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Unitarity Corrections and High Field Strengths in High Energy Hard Collisions

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Unitarity corrections to the BFKL description of high energy hard scattering are viewed in large N_c QCD in light-cone quantization. In a center of mass frame unitarity corrections to high energy hard scattering are manifestly perturbatively calculable and unrelated to questions of parton saturation. In a frame where one of the hadrons is initially at rest unitarity corrections are related to parton saturation effects and involve potential strengths $A_\mu \sim 1/g$. In such a frame we describe the high energy scattering in terms of the expectation value of a Wilson loop. The large potentials $A_\mu \sim 1/g$ are shown to be pure gauge terms allowing perturbation theory to again describe unitarity corrections and parton saturation effects. Genuine nonperturbative effects only come in at energies well beyond those energies where unitarity constraints first become important.

I. INTRODUCTION

Our object in this paper is to describe a one (hard) scale high energy scattering in QCD at those energies where unitarity corrections begin to be important. In particular, we focus on the question of the perturbative calculability of such processes. As our prototype reaction we consider high energy heavy onium-heavy onium scattering where the hard scale is given by the inverse onium radius, the only relevant transverse momentum scale for the total cross section or for a near forward elastic scattering. We could equally well focus on more directly physical processes such as the total virtual photon-virtual photon cross section [1, 2] $\gamma^*(Q) + \gamma^*(Q) \to$ hadrons, or on associated jet measurements either in electron-proton collisions [3, 4, 5] or in proton-antiproton collisions [6], however unitarity issues are especially sharp in onium-onium scattering since the onium is a bound state in QCD.

The energy dependence of an onium-onium collision is governed by the BFKL equation [7, 8, 9] , appropriate for single scale high energy collisions. If the onium is sufficiently small one can neglect diffusion and running coupling effects and use the BFKL equation up to those energies where unitarity corrections become important. We shall also use the large N_c limit of QCD where one can view the light-cone wavefunction of a high energy onium as a collection of color dipoles [10, 11, 12]. An onium-onium collision, in the BFKL approximation, consists of a scattering of a single dipole in each of the onia by means of one gluon exchange.

The BFKL approximation gives an S-matrix for onium-onium elastic scattering at impact parameter b and rapidity Y behaving as

$$S(Y,b) \sim \alpha^2 \frac{e^{(\alpha_P - 1)Y}}{[\frac{7}{2}\alpha N_c \zeta(3) Y]^{3/2}},$$

a behavior which violates the unitarity bound $|S(Y,b)| \leq 1$ when Y becomes very large. The picture of unitarity corrections is very different in different Lorentz frames in light-cone quantized QCD [13] in light-cone gauge. In the center of mass frame unitarity corrections become large when dipole densities, and corresponding field strengths

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A_μ, are relatively small. Unitarity bounds are guaranteed by multiple dipole-dipole scattering [10, 14] in much the same way as occurs for nucleus- nucleus scattering in the Glauber model. One difference, as emphasized below, is that the multiple dipole scattering must be done for a fixed wavefunction configuration with a sum over the different configurations performed as a last step. The higher number of scatterings correspond to higher orders in $1/N_c$. In the center of mass system unitarity corrections are quite straightforward and numerical simulations have already been done [14]. Unitarity limits are enforced at rapidities well below those where field strengths become large and parton saturation effects are important.

It is instructive to view unitarity corrections in a different Lorentz frame [13] where one of the onia, say the right-mover, carries almost all the available rapidity with the left-moving onium having a high enough rapidity to be relativistic but not so much that higher (gluonic) left-moving Fock components must be considered in the scattering. In this frame, called the L-frame, the scattering is of a right-moving onium with a fully developed wavefunction of many gluons colliding with a left-moving onium whose wavefunction is simply a heavy quark- antiquark pair. The S-matrix can be written as an average of Wilson loops in the right-moving state as indicated in (30). From this representation it becomes clear that unitarity corrections become important in the L-frame when $A_\mu \sim 1/g$. In this frame unitarity is associated with high potential strengths and significant parton saturation. However,it is suggested that these large values of A_μ do not yet signal nonperturbative QCD effects since the large terms in A_μ are purely gauge terms. In order to see genuine nonperturbative QCD effects one must go to rapidities well beyond (about twice as large) those rapidities where unitarity corrections first become important.

II. UNITARITY CORRECTIONS IN THE CENTER OF MASS FRAME

In this section, we review high energy heavy onium – heavy onium [10, 11, 12, 13, 14] scattering in large N_c QCD beginning at moderate energies where the two gluon exchange approximation is valid, through the energy range where the BFKL equation governs the scattering, up to energies where unitarity corrections to the BFKL equation become important,into the energy domain where unitarity corrections are very strong and the forward S-matrix elements become nearly zero (blackness), and ending at energies so high that the whole perturbative picture breaks down and high field strengths (parton saturation) occur.

Suppose $\Phi(x_{01}, z)$ is the square of the heavy quark light-cone wavefunction of the onium with $x_{01} = |\underline{x}_1 - \underline{x}_0|$ the transverse coordinate separation between the heavy quark, at $\underline{x}_0$, and the heavy antiquark, at $\underline{x}_1$ and with z the fraction of the onium's longitudinal momentum carried by the quark. Then in the two gluon exchange approximation the forward elastic high energy scattering amplitude is

$$A = \frac{i}{2}\int d^2x_{01}d^2x'_{01}\int_0^1 dzdz'\Phi(x_{01},z)\Phi(x'_{01},z')\sigma_{DD}(x_{01},x'_{01}) \tag{1}$$

where the dipole-dipole cross section, σ_{DD}, is

$$\sigma_{DD}(x_{01},x'_{01}) = 2\pi\alpha^2 x_<^2[1+\ln(x_>/x_<)] \tag{2}$$

with $x_<$ the smaller of x_{01} and x'_{01} and with $x_>$ the larger of these quantities. The onium-onium cross section is

$$\sigma = 2\,\mathrm{Im}A. \tag{3}$$

which is independent of energy at high energies. The α in (2) is $\alpha(R)$ where

$$R = \frac{1}{2}\int d^2x_{01}\int_0^1 dz x_{01}\Phi(x_{01},z) \tag{4}$$

is the average light-cone radius of the heavy quark part of the onium wavefunction. If $R\,\Lambda_{QCD}$ is sufficiently small, running coupling effects will not be important in onium-onium scattering up to energies where saturation sets in and the whole perturbative picture of high energy scattering breaks down.

If $Y = \ln(s/M^2)$ is the relative rapidity of the two colliding onia then the two gluon approximation breaks down when αY is of order 1. In this regime the BFKL equation, an equation which sums all $(\alpha Y)^n$ terms, governs high energy onium-onium scattering. In the dipole picture of high energy scattering one writes the scattering amplitude at impact parameter b as [10]

$$A(b,Y) = -i \int d^2x_{01} d^2x'_{01} \int_0^1 dz dz' \Phi(x_{01},z)\Phi(x'_{01},z') F(x_{01},x'_{01},b,Y) \tag{5}$$

where in the BFKL approximation $F = F^{(1)}$ and

$$F^{(1)} = -\frac{1}{2}\int_0^\infty \frac{dx}{x}\ \frac{dx'}{x'}\ d^2b_1 n(x_{01},x,b_1,Y/2) n(x'_{01},x',|\underline{b}-\underline{b}_1|,Y/2)\sigma_{DD}(x,x'). \tag{6}$$

$n(x_{01},x,b_1,Y/2)$ is the number density of dipoles of size x whose center is a distance b_1 from the center of the heavy quark-heavy antiquark system, x_{01}. The forward elastic amplitude is

$$A(Y) = \int d^2b A(b,Y). \tag{7}$$

For $x/b, x_{01}/b \ll 1$ [10, 14, 15]

$$n(x_{01},x,b,Y) = \frac{x_{01}}{4xb^2}\ \ln\left(\frac{16b^2}{x_{01}x}\right)\ \frac{\exp\{(\alpha_P-1)Y - a(Y)/2\ \ln^2(16b^2/x_{01}x)\}}{[\frac{7}{2}\alpha N_c\zeta(3)Y]^{3/2}} \tag{8}$$

with $\alpha_P - 1 = (4\alpha N_c/\pi)\ln 2$ and $a^{-1}(Y) = \frac{7\alpha N_c\zeta(3)Y}{\pi}$. Using (2) and (8) in (6) gives

$$F^{(1)} = -\pi\alpha^2 \frac{x_{01}x'_{01}}{b^2}\ \ln\left(\frac{16b^2}{x_{01}x'_{01}}\right) \frac{exp\{(\alpha_P-1)Y - \frac{a}{2}\ln^2(\frac{16b^2}{x_{01}x'_{01}})\}}{[\frac{7}{2}\alpha N_c\zeta(3)Y]^{3/2}} \tag{9}$$

when $x_{01}/b, x'_{01}/b \ll 1$. The resulting onium-onium cross section

$$\sigma = 16\pi R^2\alpha^2 \frac{e^{(\alpha_P-1)Y}}{\sqrt{\frac{7}{2}\alpha N_c\zeta(3)Y}} \tag{10}$$

comes from impact parameters satisfying $\ln\left(\frac{b^2}{R^2}\right) \sim \sqrt{14\alpha N_c\zeta(3)Y/\pi}$. From (3), (5), (7) and (9) one obtains

$$\frac{d\sigma}{d^2b} = 8\pi R^2\alpha^2 \ln\left(16b^2/R^2\right) \frac{\exp\{(\alpha_P-1)Y - \frac{a}{2}\ln^2(\frac{16b^2}{R^2})\}}{[\frac{7}{2}\alpha N_c\zeta(3)Y]^{3/2}b^2} \tag{11}$$

so long as $R^2/b^2 \ll 1$. Since

$$\frac{d\sigma}{d^2b} = 1 - \mathrm{Re}\ S(b), \tag{12}$$

with $S(b)$ the S-matrix for onium-onium scattering at impact parameter b, (10) and (11) violate the unitarity bound $\frac{d\sigma}{d^2b} \leq 2$ when αY becomes large.

The dipole picture of BFKL behavior is very simple. At high energies each of the onia consists of a heavy quark-antiquark pair along with a (generally) large number of soft gluons ordered in longitudinal momentum. In the large N_c limit each gluon can be viewed as a quark- antiquark pair and the onium wavefunction then can be viewed as a collection of color dipoles. In the scattering of two onia, in the BFKL approximation, a single right-moving dipole from the right-moving onium scatters off a single left-moving dipole in the left-moving onium, with the two gluon exchange cross section given in (2), leading to the parton-like expression (6).

Roughly speaking, one expects unitarity corrections to the BFKL approximation to become important when $\frac{d\sigma}{d^2b} \approx 1$. From (2) and (6) this happens at values of Y when $n \sim 1/\alpha$ and one might expect that the leading logarithmic

approximation to the onium wavefunction would start to break down. However, it is straightforward to see that when $\frac{d\sigma}{d^2b} = 1$

$$x^2 n \sim \frac{x}{\alpha b}. \tag{13}$$

Since $x/b \ll 1$ for those dipole sizes dominating the integral in (6) the onia wavefunctions are still quite dilute when unitarity corrections begin to become important. The situation is somewhat analogous to the scattering of two large, but dilute, "nuclei." If the nuclei have atomic number A and radius R then, in the single scattering approximation

$$\sigma_{AA} = A^2 \sigma_0 = 4\pi R^2 \rho^2 \left(\frac{4\pi}{9} \frac{R^4}{\sigma_0^2} \right), \tag{14}$$

where σ_0 is the nucleon-nucleon cross section and ρ is the packing fraction defined by

$$\rho = \frac{A\sigma_0^{3/2}}{(4/3)\pi R^3}. \tag{15}$$

From (14) one sees that the unitarity limit, $4\pi R^2$, is reached for σ_{AA} for very small packing fraction if $R^2/\sigma_0 \gg 1$. In this nuclear example it is clear how to impose unitarity. Since the nucleus is dilute one need only do multiple scattering quantum mechanically to obtain a correct forward scattering amplitude.

The unitarity limit is reached for onium-onium scattering in the center of mass frame also when the dipole densities in the onia are small so that the two dipoles involved in the scattering can be viewed as not interacting with any other dipoles. It would seem that unitarity can be imposed simply by including two and more scatterings of left-moving dipoles with a similar number of right-moving dipoles. And this is indeed the case [10, 14]. There are, however, two differences from the "nuclear" example. (i) The dipoles have various sizes whereas the nucleons in our nuclear example all have the same size. This is a technical problem which makes an analytical discussion of the problem difficult but does not present any problems of principle [10]. (ii) The wavefunction of an onium has large fluctuations in the number of dipoles present. Thus the single scattering term in the S-matrix is dominated by very different parts of the wavefunction than is the double scattering term which in turn samples a very different part of the wavefunction than the triple scattering term, etc. This causes the normal multiple scattering series (multiple pomeron exchange) to diverge factorially [10, 14]. This means that one must sum the multiple scatterings of the dipoles in the colliding onia for a fixed wavefunction configuration and only after this sum has been done can the average over the different configurations of the onia wavefunctions be done. Schematically,

$$S(Y,b) = \sum_{\phi} e^{-f(\phi)} P_{\phi}(Y) \tag{16}$$

where ϕ labels configuration of dipoles in each of the colliding onia. That is ϕ labels the number of dipoles in each onium along with the size of each of the dipoles and the impact parameter of the center of each of the dipoles with respect to the center of the heavy quark-antiquark pair. P_ϕ gives the probability of configuration ϕ while $f(\phi)$ is proportional to the probability of a dipole-dipole scattering in the configuration ϕ. b is the impact parameter of the onium-onium collision. If one expands the exponential in (16) and interchanges the sum with ϕ

$$S(Y,b) = \sum_{n=1}^{\infty} (-1)^n S_n(Y,b) \tag{17}$$

with

$$S_n = \sum_{\phi} \frac{1}{n!} [f(\phi)]^n P_{\phi}(Y) \tag{18}$$

gives the usual multiple scattering series. However, simple models [10] and numerical calculations in QCD [14] show that $S_n \sim n!$ so that the multiple scattering series does not converge. The problem is that rare configurations which have a large value of $f(\phi)$ contribute very much to S_n, for large n, but very little to $S(Y,b)$ as given in (16).

Numerical simulations have been carried out within the dipole framework of high energy onium-onium scattering which follow the BFKL behavior given in (10) and (11) into the regime of Y-values where unitarity corrections due to multiple scattering are important. These simulations suggest that [13]

$$S(Y,b) \sim e^{-c(Y-Y(b))^2} \tag{19}$$

for Y much greater than $Y(b)$, the value of Y at which unitarity corrections become important. The behavior given in (19) is a much slower decrease of S with Y than would be expected from an eikonal picture and corresponds to the S matrix being dominated by rare configurations consisting of many fewer dipoles than the average at the Y value being considered. The behavior (19) is also expected on theoretical grounds.

If Y_0 is the rapidity at which unitarity corrections begin to be important for $b = 0$ collisions then at $Y \approx 2Y_0$ one expects perturbation theory to break down completely and a new regime of high field strength QCD to emerge. This can be seen from the following rough argument: In (6) neglect the integrations over $dxdx'$ and set $d^2b_1 = R^2$ for a $b = 0$ collision. (The neglected integrations only give logarithmic prefactors in any case.) From (6) it is clear that unitarity corrections become strong when $R^2 n(Y_0/2) \sim 1/\alpha$. However, at this value of Y_0 the onium wavefunction is still relatively dilute. Indeed, if one scatters a single low momentum left-moving dipole on a fully developed right-moving onium state of rapidity $Y_0/2$ the cross section is of order α. The interaction of the fully developed right-moving onium state with a slightly left-moving dipole is the same as the interaction with a slightly right-moving dipole which could be part of the onium wavefunction. It is only when $R^2 n \approx 1/\alpha^2$ that a given low momentum dipole in an onium wavefunction begins to strongly interact with some of the dipoles of higher rapidity. Thus in a center of mass collision at $Y = 2Y_0$ all the soft dipoles in both the right-moving and left-moving onia suffer a strong interaction and a good portion of these dipoles will appear as freed gluons in the collision leading to a high field strength initial state, $R^2 F_{\mu\nu} \sim 1/g$, of the collision. However, at such high field strengths our control over the collision is lost because perturbation theory breaks down. (This regime is usually referred to as the saturation regime because a limit to the growth of parton densities is expected to occur.) Whether there is some classical, or semiclassical, solution which governs such collisions is an interesting and challenging problem which, however,goes far beyond the purposes of the present work.

III. SCATTERING IN AN ALMOST REST SYSTEM

In this section we shall view the forward scattering of two onia in a frame where one of the onia, say the right-moving onium, has almost all the available rapidity while the left- moving onium has only enough rapidity to make it move relativistically, but not enough so that one is required to add gluons to its wavefunction [13]. Thus the left-moving onium in this almost rest system, which we shall call the L-frame, consists only of a heavy quark and a heavy antiquark, a single dipole.

A. The BFKL Approximation in the L-Frame

The BFKL approximation is straightforward to implement in the L-frame. In (6) one just makes the replacement

$$n(x'_{01}, x', |\underline{b} - \underline{b}_1|, Y/2) \to n(x'_{01}, x', |\underline{b} - \underline{b}_1|, Y) \tag{20}$$

$$n(x_{01}, x, \underline{b}_1, Y/2) \to x\delta(x - x_{01})\delta(\underline{b}_1). \tag{21}$$

Eq.(21) simply says that the left-moving onium consists of a single dipole, the heavy quark-heavy antiquark pair. The result (9) emerges as is easy to check.

However, it is useful to view the process in a somewhat different way [16, 17, 18]. Let us view the interaction of the two onia as they pass each other as due to the interaction of the left- moving heavy quark-antiquark pair with the color field of the right-moving collection of color dipoles making up the right-moving onium. Thus as the onia pass each other we consider a fixed configuration of dipoles in the right-moving onia. Each dipole gives a classical color field (We take traces over the colors in the individual dipoles later.), and because of the large N_c approximation the color fields coming from the different dipoles do not interfere. The interaction with the left-moving heavy quark-antiquark

pair with a dipole in the right-moving onium of size x' at impact parameter b_1 from the center of the left-moving heavy quark- antiquark pair is

$$W_2(\underline{\mathrm{x}}', \underline{\mathrm{x}}_{01}, \underline{\mathrm{b}}_1) = \frac{1}{N_c}\left[trP\exp\left(ig\oint A\cdot dx\right)\right]_2 \tag{22}$$

where the integration goes around a closed Wilson loop in the $x_\perp, x_-$ plane as illustrated in Fig.1. Indeed, explicit evaluation identifies W_2 with $-\frac{1}{2\pi}f(\underline{b}_1, \underline{x}', \underline{x}_{01})$ of Ref. [14]. The A_μ in (22) is the field due to the color dipole of size x' and the subscript 2 in (22) means that we are only taking the order g^2 term. The exact values of x_-^ℓ and x_-^r for the Wilson loop do not matter so long as $x_-^r - x_-^\ell \gtrsim R$ and $x_-^r > 0, x_-^\ell < 0$ with neither x_-^r or x_-^ℓ being too close to zero. (We suppose the centers of the two onia pass each other at $x_+ = x_- = 0$.) The field, not yet counting color factors, due to the dipole x', consisting of a "quark" at $\underline{x}'_a$ and an "antiquark" at $\underline{x}'_b$, is [18]

$$A_\mu(x) = g\frac{\delta(x_-)}{4\pi}\ln\left(\frac{(\underline{x}-\underline{x}'_b)^2}{(\underline{x}-\underline{x}'_a)^2}\right)\overline{\eta}_\mu \tag{23}$$

in covariant gauge and

$$A_\mu(x) = \frac{g}{4\pi}\epsilon(x_-)\left(\frac{(x-x'_a)^\perp_\mu}{(\underline{\mathrm{x}}-\underline{\mathrm{x}}'_a)^2} - \frac{(x-x'_b)^\perp_\mu}{(\underline{\mathrm{x}}-\underline{\mathrm{x}}'_b)^2}\right) \tag{24}$$

in principal-value light-cone gauge. In (23) $\overline{\eta}$ is such that $\overline{\eta}\cdot v = v_-$ for any four-vector v_μ while $x^\perp_\mu$ in (24) means that only the $\mu = 1, 2$ components are nonzero. From the forms of A_μ in (23) and (24) it is clear that the exact values of x_-^r and x_-^ℓ will not matter when evaluating the Wilson loop in (22).

When $x_{01}/b, x'_{01}/b \ll 1$ it is straightforward to check that

$$F^{(1)}(x_{01}, x'_{01}, b, Y) = \int\frac{d^2x'}{2\pi x'^2}d^2b_1 n(x'_{01}, x', |\underline{\mathrm{b}}-\underline{\mathrm{b}}_1|, Y)W_2(\underline{\mathrm{x}}', \underline{\mathrm{x}}_{01}, \underline{\mathrm{b}}_1) \tag{25}$$

agrees with (9). Eq.(25) is in fact true for all b in the leading logarithmic (BFKL) approximation. (Also, the lowest order approximation, (21), for n allows one to identify W_2 as the dipole-dipole scattering amplitude at a definite impact parameter.) One can use (25) to find out what are the typical values of the fields A_μ in the right-moving onium. From (22) and (25)

$$\frac{1}{N_c}tr\left\langle\left(g\oint A\cdot dx\right)^2\right\rangle = -2\int d^2x'_{01}\int_0^1 dz'\Phi(x'_{01}, z')F^{(1)}(x'_{01}, x_{01}, b, Y) \tag{26}$$

where the left-hand side of (26) is the expectation in the right-moving onium state of the square of the loop integral. This is now the field due to all of the dipoles in the right-moving onium. In the BFKL approximation (26) is a gauge invariant equation. Using (9) one finds

$$\frac{1}{N_c}tr\left\langle\left(g\oint A\cdot dx\right)^2\right\rangle = 4\pi\alpha^2\frac{Rx_{01}}{b^2}\ln\left(\frac{16b^2}{Rx_{01}}\right)\frac{\exp\{(\alpha_P-1)Y - \frac{a}{2}\ln^2(\frac{16b^2}{Rx_{01}})\}}{[\frac{7}{2}\alpha N_c\zeta(3)Y]^{3/2}}. \tag{27}$$

In light-cone gauge the integration of the A_μ field goes over distances x_{01} so that the integration picks out those parts of A_μ having wavelength x_{01} or greater. Define the left-hand side of (27) to be $\frac{4}{N_c}x_{01}^2g^2\sum_i\left\langle(A^i_\perp)^2\right\rangle$. (The factor of 4 is included because, according to (24) there are equal contributions coming from $x_- < 0$ and from $x_- > 0$ in the integral on the left-hand side of (27).) Then,

$$\left\langle(A^i_\perp)^2\right\rangle = \frac{\alpha R}{2N_c x_{01}b^2}\ln\left(\frac{16b^2}{Rx_{01}}\right)\frac{\exp\left[(\alpha_P-1)Y - \frac{a}{2}\ln^2(\frac{16b^2}{Rx_{01}})\right]}{[\frac{7}{2}\alpha N_c\zeta(3)Y]^{3/2}} \tag{28}$$

gives the average value for the square of a fixed color component and a fixed transverse spatial component of A_μ in light-cone gauge when the field is measured a distance b from the center of the fast moving onium and if the measurement averages over transverse distances of size x_{01}. When αY is of order 1 the field A_μ is of size g. Taking $x_{01} = 2R$ and using (11)

$$4R^2 \left\langle (A^i_\perp)^2 \right\rangle = \frac{1}{8\pi\alpha N_c} \frac{d\sigma}{d^2b} \tag{29}$$

so that the unitarity limit, in the BFKL approximation, is reached when typical field strengths reach $A \sim (g\sqrt{N_c})^{-1}$. Hence, when light-cone field strengths reach values of the order of $1/g$ one expects strong unitarity corrections to occur. Thus, in the L-frame the onset of unitarity corrections to the BFKL approximation occurs at exactly the same rapidity that field strengths reach size $1/g$. This is in contrast to the center of mass frame where unitarity corrections become important when field strengths are of size g^0.

B. Unitarity Corrections in the L-Frame

When field strengths reach a size $1/g$ the complications of doing calculations in the L- Frame are of two varieties. First, one can no longer expect the two gluon approximation to be valid for connecting the right-moving system of a heavy quark-antiquark pair and its accompanying right-moving gluons to the left-moving heavy quark-antiquark pair. Secondly, one can no longer expect the right-moving system to consist only of dipoles. When $A^i_\mu \sim (g\sqrt{N_c})^{-1}$ the large N_c expansion begins to break down and one can expect more complicated color structures, quadrupoles, sextupoles, etc. to become important in addition to the color dipoles corresponding to the BFKL approximation. In the center of mass system the breakdown of the large N_c expansion shows up in the multiple scattering in (16) and, more explicitly, in (17) and (18).

We continue to find it convenient to view the scattering as the interaction of the left- moving heavy quark-antiquark pair (dipole) with the color field of the right-moving system. Thus from (5), with $S(Y,b) = 1 + iA(Y,b)$

$$S(Y,b) = \frac{1}{N_c} \int d^2x_{01} \int_0^1 dz_1 \Phi(x_{01}, z_1) tr \left\langle P e^{ig \oint A_\mu dx_\mu} \right\rangle_b \tag{30}$$

where the contour of integration for dx_μ is the same as before, and shown in Fig.1, while the average in (30) means that A_μ is the field in the high momentum right-moving onium at fixed impact parameter b from the center of the right-moving heavy quark- antiquark pair and with averages being done over all the numbers and distributions of gluons (dipoles, quadrupoles, etc.) of the right-moving system making up the wavefunction of the onium. That the Wilson loop correctly expresses the scattering amplitude is apparent in covariant gauge where the interaction with the left-moving quark-antiquark pair is causal and occurs with the horizontal lines shown in Fig. 1. Since the left-moving heavy quark-antiquark pair is frozen, in transverse coordinates, during the passage through the field due to the right-moving system, the integral in (30) just represents the eikonal interaction of that field with the heavy quark at $\underline{x}_0$ and with the heavy antiquark at $\underline{x}_1$.

In the BFKL approximation the fields from the right-moving system come from independent dipoles and (30) is replaced by (25). In general (30) shows that when one reaches rapidities where $A^i_\mu \sim (g\sqrt{N_c})^{-1}$, and unitarity corrections become strong, what is being determined in the scattering is the expectation of a Wilson loop. The expectations of the various sized Wilson loops contain the essential information of QCD. Very high energy scattering allows one to test QCD in a regime where g is small, corresponding to the small onium size, but where gA is not small.

Suppose we take $Y = Y_0$, the rapidity at which unitarity corrections begin to be important for a zero impact parameter onium-onium collision. At such rapidities gA_μ is of order 1 in (30), and one might expect that essential nonperturbative physics would be necessary to evaluate A. However, at a corresponding rapidity we found that perturbation theory works well in the center of mass system where gA_μ is of order g. How does one reconcile this? The resolution is that the field $A_\mu \sim 1/g$ in (30) when unitarity corrections become important, but that the large part of this field is purely gauge. The potential A_μ comes essentially from a superposition of independent fields of the type shown in (24). The potential given in (24) leads to a field strength

$$F_{+i} = \frac{g}{2\pi} \delta(x_-) \left[\frac{(x - x'_a)_i}{(\underline{x} - \underline{x}'_a)^2} - \frac{(x - x'_b)_i}{(\underline{x} - \underline{x}'_b)^2} \right]. \tag{31}$$

In reality the $\delta(x_-)$ should be smeared out [17, 18] over an extent $\Delta x_- \propto Re^{-y_{ab}}$ where y_{ab} is the smallest of the rapidities of the gluons making up the "quark a" and the "antiquark b" which is the source of (24) and (31). Thus, although the potential A_μ is of size $1/g$ when $Y = Y_0$ the F_{+i}'s which come from the different dipoles which serve as sources for A have differing longitudinal extent Δx_- so that there is no fixed coherence distance Δx_- over which F_{+i} is of size $1/g$. This means that the $1/g$ part of A_μ is purely a gauge. (The largest number of sources having a fixed coherence length are those dipoles which are (partly) made up of gluons having rapidity on the order of 1. When $Y = Y_0$ there is on the order of $N_0 \sim 1/\alpha$ such sources. These sources lead to a non-gauge part of A_μ of size $A_\mu \sim \sqrt{N_0} \cdot g \sim 0(1)$ which is the maximum non-gauge field present at $Y = Y_0$.) Nevertheless, the determination of the A_μ to use in (30) does involve nonlinear (gauge) interactions because gA is not small. It is not a trivial problem to evaluate A_μ for an onium state having rapidity $Y \gtrsim Y_0$. For the somewhat simpler problem of a large nucleus where $A \sim 1/g$ comes from different (non-overlapping) nucleons, the corresponding pure gauge field, which could be used in (30) to give hadron nucleus scattering, has been calculated in Refs. [17] and [18].

Thus at those rapidities where unitarity corrections begin to be important the potential A_μ becomes of size $1/g$ but this large part of the potential is purely gauge. As one further increases Y the rate of growth of A_μ will be considerably slower than for $Y < Y_0$ in keeping with (30) leading to (19). This slowing down of A_μ corresponds to "parton saturation" and in principle is perturbatively calculable. In the L-frame parton saturation is intimately related to the rate at which unitarity (blackness) sets in as Y becomes greater than Y_0 [13]. However, as we have seen in Sec.2 one can calculate unitarity corrections in the center of mass system where field strengths remain weak and saturation effects are not important.

Finally, we have argued that at values of Y on the order of Y_0, where unitarity corrections first become important, the wavefunction of a heavy onium is still a perturbative object even though $A_\mu \sim 1/g$. Thus the general probe described by the Wilson loop in (30) and corresponding to scattering by an undeveloped onium state does not require knowledge of nonperturbative QCD.

However, suppose we scatter, at zero impact parameter, a right-moving onium, having $Y \approx Y_0$, on a left-moving onium, having $Y \approx Y_0$. In the central unit of rapidity there are on the order of $1/\alpha$ gluons, corresponding to $A \sim 1/g$, which will be freed during the collision. These gluons lead to field strengths $F_{\mu\nu} \sim 1/g$ and now genuine nonperturbative QCD effects can be expected. Thus the genuine nonperturbative effects appear to be more connected with production in the collision process than with the wavefunction of a high energy hadron, or at least they show up at lower rapidities in the collision than in wavefunctions entering the collision.

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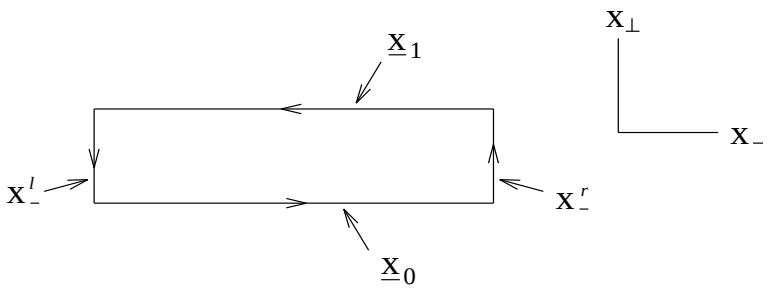
$\mathbf{x}_\perp$
$\underline{\mathbf{x}}_1$
$\mathbf{x}_-$
$\mathbf{x}_-^l$
$\mathbf{x}_-^r$
$\underline{\mathbf{x}}_0$

FIG. 1.