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QED RADIATIVE CORRECTIONS IN EXCLUSIVE ρ^0 LEPTOPRODUCTION

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Abstract

The semi-analytical approach to the model independent leptonic QED corrections to exclusive ρ^0 meson leptoproduction (i.e. electron and muon scattering experiments) is presented. The corrections to ρ^0 production at large Q^2 as well as to ρ^0 photoproduction are studied in details. The numerical results are calculated for two different experimental analyses: NMC (muoproduction at large Q^2) and ZEUS at HERA (photoproduction). It is shown that the corrections are 2-5 % for NMC and below 2 % for the ZEUS measurement. The application of the presented approach to other vector mesons production is straightforward.

1 Introduction

Elastic production of ρ^0 mesons by photons, $\gamma p \to \rho^0 p$, has been extensively studied in the fixed target experiments [1, 2, 3] and recently also at HERA ep collider [4, 5, 6], using virtual and real or quasi-real photons. In real photoproduction and small Q^2 electroproduction of ρ^0 meson characteristic features of diffractive processes are observed. It allows to describe the process in the framework of the Vector Meson Dominance Model (VMD) and pomeron exchange [7]. At large Q^2 also the perturbative QCD mechanism (exchange of two gluons between nucleon and the virtual $q\bar{q}$ pair from photon) may be appropriate. Therefore the study of the vector mesons production in DIS may help to understand the structure of the pomeron and its connection with perturbative QCD.

To determine the one photon exchange (Born) cross section in DIS from the measured cross section the radiative corrections procedure is needed [8, 9, 10]. The similar procedure should be applied to extract the Born cross section for ρ^0 meson production. The Feynman diagram in Born approximation for the reaction is shown in Fig. 1. Unfortunately the

problem of radiative corrections in the case on vector mesons production has not been fully solved [2, 4]. As the cuts used to which define the events selection depend on the experimental setup, the radiative processes should be taken into account in Monte Carlo simulations. In this paper we propose a simpler approach which allows to estimate an order of magnitude of the radiative effects in the case of vector mesons production in DIS (high Q^2) and in photoproduction. The method is based on the existing calculations of the radiative corrections for standard DIS events [8, 9, 10] and on the several, well-justified assumptions about the features of the vector mesons production. The assumptions are the following:

- The main contribution to the radiative corrections proceeds from the virtual (closed loops) and real radiations from leptons. It means that we can neglect the radiation from hadron (lower) part of the diagram in Fig. 1.
- The radiative processes connected with leptons are exactly the same as in the "standard" DIS events. The events with ρ^0 are a subsample of the DIS events observed in the experiments, selected in the off-line analysis by extra cuts and selections.
- The ρ^0 cross section can be factorized as a model-independent lepton part (the flux of virtual or quasi-real photons) and hadron part dependent on the mechanism of the vector mesons production (different models of the production).
- In both kinematical regions (small or large Q^2) the energy-momentum transver, t, from the photon to ρ^0 is assumed to be small (it can be related to p_T^2 of ρ^0). This assumption is justified by all diffractive models [7] [1] and also confirmed experimentally [2, 4, 5, 6].

The paper is organized as follows. In Section 2 the kinematics are defined and cross sections formulae are given. Sections 3, 4 and 5, together with appendices A and B contain a description of the application of the standard DIS radiative corrections procedure for the vector mesons production for large Q^2 and for photoproduction (quasi-real photons), respectively. A complete set of formulae is given for each kinematical regime. In Section 6 and 7 the numerical results are given for the NMC quasi-elastic ρ^0 cross section measurement [2] and for the ZEUS ρ^0 photoproduction analysis. Finally a summary is given in Section 8.

2 Basic Definitions and Kinematics of exclusive vector mesons production

As an example of vector meson we will consider ρ^0. The exclusive ρ^0 meson leptoproduction reaction is (Fig. 1)

$$l + N \to l + N + \rho^0. \tag{1}$$

The definitions of kinematical variables are the following:

[1] There are also some models where t is expected to be relatively large [11]; however large t is not observed in the experiments considered in this paper.

k, k'	- four-momenta of the incident and scattered leptons (electrons or muons)
p	- four-momentum of the target nucleon
v	- four-momentum of the vector meson (ρ^0)
$q = k - k'$	- four-momentum of the virtual photon
$Q^2 = -q^2$	- an invariant mass squared of the virtual photon
$\nu = (p \cdot q)/M$	- an energy of the virtual photon in the laboratory system (M is the proton mass)
$x = Q^2/(2M\nu)$	- Bjorken scaling variable
$y = (p \cdot q)/(p \cdot k)$	- fraction of the lepton energy lost in laboratory system
$W^2 = (p + q)^2$	- total energy squared in the $\gamma^\star N$ system
$t = (q - v)^2$	- four-momentum transfer between the virtual photon and the vector meson (ρ^0)
$M_X^2 = (p + q - v)^2$	- missing mass squared of the undetected recoiling system
$I = (M_X^2 - M^2)/W^2$	- Inelasticity

It is also convenient to introduce S variable defined as $S = 2(p \cdot k)$. Hence we have $Q^2 = xyS$.

Inelasticity I is equal to 0 for exclusive ρ^0 production and the cuts on inelasticity were used to select exclusive ρ^0 sample in the NMC analysis [2].

The cross section for DIS in Born approximation can be expressed in the following way

$$\sigma^B(y, Q^2) \equiv \frac{\mathrm{d}^2\sigma^B(lp \to lX)}{\mathrm{d}Q^2\mathrm{d}y} = \Gamma_T^B \sigma_{\gamma^\star}^B(y, Q^2), \tag{2}$$

where

$$\Gamma_T^B = \Gamma_T^B(\mathcal{S}_1^B, \mathcal{S}_2^B) = \frac{\alpha}{2\pi Q^2}\frac{y}{x}(1 - x)\left(\frac{\mathcal{S}_1^B}{Sx} + \frac{\mathcal{S}_2^B}{S(yS + 4M^2x)}\right) \tag{3}$$

and

$$\sigma_{\gamma^\star}^B(y, Q^2) \equiv \frac{\mathrm{d}^2\sigma^B(\gamma^\star p \to X)}{\mathrm{d}Q^2\mathrm{d}y} = \sigma_T + \varepsilon^B\sigma_L. \tag{4}$$

Introducing the function $R = \sigma_L/\sigma_T$ the formula (4) can be rewritten as follows:

$$\sigma_{\gamma^\star}^B(y, Q^2) = \sigma_T(1 + \varepsilon^B R), \tag{5}$$

where

$$\frac{1}{\varepsilon^B} = 1 + \frac{\mathcal{S}_1^B}{\mathcal{S}_2^B}(4M^2 + \frac{y}{x}S). \tag{6}$$

The functions $\mathcal{S}_1^B$ and $\mathcal{S}_2^B$ are defined as:

$$\begin{aligned} \mathcal{S}_1^B(y, Q^2) &= Q^2 - 2m_l^2 \\ \mathcal{S}_2^B(y, Q^2) &= 2\left[(1 - y)S^2 - M^2Q^2\right]. \end{aligned} \tag{7}$$

Here σ_T and σ_L denote the total cross sections for photoabsorption of the longitudinaly and transversaly polarized virtual photon respectively, and can be expressed in terms of nucleon (proton) electromagnetic structure functions

$$\begin{aligned}
\sigma_T &= \frac{4\pi^2\alpha}{Q^2} 2xF_1(x,Q^2), \\
\sigma_L &= \frac{4\pi^2\alpha}{Q^2}\left[(1+\frac{4M^2x^2}{Q^2})F_2(x,Q^2) - 2xF_1(x,Q^2)\right], \\
R &= \frac{\sigma_L}{\sigma_T} = \frac{1+\frac{4M^2x^2}{Q^2}}{2x}\frac{F_2}{F_1} - 1.
\end{aligned} \tag{8}$$

The similar set of definitions of the cross sections for ρ^0 meson production can be introduced. We have respectively

$$\sigma^B_{\rho^0}(y,Q^2) \equiv \frac{\mathrm{d}^2\sigma^B(lp \to lp\rho^0)}{\mathrm{d}Q^2\mathrm{d}y} = \Gamma^B_T \sigma^B_{\gamma^\star\rho^0}(y,Q^2). \tag{9}$$

The $\sigma^B_{\gamma^\star\rho^0}$ is a cross section for the exclusive ρ^0 meson production in virtual photon - nucleon reaction.

$$\sigma^B_{\gamma^\star\rho^0}(y,Q^2) \equiv \frac{\mathrm{d}^2\sigma^B(\gamma^\star p \to p\rho^0)}{\mathrm{d}Q^2\mathrm{d}y} = \sigma^{\rho^0}_T(1+\varepsilon^B R_{\rho^0}). \tag{10}$$

The cross sections given above corespond to the Born approximation (ie. are one-gamma exchange approximation). The measured cross sections contain also the contributions from the radiative processes and therefore differ from the Born ones. The measured cross sections will be written without B superscript

$$\begin{aligned}
\sigma(y,Q^2) &\equiv \frac{\mathrm{d}^2\sigma_{\mathrm{meas}}(lp \to lX)}{\mathrm{d}Q^2\mathrm{d}y}, \\
\sigma_{\rho^0}(y,Q^2) &\equiv \frac{\mathrm{d}^2\sigma_{\mathrm{meas}}(lp \to lp\rho^0)}{\mathrm{d}Q^2\mathrm{d}y}.
\end{aligned} \tag{11}$$

3 Radiative corections for exclusive production of vector mesons in deep inelastic scattering.

To estimate the effect of radiative processes we apply the so-called Dubna radiative corrections scheme, (D scheme), calculated by A.A. Akhundov et al.[8, 10]. The detailed description of the calculations can be found in the original paper ([8]) and also in review articles [10, 9]. The notation used here is based on [9].

The measured cross section for DIS is now expressed as follows

$$\sigma(y,Q^2) = \sigma^B(y,Q^2)\left(\mathrm{e}^{-\delta_R(y,Q^2)} + \delta^{VR}(y,Q^2)\right) + \sigma_{\mathrm{in.tail}}(y,Q^2) - \sigma^{IR}(y,Q^2). \tag{12}$$

The δ_R in eq. (12) is responsible for those parts of the *soft* and *hard collinear* photon emissions which could be resummed to all orders using the covariant exponentiation procedure, [8, 10, 9]. It reads

$$\delta_R = -\frac{\alpha}{\pi}\left(\ln\frac{Q^2}{m^2} - 1\right)\ln\frac{y^2(1-x)^2}{(1-yx)(1-y(1-x))}, \tag{13}$$

where m is the lepton mass. The δ^{VR} correction factor in eq.(12) is a remnant of the exponentiation and of the subtraction procedure used to disentangle the infrared divergent terms from the $\sigma_{\mathrm{in.tail}}$ cross section. It thus contains the vertex correction and is given by

$$\delta^{VR} = \delta_{\mathrm{vtx}} - \frac{\alpha}{2\pi}\ln^2\frac{(1-yx)}{(1-y(1-x))} + \Phi\left[\frac{(1-y)}{(1-yx)(1-y(1-x))}\right] - \Phi(1), \tag{14}$$

where $\Phi(x)$ denote the Spence function and δ_{vtx} is given by the formula

$$\delta_{\mathrm{vtx}} \;=\; \frac{2\alpha}{\pi}\left(-1 + \frac{3}{4}\ln\frac{Q^2}{m^2}\right). \tag{15}$$

The $\sigma_{\mathrm{in.tail}}$ in eq. (12) describes the inelastic radiative tail for the *lepton current correction* only

$$\begin{aligned}\sigma_{\mathrm{in.tail}}(y,Q^2) \equiv \frac{\mathrm{d}^2\sigma_{\mathrm{in.tail}}}{\mathrm{d}Q^2\mathrm{d}y} &= \frac{2\alpha^3}{S}\iint \mathrm{d}Q_h^2\mathrm{d}M_h^2\frac{1}{Q_h^4}\Big\{2F_1(x_h,Q_h^2)\mathcal{S}_1(y,Q^2,y_h,Q_h^2) \\ &+ F_2(x_h,Q_h^2)\frac{1}{yS}\mathcal{S}_2(y,Q^2,y_h,Q_h^2)\Big\}.\end{aligned} \tag{16}$$

In this formula M_h is the invariant mass of the final hadronic system and the variables bearing a subscript 'h' refer to virtual photon–target vertex in contrast to the variables measured in the inclusive electroproduction experiment. For their definitions see Appendix A where also the explicit expressions for the radiator functions $\mathcal{S}_i$ are given.

For our purpose it is more convienient to rewrite formula for $\sigma_{\mathrm{in.tail}}$ in the similar way as for σ^B in section 2 (eq. (2)):

$$\sigma_{\mathrm{in.tail}}(x,Q^2) = \frac{\alpha}{\pi}\iint \mathrm{d}Q_h^2\mathrm{d}M_h^2\Gamma'_T\sigma'_{\gamma^\star}. \tag{17}$$

In order to simplify the notation, the "prim" denotes that Γ_T and $\sigma_{\gamma^\star}$ depend not only on Q^2,y but also on Q_h^2 and y_h
Γ'_T and $\sigma'_{\gamma^\star}$ have the forms as in the equations (3) ,(5) and (6) but now with the different radiator functions $\mathcal{S}_1$ and $\mathcal{S}_2$ (see Appendix A and [9, 10]):

$$\begin{aligned}\Gamma_T &= \Gamma_T(\mathcal{S}_1,\mathcal{S}_2) \\ \sigma_{\gamma^\star} &= \sigma_T(1+\varepsilon R)\end{aligned} \tag{18}$$

and finally

$$\varepsilon = \varepsilon(\mathcal{S}_1, \mathcal{S}_2) \tag{19}$$

It is well known that the $\sigma_{\text{in.tail}}$ cross section is infrared divergent. To regularize it a simple trick ('fixation procedure') [8, 10] is used. It consists in adding to $\sigma_{\text{in.tail}}$ and subtracting from it an extra term

$$\sigma^{IR}(x, Q^2) = \frac{\mathrm{d}^2\sigma^{IR}}{\mathrm{d}Q^2\mathrm{d}x} = \sigma^B F^{IR}, \tag{20}$$

where

$$F^{IR} = \iint \mathrm{d}Q_h^2 \mathrm{d}M_h^2 \mathcal{F}^{IR}(y, Q^2, y_h, Q_h^2). \tag{21}$$

In the added term an integration over a full photon phase space was carried out, resulting in the above given expressions for δ_R and δ^{VR}. The subtracted term appears explicitly in eq. (12) so that the difference $\sigma_{\text{in.tail}} - \sigma^{IR}$ is finite over the full kinematic domain of Q_h^2 and M_h^2. The function $\mathcal{F}^{IR}$ is given in Appendix A.

The magnitude of the radiative effects in the measured cross section will be characterized by the so called radiative correction factor, $\eta(x, y)$, defined as follows

$$\eta(x, y) = \frac{\sigma^B}{\sigma_{\text{meas}}}\,. \tag{22}$$

Combining formulae in eqs. (12), (16) and (20) the factor η can be expressed as follows

$$\frac{1}{\eta} = \mathrm{e}^{-\delta_R} + \delta^{VR} + \frac{\sigma_{\text{in.tail}}}{\sigma^B} - F^{IR}. \tag{23}$$

To simplify the notation the y and Q^2 dependence in eq. (23) is not shown explicitely. Finally the vacuum polarisation was taken into account via the 'running' $\alpha(Q^2)$ which in the $Q^2 \gg {m_f}^2$ approximation (m_f stands for the lepton and quark masses) is

$$\alpha(Q^2) = \frac{\alpha}{1 + \sum_f c_f Q_f^2 \delta_{\text{vac}}^f}, \tag{24}$$

where c_f and Q_f are the colour factor and the electric charge of fermions f ($c_f = Q_f = 1$ for leptons); 'f' runs over all leptons and quarks.
δ_{vac}^f in the formula (24) is equal to

$$\delta_{\text{vac}}^f = \frac{2\alpha}{\pi}\left(-\frac{5}{9} + \frac{1}{3}\ln\frac{Q^2}{m_f^2}\right). \tag{25}$$

As it was said above this formula holds for $Q^2 \gg m_f^2$. The full formula is discussed in the case of photoproduction (section 4) and is given in Appendix B.

Now we can introduce in the same way the η factor for exclusive vector meson production. Again as an example ρ^0 meson is considered. The η_{ρ^0} radiative correction factor is given in the formula similar to eq. (23)

$$\frac{1}{\eta_{\rho^0}} = \mathrm{e}^{-\delta_R} + \delta^{VR} + \frac{\sigma_{\text{in.tail}}^{\rho^0}}{\sigma_{\rho^0}^B} - F^{IR} \tag{26}$$

The δ_R and δ^{VR} are calculated exactly in the same way as for inclusive DIS. The important differences are connected with the real-photons radiation part (inelastic tail) which is described by different formula. It is not very suprising because this term depends on the structure functions (or cross sections) and therefore on the process measured in the experiment. The limits of the integrations in eqs. (16) and (20) depend on the selection criteria used in the analysis to extract the exclusive ρ^0 sample. Therefore the last term in eq. (26), F^{IR}, is also affected, though the formula for the integrand function $\mathcal{F}^{IR}$ is universal. We will come back to this question in the section 6. Let us consider now the inelastic tail term in formula (26). Taking into account eqs. (9) and (16) the ratio $\sigma^{\rho^0}_{\text{in.tail}}/\sigma^B_{\rho^0}$ can be written as follows

$$\frac{\sigma^{\rho^0}_{\text{in.tail}}}{\sigma^B_{\rho^0}} = \frac{\alpha}{\pi}\iint \mathrm{d}Q_h^2 \mathrm{d}M_h^2 \frac{\Gamma_T{}' \sigma^{\rho^0\prime}_{\gamma^\star}}{\Gamma^B_T \sigma^{B\rho^0}_{\gamma^\star}}. \tag{27}$$

The "prim" again means the hidden dependence on "h" 's variables. According eq. (10) $\sigma^{\rho^0}_T$ is equal to

$$\sigma^{\rho^0}_T = \frac{\sigma^B_{\gamma^\star\rho^0}}{(1+\varepsilon^B R_{\rho^0})} \tag{28}$$

and therefore $\sigma^{\rho^0\prime}_{\gamma^\star}$ can be rewritten as

$$\sigma^{\rho^0\prime}_{\gamma^\star} = \sigma^B_{\gamma^\star\rho^0}{}' \frac{(1+\varepsilon' R_{\rho^0})}{(1+\varepsilon^{B\prime} R_{\rho^0})}. \tag{29}$$

Putting above formula to eq. (27) and then to formula (26) we obtain finally our "master" formula for η_{ρ^0} radiative correction factor in the large Q^2 regim:

$$\frac{1}{\eta_{\rho^0}} = \mathrm{e}^{-\delta_R} + \delta^{VR} + \frac{\alpha}{\pi}\iint \mathrm{d}Q_h^2 \mathrm{d}M_h^2 \frac{\Gamma'_T}{\Gamma^B_T}\frac{(1+\varepsilon' R_{\rho^0})}{(1+\varepsilon^{B\prime} R_{\rho^0})}\frac{\sigma^B_{\gamma^\star\rho^0}{}'}{\sigma^B_{\gamma^\star\rho^0}}. \tag{30}$$

As in the previous formulae the hidden notation is used; the y and Q^2 dependence is not shown explicitely as well as ' denotes the "h" variables dependence over which the integrations is performed. The photon fluxes (Γ'_T and Γ^B_T) and $\varepsilon's$ ($\varepsilon^{B\prime}$ and ε') are expressed in terms of radiator functions $\mathcal{S}^B_1, \mathcal{S}^B_2, \mathcal{S}_1, \mathcal{S}_2$ (see Appendix A.). R_{ρ^0} and the cross section $\sigma^B_{\gamma^\star\rho^0}$ were measured in the experiment for different y and Q^2 and in the numerical calculations the fit to data was used (see [2]). The numerical calculations for NMC analysis are discussed in the section 6.

4 Radiative corections for exclusive vector meson photoproduction

In this section we present the application of the radiative corrections procedure to the small Q^2 region. As an example ZEUS ρ^0 photoproduction at HERA will be considered (section 7) , where the considered events have very small Q^2, nearly 0 ($\approx 10^{-9}$ [4]).

The starting point is the cross section formula (9). Due to the fact that photon emitted by electron is nearly real, the electron is practically not distorted and therefore not observed in the experiment (escapes through the pipe). Also proton is not observed in this case. Therefore to estimate the ρ^0 cross section the theoretical input about production mechanism is needed. In the ZEUS analysis the Vector Meson Dominance model (VMD) relations about transverse and longitudinal cross sections in the limit of Q^2 close to 0 were assumed ([4, 7])

$$\begin{aligned}\sigma_{L\rho^0}(W,Q^2) &= \frac{Q^2}{m^2_{\rho^0}}\sigma_{T\rho^0}(W,Q^2),\\ \sigma_{T\rho^0}(W,Q^2) &= \frac{\sigma_{\gamma\rho^0}(W)}{\left(1+\frac{Q^2}{m^2_{\rho^0}}\right)^2},\end{aligned} \tag{31}$$

where $\sigma_{\gamma\rho^0}(W)$ is the cross section for elastic photoproduction ($Q^2=0$) of ρ^0 meson. Substituting the VMD predictions (eq. (31)) to eq. (10) we obtain the following expresion for the $\sigma^B_{\gamma^*\rho^0}$:

$$\sigma^B_{\gamma^*\rho^0} = \sigma_{T\rho^0}(W,Q^2)\left(1+\varepsilon^B\frac{Q^2}{m^2_{\rho^0}}\right) = \sigma_{\gamma\rho^0}(W)\frac{\left(1+\varepsilon^B\frac{Q^2}{m^2_{\rho^0}}\right)}{\left(1+\frac{Q^2}{m^2_{\rho^0}}\right)^2}. \tag{32}$$

Finally combining above expresion together with eq. (10) we find that the differential cross section in Born approximation for ρ^0 production is expressed as follows

$$\sigma^B_{\rho^0}(y,Q^2) = \Gamma^B_T\sigma_{\gamma\rho^0}(W)\frac{\left(1+\varepsilon^B\frac{Q^2}{m^2_{\rho^0}}\right)}{\left(1+\frac{Q^2}{m^2_{\rho^0}}\right)^2} = \Phi^B(y,Q^2)\sigma_{\gamma\rho^0}(W). \tag{33}$$

Φ is so-called effective photon flux. It reads in Born approximation:

$$\begin{aligned}\Phi^B(y,Q^2) &\equiv \Phi(\mathcal{S}^B_1,\mathcal{S}^B_2) = \Gamma^B_T\frac{\left(1+\varepsilon^B\frac{Q^2}{m^2_{\rho^0}}\right)}{\left(1+\frac{Q^2}{m^2_{\rho^0}}\right)^2}\\ &= \frac{\alpha}{2\pi Q^2}\left[\frac{1+(1-y)^2}{y} - \frac{2(1-y)}{y}\left(\frac{Q^2_{min}}{Q^2}-\frac{Q^2}{m^2_{\rho^0}}\right)\right.\\ &+ \left.\epsilon\left(\frac{y}{2}\left(1-\frac{Q^2}{m^2_{\rho^0}}\right) - 2(1-y)\frac{Q^2_{min}}{Q^2}\right)\right]\frac{1}{1+\epsilon}\frac{1}{\left(1+\frac{Q^2}{m^2_{\rho^0}}\right)^2}.\end{aligned} \tag{34}$$

In this equation Q^2_{min} is the minimal value of Q^2 allowed by kinematics ($10^{-9} GeV^2$ in ZEUS), m_{ρ^0} and m_e are mass of ρ^0 meson and electron, respectively. The ϵ parameter is equal to $\frac{4M^2Q^2}{S^2y^2}$ and is smaller than 10^{-6} in ZEUS photoproduction events selection and can be neclected. This simplify the effective photon flux formula to the formula used in ZEUS paper [4]

$$\Phi^B(y,Q^2) = \frac{\alpha}{2\pi Q^2}\left[\frac{1+(1-y)^2}{y} - \frac{2(1-y)}{y}\left(\frac{Q^2_{min}}{Q^2} - \frac{Q^2}{m^2_{\rho^0}}\right)\right]\frac{1}{\left(1+\frac{Q^2}{m^2_{\rho^0}}\right)^2}. \tag{35}$$

The photoproduction cross section $\sigma_{\gamma\rho^0}(W)$ at $Q^2 = 0$ is obtained as a ratio of the corresponding ep cross section integrated over y and Q^2 ranges covered by the ZEUS measurement and of the effective photon flux integrated over the kinematical range:

$$\sigma_{\gamma\rho^0} = \frac{\sigma_{\rho^0}}{\Phi_{int}}. \tag{36}$$

σ_{ρ^0} is determined from data as follows:

$$\begin{aligned} \sigma_{\rho^0} &= \frac{N_{\pi^+\pi^-}}{LA}, \\ \Phi_{int} &= \iint \mathrm{d}Q^2\mathrm{d}y\,\Phi^B(Q^2,y). \end{aligned} \tag{37}$$

$N_{\pi^+\pi^-}$ is the number of observed events with pions from ρ^0, A and L are acceptance and luminosity, respectively, see [4]. This procedure determines averaged cross section for elastic ρ^0 meson photoproduction over a given W range.[2] To calculate radiative correction factor η for the measured average cross section $\sigma_{\gamma\rho^0}$ the same method as for the large Q^2 ρ^0 meson production can be used, however now in addition the integration over W (or y) and Q^2 ranges where the events are accepted and contributed to the averaged cross section, must be performed. As a result we obtain only one number for η (or equivalently "new", corrected $\sigma_{\gamma\rho^0}$) integrated and averaged over measured kinematical region. The measured cross section σ_{ρ^0} contains not only pions pair from Born ρ^0 production process (Fig. 1) but also from radiative events. Therefore the measured cross section σ_{ρ^0} can be written as

$$\sigma_{\rho^0} = \sigma^B_{\rho^0} + \sigma^R_{\rho^0}, \tag{38}$$

where $\sigma^B_{\rho^0}$ is an integrated Born cross section from eq. (33) and $\sigma^R_{\rho^0}$ is the radiative part of the measured cross section also integrated over W (y) and Q^2. Therefore the corrected formula for $\sigma_{\gamma\rho^0}$ can be written as

$$\sigma^{\mathrm{corr}}_{\gamma\rho^0} = \frac{\sigma^B_{\rho^0}}{\Phi_{int}} = \frac{\sigma_{\rho^0} - \sigma^R_{\rho^0}}{\Phi_{int}}, \tag{39}$$

[2] The integration over Q^2 and y is experimental analysis dependent; if it is possible to measure Q^2 and y dependent cross section for small Q^2 (or $Q^2 \approx 0$) these integrations in the formulae presented in this section are not needed.

where the radiative part of the cross section $\sigma^R_{\rho^0}$ is given by the similar formula as in eq. (12):

$$\sigma^R_{\rho^0} = \iint \mathrm{d}Q^2\mathrm{d}y \left(\sigma^B_{\rho^0}\left(\delta^{VR} - \delta_R + \delta_{\mathrm{anom}}\right) + \sigma_{\mathrm{in.tail}} - \sigma^{IR}\right). \tag{40}$$

$\sigma_{\mathrm{in.tail}}$ is now expressed in terms of effective flux (now *not* in Born approximation) and $\sigma_{\gamma\rho^0}$

$$\sigma_{\mathrm{in.tail}}(y, Q^2) = \frac{\alpha}{\pi}\iint \mathrm{d}Q^2_h\mathrm{d}M^2_h\Phi'\sigma_{\gamma\rho^0}(M_h), \tag{41}$$

where the effective flux now depends on "h" indexed variables and full radiative functions $\mathcal{S}_1$ and $\mathcal{S}_2$

$$\Phi' = \Phi'(\mathcal{S}_1, \mathcal{S}_2). \tag{42}$$

The δ^{VR}, δ_R and σ^{IR} have the same meaning as in eq. (12) and in section 2, but the formulae for these quantities are slightly different due to the fact that we consider now very small Q^2 and all quantities must be calculated exactly keeping all terms, including that with electron's mass. In the formulae in section 2. the lepton mass terms (like $\frac{m_l^2}{Q^2}$) were neglected.
The correct formulae with electron's mass terms for δ^{VR}, δ_R and for vertex and vacuum polarization corrections are collected in Appendix B. For detailed calculations the reader is referred to the original papers [8, 10].
The new contribution, δ_{anom} , is present in eq. (40). This correction is important only for real (or quasi-real) photons (photoproduction) and takes into account the contribution from anomalous magnetic moment of electron. The calculations and the formula for δ_{anom} are discussed in the next section in more details.

Now it is possible to formulate the expresion for the corrected cross section for elastic ρ^0 meson photoproduction

$$\begin{aligned}\sigma^{\mathrm{corr}}_{\gamma\rho^0} &= \sigma_{\gamma\rho^0}\Bigg\{1 - \iint \mathrm{d}Q^2\mathrm{d}y\left[\frac{\Phi^B\sigma_{\gamma\rho^0}(W)(\delta^{VR} - \delta_R + \delta_{\mathrm{anom}})}{\sigma_{\rho^0}}\right.\\ &+ \left.\frac{\alpha}{\pi}\iint \mathrm{d}Q^2_h\mathrm{d}M^2_h\frac{\Phi'\sigma_{\gamma\rho^0}(M_h)}{\sigma_{\rho^0}} - \mathcal{F}^{IR}\frac{\Phi^B\sigma_{\gamma\rho^0}(W)}{\sigma_{\rho^0}}\right]\Bigg\}.\end{aligned} \tag{43}$$

This equation leads to our second "master" formula for radiative correction factor η, now for the photoproduction limit

$$\begin{aligned}\eta = \frac{\sigma^{\mathrm{corr}}_{\gamma\rho^0}}{\sigma_{\gamma\rho^0}} &= 1 - \iint \mathrm{d}Q^2\mathrm{d}y\left[\frac{\Phi^B\sigma_{\gamma\rho^0}(W)}{\sigma_{\rho^0}}(\delta^{VR} - \delta_R + \delta_{\mathrm{anom}})\right.\\ &+ \left.\frac{\alpha}{\pi}\iint \mathrm{d}Q^2_h\mathrm{d}M^2_h\frac{\Phi'\sigma_{\gamma\rho^0}(M_h)}{\Phi^B\sigma_{\gamma\rho^0}(W)} - \mathcal{F}^{IR}\right].\end{aligned} \tag{44}$$

The cross section $\sigma_{\gamma\rho^0}(W))$ (and $\sigma_{\gamma\rho^0}(M_h))$) on the right hand side of the eq. (44) results from a fit to the existing data; in the present paper the same parametrization was used as in the ZEUS analysis [4, 13]. σ_{ρ^0} is the averaged cross section measured in the experiment. The limits of integrations and the experimental cuts used in our numerical calculations will be discused in the section 7.
The anomalous electron's magnetic moment contribution is presented in the next section.

5 The anomalous magnetic moment's radiative correction δ_{anom}

In this section we consider in more details the effect proceeding from anomalous magnetic moment of the electron. The cross section contribution from the anomalous magnetic moment of electron in the case of DIS is (see e.g. [10])

$$\frac{\mathrm{d}^2\sigma_{\mathrm{anom}}}{\mathrm{d}y\mathrm{d}Q^2} = \frac{2\alpha^3 m_e^2 \nu_{anom}}{SQ^6 x}\left[2\beta^2 y^2 x F_1(x,Q^2) - (2-y)^2 F_2(x,Q^2)\right], \tag{45}$$

where

$$\begin{aligned} \nu_{\mathrm{anom}} &= -\frac{1}{\beta}\ln\frac{\beta+1}{\beta-1}, \\ \beta &= \sqrt{1+\frac{4m_e^2}{Q^2}}. \end{aligned} \tag{46}$$

The anomalous magnetic moment's contribution to the cross section is of order of $1/Q^6$ in contrast to the Born and other radiative cross sections which are of order $1/Q^4$. It means that this effect is important only for very small Q^2, where the ratio m_e^2/Q^2 is large. It is exactly the photoproduction case.
It is easy to show that the formula (45) can be rewritten in the following form:

$$\frac{\mathrm{d}^2\sigma_{\mathrm{anom}}}{\mathrm{d}y\mathrm{d}Q^2} = \delta_{\mathrm{anom}}(1-\varepsilon_R)\sigma^B(y,Q^2), \tag{47}$$

where ε_R and δ_{anom} are given by

$$\begin{aligned} \varepsilon_R &= R\frac{(\varepsilon^B-\varepsilon)}{1+\varepsilon^B R}, \\ \delta_{\mathrm{anom}} &= \frac{\alpha^2 m_e^2 y^2}{2\pi Q^6 \Gamma_T^B}\nu_{\mathrm{anom}}\left[y\beta^2 - \frac{(2-y)^2 S}{4M^2 x + yS}\right] \end{aligned} \tag{48}$$

and $\varepsilon^B, \Gamma_T^B$ and R are given by (6), (3) and (8).
The ε parameter is defined as:

$$\frac{1}{\varepsilon} = 1 - (4M^2 + \frac{y}{x}S)\frac{Q^2+4m_e^2}{S^2(2-y)^2}. \tag{49}$$

For $Q^2 \simeq 0$, ε_R is equal to:

$$\varepsilon_R = R\frac{Q^2}{4m_e^2} \simeq 0 \tag{50}$$

what simplifies the formula (47) to:

$$\frac{\mathrm{d}^2\sigma_{\mathrm{anom}}}{\mathrm{d}y\mathrm{d}Q^2} = \delta_{\mathrm{anom}}\sigma^B(y,Q^2). \tag{51}$$

Now it is clear that the contribution from anomalous magnetic moment is a multiplicative correction to the Born cross section and changing the Born cross section for DIS to the Born cross section for ρ^0 meson production cross section we obtain the $\delta_{\rm anom}$ term in our master formula (44).

6 Numerical results for the radiative correction factor for the NMC ρ^0 meson production analysis

In the NMC paper ([2]) the results on exclusive ρ^0 and ϕ production in DIS of muons on deuterium, carbon and calcium target is reported. The experiment was carried out at CERN by NMC collaboration using a 200 GeV muon beam. To select the ρ^0 meson sample the following kinematical cuts were used in the analysis ([2]): $Q^2 \geq 2GeV^2$, ν between 40 and 190 GeV and $y_{max} = 0.9$. The cuts for minimum energy of scattered muon, minimum hadron energy , minimum $m^2_{e^+e^-}$ (where $m^2_{e^+e^-}$ is the invariant mass for a pair of tracks calculated assuming the electron mass for both particles) and on the invariant mass of pion's pair ($m_{\pi^+\pi^-}$) were also used in the experimental analysis [2]. And finally the inelasticity I cut was applied:

$$-0.1 < I < 0.08 \tag{52}$$

$I = 0$ is the signal of exclusive ρ^0 production. The integration variable M_h^2 can be expresed in terms of the inelasticity I:

$$M_h^2 \;=\; M^2 + 2M\nu(1-I) - Q_h^2. \tag{53}$$

The integration limits for Q_h^2 are given by the following relations:

$$\begin{aligned} Q^2_{hmin} &= Q^2 + 2\nu I(\nu - \sqrt{\nu^2+Q^2}), \\ Q^2_{hmax} &= min\left\{2m\nu, Q^2 + 2\nu I(\nu - \sqrt{\nu^2+Q^2}\right\}). \end{aligned} \tag{54}$$

The events with inelasticity I differs from 0 can be considered as the radiative events or the events with so-called smearing effect [2]. In our aproach we are not able to take into account the smearing effect and the all events in the specified range of I are treated as a radiative events. This is not fully correct and therefore our estimates should be rather considered as an upper limit of the correction.
The real radiative processes are allowed for $I > 0$. Therefore in our calculations the cut used for I was:

$$0 < I < 0.08 \tag{55}$$

The most restricted range of inelasticity:

$$-0.05 < I < 0.0 \tag{56}$$

was also applied in the NMC analysis for the smaller sample of data; in that case there is no room for radiative corrections (when smearing is neglected) from real photons but still virtual corrections are taken into account.

The data were parametrized according to:

$$\sigma^B_{\gamma^\star\rho^0} = \sigma_0 \left(\frac{Q_0^2}{Q^2}\right)^\beta \tag{57}$$

where Q_0^2 was set equal to the average Q^2 of the data sample (for details see NMC paper [2]). The parameter σ_0 depends weakly on A of the target (deuterium, carbon or calcium) in contrast to power β which is practically independent on the target. Fortunately in our master formula for radiative correction factor η_{ρ^0}, eq. (30), the ratio of the cross sections $(\sigma^B_{\gamma^\star\rho^0}{}' \ / \ \sigma^B_{\gamma^\star\rho^0})$ is used and therefore the σ_0 cancels. It means that the radiative corrections are the same for all used targets. The numerical results for factor η_{ρ^0} are presented in Fig. 2 for different ν and different Q^2 bins. The β factor and the R_{ρ^0} are taken from NMC analysis, [2] and equal to be 2.02 and 2.0, respectively. The effect of the radiative corrections is varying between 2 and 5 %. To estimate more precisely the radiative corrections (e.g. taking into account the smearing effect) the Monte Carlo simulations are needed.

7 Numerical results for the radiative correction factor for the ZEUS ρ^0 meson photoproduction analysis

In the ref. [4] the measurement of the elastic ρ^0 photoproduction cross section at mean W of 70 GeV is reported by ZEUS collaboration. In the analysis only the momenta of the final state pions were measured. Events in which the scattered electron was detected in the ZEUS calorimeter were rejected. It restricts Q^2 to the values smaller than $Q^2_{max} \simeq 3 - 4GeV^2$. The minimal value of Q^2 is determined by electron mass and the range of y covered by the measurement:

$$Q^2_{min} = m_e \frac{y^2}{(1-y)} \tag{58}$$

and is close to 10^{-9}. The median Q^2 is approximately $10^{-4}GeV^2$. There were several experimental cuts used in the analysis (see [4]). In our calculations we used the cut $60 < W < 80$ GeV (which can be simply related to y cut) together with the Q^2 limits. The integration limits in the second double integral are defined by the set of the following equations:

$$\begin{aligned}
Q_{hmin} &= Q^2 + \frac{S_{lh}S_-}{2M^2} \\
Q_{hmax} &= min\left\{Q^2 + \frac{S_{lh}S_+}{2M^2}, S_h\right\} \\
S_{lh} &= S_l - S_h \\
S_- &= S_l - \sqrt{S_l^2 + 4M^2Q^2} \\
S_+ &= S_l + \sqrt{S_l^2 + 4M^2Q^2} \\
M_h^2 &= M^2 + S_h - Q_h^2 \\
S_l &= Sy \\
S_h &< S_L
\end{aligned} \tag{59}$$

The numerical integrations in eq.(44) are complicated. The four-dimensional integral with numerically divergent part (two infrared divergent parts which should cancel numerically) and with the integrand function varying several orders of magnitude is performed (very small Q^2, photoproduction limit). To simplify the problem part of integrations (over Q_h^2) was done analytically using *Mathematica* system; it allowed to control much better the divergences cancelation and to decrease computation time. The ρ^0 cross section was parametrized as in [4] and in [13]. The η_{ρ^0} radiative correction factor was estimated to be smaller than 2%. The nontrivial cancelation between virtual and real corrections were observed.

8 Concluding remarks

In the present paper the method of calculations of the radiative corrections for vector mesons exclusive production was presented. The numerical calculations were done for ρ^0 meson production for two different experimental analysis: NMC (fixed target 200 GeV muon beam, with virtuality of photons Q^2 higher than $2GeV^2$) and ZEUS at HERA (electron-proton collider, using quasi-real photons with space-like virtuality Q^2 between 10^{-8} and $10^{-2}GeV^2$, at average centre-of-mass energy (W) of 180 GeV). The large as well as small, close to 0, Q^2 ranges are considered. The method of calculations is based on the approach used in the calculations of the radiative corrections for DIS (so-called Dubna radiative correction scheme). The method can be applied to different experimental analysis like ZEUS or H1 large Q^2 ρ^0 production at HERA measurements ([6]). Our calculations can be also usefull in estimation of radiative correction effects in ϕ meson production or J/Ψ production (e.g. [14]). The presented method is based on semianalytical approach and sometime it can be difficult to take into account all experimental cuts in calculations. In that cases the Monte Carlo approach for radiative corrections should be applied.

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9 Appendix A

Below the exact expressions for certain functions in the D scheme will be given. The 'radiator' functions $\mathcal{S}_i$ and the function $\mathcal{F}^{IR}$ are:

$$\begin{aligned}
\mathcal{S}_1(y,Q^2,y_h,Q_h^2) &= \left\{\frac{1}{\sqrt{C_2}}\left[\frac{Q_h^2-Q^2}{2}+\frac{(Q^2+2m^2)(Q_h^2-2m^2)}{Q_h^2-Q^2}\right]\right. \\
&\quad \left.-m^2(Q_h^2-2m^2)\frac{B_2}{C_2^{3/2}}\right\}-\Big\{S\leftrightarrow -X\Big\}+\frac{1}{\sqrt{A_2}}, \\
\mathcal{S}_2(y,Q^2,y_h,Q_h^2) &= \left\{\frac{1}{\sqrt{C_2}}[M^2(Q_h^2+Q^2)-XS_h]\right. \\
&\quad +\frac{1}{(Q_h^2-Q^2)\sqrt{C_2}}\Big[Q_h^2[S(S-S_h)+X(X+S_h)-2M^2(Q_h^2+2m^2)] \\
&\quad +2m^2[(S-S_h)(X+S_h)+SX]\Big] \\
&\quad \left.-2m^2\frac{B_2}{C_2^{3/2}}[S(S-S_h)-M^2Q_h^2]\right\}-\Big\{S\leftrightarrow -X\Big\}-\frac{2M^2}{\sqrt{A_2}}, \\
\mathcal{F}^{IR}(y,Q^2,y_h,Q_h^2) &= \frac{Q^2+2m^2}{Q^2-Q_h^2}\Big(\frac{1}{\sqrt{C_1}}-\frac{1}{\sqrt{C_2}}\Big)\;-m^2\Big(\;\frac{B_1}{C_1^{3/2}}+\frac{B_2}{C_2^{3/2}}\Big)\;,
\end{aligned}$$

where

$$\begin{aligned}
A_2 &= \lambda_l \equiv A_1, \\
\lambda_l &= S_l^2+4M^2Q^2 \\
B_2 &= 2M^2Q^2(Q^2-Q_h^2)+X(S_lQ_h^2-S_hQ^2) \\
&\quad + SQ^2(S_l-S_h)\equiv -B_1(S\leftrightarrow -X), \\
C_2 &= [XQ_h^2-Q^2(S-S_h)]^2+4m^2[(S_l-S_h)(S_lQ_h^2-S_hQ^2) \\
&\quad - M^2(Q_h^2-Q^2)^2]\equiv C_1[S\leftrightarrow -X],
\end{aligned}$$

with

$$\begin{aligned}
X=S(1-y) &= 2ME' \\
S_h &= Sy_h
\end{aligned}$$

and the hadron defined variables, x_h, y_h are given by the following equations

$$\begin{aligned} M_X^2 &= M^2 + S y_h (1 - x_h) \\ Q_h^2 &= S x_h y_h \end{aligned}$$

10 Appendix B

Below the exact (with electron's mass terms) expressions for $\delta_{\rm vac}, \delta_R, \delta^{VR}$ and $\delta_{\rm vtx}$ are given:

$$\begin{aligned} \delta_{\rm vac}^{e,\mu} &= \frac{2\alpha}{\pi}\left[\frac{-5}{9} + \frac{4m_{e,\mu}^2}{3Q^2} + \frac{1}{3}\sqrt{1+\frac{4m_{e,\mu}^2}{Q^2}}\left(1 - \frac{2m_{e,\mu}^2}{Q^2}\right)\ln\left(\frac{\sqrt{1+4m_{e,\mu}^2/Q^2}+1}{\sqrt{1+4m_{e,\mu}^2/Q^2}-1}\right)\right] \\ \delta_R &= \frac{-\alpha}{\pi}\ln\frac{(W^2-M^2)^2}{m^2W^2}\left(\frac{1+\beta^2}{2\beta}L_\beta - 1\right) \\ \delta^{VR} &= \delta_{\rm vtx} - \frac{\alpha}{\pi}\frac{1+\beta^2}{2\beta}\left[L_\beta \ln\frac{4\beta^2}{\beta^2-1} + \Phi(\frac{1+\beta}{1-\beta}) - \Phi(\frac{1-\beta}{1+\beta})\right] \\ &\quad + \frac{\alpha}{\pi}\left[\frac{1}{2\beta_1}\ln\frac{1+\beta_1}{1-\beta_1} + \frac{1}{2\beta_2}\ln\frac{1+\beta_2}{1-\beta_2} + S_\Phi\right] \\ \delta_{\rm vtx} &= \frac{2\alpha}{\pi}\left(-1 + \frac{3}{4}\beta L_\beta\right) \\ S_\Phi &= \frac{1}{2}\left(Q^2+2m^2\right)\int_0^1 \frac{d\alpha}{\beta_\alpha(-k_\alpha^2)}\ln\frac{1-\beta_\alpha}{1+\beta_\alpha} \end{aligned}$$

where:

$$\begin{aligned} \beta &= \sqrt{1+\frac{4m^2}{Q^2}} \\ \beta_1 &= \sqrt{1-\frac{4m^2M^2}{(S-Q^2)^2}} \\ \beta_2 &= \sqrt{1-\frac{4m^2M^2}{[S(1-y)+Q^2]^2}} \\ L_\beta &= \ln\frac{\beta+1}{\beta-1} \end{aligned}$$

and finally:

$$\begin{aligned} \beta_\alpha &= \frac{\mid \mathbf{k}_\alpha \mid}{k^0_\alpha} \\ k_\alpha &= k\alpha + k'(1-\alpha) \\ -k^2_\alpha &= m^2 + Q^2\alpha(1-\alpha) \end{aligned}$$

Figure Captions

1. Diagram of the exclusive leptoproduction (muon and electron) of the vector meson.

2. The η_{ρ^0} correction factor from eq.(30) for different ν and Q^2 bins calculated for NMC ([2]).

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