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Next-To-Leading Logarithmic Results in $B \to X_s\gamma$ [a]

CHRISTOPH GREUB
Deutsches Elektronen-Synchroton DESY,
22603 Hamburg, Germany

TOBIAS HURTH
Institute for Theoretical Physics, SUNY at Stony Brook,
Stony Brook, New York 11794-3840, USA

We give a brief review of the next-to-leading logarithmic results in $B \to X_s\gamma$. Combining the results of different groups, a practically complete next-to- leading-logarithmic prediction of the inclusive decay rate was recently presented. The theoretical uncertainty in the decay rate is now less than half of the error in the previously leading-logarithmic result. Therefore, the inclusive $B \to X_s\gamma$ mode will provide important tests of the SM and its extensions when more precise experimental data is available.

Rare B meson decays provide an alternative approach in the search for new physics: The $B \to X_s\gamma$ decay in particular does not arise at the tree level in the standard model (SM) but is induced by one-loop W-exchange diagrams. Therefore nonstandard contributions (charged scalar exchanges, SUSY one-loop diagrams etc.) are not suppressed by an extra factor $\alpha/4\pi$ relative to the standard model amplitude which implies the high sensitivity of this decay for new physics. However, even within the SM, the $B \to X_s\gamma$ decay is also important for constraints on the Cabibbo-Kobayashi-Maskawa matrix elements which involve the top-quark. For both these reasons, precise experimental and theoretical work on these decays is required.

The experimental status of this decay can be summarized as follows: Following the first observation of the exclusive $B \to K^*\gamma$ mode [1], the first evidence for a penguin decay ever, the CLEO collaboration measured the inclusive $B \to X_s\gamma$ branching ratio to be $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$, where the first error is statistical and the second is systematic [2]. In fact, there are two separate CLEO analyses. The first one measures the inclusive photon spectrum from B-decay near the end point. The second technique constructs the inclusive rate by summing up the possible exclusive final states. The branching ratio

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stated above is the average of the two measurements, taking into account the correlation between the two techniques. In the upcoming years much more precise measurements are expected from the upgraded CLEO detector, as well as from the B-factories presently under construction at SLAC and KEK. In view of the expected high luminosity of the B-factories, experimental accuracy below 10% seems to be possible.

The inclusive $B \to X_s\gamma$ mode in contrast to exclusive decay modes is theoretically clean in the sense that no specific model is needed to describe the final hadronic state. Indeed, heavy quark effective theory tells us that the decay width $\Gamma(B \to X_s\gamma)$ is well approximated by the partonic decay rate $\Gamma(b \to X_s\gamma)$ which can be analyzed in renormalization group improved perturbation theory. The class of non-perturbative effects which scales like $1/m_b^2$ is expected to be well below 10% [3]. This numerical statement is supposed to hold also for the recently discovered non-perturbative contributions which scale like $1/m_c^2$ [4]. Therefore, we focus on the dominant partonic decay rate in the following.

It is well-known that the QCD corrections enhance the partonic decay rate $\Gamma(b \to s\gamma)$ by more than a factor of two. These QCD effects can be attributed to logarithms of the form $\alpha_s^n(m_b)\,\log^m(m_b/M)$, where $M = m_t$ or $M = m_W$ and $m \leq n$ (with $n = 0, 1, 2, ...$). In order to get a reasonable result at all, one has to resum at least the leading-log (LL) series ($m = n$). Working to next-to-leading-log (NLL) precision means that one is also resumming all the terms of the form $\alpha_s(m_b)\;(\alpha_s^n(m_b)\,\ln^n(m_b/M))$.

An appropriate framework to achieve the necessary resummations is an effective low-energy theory, obtained by integrating out the heavy particles which in the SM are the top quark and the W-boson. The effective Hamiltonian relevant for $b \to s\gamma$ and $b \to sg$ in the SM and most of its extensions reads

$$H_{eff}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}}\,\lambda_t \sum_{i=1}^{8} C_i(\mu)\,O_i(\mu) \quad , \tag{1}$$

where $O_i(\mu)$ are the relevant operators, $C_i(\mu)$ are the corresponding Wilson coefficients, which contain the complete top- and W- mass dependence, and $\lambda_t = V_{tb}V_{ts}^*$ with V_{ij} being the CKM matrix elements [b]. Neglecting operators with dimension > 6 which are suppressed by higher powers of $1/m_{W/t}$ and using the equations of motion for the operators, one arrives at the following basis of dimension 6 operators [5]

$$O_1 \;=\; (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})\;(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})\;,$$

[b] The CKM dependence globally factorizes, because we work in the approximation $\lambda_u = 0$.

$$
\begin{aligned}
O_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})\ (\bar{s}_{L\beta}\gamma_\mu c_{L\beta})\ , \\
O_7 &= (e/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b(\mu)R + m_s(\mu)L)\,b_\alpha\ F_{\mu\nu}\ , \\
O_8 &= (g_s/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b(\mu)R + m_s(\mu)L)\,(\lambda^A_{\alpha\beta}/2)\,b_\beta\ G^A_{\mu\nu} \quad . \qquad (2)
\end{aligned}
$$

Because the Wilson coefficients of the penguin induced Four-Fermi operators $O_3,..O_6$ are very small, we do not list them here. The perturbative QCD corrections for the $b \to s\gamma$ decay rate are twofold:
1 • The corrections to the Wilson coefficients $C_i(\mu)$ at the scale $\mu \approx m_b$.
2 • The corrections to the matrix elements of the operators O_i also at the low-energy scale $\mu \approx m_b$.
Only the sum of the two contributions is renormalization scheme independent and in fact, from the μ-independence of the effective Hamiltonian, one can derive a renormalization group equation (RGE) for the Wilson coefficients $C_i(\mu)$:

$$
\mu\frac{d}{d\mu}C_i(\mu) = \gamma_{ji}\,C_j(\mu) \quad , \qquad (3)
$$

where the (8×8) matrix γ is the anomalous dimension matrix of the operators O_i. The standard procedure to calculate the two contributions involves the following three steps:
ad 1a • One has to match the full standard model theory with the effective theory at the scale $\mu = \mu_W$, where μ_W denotes a scale of order m_W or m_t. At this scale, the matrix elements of the operators in the effective theory lead to the same logarithms as the full theory calculation. Consequently, the Wilson coefficients $C_i(\mu_W)$ only pick up small QCD corrections, which can be calculated in fixed-order perturbation theory. In the LL (NLL) program, the matching has to be worked out to order α_s^0 (α_s^1).
ad 1b • Solving the RGE (3) and using the $C_i(\mu_W)$ of Step 1a as initial conditions, one performs the evolution of these Wilson coefficients from $\mu = \mu_W$ down to $\mu = \mu_b$, where μ_b is of the order of m_b. As the matrix elements of the operators evaluated at the low scale μ_b are free of large logarithms, the latter are contained in resummed form in the Wilson coefficients. For a LL (NLL) calculation, this RGE step has to be performed using the anomalous dimension matrix γ_{ji} up to order α_s^1 (α_s^2).
ad 2 • The corrections to the matrix elements of the operators $\langle s\gamma|O_i(\mu)|b\rangle$ at the scale $\mu = \mu_b$ have to be calculated to order α_s^0 (α_s^1) in the LL (NLL) calculation.

Until recently, only the leading logarithmic (LL) perturbative QCD corrections had been calculated [6] systematically. The error in these calculations is dominated by a large renormalization scale dependence at the $\pm 25\%$ level. The measurement of the CLEO collaboration [2] overlaps with the estimates

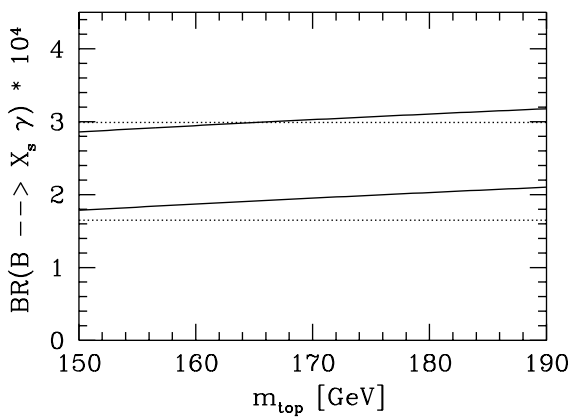

Figure 1: Branching ratio for $B \to X_s\gamma$ as a function of m_t based on LL calculations. The upper (lower) solid curve is for $\mu = m_b/2$ ($\mu = 2m_b$). The dotted curves show the CLEO $1-\sigma$ bounds[2]. The other input parameters are taken at their central values.

based on leading logarithmic calculations (or with some next-to-leading effects partially included) and the experimental and theoretical errors are comparable (see Figure 1)[7,8,9]. However, in view of the expected increase in the experimental precision in the near future, it is clear that a systematic inclusion of the NLL corrections becomes necessary. Already the large μ dependence of the leading-log result ($\pm 25\%$) indicates the importance of the NLL series. This ambitious NLL enterprise was recently completed. All three steps (1a,1b,2) to NLL precision involve rather difficult calculations. The most difficult part in Step 1a is the two-loop (or order α_s) matching of the dipole operators O_7 and O_8. It involves two-loop diagrams both in the full and in the effective theory. It was worked out by Adel and Yao[10] some time ago. Using a different method, Greub and Hurth recently presented a detailed re-calculation of this step, confirming the former result [11]. Step 2 basically consists of Bremsstrahlung corrections and virtual corrections. While the Bremsstrahlung corrections (together with some virtual corrections needed to cancel infrared singularities) were worked out some time ago by Ali and Greub[7] and have been confirmed and extended by Pott[12], a complete analysis of the virtual corrections (up to the contributions of the Four-Fermi operators with very small coefficients) was presented by Greub, Hurth and Wyler[13]. This calculation involves two- loop diagrams where the full charm dependence has to be taken into account. The main result of this analysis consists in a drastic reduction of the renormalization

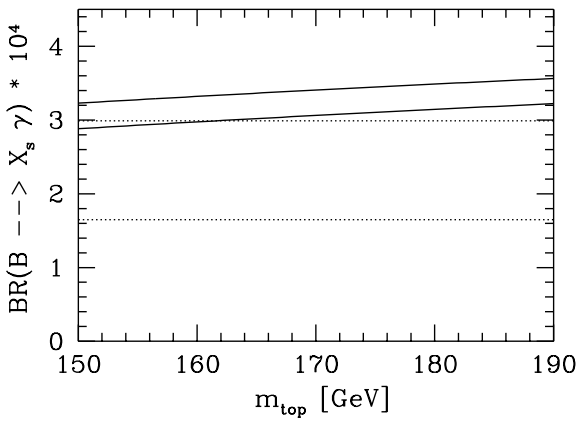

Figure 2: Branching ratio for $B \to X_s\gamma$ as a function of m_t based on the NLL calculation, not including the NLL corrections to the Wilson coefficient C_7.

scale uncertainty from about $\pm 25\%$ to about $\pm 5\%$. Moreover, the central value was shifted outside the 1σ bound of the CLEO measurement (see Figure 2). However, at that time, the essential coefficient $C_7(\mu_b)$ was only known to leading-log precision. It was therefore unclear how much the overall normalization will be changed when the NLL value for $C_7(\mu_b)$ is used. Recently, the order α_s^2 anomalous matrix (Step 1b) has been completely worked out by Chetyrkin, Misiak and Münz [14]. The extraction of some of the elements in the $O(\alpha_s^2)$ anomalous dimension matrix involves pole parts of three-loop diagrams. Using the matching result (Step 1a), these authors obtained the next-to-leading correction to the Wilson coefficient $C_7(\mu_b)$ which is the only relevant one for the $b \to X_s\gamma$ decay rate. Numerically, the LL and the NLL value for $C_7(\mu_b)$ are rather similar; the NLL corrections to the Wilson coefficient $C_7(\mu_b)$ lead to a change of the $b \to X_s\gamma$ decay rate which does not exceed 6% [14]: The new contributions can be split into a part which is due to the order α_s corrections to the matching (Step 1a) and into a part stemming from the improved anomalous dimension matrix (Step 1b). While individually these two parts are not so small (in the NDR scheme, which was used in [14]), they almost cancel when combined as illustrated in [14]. This shows that all the three different pieces are numerically equally important. However, strictly speaking the relative importance of different NLO-corrections at the scale $\mu = \mu_b$, namely the order α_s corrections to the matrix elements of the operators (Step 2) and the improved Wilson coefficient C_7 (Step 1 a+b), is a renormalization-scheme

dependent issue; so we stress that the discussion above was done within the naive dimensional regularization scheme (NDR).

Combining the NLL calculations of all the three steps (1a+b,2), the first complete theoretical prediction to NLL pecision for the $b \to X_s + \gamma$ branching ratio was presented in [14]: $BR(B \to X_s\gamma) = (3.28 \pm 0.33) \times 10^{-4}$. The theoretical error has two dominant sources: The μ dependence is reduced to 5% as mentioned above. Another 5% uncertainty stems from the m_c/m_b dependence.

Summing up, the present NLL-prediction for the $B \to X_s\gamma$ decay is still in agreement with the CLEO measurement at the 2σ-level. The theoretical error is half of the uncertainty in the previous leading logarithmic prediction. Clearly, the inclusive $B \to X_s + \gamma$ mode will provide an interesting test of the SM and its extensions as soon as more precise experimental data are available.

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