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Resummed heavy quark production cross sections to next-to-leading logarithm

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Abstract

We study how next-to-leading logarithms modify predictions from leading logarithmic soft-gluon resummation in the heavy quark production cross section near threshold. Numerical results are presented for top quark production at the Fermilab Tevatron and bottom quark production at fixed-target energies.

1 Introduction

The calculation of hadronic cross sections in the elastic limit (*i.e.*, near threshold) in perturbative QCD involves contributions from the emission of soft gluons. In n-th order QCD one encounters leading logarithmic (LL) contributions proportional to $(-\alpha_s^n/n!)[\ln^{2n-1}((1-z)^{-1})/(1-z)]_+$. There are also terms with next-to-leading logarithmic (NLL) contributions proportional to $(-\alpha_s^n/n!)[\ln^{2n-2}((1-z)^{-1})/(1-z)]_+$. These "plus" distributions are large near threshold, $z = 1$, (the precise definition of z will be given in the next section) and resummation techniques were originally developed to sum them in Drell-Yan production [1]. As the Drell-Yan process involves electroweak interactions it has a rather simple color structure. Gluons are only radiated from the incoming quark-antiquark pair in the hadronic collision process so the amplitude is a color singlet.

The first resummation of LL terms (as well as some NLL terms) in the heavy (top) quark production cross section was discussed [2, 3] prior to the discovery of the top quark at the Fermilab Tevatron [4]. The analysis was based on the fact that the LL terms are identical to those in the Drell-Yan process and, even though a cutoff was used to define the resummed perturbation series, this paper pointed out the importance of incorporating resummation effects in threshold production of heavy quarks in QCD. Recently other groups have applied more sophisticated LL resummation methods to the top quark production cross section, *c.f.* Refs. [5, 6], which avoid the use of an explicit cutoff, and lead to slightly different values for the top quark cross section. At present the experimental results cannot discriminate between them and there is an ongoing discussion as to which method is theoretically superior. The quark-antiquark annihilation channel is the dominant partonic process for top quark production in $p\overline{p}$ collisions at center of mass energy $\sqrt{S} = 1.8$ TeV. However the gluon-gluon channel is more important in bottom and charm production near threshold in fixed-target experiments with proton and pion beams. The LL resummation method of [2] has also been applied to b-quark production at HERA-B [7] and more generally for fixed-target bottom and charm production by hadron beams [8].

The order α_s^2 corrections to the Drell-Yan process in [9] have allowed a check of the NLL terms in the resummation formulae, thus the theory is in excellent shape. However, this information cannot be used in heavy-quark resummation since some of the NLL terms are a consequence of the more

complicated color structure in heavy quark production. There is no exact calculation of the heavy-quark production cross section at order α_s^4. Even though the NLL terms at order α_s^3 for heavy quark production near threshold were available in 1990 [10], at the time of the heavy quark resummation analysis [2] it was not clear how to resum them. Therefore all work has concentrated on the LL terms. Now this situation has changed. A resummation formalism which correctly incorporates NLL resummation near threshold has recently been presented by Kidonakis and Sterman [11, 12]. The authors analytically compared the order-by-order expansion of their results with the known NLL terms [10] and found agreement at threshold in both the quark-antiquark channel and the gluon-gluon channel.

In this paper we will apply this new NLL resummation formalism to calculate the top quark production cross section at the Fermilab Tevatron and the bottom quark production cross section at fixed target energies. We are particularly interested in the size and therefore the phenomenological importance of the NLL terms with respect to the LL terms. To determine the relative importance of the NLL terms, we will use the cutoff method proposed originally in [2]. There are several reasons for this, which we now discuss. First, as already stated, we are primarily interested in the corrections the NLL terms make to the LL terms. Next, by varying the cutoff the failure of the perturbative expansion and the onset of the nonperturbative region may be studied directly, bypassed in the newer methods. Finally, given the complexity of the formulae it is best to use the simplest approach to incorporate the NLL terms in a first test of their magnitude. An analysis of the NLL terms is very important since the present LL resummations are based either on neglecting them entirely [5] or retaining only some of them [6]. These terms are not universal and could possibly be of importance in several areas of perturbative QCD besides the analysis of the heavy quark cross section, including the production of large transverse energy jets and the production of supersymmetric particles. The conclusions of our study should not be generalized to these other reactions since each process is different and requires a separate study.

2 General Formalism

Here we present some basic formulae which we need for our analysis, based on the theory developed by Kidonakis and Sterman [11, 12]. For heavy quark production to be kinematically allowed, the square of the partonic center of mass energy, $s = x_a x_b S$, must be larger than the threshold value of the invariant mass of the heavy quark-antiquark pair, $Q^2 = 4m^2$. When the heavy quarks are produced with zero velocity, the true threshold and the partonic threshold are equivalent. In either case, the plus distributions which must be resummed are functions of

$$z = \frac{Q^2}{s}. \tag{2.1}$$

Our calculations are based on the factorization of soft gluons from high-energy partons in perturbative QCD [13, 14]. All the singular distributions in the heavy quark (Q) production cross section can be expressed in the form

$$\begin{aligned} \frac{d\sigma_{h_1 h_2}}{dQ^2 d\cos\theta^* dy} &= \sum_{ab}\sum_{IJ}\int \frac{dx_a}{x_a}\frac{dx_b}{x_b}\phi_{a/h_1}(x_a,Q^2)\phi_{b/h_2}(x_b,Q^2) \\ &\quad \times\delta\Big(y-\frac{1}{2}\ln\frac{x_a}{x_b}\Big)\hat{\sigma}_{ab}^{(IJ)}\Big(\frac{Q^2}{x_a x_b S},\theta^*,\alpha_s(Q^2)\Big)\,, \end{aligned} \tag{2.2}$$

where x_a, x_b are the momentum fractions of the incoming partons, y is the rapidity of the $Q\overline{Q}$ pair, and θ^* is the scattering angle in the pair center of mass system. The ϕ's are parton densities, evaluated at the mass factorization scale Q^2, in the DIS or $\overline{\rm MS}$ mass factorization scheme. The hard scattering functions $\hat{\sigma}$, which depend on the color structure (IJ) of the interaction as well as on the partonic channel ab and the mass factorization scheme, contain all the plus distributions in the threshold region. In [11] it was shown that up to NLL it is possible to pick a color basis in which moments of the functions $\hat{\sigma}_{ab}^{(IJ)}(z,\theta^*,\alpha_s(Q^2))$ exponentiate with respect to z so that

$$\begin{aligned} \tilde{\sigma}_{ab}^{(IJ)}(n,\theta^*,Q^2) &= \int_0^1 dz z^{n-1}\hat{\sigma}_{ab}^{(IJ)}(z,\theta^*,\alpha_s(Q^2)) \\ &= h_I(\theta^*,Q^2)h_J^*(\theta^*,Q^2)e^{E_{IJ}(n,\theta^*,Q^2)}\,. \end{aligned} \tag{2.3}$$

The hard scattering prefactor is a product of the contributions from the amplitude h_I and its complex conjugate h_J^*. Therefore I refers to the decomposition of the hard scattering amplitude into a color basis, *e.g.* into an

s-channel singlet and octet. The functions h_I and h_J^* have no collinear or soft divergences at the partonic threshold since these terms have been factored into the exponent E_{IJ} in eq. (2.3). The exponential in (2.3), called S_{IJ} in [11, 12], satisfies a renormalization group equation with an anomalous dimension matrix Γ_{IJ}.

The exponents of eq. (2.3) are given by

$$E_{IJ}^{(ab)}(n,\theta^*,Q^2) = -\int_0^1 dz \frac{z^{n-1}-1}{1-z}$$
$$\times\Big\{(2-r)\int_0^z \frac{dy}{1-y}\, g_1^{(ab)}[\alpha_s((1-y)^{2-r}(1-z)^r Q^2)] + g_2^{(ab)}[\alpha_s((1-z)Q^2)]$$
$$+g_3^{(I)}[\alpha_s((1-z)^2Q^2),\theta^*] + g_3^{(J)*}[\alpha_s((1-z)^2Q^2),\theta^*]\Big\}, \qquad (2.4)$$

where g_1, g_2 and g_3 are functions of the running coupling constant α_s with z- (and y)-dependent arguments. The parameter r, 1 in the DIS scheme and 0 in the $\overline{\rm MS}$ scheme, effectively changes the lower limit of the y-integral in the $\overline{\rm MS}$ scheme (*c.f.* [15] and our later discussion). As it stands, eq. (2.4) is ill-defined since it incorporates arbitrarily soft gluon radiation at the point where the QCD perturbation expansion diverges. There are different opinions as to how to define the exponent in a manner which clearly separates the perturbative and non-perturbative regions. We will return to this point shortly.

To go to NLL in the exponents E_{IJ}, we need g_1 up to and including $O(\alpha_s^2)$ and g_2 and $g_3^{(I)}$ to $O(\alpha_s)$. Refs. [5, 6] do not include the NLL g_3 term, treated numerically for the first time here.

The functions g_1 and g_2 depend on the mass factorization scheme and the identity of the incoming partons but are essentially independent of the color structure. The function g_1 is scheme dependent because of the r dependence of the y integration. It also depends on the identity of the incoming partons so that

$$g_1^{(ab)}[\alpha_s] = (C_a + C_b)\left(\frac{\alpha_s}{\pi} + \frac{1}{2}K\left(\frac{\alpha_s}{\pi}\right)^2\right), \qquad (2.5)$$

where $C_q = C_{\overline{q}} = C_F$ and $C_g = C_A$. The constant K is (*c.f.* [16])

$$K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n_{\rm f}, \qquad (2.6)$$

where $n_{\rm f}$ is the number of quark flavors. The g_2 contribution is only nonzero

in the DIS scheme [15],

$$g_2^{(q\overline{q})\,\mathrm{DIS}}[\alpha_s] = -\frac{3}{2} C_F \, \frac{\alpha_s}{\pi} \,, \tag{2.7}$$

Otherwise, $g_2^{(q\overline{q})\,\overline{\mathrm{MS}}} = g_2^{gg} = 0$.

The functions $g_3^{(I)}$ depend on the color structure in the hard scattering but are mass factorization scheme independent. In [11, 12] it is shown that they are the eigenvalues of the anomalous dimension matrix Γ_{IJ} which appears in the renormalization group equation for the function S_{IJ}. The precise relation between the $g_3^{(I)}$ and the eigenvalues of Γ_{IJ} is

$$g_3^{(I)}[\alpha_s((1-z)^2Q^2), \theta^*] = -\lambda_I[\alpha_s((1-z)^2Q^2), \theta^*] \,, \tag{2.8}$$

showing that $g_3^{(I)}$ and λ_I are functions of both the running coupling constant α_s (evaluated at a z-dependent scale) and the angle θ^* between the directions of the incoming parton and outgoing heavy quark. As can be seen below, although the eigenvalues are complex, the total g_3 contribution to $E_{IJ}^{(ab)}$ in eq. (2.4) is real.

Consider first the reaction channel $q(p_a) + \overline{q}(p_b) \to \overline{Q}(p_i) + Q(p_j)$ where Γ_{IJ} is two-dimensional. In the s-channel singlet-octet basis the color decomposition is into $\delta^{ab}\delta^{ij}$ (singlet) and $-\delta^{ab}\delta^{ij}/(2N) + \delta^{aj}\delta^{bi}/2$ (octet). In this basis the components of the matrix Γ_{IJ} are [12]

$$\begin{aligned}
\Gamma_{11} &= -\frac{\alpha_s}{\pi} C_F (L_\beta + 1 + i\pi), \\
\Gamma_{21} &= \frac{2\alpha_s}{\pi} \ln\left(\frac{u_1}{t_1}\right), \\
\Gamma_{12} &= \frac{\alpha_s}{\pi} \frac{C_F}{C_A} \ln\left(\frac{u_1}{t_1}\right), \\
\Gamma_{22} &= \frac{\alpha_s}{\pi} \left\{ C_F \left[4 \ln\left(\frac{u_1}{t_1}\right) - L_\beta - 1 - i\pi \right] \right. \\
&\quad \left. + \frac{C_A}{2} \left[-3 \ln\left(\frac{u_1}{t_1}\right) - \ln\left(\frac{m^2 s}{t_1 u_1}\right) + L_\beta + i\pi \right] \right\} .
\end{aligned} \tag{2.9}$$

where

$$L_\beta = \frac{1 - 2m^2/s}{\beta} \left[\ln\left(\frac{1-\beta}{1+\beta}\right) + i\pi \right] \,, \tag{2.10}$$

and $\beta^2 = 1 - 4m^2/s$. The Mandelstam invariants for the reaction are $s = (p_a + p_b)^2$, $t = t_1 + m^2 = (p_a - p_i)^2$, and $u = u_1 + m^2 = (p_b - p_i)^2$.

Γ_{IJ} is diagonalized in this basis when the parton-parton c.m. scattering angle $\theta^* = 90^\circ$ ($u_1 = t_1 = -s/2$ at threshold) with eigenvalues

$$\begin{aligned} \lambda_1 &= \lambda_{\rm singlet} = -\frac{\alpha_s}{\pi} C_F (L_\beta + 1 + i\pi)\,, \qquad (2.11) \\ \lambda_2 &= \lambda_{\rm octet} = \frac{\alpha_s}{\pi}\Big\{ - C_F (L_\beta + 1 + i\pi) + \frac{C_A}{2}\Big[L_\beta - \ln\Big(\frac{m^2 s}{t_1^2}\Big) + i\pi\Big]\Big\}\,. \end{aligned}$$

It is also diagonalized at partonic threshold $s = 4m^2$ for arbitrary θ^*.

In the partonic channel $g(p_a) + g(p_b) \to \overline{Q}(p_i) + Q(p_j)$ the anomalous dimension matrix is three dimensional. In the color basis $\delta^{ab}\,\delta_{ji}$, $d^{abc}\,T^c_{ji}$ and $if^{abc}\,T^c_{ji}$ the components of the matrix Γ_{IJ} are [12]

$$\begin{aligned} \Gamma_{11} &= -\frac{\alpha_s}{\pi}[C_F(L_\beta + 1) + C_A i\pi], \\ \Gamma_{21} &= 0\,, \\ \Gamma_{31} &= \frac{2\alpha_s}{\pi}\ln\left(\frac{u_1}{t_1}\right), \\ \Gamma_{12} &= 0\,, \\ \Gamma_{22} &= \frac{\alpha_s}{\pi}\left\{-C_F(L_\beta + 1) + \frac{C_A}{2}\left[-\ln\left(\frac{m^2 s}{t_1 u_1}\right) + L_\beta - i\pi\right]\right\}, \\ \Gamma_{32} &= \frac{N^2 - 4}{4N}\Gamma_{31}\,, \\ \Gamma_{13} &= \frac{1}{2}\Gamma_{31}\,, \\ \Gamma_{23} &= \frac{C_A}{4}\Gamma_{31}\,, \\ \Gamma_{33} &= \Gamma_{22}\,. \qquad (2.12) \end{aligned}$$

At $\theta^* = 90^\circ$ the anomalous dimension matrix becomes diagonal with eigenvalues

$$\begin{aligned} \lambda_1 &= -\frac{\alpha_s}{\pi}\left[C_F(L_\beta + 1) + C_A i\pi\right], \\ \lambda_2 &= \frac{\alpha_s}{\pi}\Big\{ - C_F(L_\beta + 1) + \frac{C_A}{2}\Big[L_\beta - \ln\Big(\frac{m^2 s}{t_1^2}\Big) - i\pi\Big]\Big\}, \\ \lambda_3 &= \lambda_2\,. \qquad (2.13) \end{aligned}$$

We note that Γ_{IJ} is also diagonal at partonic threshold for arbitrary θ^*.

3 Heavy-quark production cross sections

As we have already stated, we are primarily interested in the magnitude of the NLL terms, particularly the g_3 contribution, relative to the LL results of [2, 3, 7, 8]. Therefore we use a similar cutoff scheme, modifying the definitions of the exponents in eqs. (2.3) and (2.4) to work directly in momentum space and avoid working in Mellin space. This method, introduced in [2], exploits the correspondence between the moment variable n and logarithmic terms in the momentum space variable $1 - z$. To directly compare with other recent LL resummation schemes in Refs. [5, 6] would, in principle, require three separate studies since each group of authors defines eq. (2.4) differently. Therefore, in this paper we only consider the approach based on [2].

First, we identify

$$1 - z = \frac{s_4}{2m^2}\,, \tag{3.1}$$

where s_4 is the invariant mass of the heavy-quark + gluon system which recoils against the detected heavy antiquark in inclusive $\overline{Q}$ production. The invariants satisfy $s + t_1 + u_1 = s_4$ so that in the elastic limit $s_4 \to 0$ or $z \to 1$. Note that eq. (3.1) differs from the identification $1 - z = s_4/m^2$ in Refs. [2, 5]. The factor of two is necessary to compare the NLL terms [12] with the expressions previously given in [10]. With the identification in eq. (3.1) all the NLL terms in the $q\overline{q}$ channel are reproduced, not only exactly at threshold but also at finite s_4 except for one NLL term in C_F^2 which has a different coefficient. (This small difference is due to the fact that the $Q\overline{Q}$ invariant mass and the Qg invariant mass are different invariants). A similar observation holds for the gg channel.

Further we exploit the fact that a heavy quark-antiquark pair will be preferentially produced back-to-back in the parton-parton center-of-mass frame near threshold. We also choose the case of $\theta^* = 90°$ to use the results of eqs. (2.11) and (2.13). This choice avoids both the diagonalization of the three dimensional matrix in the gluon-gluon channel and an extra numerical integration over θ^*.

The resummation is done using eqs. (3.17) and (3.19) of [2] where the "plus" distributions are removed through integration by parts. Therefore

the resummed partonic cross section is defined as

$$\sigma_{ab}(s,m^2) = -\int_{s_{\rm cut}}^{s-2ms^{1/2}} ds_4 f_{ab}\left(\frac{s_4}{2m^2}\right)\frac{d\overline{\sigma}_{ab}^{(0)}(s,s_4,m^2)}{ds_4}\,, \tag{3.2}$$

where $ab = q\overline{q}$ or gg and $s_{\rm cut}$ will be defined below. The other inputs needed here are the function f_{ab}, in either the DIS scheme or the $\overline{\rm MS}$ scheme and the differential of the Born cross section. The exponential function $f_{ab}(s_4/2m^2)$ is given by a momentum space version of eq. (2.4). This is best illustrated by examining eqs. (3.31) and (3.35) in [2]. There we showed how to include gluons with an invariant mass $s_4' > s_4$ and integrate over their transverse momenta up to the kinematic limit. The corresponding result follows by first replacing z in eq. (2.4) by z'. Next we replace $z'^{n-1}-1$ in the numerator of eq. (2.4), by -1 and introduce the variables $\omega' = 1-z' = s_4'/(2m^2)$ and $\xi = (1-y)(1-z')Q^2/\Lambda^2$. By explicitly leaving Q^2 in the following discussion, we can later study the scale dependence of the calculation. Note that α_s in eqs. (2.5)-(2.13) is a function of $s_4'/2m^2$.

We first discuss the $q\overline{q}$ channel. There is only one kinematic structure in this channel so that the singlet and octet eigenvalues in eq. (2.11) multiply the same function. At this order in perturbation theory however, only the octet component contributes, *c.f.* the discussion in [11]. Therefore in the DIS scheme at $\theta^* = 90^\circ$, the exponential function is

$$f_{q\overline{q}}^{\rm DIS}\Big(\frac{s_4}{2m^2}\Big) = \exp[E_{q\overline{q}}^{\rm DIS} + E_{q\overline{q}}(\lambda_2)]\,, \tag{3.3}$$

where

$$\begin{aligned} E_{q\overline{q}}^{\rm DIS} &= E_{q\overline{q}}^{\rm DIS}(g_1) + E_{q\overline{q}}^{\rm DIS}(g_2) \\ &= \int_{\omega_0}^1 \frac{d\omega'}{\omega'}\Big\{\int_{\omega'^2Q^2/\Lambda^2}^{\omega' Q^2/\Lambda^2}\frac{d\xi}{\xi}\Big[\frac{2C_F}{\pi}\Big(\alpha_s(\xi)+\frac{1}{2\pi}\alpha_s^2(\xi)K\Big)\Big] \\ &\qquad -\frac{3}{2}\frac{C_F}{\pi}\alpha_s\Big(\frac{\omega' Q^2}{\Lambda^2}\Big)\Big\}\,. \end{aligned} \tag{3.4}$$

Since our calculation is not done in moment space, the ω' integral is cut off at $\omega_0 = s_4/2m^2$. Because the running coupling constant diverges when $\omega'^2Q^2/\Lambda^2 \sim 1$, the minimum cutoff in eq. (3.2) is $s_{\rm cut} = s_{4,\rm min} \sim 2m^2\Lambda/Q$. If we choose the scale $Q^2 = m^2$, $s_{\rm cut} \sim 2m\Lambda$. In general we choose a larger value to stay away from the point of divergence. Once we have chosen a reasonable

cutoff, consistent with the sum of the first few terms in the perturbative expansion, we can study the size of the NLL terms with respect to the LL terms (the order-by-order expansion does not require a cutoff).

We use the two-loop running coupling constant,

$$\begin{aligned}
\alpha_s(\xi) &= \frac{1}{a\ln\xi} + \frac{b}{a}\frac{\ln(\ln\xi)}{\ln^2(\xi)}\,, \\
a &= \frac{11C_A - 4T_f n_{\rm f}}{12\pi}\,, \\
b &= -6\frac{17C_A^2 - (6C_F + 10C_A)T_f n_{\rm f}}{(11C_A - 4T_f n_{\rm f})^2}\,,
\end{aligned} \tag{3.5}$$

with $C_A = 3$, $C_F = 4/3$ and $T_f = 1/2$. It is obvious that when $\xi < 1$, $\ln\xi$ is negative so that $\ln(\ln\xi)$ has a cut and needs a precise definition. We use the cutoff on the ω' variable to stop the integration before that point. The exact cutoff depends on the quark mass, the scale and $\Lambda_{n_{\rm f}}$, the QCD scale parameter for $n_{\rm f}$ light quark flavors, in the parton densities. Both the g_1 integrals over ω' and ξ and the g_2 integral over ω' as well as the contributions to the g_3 integration which only contain s_4' in the running coupling constant can be done analytically, as shown in the appendix.

In the $\overline{\rm MS}$ scheme,

$$f_{q\overline{q}}^{\overline{\rm MS}}\Big(\frac{s_4}{2m^2}\Big) = \exp[E_{q\overline{q}}^{\overline{\rm MS}} + E_{q\overline{q}}(\lambda_2)]\,, \tag{3.6}$$

where now

$$E_{q\overline{q}}^{\overline{\rm MS}} = E_{q\overline{q}}(g_1) = \int_{\omega_0}^1 \frac{d\omega'}{\omega'} \int_{\omega'^2 Q^2/\Lambda^2}^{Q^2/\Lambda^2} \frac{d\xi}{\xi}\Big[\frac{2C_F}{\pi}\Big(\alpha_s(\xi) + \frac{1}{2\pi}\alpha_s^2(\xi)K\Big)\Big]\,. \tag{3.7}$$

Note the difference between the upper limits of the ξ-integrations introduced by the mass factorization scheme in eqs. (3.4) and (3.7).

The color-dependent g_3 contribution in eq. (2.11) leads to

$$E_{q\overline{q}}(\lambda_i) = -\int_{\omega_0}^1 \frac{d\omega'}{\omega'}\Big\{\lambda_i\Big[\alpha_s\Big(\frac{\omega'^2 Q^2}{\Lambda^2}\Big), \theta^* = 90^\circ\Big] + \lambda_i^*\Big[\alpha_s\Big(\frac{\omega'^2 Q^2}{\Lambda^2}\Big), \theta^* = 90^\circ\Big]\Big\}\,. \tag{3.8}$$

in both mass factorization schemes, where $i = 1, 2$.

The differential function in eq. (3.2) follows from the kinematic behaviour of the Born cross section, represented by

$$F^B_{q\overline{q}}(s,t_1,u_1) = \frac{t_1^2+u_1^2}{s^2} + \frac{2m^2}{s}\,. \tag{3.9}$$

Substituting $t_1 = -1/2\{s-s_4-[(s-s_4)^2-4sm^2]^{1/2}\cos\theta^*\}$ and $u_1 = -1/2\{s-s_4+[(s-s_4)^2-4sm^2]^{1/2}\cos\theta^*\}$, and taking $\theta^* = 90°$, we define, analogous to eq. (2.20) in [2]

$$\overline{F}^{(0)}_{q\overline{q}} = \frac{[(s-s_4)^2-4sm^2]^{1/2}}{2s^2} F^B_{q\overline{q}}\,. \tag{3.10}$$

Differentiating with respect to s_4, we find

$$\frac{d\overline{F}^{(0)}_{q\overline{q}}}{ds_4} = -\frac{1}{4s^4}\frac{s-s_4}{\sqrt{(s-s_4)^2-4sm^2}}[3(s-s_4)^2-4sm^2]\,. \tag{3.11}$$

The integrand in eq. (3.2) becomes, for both factorization schemes,

$$\begin{aligned} f_{q\overline{q}}\left(\frac{s_4}{2m^2}\right)\frac{d\overline{\sigma}^{(0)}_{q\overline{q}}(s,s_4,m^2)}{ds_4} &= \pi\alpha_s^2 K_{q\overline{q}} N C_F \frac{d\overline{F}^{(0)}_{q\overline{q}}}{ds_4} \\ &\quad\times \exp[E_{q\overline{q}} + E_{q\overline{q}}(\lambda_2)]\,, \end{aligned} \tag{3.12}$$

where $K_{q\overline{q}} = 1/N^2$ is a color average factor. We note that near threshold $d\overline{\sigma}^{(0)}_{q\overline{q}}/ds_4$ is approximately a factor of two smaller than $d\sigma^{(0)}_{q\overline{q}}/ds_4$ in eq. (3.20) of Ref. [2] where the angle θ^* was analytically integrated before differentiating with respect to s_4.

The treatment of the gluon-gluon channel is very similar but now we have three distinct color structures. However, only two of them are independent. Therefore we define f_{gg} for each eigenvalue so that $f_{gg,i} = \exp(E_{gg} + E_{gg}(\lambda_i))$ with a one-to-one correspondence between $E_{gg}(\lambda_i)$ and the eigenvalues of eq. (2.13) so that

$$E_{gg}(\lambda_i) = -\int_{\omega_0}^1 \frac{d\omega'}{\omega'}\Big\{\lambda_i\Big[\alpha_s\Big(\frac{\omega'^2Q^2}{\Lambda^2}\Big),\theta^*=90°\Big] + \lambda_i^*\Big[\alpha_s\Big(\frac{\omega'^2Q^2}{\Lambda^2}\Big),\theta^*=90°\Big]\Big\}\,, \tag{3.13}$$

where $i = 1, 2$. In the $\overline{\rm MS}$ scheme, we take

$$E_{gg} = \int_{\omega_0}^1 \frac{d\omega'}{\omega'} \int_{\omega'^2 Q^2/\Lambda^2}^{Q^2/\Lambda^2} \frac{d\xi}{\xi} \Big[\frac{2C_A}{\pi} \Big(\alpha_s(\xi) + \frac{1}{2\pi} \alpha_s^2(\xi) K \Big) \Big] , \tag{3.14}$$

where K is defined as in eq. (2.6). This choice of K requires some justification, which we have not found explicitly in the literature, due to the fact that the discussions of the corrections to the Drell-Yan process are made in the DIS scheme. However, it should be possible to prove rigorously that (3.14) holds [17].

Because there are two kinematical structures, $d\overline{\sigma}_{gg}^{(0)}/ds_4$ has two contributions:

$$\begin{aligned} F^B_{gg,I}(s,t_1,u_1) &= \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s}{t_1 u_1} \Big(1 - \frac{m^2 s}{t_1 u_1} \Big) \\ F^B_{gg,II}(s,t_1,u_1) &= \Big(1 - \frac{4t_1 u_1}{s^2} \Big) F^B_{gg,I}(s,t_1,u_1) \, . \end{aligned} \tag{3.15}$$

The second kinematical structure, proportional to the first, comes from the nonabelian part of the cross section. As in eq. (3.10), we define

$$\overline{F}^{(0)}_{gg,i} = \frac{[(s-s_4)^2 - 4sm^2]^{1/2}}{2s^2} F^B_{gg,i} \, . \tag{3.16}$$

The differential with respect to s_4 of the two components yields

$$\begin{aligned} \frac{d\overline{F}^{(0)}_{gg,I}(s,s_4,m^2)}{ds_4} &= -\frac{1}{2s^2\sqrt{(s-s_4)^2 - 4sm^2}} \Big[2(s-s_4) - \frac{16m^2 s}{s-s_4} \\ &\quad + \frac{320 m^4 s^2}{(s-s_4)^3} - \frac{1024 m^6 s^3}{(s-s_4)^5} \Big] \end{aligned} \tag{3.17}$$

$$\begin{aligned} \frac{d\overline{F}^{(0)}_{gg,II}(s,s_4,m^2)}{ds_4} &= \frac{d\overline{F}^{(0)}_{gg,I}(s,s_4,m^2)}{ds_4} \\ &\quad - \frac{1}{2s^2\sqrt{(s-s_4)^2 - 4sm^2}} \left[\frac{512 m^6 s}{(s-s_4)^3} - \frac{6(s-s_4)^3}{s^2} - \frac{64 m^4}{s-s_4} \right] \end{aligned} \tag{3.18}$$

Inserting the remaining color factors yields the gg integrand in eq. (3.2).

$$\begin{aligned} f_{gg}\Big(\frac{s_4}{2m^2}\Big) \frac{d\overline{\sigma}^{(0)}_{gg}(s,s_4,m^2)}{ds_4} &= \frac{\pi}{8} K_{gg} N C_F \alpha_s^2 \Big\{ \Big[2C_F f_{gg,1}\Big(\frac{s_4}{2m^2}\Big) \\ &\quad + C_F(N^2-4) f_{gg,2}\Big(\frac{s_4}{2m^2}\Big) \Big] \frac{d\overline{F}^{(0)}_{gg,I}}{ds_4} + 4C_A f_{gg,2}\Big(\frac{s_4}{2m^2}\Big) \frac{d\overline{F}^{(0)}_{gg,II}}{ds_4} \Big\} \, , \end{aligned} \tag{3.19}$$

where $K_{gg} = (N^2-1)^{-2}$ is a color average factor. One can numerically compare the coefficients of the color terms in $d\sigma_{gg}^{(0)}/ds_4$ in eq. (3.21) of Ref. [2] with the color terms near threshold in $d\overline{\sigma}_{gg}^{(0)}/ds_4$ in eq. (3.19). Those in Ref. [2] are integrated over the angle θ^* before differentiation with respect to s_4 and are several times larger than those found here. Note that eq. (3.19) contains several terms which can have different signs, making it a more complicated function of s_4. Therefore we do not expect our gg results to agree with results obtained without attention to the color decomposition [2, 3, 7, 8].

Because the ω' integral is cut off at $s_4/(2m^2)$, the partonic cross section in eq. (3.2) must also have a lower limit of $s_{\rm cut}$ for the integrals to be finite. In earlier work up to LL [2, 3, 7, 8], the lower limit of the partonic cross section, s_0, was taken to be $s_0/m^2 = (\mu_0/\mu)^2$ in the DIS scheme and $s_0/m^2 = (\mu_0/\mu)^3$ in the $\overline{\rm MS}$ scheme where $\Lambda_{n_f} \ll \mu_0 \ll \mu$ and μ is the renormalization scale. It was assumed that the integral over the nonperturbative region below s_0 was negligible compared to the integral over the perturbative region. The final resummed cross section was obtained by comparison with the order-by-order approximate cross section up to $O(\alpha_s^4)$ as a function of μ_0. Since each term in the perturbative result is positive, μ_0 was chosen such that the resummed cross section was slightly larger than the sum of the approximate cross sections. It was found that for $\mu = m$ in the DIS scheme $\mu_0 \approx (0.05 - 0.1)m$. Larger values were needed in the $\overline{\rm MS}$ scheme because of the cubic power of μ_0/μ. In the $q\overline{q}$ channel, $\mu_0 \approx (0.1-0.2)m$, while $\mu_0 \approx 0.35m$ in the gg channel. The μ_0 needed in the gg channel was larger because of the color factor C_A in E_{gg}, see eq. (3.14). As before, we will use the cutoff method since increasing $s_{\rm cut}$ away from $s_{4,\rm min}$ reduces the scale and scheme dependence of the resummed perturbative cross section, as we discuss in detail in the next section and was also shown previously for the LL resummation [2, 3]. In the next section, we will calculate the μ_0 appropriate for our chosen $s_{\rm cut}$.

4 Numerical results

We first present our results for top quark production. In fig. 1(a) we plot the exponents contributing to $f_{q\overline{q}}^{\rm DIS}$ in eq. (3.3) at $\theta^* = 90°$ as a function of $s_4/2m^2$ with $m = 175$ GeV/c^2 and $\sqrt{s} = 351$ GeV. Since only the octet component contributes to g_3 here, only $E_{q\overline{q}}(\lambda_2)$ is shown. We also show, for comparison,

the one-loop calculation of g_1, $E^{\rm DIS}$, given in [2]. The one-loop and two-loop results are quite similar until s_4 approaches $s_{4,\rm min}$. As $\omega' Q/\Lambda \to 1$, $E^{\rm DIS}_{q\overline{q}}(g_1)$ is larger since $E^{\rm DIS}$ was smoothed near the cutoff [2]. The sum $E^{\rm DIS}_{q\overline{q}} + E_{q\overline{q}}(\lambda_2)$ is always larger than $E^{\rm DIS}$ because the g_3 contribution compensates for the small, negative g_2 component ($|E^{\rm DIS}_{q\overline{q}}(g_2)|$ is shown). The power of $1-z$ in α_s determines the slope of the exponents at small $s_4/2m^2$. The g_2 contribution, linear in $1-z$, is the flattest as $s_4/2m^2 \to 0$. The g_1 component shows more growth, diverging from $E^{\rm DIS}$ near $s_{4,\rm min}$, and the g_3 contribution, quadratic in $1-z$, grows fastest. The growth of the sum is intermediate to that of $E^{\rm DIS}_{q\overline{q}}(g_1)$ and $E_{q\overline{q}}(\lambda_2)$.

To better illustrate the importance of these contributions to the partonic cross section, we show the relative enhancements as a function of $s_4/2m^2$ in fig. 1(b). The negative contribution and the slow growth of g_2 near $s_{4,\rm min}$ are reflected in the near threshold behavior of $f_{q\overline{q}}(g_1+g_2)/f_{q\overline{q}}(g_1)$ where *e.g.* $f_{q\overline{q}}(g_1) \equiv \exp(E^{\rm DIS}_{q\overline{q}}(g_1))$. In the intermediate range of $s_4/2m^2$, $E^{\rm DIS}$ is larger than $E^{\rm DIS}_{q\overline{q}}(g_1)$. Although $E_{q\overline{q}}(\lambda_2)$ dominates $f^{\rm DIS}_{q\overline{q}}$ as $s_4/2m^2 \to 1$, the enhancement in this region is not very significant. Thus the choice of the cutoff determines the importance of the NLL contributions. However, a large NLL correction is not necessarily reflected in the hadronic cross section since it is the parton luminosity relative to the equivalent $\eta = s/4m^2 - 1$ that determines the true strength of the corrections.

Similar results are seen for the exponents in the $\overline{\rm MS}$ scheme, shown in fig. 2. The g_2 term vanishes in this scheme. The change in the upper limit of the ξ integral increases the phase space of $E^{\overline{\rm MS}}_{q\overline{q}}$. Both these changes enhance the $\overline{\rm MS}$ result. The g_3 contribution is identical to that shown in fig. 1.

In fig. 3 we plot the corresponding results for the gg channel in the $\overline{\rm MS}$ scheme. Note that the real parts of the λ_i's are the same in eqs. (2.11) and (2.13) so that the strength of the g_3 contributions are the same in the $q\overline{q}$ and gg channels. However, since different color structures in the gg channel multiply different derivatives of the Born cross section, there is no single sum involving the NLL components as in the $q\overline{q}$ channel. The $E_{gg}(\lambda_1)$ component is quite small and negative near threshold, making the sum $E_{gg} + E_{gg}(\lambda_1)$ indistinguishable from E_{gg}. The $E_{gg}(\lambda_2)$ contribution is the same as in the $q\overline{q}$ channel, resulting in the larger sum $E_{gg} + E_{gg}(\lambda_2)$ shown in the dot-dashed curve.

Note that changing the scale effectively shifts the range of $s_4/2m^2$. We

found also that, for the same value of $s_4/2m^2$, the exponents are larger for smaller values of Q^2, as expected.

We have checked the energy dependence of the g_3 contributions. We find that $E_{gg}(\lambda_1)$ is always negative, decreasing the NLL correction for all s. Its energy dependence is quite strong and thus above threshold it serves to damp the resummation. On the other hand, there is very little energy dependence of $E_{IJ}(\lambda_2)$ close to threshold. As s grows large, the $\ln(m^2 s/t_1^2)$ term in eqs. (2.11) and (2.13) dominates the energy dependence, causing $E_{IJ}(\lambda_2)$ to change sign, eventually leading to a strong damping of the NLL component (and probably leading to a smooth energy variation of the resummed cross section above threshold). This logarithmic term also has the only explicit s_4' dependence of the g_3 contribution at $\theta^* = 90^\circ$ apart from the argument of α_s. Since λ_1 only depends on s_4' through α_s, these contributions have similar behaviours in s.

In fig. 4 we plot the partonic $t\bar{t}$ cross section as a function of η for the $q\overline{q}$ and gg channels. At $\eta \approx 1$ the gg contribution is numerically smaller than the corresponding $q\overline{q}$ result. The importance of the NLL contributions to the partonic cross section depends on the lower limit of the s_4 integral. We have chosen $s_{\rm cut} = 10 s_{4,\rm min}$, corresponding to $\mu_0 = 0.26m$ in the $\overline{\rm MS}$ scheme and $0.13m$ in the DIS scheme when $Q^2 = m^2$, similar to the values used previously. Changing $s_{\rm cut}$ changes the effective η range. However, note that for $\eta \geq 1$, the $q\overline{q}$ results are nearly scheme independent. The $q\overline{q}$ result in the $\overline{\rm MS}$ scheme is always larger than the DIS result. We have checked that the dependence on $s_{\rm cut}$ is rather weak in this channel. The gg channel is similar although the different color structure of the NLL terms leads to a stronger variation over all η in this channel.

To illustrate the contribution of the NLL terms to the partonic cross section, in fig. 4(b) we show the partonic cross section with only the LL contributions for the same value of $s_{\rm cut}$. At $\eta > 1$ the $q\overline{q}$ contributions with and without the NLL components are nearly the same but at $\eta \leq 0.02$ the g_3 contribution makes the partonic cross section up to a factor of two larger. Both with and without the NLL terms, the scheme dependence increases as η decreases.

We note that for fixed angle, $\theta^* = 90^\circ$, the partonic cross section in the gg channel can become negative around $\eta \sim 1 - 2$. Because $d\overline{F}^{(0)}_{gg,II}/ds_4$ is generally opposite in sign to $d\overline{F}^{(0)}_{gg,I}/ds_4$, when it grows larger than $d\overline{F}^{(0)}_{gg,I}/ds_4$

the partonic gg cross section becomes negative. In our calculation of the hadronic cross section, we exclude the negative region if it occurs. For the particular values of $s_{\rm cut}$ and Q^2 shown here, the partonic cross section is always positive. However, for other values of $s_{\rm cut}$ and Q^2 the change in sign can occur, both with and without the g_3 contribution and thus is not just a feature of the NLL contribution at this angle.

The results in fig. 4 imply that the relative importance of the NLL contributions to the hadronic cross section depend on which regions in η are weighted most heavily by the convolution with the parton densities. With equivalent parton luminosities, if $\eta \approx 0.5$, then we expect the gg contribution to be comparable to that of the $q\overline{q}$ channel while if $\eta \approx 2$ then we expect the $q\overline{q}$ channel to dominate. Thus the parton luminosity in the x space probed by the hadronic cross section determines the ultimate importance of the NLL contribution. A comparison with previous results obtained by integrating over the angle θ^* [2, 3] is inconclusive since here we work at fixed θ^*. Finally we note that previously [2, 3, 7, 8] different cutoffs were chosen in the $q\overline{q}$ and gg channels to match the order-by-order contributions, as described at the end of the previous section, whereas here we keep the same $s_{\rm cut}$ for both channels.

The hadronic cross section calculated to NLL is

$$\sigma^{\rm NLL}(S, m^2) = \sum_{ij} \int_{\tau_0}^1 d\tau \int_\tau^1 \frac{dx}{x} f_i^{h_1}(x, \mu^2) f_j^{h_2}(\frac{\tau}{x}, \mu^2) \sigma_{ij}(\tau S, m^2), \qquad (4.1)$$

where $\sigma_{ij}(\tau S, m^2)$ is the partonic cross section, eq. (3.2) and $\tau_0 = (m + \sqrt{m^2 + s_{\rm cut}})^2/S$. We evaluate the parton densities at $\mu^2 = Q^2$. Since the parton densities are only available at fixed order, the application to a resummed cross section introduces some uncertainty. We have used the MRS D$-'$ DIS densities for the $q\overline{q}$ DIS channel and the MRS D$-'$ $\overline{\rm MS}$ densities [18, 19] for both channels in this scheme[4]. Note that $\Lambda_5 = 0.1559$ GeV for both sets. The result is not strongly dependent on the parton distributions in the x range probed at this energy, $x \sim 0.2$ at $y = 0$ for $m = 175$ GeV/c^2, entering primarily through the value of Λ_5 since different sets of parton densities have a different Λ_5, changing $s_{4,\rm min}$ as well as the value of α_s. Taking α_s to only one loop, as in [2], increases α_s and thus the hadronic cross section.

[4]The x dependence of the parton densities in the two schemes is very similar.

For a more complete discussion of the parton density and α_s dependence, see [8].

In fig. 5(a) we plot the $t\bar{t}$ production cross section at $\theta^* = 90°$ as a function of top quark mass at the Fermilab Tevatron with $\sqrt{S} = 1.8$ TeV in the DIS and $\overline{\rm MS}$ schemes for the $q\overline{q}$ channel and the $\overline{\rm MS}$ scheme for the gg channel with $s_{\rm cut} = 10 s_{4,\rm min}$ and $Q^2 = m^2$. The total $\overline{\rm MS}$ cross section at $\theta^* = 90°$ is 5.3 pb at $m = 175$ GeV/c^2, similar to the angle-integrated results of Ref. [5]. Previous NLO results showed that for $m = 100$ GeV/c^2, even though $\eta_{\rm max} = 80$ the total hadronic cross section was obtained already for $\eta \approx 3$ (*c.f.* fig.1 in [3]). When $m = 175$ GeV/c^2, $\eta_{\rm max} = 26$. Comparing figs. 4a and 5a, we can infer that in this case the important η region is $\eta \approx 0.3$. At this η the $q\overline{q}$ luminosity is six times larger than the gg luminosity.

To quantify the enhancement in the $t\bar{t}$ cross section at fixed angle due to the NLL terms, in fig. 5(b) we show our results with the same values of $s_{\rm cut}$ and Q^2 but without the NLL contributions, *i.e.* no g_3 component. The NLL contribution increases with top mass at this energy. The DIS cross section is enhanced between 25 and 30% as the top mass increases from 140 to 200 GeV/c^2. The enhancement is somewhat larger for the $\overline{\rm MS}$ scheme, between 36 and 43% in the $q\overline{q}$ channel and 47 to 55% in the gg channel. The change in the amount of enhancement is, in part, due to the energy dependence of the g_3 contribution. For our choice of $s_{\rm cut}$, the $t\bar{t}$ cross section at $m = 175$ GeV/c^2 without the NLL g_3 contribution is 3.7 pb. Thus the NLL terms enhance the total $\overline{\rm MS}$ cross section at $\theta^* = 90°$ by 43% at this mass and value of $s_{\rm cut}$ in our calculation. The relatively large contribution from the NLL terms verifies that the lower η region is most important.

We have further investigated the $s_{\rm cut}$ and scale dependence of our calculation. In Tables 1-3 we give the numerical values of the hadronic top production cross sections at LL and NLL with $s_{\rm cut} = 10 s_{4,\rm min}$, $5 s_{4,\rm min}$ and $20 s_{4,\rm min}$ respectively. For each value of $s_{\rm cut}$ we show the results for $Q^2 = m^2$, $4m^2$ and $m^2/4$. In the tables we have also indicated the equivalent value of the cutoff μ_0 defined in [2] in both the DIS and $\overline{\rm MS}$ schemes. Note that in Tables 1 and 3, the scale dependence of the gg channel is weak compared to the $q\overline{q}$ contributions. When $s_{\rm cut} = 10 s_{4,\rm min}$, the gg cross section changes 41% between $Q^2 = m^2/4$ and $4m^2$ at $m = 175$ GeV/c^2. In contrast, the $q\overline{q}$ $\overline{\rm MS}$ cross section changes 47% and the DIS cross section by 65%. At the same mass but with $s_{\rm cut} = 20 s_{4,\rm min}$, the gg cross section changes 10% between the highest and lowest scales while the $q\overline{q}$ $\overline{\rm MS}$ cross section changes 35% and the

DIS cross section 52%. On the other hand, for the lowest $s_{\rm cut}$ the gg cross section has the largest scale dependence, more than a factor of two change while the $\overline{\rm MS}$ cross section changes 53% in the $q\overline{q}$ channel.

Note that for a fixed value of $s_{\rm cut}$, the gg cross section is actually largest when $Q^2 = 4m^2$ than at lower Q^2. This result is counterintuitive since one generally expects that increasing the scale decreases the cross section. This is true in the $q\overline{q}$ channel and also in the gg channel when $\eta > 1$. At fixed s_4, the exponents are larger for lower values of the scale due to the running of α_s. However, the difference in the factors $f_{gg,1}$ and $f_{gg,2}$ at low η shifts the relative weights of $d\overline{F}^{(0)}_{gg,I}/ds_4$ and $d\overline{F}^{(0)}_{gg,II}/ds_4$ so that the partonic cross section is largest for the largest scale studied, $Q^2 = 4m^2$, when $\eta < 0.5$. In addition, changing the scale for fixed $s_{\rm cut}/s_{4,\rm min}$ effectively changes $s_{\rm cut}$ in eq. (3.2) since $s_{4,\rm min} = 2m^2\Lambda/Q$. Therefore increasing Q^2 correspondingly increases the effective s_4 integration region. This latter effect seems to be the most important because the largest scale also produces the largest gg contribution to the cross section when only the LL terms are considered.

When $s_{\rm cut} = 5s_{4,\rm min}$, the gluon contribution is enhanced and actually dominates for $Q^2 = m^2$ and $4m^2$, seen in Table 2. This is not surprising since at $s_{\rm cut} = 5s_{4,\rm min}$, $\mu_0 \approx 0.1m$ in the DIS scheme and $0.2m$ in the $\overline{\rm MS}$ scheme when $Q^2 = m^2$. At such low values of μ_0 in the gg channel, the gg cross section can become large, see *e.g.* figs. 12-14 in [2]. Additionally, as $s_{\rm cut}$ is reduced, all the contributions to the hadronic cross section increase because as $s_{\rm cut}$ approaches $s_{4,\rm min}$ the exponents grow more rapidly. The fastest growth of the exponents in this region occurs when $Q^2 = m^2/4$.

A comparison of the NLL and LL results in the tables helps to clarify where the NLL enhancements are largest. As discussed above, the enhancement for a fixed $s_{\rm cut}$ is largest for $Q^2 = 4m^2$ since the larger scale increases the effective s_4 probed at larger Q^2. The enhancement is also larger for lower $s_{\rm cut}$ due to the running of the coupling constant. Finally we note that the NLL enhancement increases with mass since $t\bar{t}$ production is closer to threshold at fixed energy with larger masses.

We conclude our discussion of top production by noting that the scheme dependence is rather strong, especially for lower $s_{\rm cut}$. The scheme dependence arises because $f^{\overline{\rm MS}}_{q\overline{q}} > f^{\rm DIS}_{q\overline{q}}$, particularly near $s_{4,\rm min}$, see figs. 1 and 2 and also *e.g.* figs. 12 and 13 in Ref. [2]. For $s_{\rm cut} = 10s_{4,\rm min}$ and $Q^2 = m^2$, the $\overline{\rm MS}$ cross section is 62-71% larger than the DIS cross section at NLL and

49-55% larger at LL. The scheme dependence is largest for small $s_{\rm cut}$ and large Q^2 because smaller values of $s_{\rm cut}$ are increasingly sensitive to the upper limit of the ξ integral in $E_{ab}(g_1)$, see the appendix. The scheme dependence increases slightly with mass at fixed energy. Thus close to threshold the scheme dependence is unavoidable. Even though the partonic cross section is only weakly scheme dependent for $\eta > 1$, as seen in fig. 4, some scheme dependence will remain until new sets of parton densities which have incorporated resummation effects before being fitted to data are available. In the absence of such densities we favor the $\overline{\rm MS}$ results as they have a more reliable theoretical basis and the $q\overline{q}$ and gg channels can be treated consistently only in this scheme.

Since α_s depends strongly on the heavy quark mass scale, we repeat our calculations for bottom quark production with $m = 4.75$ GeV/c^2 at $\sqrt{s} = 9.51$ GeV. The smaller quark mass and the larger Λ_4, 0.23 GeV for the MRS D$-'$ distributions, result in a reduced range in $s_4/2m^2$ before the exponents diverge, as shown in figs. 6-8. The exponents tend to be somewhat smaller for the bottom quark except near $s_{4,\rm min}$ due to the faster running of the coupling constant. The running of α_s also produces faster growth of the exponents at lower Q^2. Thus the scale dependence is also stronger for $b\overline{b}$ production. We note that the energy dependence of g_3 is stronger for the lighter quark mass. While $E_{gg}(\lambda_1)$ is smaller near threshold, it increases faster with energy than at the top quark mass. In general, the bottom quark cross section is more sensitive to $s_{\rm cut}$ and Q^2 than the much more massive top quarks, see also [7, 8].

The partonic cross sections with $s_{\rm cut} = 1.4 s_{4,\rm min}$ and $Q^2 = m^2$ are shown with and without the NLL g_3 terms in fig. 9. This value of $s_{\rm cut}$ corresponds to $\mu_0 \approx 0.37m$ and $0.51m$ in the DIS and $\overline{\rm MS}$ schemes, somewhat larger than those in [7, 8]. Varying $s_{\rm cut}$ we find that for $\eta < 1$, the partonic cross section can change by an order of magnitude near $\eta \sim 0.1$ and a factor of two to three at $\eta \sim 0.5$. The largest variation is in the gg channel. Of course the parton luminosity in the x range probed at a given energy determines the sensitivity of the hadronic cross section. At higher energies, away from threshold, the sensitivity to $s_{\rm cut}$ decreases. Similar trends can be seen in charm production although for $c\overline{c}$ production, $s_{4,\rm min}$ already corresponds to the μ_0 values used in [8], suggesting that a stable cross section is even more difficult to obtain for charm production.

In fig. 10 we show the $b\overline{b}$ production cross section at $\theta^* = 90°$ as a function

of beam momentum for pp interactions with $s_{\rm cut} \approx 1.4 s_{4,\rm min}$ and $Q^2 = m^2$, with and without the NLL contributions. As expected, gg fusion is dominant [7, 8]. These energies, $20 \leq \sqrt{S} \leq 45$ GeV, correspond to $3.4 \leq \eta_{\rm max} \leq 21.4$. At the HERA-B energy, $\sqrt{S} = 39.2$ GeV, we find a total $\overline{\rm MS}$ cross section at $\theta^* = 90^\circ$ of 8.6 nb at LL and 19.1 nb at NLL, more than a factor of two enhancement. Note that the $b\overline{b}$ cross section is somewhat smaller than found previously [7, 8], in part because we are only considering a fixed angle.

The sensitivity to $s_{\rm cut}$ and the scale is detailed in Tables 4-6 for $s_{\rm cut} = 1.2$, 1.4, and $1.6 s_{4,\rm min}$ and $Q^2 = m^2$, $4m^2$ and $m^2/4$. The same trends can be observed as we remarked upon for $t\overline{t}$ production although the enhancements are generally larger for $b\overline{b}$ production. We note that for the values of $s_{\rm cut}$ chosen here, the NLL enhancement and scheme dependence decrease with increasing energy since the higher energy probes the region where the corrections are less important. There seems to be a general decrease of the scheme dependence with energy for the bottom quarks. However, it is difficult to fairly compare the scheme dependence of top and bottom production since our values of $s_{\rm cut}$ do not correspond to the same μ_0/m ratio in both cases. Our previous work on LL resummation favored a fixed ratio to describe production of all heavy quarks [7, 8]. Thus, while our bottom quark results are illustrative only, they clearly indicate that a full NLL calculation including all angles is needed to clarify bottom quark resummation.

5 Conclusions

We have investigated the numerical importance of the NLL terms in the resummation of subleading soft gluon contributions near threshold for heavy quark production. We have shown that, for the resummation method of [2] and at $\theta^* = 90^\circ$ these contributions are either numerically small or there are cancellations between them. Therefore the inclusion of the NLL terms leads to only a moderate change in the top quark production cross section at $\theta^* = 90^\circ$. Bottom production is much more sensitive to $s_{\rm cut}$, making definitive statements about the resummed cross sections more difficult, particularly in a model without a calculable perturbative cutoff.

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A Appendix

Here we present the analytical results for the g_1, g_2 and g_3 (where appropriate) integrals. As previously discussed, we use the two-loop running coupling constant,

$$\begin{aligned}\alpha_s(\xi) &= \frac{1}{a\ln\xi} + \frac{b}{a}\frac{\ln(\ln\xi)}{\ln^2(\xi)}\,, \\ a &= \frac{11C_A - 4T_f n_{\rm f}}{12\pi}\,, \\ b &= -6\frac{17C_A^2 - (6C_F + 10C_A)T_f n_{\rm f}}{(11C_A - 4T_f n_{\rm f})^2}\,,\end{aligned} \tag{A.1}$$

with $C_A = 3$, $C_F = 4/3$ and $T_f = 1/2$. It is obvious that when $\xi < 1$, $\ln\xi$ is negative so that $\ln(\ln\xi)$ has a cut and needs a precise definition. We use the cutoff on the ω' variable to stop the integration at that point.

In general,

$$\begin{aligned}E_{ab}^{\rm sch} = \int_{\omega_0}^1 \frac{d\omega'}{\omega'}\Big\{\int_{\xi_L}^{\xi_U^{\rm sch}} \frac{d\xi}{\xi}\Big[\gamma_{ab}\Big(\alpha_s(\xi) + \frac{K}{2\pi}\alpha_s^2(\xi)\Big)\Big] - \delta_{ab}^{\rm sch}\alpha_s\Big(\frac{\omega' Q^2}{\Lambda^2}\Big) \\ -\Big\{\lambda_{i,ab}\Big[\alpha_s\Big(\frac{\omega'^2 Q^2}{\Lambda^2}\Big), \theta^* = 90^\circ\Big] + \lambda_{i,ab}^*\Big[\alpha_s\Big(\frac{\omega'^2 Q^2}{\Lambda^2}\Big), \theta^* = 90^\circ\Big]\Big\}\Big\}\end{aligned} \tag{A.2}$$

where $\xi_L = \omega'^2 Q^2/\Lambda^2$, $\xi_U^{\rm DIS} = \omega' Q^2/\Lambda^2$, $\xi_U^{\overline{\rm MS}} = Q^2/\Lambda^2$, $\gamma_{q\overline{q}} = 2C_F/\pi$, $\gamma_{gg} = 2C_A/\pi$ and $\delta_{ab}^{\rm sch} = 3C_F/2\pi$ for $q\overline{q}$ in the DIS scheme and 0 otherwise. We will give the results for single integrals over ω' first. For convenience, we define

$$L_0 = \ln\Big(\frac{Q^2}{\Lambda^2}\Big)\;; \quad L_1 = \ln\Big(\frac{\omega_0 Q^2}{\Lambda^2}\Big)\;; \quad L_2 = \ln\Big(\frac{\omega_0^2 Q^2}{\Lambda^2}\Big)\;. \tag{A.3}$$

Note that the contributions dependent on L_2 set the lower limit on $s_{\rm cut}$ since the value of ω_0 where $\omega_0 \sim \Lambda/Q$ is the lowest ω_0 at which the integrals diverge.

Then, in the DIS scheme, the g_2 result is

$$E_{q\overline{q}}^{\rm DIS}(g_2) = -\frac{3}{2}\frac{C_F}{\pi a}\Big[\ln L_0 - \frac{b}{L_0}(\ln L_0 + 1) - \ln L_1 + \frac{b}{L_1}(\ln L_1 + 1)\Big] \ . \quad \text{(A.4)}$$

For the terms in $\lambda_{i,ab}$ which only contain an s_4' dependence in the running coupling constant, the g_3 integral can also be done analytically. In these cases,

$$E_{ab}(g_3) \propto \frac{1}{2a}\Big[\ln L_0 - \frac{b}{L_0}(\ln L_0 + 1) - \ln L_2 + \frac{b}{L_2}(\ln L_2 + 1)\Big] \ . \quad \text{(A.5)}$$

Thus the g_3 contributions diverge at a larger value of ω_0 than do the g_2.

After the ξ integral, the g_1 exponent is

$$\begin{aligned} E_{ab}^{\rm sch}(g_1) \ &= \ \frac{\gamma_{ab}}{a}\int_{\omega_0}^1 \frac{d\omega'}{\omega'}\Big[\Big\{\ln(\ln\xi) - \frac{b}{\ln\xi}(\ln(\ln\xi)+1)\Big\} - \frac{K}{2\pi a\ln\xi}\Big\{1 \quad \text{(A.6)} \\ &\quad + \frac{b}{2\ln\xi}(2\ln(\ln\xi)+1) + \frac{b^2}{27\ln^2\xi}(9\ln^2(\ln\xi) + 6\ln(\ln\xi)+2)\Big\}\Big]\Big|_{\xi_L}^{\xi_U^{\rm sch}} \ . \end{aligned}$$

In the DIS scheme the final result is

$$\begin{aligned} E_{ab}^{\rm DIS}(g_1) = \frac{\gamma_{ab}}{a}\Big[&\frac{1}{2}L_0(\ln L_0 - 1) - \frac{b}{4}\ln L_0(\ln L_0 + 2) \\ &- L_1(\ln L_1 - 1) + \frac{b}{2}\ln L_1(\ln L_1 + 2) \\ &+ \frac{1}{2}L_2(\ln L_2 - 1) - \frac{b}{4}\ln L_2(\ln L_2 + 2) \qquad \text{(A.7)} \\ &- \frac{K}{2\pi a}\Big\{\frac{1}{2}\ln L_0 - \frac{b}{4L_0}(2\ln L_0 + 3) - \frac{b^2}{216L_0^2}(18\ln^2 L_0 + 30\ln L_0 + 19) \\ &- \ln L_1 + \frac{b}{2L_1}(2\ln L_1 + 3) + \frac{b^2}{108L_1^2}(18\ln^2 L_1 + 30\ln L_1 + 19) \\ &+ \frac{1}{2}\ln L_2 - \frac{b}{4L_2}(2\ln L_2 + 3) - \frac{b^2}{216L_2^2}(18\ln^2 L_2 + 30\ln L_2 + 19)\Big\}\Big] \ . \end{aligned}$$

The result is somewhat simpler in the $\overline{\rm MS}$ scheme since $\xi_U^{\overline{\rm MS}}$ does not depend on ω'. In this case,

$$E_{ab}^{\overline{\rm MS}}(g_1) = \frac{\gamma_{ab}}{a}\Big[-\ln\omega_0\Big\{\ln L_0 - \frac{b}{L_0}(\ln L_0 + 1)\Big\}$$

$$
\begin{aligned}
&-\frac{1}{2}L_0(\ln L_0 - 1) + \frac{b}{4}\ln L_0(\ln L_0 + 2) \\
&+\frac{1}{2}L_2(\ln L_2 - 1) - \frac{b}{4}\ln L_2(\ln L_2 + 2) \qquad \text{(A.8)} \\
&-\frac{K}{2\pi a}\Big\{ -\frac{\ln\omega_0}{L_0}\Big[1 + \frac{b}{2L_0}(2\ln L_0 + 1) + \frac{b^2}{27L_0^2}(9\ln^2 L_0 + 6\ln L_0 + 2)\Big] \\
&-\frac{1}{2}\ln L_0 + \frac{b}{4L_0}(2\ln L_0 + 3) + \frac{b^2}{216L_0^2}(18\ln^2 L_0 + 30\ln L_0 + 19) \\
&+\frac{1}{2}\ln L_2 - \frac{b}{4L_2}(2\ln L_2 + 3) - \frac{b^2}{216L_2^2}(18\ln^2 L_2 + 30\ln L_2 + 19)\Big\}\Big] \ .
\end{aligned}
$$

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Figure Captions

Figure 1. (a) We show the contributions to $f_{q\overline{q}}^{\rm DIS}$ in top quark production for $m = 175$ GeV/c^2 and $\sqrt{s} = 351$ GeV as a function of $s_4/2m^2$. The solid curve shows $\ln(f_{q\overline{q}}^{\rm DIS})$ for the octet component at 90°, the dot-dashed curve, $E_{q\overline{q}}(\lambda_2)$, the dashed curve, $E_{q\overline{q}}^{\rm DIS}(g_1)$, and the dot-dot-dashed curve, $|E_{q\overline{q}}^{\rm DIS}(g_2)|$. The dotted curve shows $E^{\rm DIS}$ as defined in eq. (3.30) of Ref. [2]. (b) The ratios of the exponential factors are given to show the enhancement relative to $f_{q\overline{q}}(g_1)$ where $f_{q\overline{q}}(g_1) \equiv \exp(E_{q\overline{q}}^{\rm DIS}(g_1))$. The solid curve shows $f_{q\overline{q}}^{\rm DIS}/f_{q\overline{q}}(g_1)$, the dashed curve, $f_{q\overline{q}}(g_1+g_2)/f_{q\overline{q}}(g_1)$, and the dotted curve, $f_{q\overline{q}}(g_1+g_3(\lambda_2))/f_{q\overline{q}}(g_1)$. The ratio $f_{q\overline{q}}(g_1)/\exp(E^{\rm DIS})$, given in the dot-dashed curve, shows the enhancement of g_1 from eq. (2.5) relative to the one-loop value in [2].

Figure 2. (a) We show the contributions to $f_{q\overline{q}}^{\overline{\rm MS}}$ in top quark production for $m = 175$ GeV/c^2 and $\sqrt{s} = 351$ GeV as a function of $s_4/2m^2$. The solid curve shows $\ln(f_{q\overline{q}}^{\overline{\rm MS}})$ for the octet component at 90°, the dot-dashed curve, $E_{q\overline{q}}(\lambda_2)$, and the dashed curve, $E_{q\overline{q}}^{\overline{\rm MS}}(g_1)$. The dotted curve shows $E^{\overline{\rm MS}}$ as defined in eq. (3.34) of Ref. [2]. (b) The ratios of the exponential factors are given to show the enhancements. The solid curve shows $f_{q\overline{q}}^{\overline{\rm MS}}/f_{q\overline{q}}(g_1)$ while the dashed curve is the ratio $f_{q\overline{q}}(g_1)/\exp(E^{\overline{\rm MS}})$ where $f_{q\overline{q}}(g_1) \equiv \exp(E_{q\overline{q}}^{\overline{\rm MS}}(g_1))$.

Figure 3. (a) We show the contributions to f_{gg} in top quark production for $m = 175$ GeV/c^2 and $\sqrt{s} = 351$ GeV as a function of $s_4/2m^2$. The solid curve shows E_{gg}; the dashed and dotted curves give the contributions from the eigenvalues, $E_{gg}(\lambda_2)$ and $|E_{gg}(\lambda_1)|$ respectively. The dot-dashed curve shows the sum $E_{gg} + E_{gg}(\lambda_2)$; the sum $E_{gg} + E_{gg}(\lambda_1)$ is indistinguishable from the solid curve. The dot-dot-dashed curve shows $E^{\overline{\rm MS}}$ for the gluon from Ref. [2]. (b) The ratios of the exponential factors are given to show the enhancements relative to $f_{gg}(g_1) \equiv \exp(E_{gg})$. The solid curve shows $f_{gg}(g_1+g_3(\lambda_1))/f_{gg}(g_1)$ and the dashed curve, $f_{gg}(g_1+g_3(\lambda_2))/f_{gg}(g_1)$. The ratio of $f_{gg}(g_1)$ to $\exp(E^{\overline{\rm MS}})$, given in the dot-dashed curve, compares the g_1 from eq. (2.5) relative to the one-loop value in [2].

Figure 4. The partonic top quark production cross section is shown as a function of $\eta = (s - 4m^2)/(4m^2)$ for the $q\overline{q}$ channel in the DIS (solid) and $\overline{\rm MS}$ (dashed) schemes and the gg (dot-dashed) channel. We use $s_{\rm cut} \approx 10 s_{4,\rm min}$ with (a) and without (b) the NLL g_3 contributions respectively.

Figure 5. The hadronic $t\overline{t}$ production cross section at $\theta^* = 90^\circ$ is given as a function of top quark mass for $p\overline{p}$ collisions at the Tevatron energy, $\sqrt{S} = 1.8$ TeV. We use the MRS D$-'$ parton densities in the DIS scheme for $q\overline{q}$ annihilation (dashed) and in the $\overline{\rm MS}$ scheme for the $q\overline{q}$ (dot-dashed) and gg (dotted) channels. The sum of the $q\overline{q}$ and gg channels in the $\overline{\rm MS}$ scheme is given in the solid curve. The results are given for $s_{\rm cut} \approx 10 s_{4,\rm min}$ with the NLL g_3 contributions in (a) and with these excluded in (b).

Figure 6. We show the contributions to $f_{q\overline{q}}^{\rm DIS}$ and the enhancement factors in bottom quark production for $m = 4.75$ GeV$/c^2$ and $\sqrt{s} = 9.51$ GeV. The labels are as in fig. 1.

Figure 7. We show the contributions to $f_{q\overline{q}}^{\overline{\rm MS}}$ and the enhancement factors in bottom quark production for $m = 4.75$ GeV$/c^2$ and $\sqrt{s} = 9.51$ GeV. The labels are as in fig. 2.

Figure 8. We show the contributions to f_{gg} and the enhancement factors in bottom quark production for $m = 4.75$ GeV$/c^2$ and $\sqrt{s} = 9.51$ GeV. The labels are as in fig. 3.

Figure 9. The partonic bottom quark production cross section is shown as a function of η. The labels are as in fig. 4. We use $s_{\rm cut} \approx 1.4 s_{4,\rm min}$ with the NLL g_3 contributions in (a) and without them in (b).

Figure 10. The hadronic $b\overline{b}$ production cross section at $\theta^* = 90^\circ$ is given as a function of beam momentum for pp interactions. We use the MRS D$-'$

parton densities in the DIS scheme for $q\overline{q}$ annihilation (dashed) and in the $\overline{\rm MS}$ scheme for the $q\overline{q}$ (dot-dashed) and gg (dotted) channels. The $\overline{\rm MS}$ sum is given in the solid curve. The results are given for $s_{\rm cut} \approx 1.4 s_{4,\rm min}$ with the NLL g_3 corrections in (a) and without them in (b).

	NLL			LL		
m (GeV/c^2)	$\sigma_{q\overline{q}}^{\rm DIS}$ (pb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (pb)	σ_{gg} (pb)	$\sigma_{q\overline{q}}^{\rm DIS}$ (pb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (pb)	σ_{gg} (pb)
	$Q^2 = m^2 \Rightarrow \mu_0^{\rm DIS} = 0.13m, \mu_0^{\overline{\rm MS}} = 0.26m$					
150	4.91	8.04	3.99	3.91	5.85	2.68
155	4.21	6.94	3.21	3.32	5.01	2.15
160	3.63	5.99	2.61	2.87	4.33	1.75
165	3.13	5.20	2.12	2.47	3.74	1.40
170	2.71	4.51	1.73	2.12	3.23	1.14
175	2.34	3.92	1.41	1.84	2.80	0.93
180	2.03	3.41	1.15	1.59	2.43	0.75
185	1.77	2.98	0.94	1.37	2.11	0.62
190	1.53	2.59	0.77	1.19	1.84	0.50
195	1.33	2.27	0.63	1.03	1.60	0.41
200	1.16	1.99	0.52	0.90	1.39	0.34
	$Q^2 = 4m^2 \Rightarrow \mu_0^{\rm DIS} = 0.19m, \mu_0^{\overline{\rm MS}} = 0.41m$					
150	3.83	6.60	4.94	3.06	4.74	3.11
155	3.30	5.71	3.95	2.61	4.07	2.49
160	2.83	4.92	3.22	2.24	3.50	2.01
165	2.44	4.27	2.61	1.93	3.03	1.62
170	2.10	3.69	2.13	1.65	2.61	1.32
175	1.82	3.21	1.74	1.43	2.26	1.08
180	1.56	2.78	1.42	1.23	1.95	0.88
185	1.36	2.42	1.16	1.06	1.69	0.71
190	1.18	2.11	0.96	0.92	1.47	0.59
195	1.03	1.84	0.79	0.80	1.28	0.48
200	0.89	1.61	0.65	0.69	1.11	0.39
	$Q^2 = m^2/4 \Rightarrow \mu_0^{\rm DIS} = 0.094m, \mu_0^{\overline{\rm MS}} = 0.16m$					
150	6.20	9.53	3.51	4.97	7.13	2.51
155	5.36	8.28	2.84	4.26	6.12	2.03
160	4.62	7.17	2.29	3.67	5.29	1.62
165	4.00	6.24	1.85	3.16	4.57	1.31
170	3.46	5.41	1.51	2.73	3.97	1.06
175	3.00	4.72	1.23	2.36	3.44	0.86
180	2.61	4.11	1.00	2.04	2.98	0.70
185	2.27	3.59	0.82	1.77	2.60	0.57
190	1.99	3.16	0.68	1.54	2.26	0.47
195	1.73	2.75	0.55	1.34	1.97	0.38
200	1.51	2.42	0.46	1.16	1.72	0.31

Table 1: The hadronic $t\overline{t}$ production cross sections in $p\overline{p}$ collisions at $\sqrt{S} =$ 1.8 TeV for $s_{\rm cut} = 10 s_{4,\rm min}$ and $\theta^* = 90°$. The effective μ_0 from [2] is given for each Q^2.

	NLL			LL		
m (GeV/c^2)	$\sigma_{q\overline{q}}^{\rm DIS}$ (pb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (pb)	σ_{gg} (pb)	$\sigma_{q\overline{q}}^{\rm DIS}$ (pb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (pb)	σ_{gg} (pb)
	$Q^2 = m^2 \Rightarrow \mu_0^{\rm DIS} = 0.094m, \mu_0^{\overline{\rm MS}} = 0.21m$					
150	5.43	10.81	17.31	4.13	7.07	9.67
155	4.65	9.29	14.13	3.55	6.09	7.89
160	4.01	8.08	11.66	3.05	5.25	6.44
165	3.47	7.02	9.54	2.63	4.55	5.27
170	2.99	6.10	7.93	2.25	3.93	4.30
175	2.59	5.32	6.53	1.95	3.41	3.54
180	2.25	4.63	5.38	1.69	2.96	2.90
185	1.95	4.04	4.47	1.46	2.58	2.41
190	1.70	3.56	3.68	1.26	2.24	1.99
195	1.48	3.09	3.06	1.10	1.96	1.65
200	1.28	2.71	2.55	0.96	1.71	1.37
	$Q^2 = 4m^2 \Rightarrow \mu_0^{\rm DIS} = 0.13m, \mu_0^{\overline{\rm MS}} = 0.33m$					
150	4.15	8.80	26.50	3.20	5.68	14.01
155	3.55	7.58	21.69	2.73	4.86	11.32
160	3.06	6.55	17.85	2.34	4.19	9.22
165	2.63	5.68	14.66	2.01	3.63	7.57
170	2.28	4.95	12.03	1.73	3.12	6.26
175	1.96	4.29	10.03	1.49	2.71	5.13
180	1.70	3.74	8.24	1.28	2.34	4.23
185	1.47	3.26	6.86	1.11	2.04	3.51
190	1.28	2.85	5.65	0.96	1.77	2.88
195	1.11	2.48	4.74	0.83	1.54	2.38
200	0.96	2.17	3.89	0.72	1.34	1.98
	$Q^2 = m^2/4 \Rightarrow \mu_0^{\rm DIS} = 0.067m, \mu_0^{\overline{\rm MS}} = 0.13m$					
150	7.10	13.16	12.44	5.41	8.84	7.42
155	6.13	11.44	10.21	4.65	7.63	6.04
160	5.30	9.91	8.35	3.99	6.58	4.91
165	4.58	8.65	6.86	3.44	5.69	4.03
170	3.95	7.49	5.64	2.97	4.94	3.28
175	3.44	6.57	4.65	2.57	4.29	2.71
180	2.98	5.72	3.84	2.23	3.75	2.25
185	2.60	5.02	3.20	1.93	3.25	1.84
190	2.27	4.40	2.65	1.68	2.84	1.52
195	1.98	3.85	2.19	1.46	2.48	1.25
200	1.73	3.38	1.82	1.28	2.17	1.04

Table 2: The hadronic $t\overline{t}$ production cross sections in $p\overline{p}$ collisions at $\sqrt{S} =$ 1.8 TeV for $s_{\rm cut} = 5s_{4,\rm min}$ and $\theta^* = 90°$. The effective μ_0 from [2] is given for each Q^2.

	NLL			LL		
m (GeV/c^2)	$\sigma_{q\overline{q}}^{\rm DIS}$ (pb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (pb)	σ_{gg} (pb)	$\sigma_{q\overline{q}}^{\rm DIS}$ (pb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (pb)	σ_{gg} (pb)
	$Q^2 = m^2 \Rightarrow \mu_0^{\rm DIS} = 0.19m, \mu_0^{\overline{\rm MS}} = 0.33m$					
150	4.40	6.30	1.61	3.61	4.90	1.24
155	3.78	5.43	1.29	3.10	4.23	0.98
160	3.25	4.69	1.03	2.66	3.64	0.78
165	2.80	4.06	0.83	2.28	3.13	0.63
170	2.42	3.53	0.67	1.97	2.71	0.50
175	2.10	3.06	0.54	1.70	2.34	0.40
180	1.82	2.67	0.44	1.47	2.03	0.33
185	1.58	2.32	0.35	1.27	1.76	0.26
190	1.37	2.02	0.29	1.10	1.53	0.21
195	1.19	1.76	0.23	0.96	1.33	0.17
200	1.04	1.55	0.19	0.83	1.16	0.14
	$Q^2 = 4m^2 \Rightarrow \mu_0^{\rm DIS} = 0.27m, \mu_0^{\overline{\rm MS}} = 0.52m$					
150	3.55	5.36	1.70	2.91	4.11	1.24
155	3.05	4.61	1.36	2.48	3.52	0.99
160	2.62	3.98	1.09	2.13	3.03	0.79
165	2.25	3.44	0.87	1.83	2.61	0.63
170	1.94	2.97	0.71	1.57	2.25	0.51
175	1.68	2.58	0.57	1.35	1.94	0.41
180	1.45	2.24	0.46	1.17	1.68	0.33
185	1.26	1.95	0.38	1.01	1.46	0.27
190	1.09	1.69	0.31	0.87	1.26	0.22
195	0.95	1.48	0.25	0.75	1.09	0.18
200	0.82	1.28	0.20	0.69	0.96	0.14
	$Q^2 = m^2/4 \Rightarrow \mu_0^{\rm DIS} = 0.13m, \mu_0^{\overline{\rm MS}} = 0.21m$					
150	5.27	7.10	1.57	4.40	5.68	1.25
155	4.54	6.14	1.25	3.77	4.89	1.00
160	3.92	5.31	0.99	3.25	4.23	0.80
165	3.39	4.62	0.80	2.80	3.65	0.64
170	2.94	4.02	0.64	2.41	3.16	0.51
175	2.55	3.49	0.52	2.08	2.74	0.41
180	2.22	3.05	0.42	1.81	2.38	0.33
185	1.93	2.67	0.34	1.57	2.07	0.27
190	1.68	2.33	0.28	1.37	1.80	0.22
195	1.47	2.04	0.23	1.18	1.57	0.18
200	1.28	1.78	0.18	1.03	1.37	0.14

Table 3: The hadronic $t\overline{t}$ production cross sections in $p\overline{p}$ collisions at $\sqrt{S} =$ 1.8 TeV for $s_{\rm cut} = 20 s_{4,\rm min}$ and $\theta^* = 90°$. The effective μ_0 from [2] is given for each Q^2.

	NLL			LL		
$p_{\rm lab}$ (GeV/c)	$\sigma_{q\overline{q}}^{\rm DIS}$ (nb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (nb)	σ_{gg} (nb)	$\sigma_{q\overline{q}}^{\rm DIS}$ (nb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (nb)	σ_{gg} (nb)
	$Q^2 = m^2 \Rightarrow \mu_0^{\rm DIS} = 0.37m, \mu_0^{\overline{\rm MS}} = 0.51m$					
200	0.0049	0.0081	0.034	0.0021	0.0031	0.014
300	0.049	0.091	0.410	0.023	0.037	0.172
400	0.169	0.326	1.54	0.083	0.140	0.658
500	0.373	0.734	3.58	0.190	0.325	1.56
600	0.653	1.30	6.52	0.344	0.591	2.90
700	1.00	2.01	10.44	0.537	0.927	4.64
800	1.40	2.84	15.00	0.764	1.32	6.76
900	1.84	3.72	20.23	1.02	1.77	9.20
1000	2.31	4.70	26.18	1.30	2.25	11.86
	$Q^2 = 4m^2 \Rightarrow \mu_0^{\rm DIS} = 0.52m, \mu_0^{\overline{\rm MS}} = 0.82m$					
200	0.0036	0.0079	0.041	0.0016	0.0029	0.015
300	0.036	0.082	0.482	0.017	0.032	0.183
400	0.124	0.286	1.81	0.060	0.118	0.702
500	0.273	0.636	4.32	0.138	0.269	1.69
600	0.479	1.12	8.01	0.248	0.484	3.20
700	0.732	1.72	12.87	0.387	0.756	5.15
800	1.02	2.41	18.86	0.551	1.07	7.60
900	1.35	3.17	25.91	0.735	1.43	10.42
1000	1.70	4.01	33.68	0.941	1.83	13.69
	$Q^2 = m^2/4 \Rightarrow \mu_0^{\rm DIS} = 0.26m, \mu_0^{\overline{\rm MS}} = 0.32m$					
200	0.0041	0.0054	0.026	0.0019	0.0022	0.012
300	0.050	0.076	0.354	0.024	0.033	0.167
400	0.188	0.300	1.37	0.096	0.139	0.665
500	0.433	0.713	3.20	0.232	0.341	1.59
600	0.783	1.30	5.78	0.429	0.640	2.94
700	1.21	2.04	9.11	0.682	1.03	4.69
800	1.72	2.92	12.96	0.985	1.49	6.79
900	2.27	3.90	17.37	1.33	2.02	9.16
1000	2.87	4.95	22.29	1.70	2.60	11.81

Table 4: The hadronic $b\overline{b}$ production cross sections in pp interactions for $s_{\rm cut} = 1.4 s_{4,\rm min}$, $m = 4.75$ GeV/c^2 and $\theta^* = 90°$.

	NLL			LL		
$p_{\rm lab}$ (GeV/c)	$\sigma_{q\overline{q}}^{\rm DIS}$ (nb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (nb)	σ_{gg} (nb)	$\sigma_{q\overline{q}}^{\rm DIS}$ (nb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (nb)	σ_{gg} (nb)
	$Q^2 = m^2 \Rightarrow \mu_0^{\rm DIS} = 0.34m, \mu_0^{\overline{\rm MS}} = 0.49m$					
200	0.0094	0.020	0.248	0.0029	0.0049	0.051
300	0.087	0.206	2.75	0.029	0.055	0.594
400	0.287	0.700	9.82	0.103	0.199	2.19
500	0.617	1.54	22.43	0.232	0.452	5.06
600	1.06	2.67	40.53	0.413	0.811	9.21
700	1.59	4.05	63.18	0.639	1.26	14.67
800	2.20	5.62	89.85	0.902	1.78	20.84
900	2.85	7.32	120.6	1.20	2.37	28.00
1000	3.56	9.18	154.2	1.51	2.99	36.60
	$Q^2 = 4m^2 \Rightarrow \mu_0^{\rm DIS} = 0.48m, \mu_0^{\overline{\rm MS}} = 0.77m$					
200	0.0063	0.017	0.301	0.0019	0.0042	0.057
300	0.059	0.171	3.41	0.020	0.045	0.665
400	0.193	0.575	12.56	0.071	0.159	2.51
500	0.416	1.25	29.35	0.159	0.356	5.96
600	0.716	2.16	54.00	0.284	0.635	11.10
700	1.08	3.30	85.96	0.442	0.985	17.73
800	1.49	4.58	125.7	0.626	1.39	26.18
900	1.95	5.97	170.1	0.829	1.84	35.82
1000	2.45	7.47	220.3	1.05	2.34	46.82
	$Q^2 = m^2/4 \Rightarrow \mu_0^{\rm DIS} = 0.24m, \mu_0^{\overline{\rm MS}} = 0.31m$					
200	0.0099	0.016	0.201	0.0030	0.0041	0.046
300	0.106	0.200	2.35	0.035	0.056	0.571
400	0.369	0.735	8.33	0.132	0.218	2.10
500	0.813	1.66	18.63	0.306	0.516	4.80
600	1.41	2.94	32.98	0.557	0.948	8.63
700	2.14	4.51	50.25	0.868	1.49	13.36
800	2.97	6.33	70.60	1.24	2.13	19.03
900	3.85	8.26	92.73	1.65	2.85	25.16
1000	4.78	10.32	116.5	2.10	3.63	32.12

Table 5: The hadronic $b\overline{b}$ production cross sections in pp interactions for $s_{\rm cut} = 1.2 s_{4,\rm min}$, $m = 4.75$ GeV/c^2 and $\theta^* = 90°$.

	NLL			LL		
$p_{\rm lab}$ (GeV/c)	$\sigma_{q\overline{q}}^{\rm DIS}$ (nb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (nb)	σ_{gg} (nb)	$\sigma_{q\overline{q}}^{\rm DIS}$ (nb)	$\sigma_{q\overline{q}}^{\overline{\rm MS}}$ (nb)	σ_{gg} (nb)
	$Q^2 = m^2 \Rightarrow \mu_0^{\rm DIS} = 0.39m, \mu_0^{\overline{\rm MS}} = 0.54m$					
200	0.0035	0.0050	0.015	0.0017	0.0023	0.0071
300	0.037	0.061	0.187	0.019	0.029	0.094
400	0.131	0.225	0.723	0.072	0.111	0.373
500	0.295	0.517	1.72	0.166	0.262	0.910
600	0.525	0.932	3.20	0.303	0.482	1.70
700	0.811	1.46	5.14	0.478	0.765	2.76
800	1.14	2.06	7.46	0.685	1.10	4.05
900	1.51	2.74	10.18	0.918	1.48	5.54
1000	1.92	3.48	13.20	1.17	1.90	7.24
	$Q^2 = 4m^2 \Rightarrow \mu_0^{\rm DIS} = 0.56m, \mu_0^{\overline{\rm MS}} = 0.85m$					
200	0.0028	0.0054	0.018	0.0013	0.0023	0.0079
300	0.029	0.059	0.216	0.015	0.027	0.099
400	0.102	0.209	0.825	0.054	0.097	0.388
500	0.228	0.471	1.99	0.126	0.227	0.945
600	0.402	0.837	3.72	0.227	0.409	1.78
700	0.620	1.29	6.02	0.356	0.644	2.93
800	0.874	1.83	8.85	0.509	0.920	4.31
900	1.16	2.43	12.17	0.684	1.23	5.93
1000	1.46	3.06	15.94	0.873	1.58	7.88
	$Q^2 = m^2/4 \Rightarrow \mu_0^{\rm DIS} = 0.28m, \mu_0^{\overline{\rm MS}} = 0.34m$					
200	0.0025	0.0029	0.010	0.0013	0.0014	0.0057
300	0.033	0.045	0.159	0.018	0.023	0.090
400	0.133	0.188	0.646	0.076	0.101	0.376
500	0.315	0.461	1.57	0.188	0.255	0.940
600	0.580	0.866	2.93	0.356	0.489	1.78
700	0.915	1.38	4.69	0.576	0.801	2.89
800	1.31	1.99	6.81	0.837	1.17	4.25
900	1.76	2.70	9.26	1.14	1.60	5.85
1000	2.24	3.46	11.92	1.47	2.08	7.58

Table 6: The hadronic $b\overline{b}$ production cross sections in pp interactions for $s_{\rm cut} = 1.6 s_{4,\rm min}$, $m = 4.75$ GeV/c^2 and $\theta^* = 90°$.