary to sum the series of

Figure 3, or in other words to solve a Lippmann-Schwinger equation*, rather than taking the three-nucleon S-matrix as the effective potential itself.

5. More Nucleons

The case of $N \geq 4$ nucleons can be dealt with in a similar fashion. The most general N-nucleon irreducible graph consists of c connected parts, to each of which is attached $N_1, N_2, \cdots, N_c$ incoming nucleon lines and an equal number of outgoing nucleon lines, plus d disconnected single lines, with $\sum_r N_r + d = N$. According to Eq. (7), the leading connected N_r-nucleon-irreducible tree graphs with N_r incoming and N_r outgoing nucleon lines is of order Q^{2-N_r}, and each extra connected part or single line contributes a three-dimensional delta function to the whole kernel, so the whole graph is of order $Q^{\nu'}$, where

$$\nu' = \sum_{r=1}^{c}(2 - N_r) - 3(c + d) = -c - 2d - N\ . \tag{26}$$

The leading contributions to the N-nucleon-irreducible kernel are hence those with the maximum number $d = N-2$ of disconnected single lines, plus a single two-nucleon connected piece. Again, the Lippmann-Schwinger equation can be used to calculate the N-nucleon S-matrix, with the potential in the kernel being given by a sum of the $N(N-1)/2$ two-nucleon potentials V_{ij}.

*Because the Lippman-Schwinger integral equation has a disconnected kernel, it cannot be solved by numerical methods that approximate the kernel as a finite matrix. As is well known, for three[11] or more[12] particles it is possible to replace the Lippmann-Schwinger equation by an integral equation with a connected and hence square-integrable kernel, which can be solved by ordinary numerical methods. The leading term in this kernel is calculated using the potential (25), with the connected three-body graphs of Figure 2 providing a higher-order correction.

In a way it is a pity that the specifically nonlinear terms in the pion-nucleon interaction are not manifested except as higher-order corrections.

6. First Corrections*

There are several sources of corrections to the leading terms discussed above, but the first corrections with just one extra power of Q are quite limited. Each added loop and each decrease in the number of disconnected pieces of the effective potential contributes two extra powers of Q, so we must calculate the first corrections to the effective potential still using only tree graphs, and still as a sum of two-body terms. However now we must calculate the two-nucleon effective potential including one vertex with $d_i + \frac{1}{2}n_i - 2 = +1$, as well as arbitrary numbers of vertices satisfying Eq. (8) that are constructed from the terms in the Lagrangian shown explicitly in Eq. (1). Parity conservation does not permit any interactions with four nucleon fields and one derivative, and Lorentz invariance does not allow any purely pionic interactions with three spacetime derivatives. This leaves just one-nucleon interactions with $n_i = 2$ and $d_i = 2$. These include the terms

$$D^{-2}(\vec{\nabla}\pi)^2\,\bar{N}N \tag{27}$$

and

$$D^{-2}\epsilon_{ijk}\epsilon_{abc}\partial_i\pi_a\,\partial_j\pi_b\,\bar{N}\sigma_k t_c N \tag{28}$$

as well as the interaction terms in the chiral-invariant completion of the

*The work of this Section was done in collaboration with C. Ordóñez and U. van Kolck.

nucleon kinetic energy term (9):

$$\bar{N}\left[\vec{\nabla} + 2iD^{-1}\mathrm{t}\cdot(\pi\times\vec{\nabla}\pi)\right]^2 N/2m_N\,. \tag{29}$$

In addition there are terms with the chiral symmetry transformation properties of the quark mass terms: one term proportional to $m_u + m_d \propto m_\pi^2$ that transforms as the fourth component of a chiral four-vector*

$$D^{-1}\pi^2\bar{N}N \tag{30}$$

and another term proportional to $m_u - m_d$ that transforms as the third component of a different chiral four-vector**

$$D^{-1}\pi_3\bar{N}(\mathrm{t}\cdot\pi)N\,. \tag{31}$$

However, apart from any questions of power counting, the only tree graphs that can contribute to the *two*-nucleon effective potential are a direct nucleon-nucleon interaction and one-pion exchange. The interactions (27) - (31) all involve at least two pion fields, so none of them can contribute to the two-nucleon effective potential in the tree approximation. In particular, the absence of any contribution in this order from the isospin-violating term (31) provides some justification for an approximation that seems to be nearly universal among nuclear physicists, that isospin violation arises solely from photon exchange and from the effect of nucleon mass differences in the Lippmann-Schwinger equations (both effects that have not been considered here), but

*This is the so-called sigma-term.

**The presence of this term in pion-nucleon scattering was pointed out in ref. 13.

not from effects of quark mass differences on one-pion exchange or other strong interactions among nucleons.

There is another possible term in the Lagrangian that appears to contain a term linear in the pion field:

$$D^{-1}\bar{N}\,(\mathbf{t}\cdot\dot{\boldsymbol{\pi}})\;\vec{\sigma}\cdot\left[\vec{\nabla}+2iD^{-1}\mathbf{t}\cdot(\boldsymbol{\pi}\times\vec{\nabla}\boldsymbol{\pi})\right]N.$$

but by integration by parts and using the nucleon field equation this can be written in terms of other chiral-invariant terms of the same order, all of which involve at least two powers of the pion field. We conclude that *there are no corrections to the leading terms in the interactions of two or more nucleons that are smaller than the leading terms by just one factor Q.*

There are various corrections of order Q^2, arising from the following sources:

- Two-nucleon effective potentials calculated from either (i) a single interaction having four nucleon fields and two derivatives or one quark mass factor, or (ii) a one-pion-exchange graph with a correction to one of the pion-nucleon vertices involving three derivatives or one derivative and one quark mass factor.

- Two-nucleon effective potentials calculated from one-loop graphs, with all interactions satisfying (8).

- Three-nucleon effective potentials calculated from tree graphs with all interactions satisfying (8), as shown in Figure 2.

It will be interesting to see if more of the observed features of nuclear forces emerge in these corrections of order Q^2. Such calculations are being pursued by C. Ordóñez and U. van Kolck.

I am grateful for discussions with many colleagues, especially S. Coleman, J. Negele, C. Ordóñez, J. Polchinski, E. F. Redish, U. van Kolck, and E. Witten.

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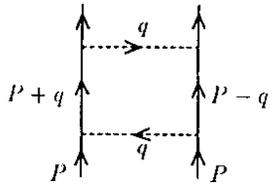

Figure 1. A graph for nucleon-nucleon scattering that violates the simple power-counting rules. (Solid lines are nucleons; dashed lines are pions.)

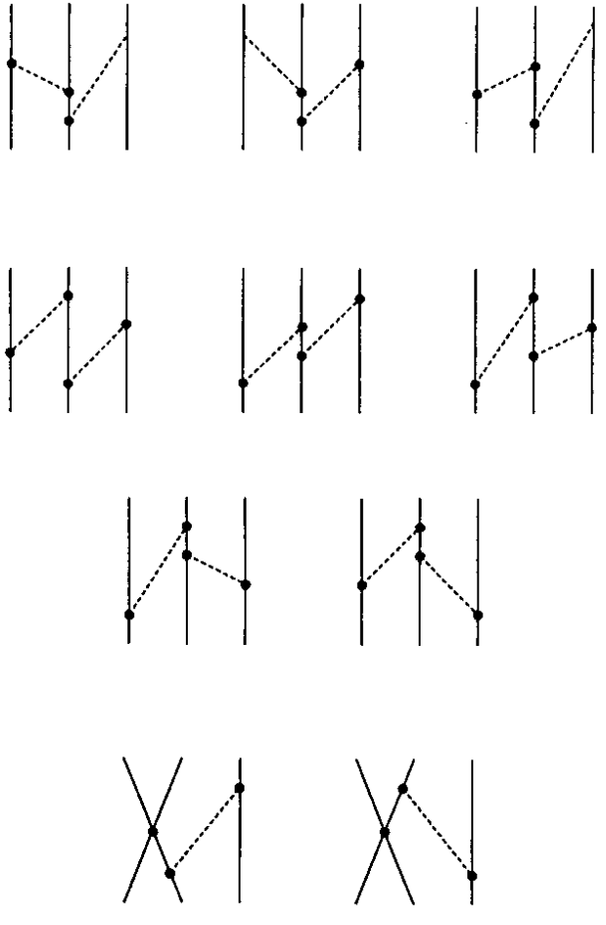

Figure 2. Graphs for the three-nucleon effective potential. Other graphs involving nonlinear pion interactions all cancel.

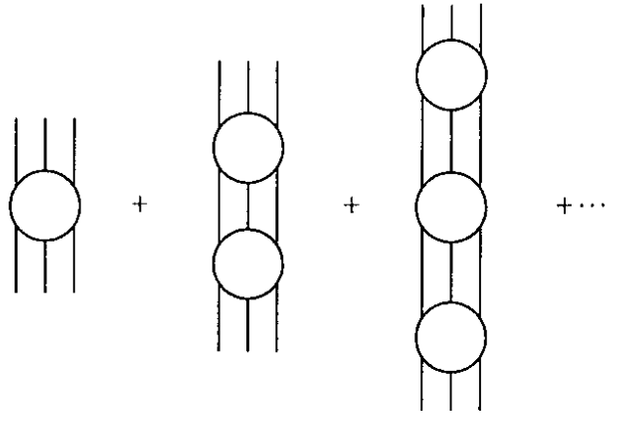

Figure 3. Sum of graphs for the 3-nucleon S-matrix. Circles here denote sums of 3-nucleon-irreducible subgraphs, shown in Figure 4.

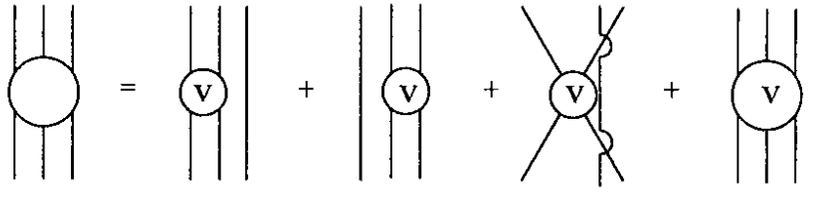

Figure 4. Sum of graphs for the 3-nucleon-irreducible subgraphs in Figure 3. Here circles marked "V" denote effective potentials, i.e., connected three-nucleon-irreducible graphs.

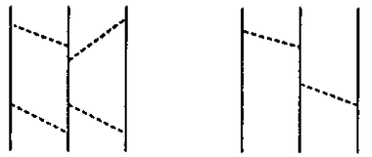

Figure 5. Two connected graphs for the three-nucleon S-matrix, each with a single 3-nucleon intermediate state.