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DISORDER OPERATORS, QUANTUM DOUBLES, AND HAAG DUALITY IN $1+1$ DIMENSIONS*

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1 INTRODUCTION AND PREREQUISITES

Since the notion of the 'quantum double' was coined by Drinfel'd in his famous ICM lecture [8] there have been several attempts aimed at a clarification of its relevance to two dimensional quantum field theory. The quantum double appears implicitly in the work [3] on orbifold constructions in conformal field theory, where conformal quantum field theories (CQFTs) are considered whose operators are fixpoints under the action of a symmetry group on another CQFT. Whereas the authors emphasize that 'the fusion algebra of the holomorphic G-orbifold theory naturally combines both the representation and class algebra of the group G' the relevance of the double is fully recognized only in [4]. The quantum double also appears in the context of integrable quantum field theories, e.g. [1], as well as in certain lattice models (e.g. [18]). Common to these works is the role of disorder operators or 'twist fields' which are 'local with respect to $\mathcal{A}$ up to the action of an element $g \in G$' [3].

In this note, which is a compressed version of [12], we will use the methods of algebraic quantum field theory [10, 11] to demonstrate the role of the quantum double as a hidden symmetry in *every* quantum field theory with group symmetry in $1+1$ dimensions fulfilling (besides the usual assumptions like locality) only two technical assumptions (Haag duality and split property, see below) but independent of conformal covariance or exact integrability. As in [5] we will consider a quantum field theory to be specified by a net of von Neumann algebras, i.e. a map $\mathcal{O} \mapsto \mathcal{F}(\mathcal{O})$ which assigns to any bounded region in $1+1$ dimensional Minkowski space a von Neumann algebra (i.e. an algebra of bounded operators closed under hermitian conjugation and weak limits) on the common Hilbert space $\mathcal{H}$ such that isotony holds:

$$\mathcal{O}_1 \subset \mathcal{O}_2 \;\Rightarrow\; \mathcal{F}(\mathcal{O}_1) \subset \mathcal{F}(\mathcal{O}_2). \tag{1.1}$$

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The quasilocal algebra $\mathcal{F} = \overline{\bigcup_{\mathcal{O}\in\mathcal{K}} \mathcal{F}(\mathcal{O})}^{\|\cdot\|}$, $\mathcal{K}$ being the set of all double cones (intersections of forward and backward lightcones), is assumed to be irreducible: $\mathcal{F}' = \mathbb{C}\mathbf{1}$.[†] In order to simplify the exposition we restrict ourselves in this note to pure Bose fields (for the case of general Bose-Fermi commutation relations see [12]):

$$\mathcal{F}(\mathcal{O}) \subset \mathcal{F}(\mathcal{O}')'. \tag{1.2}$$

Poincaré covariance is implemented by assuming the existence of a (strongly continuous) unitary representation on $\mathcal{H}$ of the Poincaré group $\mathcal{P}$ such that

$$\alpha_{(\Lambda,a)}(\mathcal{F}(\mathcal{O})) = Ad\,U(\Lambda,a)(\mathcal{F}(\mathcal{O})) = \mathcal{F}(\Lambda\mathcal{O}+a). \tag{1.3}$$

The spectrum of the generators of the translations (momenta) is required to be contained in the closed forward lightcone and the existence of a unique vacuum vector Ω invariant under $\mathcal{P}$ is assumed. Covariance under the conformal group, however, is *not* required.

Our last postulate (for the moment) concerns the inner symmetries of the theory. There shall be a compact group G, represented in a strongly continuous fashion by unitary operators on $\mathcal{H}$ leaving invariant the vacuum such that the automorphisms $\alpha_g(F) = Ad\,U(g)(F)$ of $\mathcal{B}(\mathcal{H})$ respect the local structure:

$$\alpha_g(\mathcal{F}(\mathcal{O})) = \mathcal{F}(\mathcal{O}). \tag{1.4}$$

The action may be assumed faithful, i.e. $\alpha_g \neq \mathrm{id}\ \forall g \neq e$. (Compactness of G need in fact not be postulated, as it is known to follow from the split property which will be introduced later. For the sake of simplicity we assume in this note that the group G commutes with the Poincaré group, see [12] and [13, Appendix] for further discussion.) The observables are now defined as the gauge invariant operators:

$$\mathcal{A}(\mathcal{O}) = \mathcal{F}(\mathcal{O})^G = \mathcal{F}(\mathcal{O}) \cap U(G)'. \tag{1.5}$$

This framework was the starting point for the investigations in [5] where in particular properties of the observable net (1.5) and its representations on the sectors in $\mathcal{H}$, i.e. the G-invariant subspaces, were studied. One important notion examined in [5] was that of *duality* designating a maximality property in the sense that the local algebras cannot be enlarged (on the same Hilbert space) without violating spacelike commutativity. The postulate of duality for the fields consists in strengthening the locality postulate (1.2) to

$$\mathcal{F}(\mathcal{O}) = \mathcal{F}(\mathcal{O}')', \tag{1.6}$$

which means that $\mathcal{F}(\mathcal{O}')$, the von Neumann algebra generated by all $\mathcal{F}(\mathcal{O}_1)$, $\mathcal{O}' \supset \mathcal{O}_1 \in \mathcal{K}$ contains all operators commuting with $\mathcal{F}(\mathcal{O})$. (This can easily be generalized to the case with fermions.) Duality has been proved to hold for free massive and massless scalar and Dirac fields in all dimensions as well as for several interacting theories $(P(\phi)_2, Y_2)$.

From this it has been derived [5, Theorem 4.1] (for $\geq 2+1$ dimensions) that duality holds for the observables when restricted to a simple sector $\mathcal{H}_1$:

$$\mathfrak{A}(\mathcal{O}) \equiv \mathcal{A}(\mathcal{O}) \restriction \mathcal{H}_1 \implies \mathfrak{A}(\mathcal{O})' = \mathfrak{A}(\mathcal{O}')\ \ \forall \mathcal{O}. \tag{1.7}$$

[†] In general $\mathcal{M}' = \{X \in \mathcal{B}(\mathcal{H}) | XY = YX\ \forall Y \in \mathcal{M}\}$ denotes the algebra of all bounded operators commuting with all operators in $\mathcal{M}$.

A sector $\mathcal{H}_1$ is called simple if the group G acts on it via multiplication with a character

$$U(g) \restriction \mathcal{H}_1 = \chi(g) \cdot \mathbf{1} \restriction \mathcal{H}_1. \tag{1.8}$$

Clearly the vacuum sector is simple. Furthermore it has been shown [5, Theorem 6.1] that the irreducible representations of the observables on the charge sectors in $\mathcal{H}$ are strongly locally equivalent to the vacuum representation in the sense that for any representation $\pi(A) = A \restriction \mathcal{H}_\pi$ and any $\mathcal{O} \in \mathcal{K}$

$$\pi \restriction \mathcal{A}(\mathcal{O}') \cong \pi_0 \restriction \mathcal{A}(\mathcal{O}'). \tag{1.9}$$

The fundamental facts (1.7) and (1.9), which have come to be called Haag duality and the DHR criterion respectively, were taken as starting points in [6] where a more ambitious approach to the theory of superselection sectors was advocated and developed to a large extent. The basic idea was that the physical content of any quantum field theory should reside in the observables and their vacuum representation whereas all other physically relevant representations as well as unobservable charged fields interpolating between those and the vacuum sector should be constructed from the observable data. The vacuum representation was postulated to satisfy (1.7), while (1.9) was chosen as a selection criterion for a class of interesting representations. It may be considered as one of the triumphs of the algebraic approach that it has finally been possible to prove [7, and references given there] the existence of a compact group G 'describing' the structure of the DHR sectors and of an essentially unique net of field algebras acted upon by G and generating the charged sectors from the vacuum.

In 1+1 dimensions part of the analysis breaks down due to the topological pecularity that the spacelike complement of a bounded (connected) region consists of two connected components as a consequence of which the permutation group governing the statistics is replaced by the braid group. The algebraic formalism of [6] was adapted to this situation in [9], see also [13]. It is still not known by which structure the compact group appearing in the higher dimensional situation has to be replaced if a completely general solution to this question exists at all. Even though in $1+1$ dimensions one can not conclude the existence of a field net with group symmetry it appears interesting to study nets of observables arising as fixpoint nets ('orbifold theories'). This is the aim of the research to be reported here which in particular leads to a complete understanding of another peculiarity in $1+1$ dimensions as will be discussed in the last section.

2 SPLIT PROPERTY, DISORDER OPERATORS, AND NONLOCAL FIELD EXTENSIONS

We begin by introducing some notation. For any double cone $\mathcal{O} \in \mathcal{K}$ we designate the left and right spacelike complement by $W_{LL}^{\mathcal{O}}$ and $W_{RR}^{\mathcal{O}}$, respectively. Furthermore we write $W_L^{\mathcal{O}}$ and $W_R^{\mathcal{O}}$ for ${W_{RR}^{\mathcal{O}}}'$ and ${W_{LL}^{\mathcal{O}}}'$. These regions are wedge shaped, i.e. translates of the standard wedges $W_L = \{x \in \mathbb{R}^2 \mid x^1 < -|x^0|\}$ and $W_R = \{x \in \mathbb{R}^2 \mid x^1 > |x^0|\}$. With these definitions we have $\mathcal{O} = W_L^{\mathcal{O}} \cap W_R^{\mathcal{O}}$ and $\mathcal{O}' = W_{LL}^{\mathcal{O}} \cup W_{RR}^{\mathcal{O}}$ which graphically looks as in Figure 1. In analogy to ideas in statistical mechanics we introduce the notion of a family of *disorder operators* which consists, for any $\mathcal{O} \in \mathcal{K}$ and any $g \in G$, of two unitary operators $U_L^{\mathcal{O}}(g)$ and $U_R^{\mathcal{O}}(g)$ verifying

$$\begin{array}{ccccc} \mathrm{Ad}\, U_L^{\mathcal{O}}(g) \restriction \mathcal{F}(W_{LL}^{\mathcal{O}}) & = & \alpha_g & = & \mathrm{Ad}\, U_R^{\mathcal{O}}(g) \restriction \mathcal{F}(W_{RR}^{\mathcal{O}}), \\ \mathrm{Ad}\, U_L^{\mathcal{O}}(g) \restriction \mathcal{F}(W_{RR}^{\mathcal{O}}) & = & \mathrm{id} & = & \mathrm{Ad}\, U_R^{\mathcal{O}}(g) \restriction \mathcal{F}(W_{LL}^{\mathcal{O}}). \end{array} \tag{2.1}$$

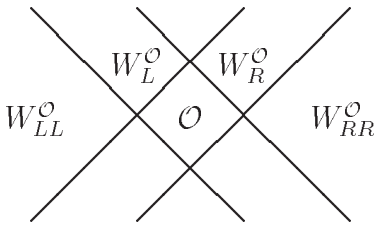

Figure 1. Wedges associated to a double cone

A disorder operator thus interpolates between the action of an unbroken global symmetry on one wedge and the trivial action on a wedge properly contained in the spacelike complement of the first one.

In general it is not obvious that disorder operators exist. Therefore we introduce as another axiom the *split property for wedges* which formalizes a strong form of statistical independence of spacelike separated regions. A net of field algebras has this property if for every double cone $\mathcal{O}$ the von Neumann algebra $\mathcal{F}(W_{LL}^{\mathcal{O}}) \vee \mathcal{F}(W_{RR}^{\mathcal{O}})$ is algebraically isomorphic to the tensor product $\mathcal{F}(W_{LL}^{\mathcal{O}}) \otimes \mathcal{F}(W_{RR}^{\mathcal{O}})$. It is known that free massive scalar and Dirac fields satisfy this property and it seems reasonable to expect this to be the case in every well behaved massive theory. For a discussion of related properties and for further references we refer to the detailed review [17].

Using essentially the same methods as in [2] one can show that the split property implies the existence of disorder operators $U_L^{\mathcal{O}}(g)$, $U_R^{\mathcal{O}}(g)$ for all $\mathcal{O} \in \mathcal{K}$, $g \in G$. Besides the above defining equations these operators have the following additional properties:

$$\big[U_L^{\mathcal{O}}(g), U_R^{\mathcal{O}}(h)\big] = 0, \quad U_L^{\mathcal{O}}(g)\; U_R^{\mathcal{O}}(g) = U(g). \tag{2.2}$$

We thus have, for each double cone $\mathcal{O}$, a 'factorization' of the global symmetry group into two commuting representations which are localized along half lines. An immediate consequence of (2.2) and the representation property is

$$U(g)\, U_{L/R}^{\mathcal{O}}(h)\, U(g)^* = U_{L/R}^{\mathcal{O}}(ghg^{-1}), \tag{2.3}$$

which expresses covariance of the disorder operators under global gauge transformations. Arguing that in view of (2.1) the operators $U_L^{\mathcal{O}}(g), U_R^{\mathcal{O}}(g)$ are associated to the double cone $\mathcal{O}$ we define the following extension of the field algebras:

$$\hat{\mathcal{F}}(\mathcal{O}) = \mathcal{F}(\mathcal{O}) \vee U_L^{\mathcal{O}}(G)''. \tag{2.4}$$

Whereas this enlarged net obviously is nonlocal one can still verify isotony $\mathcal{O}_1 \subset \mathcal{O}_2 \;\Rightarrow\; \hat{\mathcal{F}}(\mathcal{O}_1) \subset \hat{\mathcal{F}}(\mathcal{O}_2)$ and Poincaré covariance. Furthermore, the extended net is not too large in the following sense:

$$\hat{\mathcal{F}}(\mathcal{O}) \wedge \mathcal{A}(\mathcal{O})' = \mathbb{C}\mathbf{1} \quad \forall \mathcal{O} \in \mathcal{K}. \tag{2.5}$$

Due to (2.3) the algebras $\hat{\mathcal{F}}(\mathcal{O})$ are stable under the extension of the global group action. This allows to define the fixpoint net

$$\hat{\mathcal{A}}(\mathcal{O}) = \hat{\mathcal{F}}(\mathcal{O}) \cap U(G)', \tag{2.6}$$

which gives rise to the following square of inclusions for each double cone $\mathcal{O}$:

$$\begin{array}{ccc} \hat{\mathcal{A}}(\mathcal{O}) & \subset & \hat{\mathcal{F}}(\mathcal{O}) \\ \cup & & \cup \\ \mathcal{A}(\mathcal{O}) & \subset & \mathcal{F}(\mathcal{O}). \end{array} \tag{2.7}$$

In this diagram one can go from the right column to the left by restricting to the invariant elements under G. Furthermore, one can show $\hat{\mathcal{F}}(\mathcal{O})$ to be isomorphic to the crossed product of $\mathcal{F}(\mathcal{O})$ by the automorphism group $\alpha_g^{\mathcal{O}} = Ad\, U_L^{\mathcal{O}}(g)$. In order to simplify the exposition from now on we assume the group G to be finite. Most of our results remain valid for compact groups, see [12]. The inclusion $\mathcal{A}(\mathcal{O}) \subset \hat{\mathcal{F}}(\mathcal{O})$, being irreducible and composed of the finite-index depth-2 inclusions $\mathcal{A}(\mathcal{O}) \subset \mathcal{F}(\mathcal{O})$ and $\mathcal{F}(\mathcal{O}) \subset \hat{\mathcal{F}}(\mathcal{O})$, is known to be of depth 2, too. This amounts to the existence of a finite dimensional Hopf algebra H acting on $\hat{\mathcal{F}}(\mathcal{O})$ such that $\mathcal{A}(\mathcal{O})$ is the fixpoint algebra $\hat{\mathcal{F}}(\mathcal{O})^H$. In the next section this structure will be analyzed quite explicitly.

3 SPONTANEOUSLY BROKEN QUANTUM DOUBLE SYMMETRY

We have already remarked that the algebra $\hat{\mathcal{F}}(\mathcal{O})$ is isomorphic to a crossed product which is equivalent to the existence of a G-gradation. This implies that every $\hat{F}$ has a unique representation of the form

$$\hat{F} = \sum_{g\in G} F(g)\, U_L^{\mathcal{O}}(g), \quad F(g) \in \mathcal{F}(\mathcal{O}). \tag{3.1}$$

Given an arbitrary function $f \in \mathbb{C}(G)$ on the group there is thus an action γ_f:

$$\gamma_f\left(\sum_{g\in G} F(g)\, U_L^{\mathcal{O}}(g)\right) = \sum_{g\in G} f(g)\, F(g)\, U_L^{\mathcal{O}}(g), \quad F(g) \in \mathcal{F}(\mathcal{O}), f \in \mathbb{C}(G). \tag{3.2}$$

In particular, for the delta-functions $\delta_g(h) = \delta_{g,h}$ we obtain the projections $\gamma_g := \gamma_{\delta_g}$. Due to (2.3) we have

$$\alpha_g \circ \gamma_h = \gamma_{ghg^{-1}} \circ \alpha_g. \tag{3.3}$$

We are now prepared to exhibit the action of the quantum double $D(G)$ on the extended algebras. Let $\mathbb{C}(G)$ be the algebra of (complex valued) functions on the finite group G and consider the adjoint action of G on $\mathbb{C}(G)$ according to $\alpha_g : f \mapsto f \circ Ad(g^{-1})$. The quantum double $D(G)$ is defined as the crossed product $D(G) = \mathbb{C}(G) \rtimes_\alpha G$ of $\mathbb{C}(G)$ by this action. In terms of generators, $D(G)$ is the $*$-algebra generated by unitary and selfadjoint, respectively, elements U_g, V_h, $g, h \in G$ with the relations

$$U_g\, U_h = U_{gh}, \quad V_g\, V_h = \delta_{g,h} V_g, \quad U_g\, V_h = V_{ghg^{-1}}\, U_g \tag{3.4}$$

and the identification $U_e = \sum_g V_g = \mathbf{1}$. The action of $D(G)$ is now defined for the basis $\{V_g U_h,\ g, h \in G\}$ by

$$\gamma_{V_g U_h}(\hat{F}) = \gamma_g \circ \alpha_h(\hat{F}) \tag{3.5}$$

and for $D(G)$ by linear extension. One easily verifies $\gamma_{ab}(x) = \gamma_a \circ \gamma_b(x)$ and $\gamma_{\mathbf{1}}(x) = x$, whereas the well-known Hopf algebra maps on $D(G)$ [4] lead to

$$\Delta(V(g)U(h)) = \sum_k V(hk)U(h) \otimes V(k^{-1})U(h) \quad \Rightarrow \quad \gamma_a(xy) = \gamma_{a_{(1)}}(x)\gamma_{a_{(2)}}(y),$$

$$S(V(g)U(h)) = V(h^{-1}g^{-1}h)U(h^{-1}) \quad \Rightarrow \quad (\gamma_a(x))^* = \gamma_{S(a^*)}(x^*), \qquad (3.6)$$
$$\varepsilon(V(g)U(h)) = \delta_{g,e} \quad \Rightarrow \quad \gamma_a(\mathbf{1}) = \varepsilon(a)\mathbf{1}.$$

(We have used the standard notation $\Delta(a) = a^{(1)} \otimes a^{(2)}$ for the coproduct.) This proves that $\gamma : D(G) \times \mathcal{M} \to \mathcal{M}$ defines an action of $D(G)$ on the local algebras $\hat{\mathcal{F}}(\mathcal{O})$. As this action is compatible with the local structure it extends to a unique action on the quasilocal algebra $\hat{\mathcal{F}}$.

In the case of an abelian group G this can be reformulated in terms of commuting actions of G and the dual group $\hat{G}$, the total symmetry group thus being $G \times \hat{G}$. It is clear that the $\hat{G}$-part of the symmetry is spontaneously broken in the sense that there are no unitary operators on $\mathcal{H}$ implementing this action. The same holds, of course, in the non-abelian case where the symmetry to be unbroken would mean that there exist operators $U(a)$ $\forall a \in D(G)$ such that

$$U(a)\, x = \gamma_{a^{(1)}}(x)\, U(a^{(2)}). \qquad (3.7)$$

Despite the partial breakdown of the symmetry one can prove that the spectrum of the action of $D(G)$ is complete in the sense that for every finite dimensional representation D_{ij} of $D(G)$ there is a multiplet $\psi_i,\ i = 1, \ldots, \dim(D)$ in each $\hat{\mathcal{F}}(\mathcal{O})$ such that

$$\gamma_a(\psi_i) = \sum_{i'=1}^{d} D_{i'i}(a)\, \psi_{i'}. \qquad (3.8)$$

Let now ψ^1, ψ^2 be $D(G)$-tensors in $\hat{\mathcal{F}}(\mathcal{O}_1)$, $\hat{\mathcal{F}}(\mathcal{O}_2)$, respectively, where $\mathcal{O}_2$ lies in the left spacelike complement of $\mathcal{O}_1$. Then one can prove the following C-number commutation relations:

$$\psi^1_i\, \psi^2_j = \sum_{i'j'} \psi^2_{j'}\, \psi^1_{i'}\ (D^1_{i'i} \otimes D^2_{j'j})(R), \qquad (3.9)$$

where D^1, D^2 are the matrices of the respective representations and

$$R = \sum_{g \in G} V_g \otimes U_g \in D(G) \otimes D(G). \qquad (3.10)$$

The operators ψ_i can be chosen as isometries fulfilling the relations

$$\psi^*_i\, \psi_j = \delta_{i,j}\mathbf{1}, \quad \sum_{i=1}^{d} \psi_i\, \psi^*_i = \mathbf{1}. \qquad (3.11)$$

Let $\psi_i,\ i = 1 \ldots \dim(D)$ be a multiplet of isometries in $\hat{\mathcal{F}}(\mathcal{O})$ transforming according to the irreducible representation D of $D(G)$. Then the maps

$$\rho(\cdot) = \sum_i \psi_i \cdot \psi^*_i, \quad \phi(\cdot) = \frac{1}{\dim(D)} \sum_i \psi^*_i \cdot \psi_i \qquad (3.12)$$

define a unital *-endomorphism of $\hat{\mathcal{F}}$ and its left inverse [6], respectively. The relative locality of $\mathcal{A}$ and $\hat{\mathcal{F}}$ implies the restriction of ρ to $\mathcal{A}$ to be localized in $\mathcal{O}$ in the sense that $\rho(A) = A\ \forall A \in \mathcal{A}(\mathcal{O}')$. If the net $\mathcal{A}$ satisfied Haag duality we could conclude by standard arguments [6] that ρ maps $\mathcal{A}(\mathcal{O}_1)$ into itself if $\mathcal{O}_1 \supset \mathcal{O}$. Despite the fact

that Haag duality does not hold for $\mathcal{A}$, this is still true, however, as follows from the $D(G)$-invariance of $\rho(x)$ for $x \in \mathcal{A}$. We can thus use the formalism of [6] to define the statistics operator which can be shown to be

$$\varepsilon(\rho_1, \rho_2) = \sum_{ijkl} \psi_i^{(2)} \psi_l^{(1)} \psi_j^{(2)*} \psi_k^{(1)*} \; (D^1_{lk} \otimes D^2_{ij})(R). \tag{3.13}$$

Computing $\lambda_\rho = \omega_\rho / d_\rho \equiv \phi_\rho(\varepsilon(\rho, \rho))$, the statistics dimension d_ρ turns out to coincide with the dimension of the representation D of $D(G)$ whereas the statistics phase ω_ρ is given by

$$\omega_\rho \, \delta_{ij} = D_{ij}(X), \quad X = \sum_g V_g \, U_g \in D(G), \tag{3.14}$$

where the central unitary element $X \in D(G)$ is just the (inverse of the) 'ribbon element' [15] of the modular Hopf algebra $D(G)$. Finally, defining the monodromy operators $\varepsilon_M(\rho_1, \rho_2) = \varepsilon(\rho_1, \rho_2)\varepsilon(\rho_2, \rho_1)$ we can compute the statistics characters [14]:

$$Y_{ij} \equiv d_i d_j \, \phi_i(\varepsilon_M(\rho_i, \rho_j)^*) = (tr_i \otimes tr_j) \circ (D^i \otimes D^j)(I^*), \tag{3.15}$$

where $I = R\sigma(R)$ is again well known in the context of modular Hopf algebras. We have thus established, for a special class of models, a correspondence between the notions of algebraic QFT and those of [15]. Yet, the framework is not exactly as in [6, 9]. The point is that one can prove [13] that our assumptions, in particular the split property for wedges, preclude the existence of nontrivial DHR sectors. That this no-go theorem does not apply in the present situation is due to the fact, to be discussed in the rest of this note, that Haag duality does not hold for the fixpoint net $\mathcal{A}$.

4 HAAG DUALITY

We will now comment on a less well known two-dimensional pecularity, namely the fact that the step [5] from (1.6) to (1.7) fails in 1+1 dimensions. This means that one cannot conclude from (twisted) duality of the fields that duality holds for the observables in simple sectors, which in fact is violated. The origin of this phenomenon is easily understood. Let $\mathcal{O} \in \mathcal{K}$ be a double cone. One can then construct gauge invariant operators in $\mathcal{F}(\mathcal{O}')$ which are obviously contained in $\mathcal{A}(\mathcal{O})'$ but not in $\mathcal{A}(\mathcal{O}')$. This is seen remarking that the latter algebra, belonging to a disconnected region, is defined to be generated by the observable algebras associated to the left and right spacelike complements of $\mathcal{O}$, respectively. This algebra does not contain gauge invariant operators constructed using fields localized in both components. The weaker property of *wedge duality* remains true, however. Let $\mathcal{H}_1$ be a simple sector. Then:

$$\mathfrak{A}(W) \equiv \mathcal{A}(W) \restriction \mathcal{H}_1 \implies \mathfrak{A}(W)' = \mathfrak{A}(W') \quad \forall W. \tag{4.1}$$

Defining now the *dual net* by

$$\mathfrak{A}^d(\mathcal{O}) = \mathfrak{A}(W_L^{\mathcal{O}}) \cap \mathfrak{A}(W_R^{\mathcal{O}}) \tag{4.2}$$

it is easy to verify that Haag duality holds for $\mathfrak{A}^d$. One would, however, like to know which additional operators are obtained in this way. Using the above methods we can actually compute the dual net in terms of $\mathcal{A}(\mathcal{O})$ and the disorder operators. One can

show that the net $\hat{\mathcal{A}}$ defined in (2.6) is local and leaves the sectors in $\mathcal{H}$ invariant so that it constitutes a local extension of $\mathfrak{A}$ in each sector. Using the formula

$$\mathcal{A}(\mathcal{O}')' = \mathcal{F}(\mathcal{O}) \vee U_L^{\mathcal{O}}(G)'' \vee U_R^{\mathcal{O}}(G)'' \tag{4.3}$$

one can in fact prove it to coincide with the dual net: $\hat{\mathcal{A}}(\mathcal{O}) \restriction \mathcal{H}_1 = \mathfrak{A}^d(\mathcal{O})$ for every simple sector $\mathcal{H}_1$. This is reminiscent of the analysis in [16] where nets of observables (in at least 2+1 dimensions) which arise as fixpoints under a group of inner symmetries from a field theory were shown to violate Haag duality whenever the symmetry is spontaneously broken in the sense that the vacuum is not invariant under the whole group. Again the observables fulfill a weaker property (*essential duality*) which allows to construct a maximal local extension satisfying Haag duality. This dual net was shown in [16] to be just the fixpoint net of the field net under the *unbroken* part of the gauge group. The analogy to the situation studied above is obvious, for here $\hat{\mathcal{A}} = \hat{\mathcal{F}}^G$ are the invariants under the unbroken part $G \subset D(G)$ of the quantum double.

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