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# Hadron Structure Functions from Lattice QCD – 1997

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**Abstract.** We review the status of lattice calculations of the deep-inelastic structure functions of the nucleon. In addition, we present some results on the pion and rho structure functions.

## INTRODUCTION

The calculation of the deep-inelastic structure functions of hadrons from *QCD* is basically a non-perturbative problem. Perturbation theory can only describe the evolution of the structure functions from one scale to the other, provided one is at high enough scales. The most promising tool to solve this problem is lattice gauge theory.

Three years ago we have initiated a program to compute the lower moments of the structure functions of the nucleon on the lattice. First results for the unpolarized structure functions $F_1$, $F_2$ and the polarized structure functions $g_1$, $g_2$ are now available [1–3]. It has been found that with the help of dedicated computers one is able to compute the lower moments of all four structure functions with satisfactory precision – at least for the non-singlet leading twist

[1] Minireview given by G. Schierholz at *DIS97*, Chicago, April 1997

contributions which we have addressed so far. It is needless to say that all these calculations have been performed in the quenched approximation, where the effect of internal quark loops has been neglected. Three or four moments are generally sufficient to determine the individual quark distribution functions [4].

Recently, we have extended our calculations to the structure functions of the pion and the rho [5]. The pion structure function is being measured at *HERA*, and data should soon become available. The structure functions of the rho are interesting because they give qualitatively new information about quark binding effects.

Lattice calculations are subject to systematic errors arising from the finite lattice spacing, the finite volume, the extrapolation to the chiral limit and, of course, quenching. Before we can trust our results entirely, it is important to examine these effects carefully. This will be a major task in the future.

The aim of this talk is twofold. In the first part we will present and discuss our results obtained over the last two years. This will be mainly about the nucleon structure functions and briefly about the structure functions of the pion and the rho. In the second part of the talk we will report on first attempts to reduce discretization errors.

## BASICS

The theoretical basis of the calculation is the operator product expansion. For large $Q^2$ the moments of the nucleon structure functions are given by

$$\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{f=u,d,g} c_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, v_n^{(f)}(\mu), \\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d,g} c_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\, v_n^{(f)}(\mu), \\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \frac{1}{2}\sum_{f=u,d,g} e_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, a_n^{(f)}(\mu), \\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{f=u,d,g} [e_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\, d_n^{(f)}(\mu) \\
&\quad - e_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, a_n^{(f)}(\mu)],
\end{aligned} \tag{1}$$

where $c_1$, $c_2$ and $e_1$, $e_2$ are the Wilson coefficients, and $v_n$, $a_n$ and $d_n$ are forward nucleon matrix elements of certain local operators $\mathcal{O}$. For details see (e.g.) ref. [1]. In parton model language

$$v_{n+1}^{(f)} = \langle x^n \rangle^{(f)} = \int_0^1 \mathrm{d}x x^n \frac{1}{2}[q_\uparrow^{(f)}(x) + q_\downarrow^{(f)}(x)],\ f = u,d, \tag{2}$$

$$v_{n+1}^{(g)} = \langle x^n \rangle^{(g)} = \int_0^1 \mathrm{d}x x^n g(x), \tag{3}$$

and

$$a_n^{(f)} = 2\Delta^{(n)}q^{(f)} = 2\int_0^1 \mathrm{d}x x^n \frac{1}{2}[q_\uparrow^{(f)}(x) - q_\downarrow^{(f)}(x)],\ f = u, d \tag{4}$$

with $\Delta^{(0)}q^{(f)} \equiv \Delta q^{(f)}$, and $q_\uparrow(x)$, $q_\downarrow(x)$ being the quark distribution functions with spin $\uparrow$, $\downarrow$ with respect to the direction of motion. In eqs. (2), (4) we have omitted the sea quark contributions because we are working in the quenched approximation. The matrix elements $d_n$ have twist three and so have no parton model interpretation.

For a spin-zero target like the pion one has $q_\uparrow(x) = q_\downarrow(x)$. For a spin-one target like the rho the quark-spin distribution will depend on the spin projection $m$ of the particle. Writing $q^m(x) = 1/2\,[q_\uparrow^m(x) + q_\downarrow^m(x)]$, one finds the new structure function [6]

$$b_1(x) = q^0(x) - q^1(x). \tag{5}$$

On the lattice one computes bare operators $\mathcal{O}(a)$, where $a$ is the lattice spacing. These operators are in general divergent and must be renormalized. One may define finite operators renormalized at the finite scale $\mu$ by

$$\mathcal{O}(\mu) = Z_{\mathcal{O}}(a\mu, g(a))\mathcal{O}(a). \tag{6}$$

The renormalization condition here must match the renormalization condition used to calculate the Wilson coefficients, so that the $\mu$-dependence drops out of the product of Wilson coefficients and $Z_{\mathcal{O}}$'s. In principle the renormalization constants can be calculated in perturbation theory. The problem is only that lattice perturbation theory converges very slowly [7].

## NUCLEON STRUCTURE FUNCTIONS

The lattice calculation of structure functions divides into two parts. Part one is the calculation of the hadron matrix elements. Part two is the determination of the renormalization constants $Z_{\mathcal{O}}$. Altogether we have considered $O(20)$ different operators so far. On the hypercubic lattice one is limited to operators with spin $\leq 4$. In the notation of eqs. (2)-(4) this means $n \leq 3$.

For nearly all of the operators the renormalization constants have been computed perturbatively to one loop order [8–10]. A non-perturbative calculation has only been started recently [11,12]. In the few cases where we can compare perturbative and non-perturbative results we see differences of not more than 5% if boosted perturbation theory [7] is applied.

The numerical calculations of the matrix elements have so far been done at one value of the coupling, $\beta \equiv 6/g^2 = 6.0$. At this coupling the lattice spacing is $a \approx 0.1\,\mathrm{fm}$. We did calculations on two volumes, $16^3 \times 32$ and $24^3 \times 32$, and for several quark masses ranging from $\approx$ 30 to 200 MeV. It is important to

**TABLE 1.** Results for the unpolarized structure functions in the chiral limit. The lattice numbers are compared with the *CTEQ3M* [13] fit of the valence quark distribution functions. For $\langle x \rangle$, which has been calculated using different procedures, we have averaged over the results in ref. [2].

| Moment | Lattice (quenched, $\mu^2 \approx 5\,GeV^2$) | Experiment ($\mu^2 = 4\,GeV^2$) |
|---|---|---|
| $\langle x \rangle^{(u)}$ | 0.410(34) | 0.284 |
| $\langle x \rangle^{(d)}$ | 0.180(16) | 0.102 |
| $\langle x \rangle^{(u)}$-$\langle x \rangle^{(d)}$ | 0.230(38) | 0.182 |
| $\langle x^2 \rangle^{(u)}$ | 0.108(16) | 0.083 |
| $\langle x^2 \rangle^{(d)}$ | 0.036(8) | 0.025 |
| $\langle x^3 \rangle^{(u)}$ | 0.020(10) | 0.032 |
| $\langle x^3 \rangle^{(d)}$ | 0.000(6) | 0.008 |
| $\langle x \rangle^{(g)}$ | 0.53(23) | 0.441 |

**TABLE 2.** Results for the polarized structure functions in the chiral limit. The lattice numbers are compared with experiment [14,15] and the $LO$ [16] fit to the valence quark distribution functions. The latter are referred to as *valence*. The data for $\Delta^{(1)}u$ and $\Delta^{(1)}d$ refer to $\mu^2 = 10\,\mathrm{GeV}^2$.

| Moment | Lattice (quenched, $\mu^2 \approx 5\,GeV^2$) | Experiment ($\mu^2 = 3-5\,GeV^2$) | |
|---|---|---|---|
| $\Delta u$ | 0.841(52) | 0.823 | *valence* |
| $\Delta d$ | −0.245(15) | −0.303 | *valence* |
| $g_A$ | 1.086(67) | 1.26 | |
| $\int_0^1 \mathrm{d}x(g_1^p - g_1^n)$ | 0.176(14) | 0.163(10)(16) | |
| $\Delta^{(1)}u$ | 0.198(8) | 0.169(18)(12) | |
| $\Delta^{(1)}d$ | −0.048(3) | −0.055(27)(11) | |
| $\int_0^1 \mathrm{d}x x^2 g_1^{(p)}$ | 0.0150(32) | 0.012(1) | |
| $\int_0^1 \mathrm{d}x x^2 g_1^{(n)}$ | −0.0012(20) | −0.004(3) | |
| $\int_0^1 \mathrm{d}x x^2 g_2^{(p)}$ | −0.0261(38) | −0.006(2) | |
| $\int_0^1 \mathrm{d}x x^2 g_2^{(n)}$ | −0.0004(22) | 0.005(8) | |

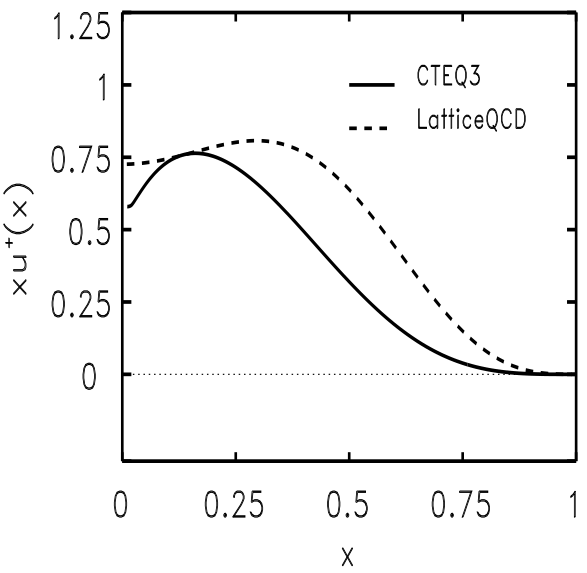


**FIGURE 1.** The $u$-quark distribution function extracted from the lattice data compared with the experimental valence quark distribution function. The figure is taken from ref. [4].

have data at many quark masses to do a reliable extrapolation to the chiral limit. The calculation of nucleon matrix elements, in particular of higher derivative operators, requires high statistics. Our calculations on the $16^3 \times 32$ lattice involved $O(1000)$ gauge field configurations, and $O(100)$ configurations on the $24^3 \times 32$ lattice. The calculations on the large volume were nearly as expensive in computer time as the high statistics calculations on the smaller volume as we went to smaller quark masses. For the calculation of the gluon distribution function we even used $O(5000)$ configurations. By comparing the results on the two volumes, we find that our calculations on the $16^3 \times 32$ lattice do not suffer from finite size effects. All results in this and the next section are for Wilson fermions.

Our results for the unpolarized structure functions are listed in table 1, and in table 2 we give the results for the polarized structure functions. Recent work on $\Delta q$ by two other groups [2] [17,18] give similar numbers to ours. We compare the lattice results directly with experiment, where possible, and otherwise with the phenomenological valence quark distribution functions, namely *CTEQ3M* [13] for the unpolarized structure functions and *LO* [16] for the polarized structure functions.

Let us now discuss the results in some detail. We begin with the unpolarized

[2] These groups also compute the sea quark contribution which one might object to in a quenched calculation.

structure functions in table 1. We see that the lattice prediction for the lowest moment, $\langle x \rangle$, is significantly larger than the lowest moment of the phenomenological valence quark distribution function, both for the $u$ and the $d$ quark. When one goes to the higher moments the agreement improves.

What does this mean for the structure functions themselves? In [4] the moments from table 1 have been converted to 'real' structure functions. In fig. 1 the result is shown for the $u$-quark distribution function. This is compared with the phenomenological curve. We see that the 'lattice' structure function is quite different in shape from the experimental one. The difference shows mainly at $x$ values around $1/2$. We do not expect that the missing internal quark loops will have an effect at such large values of $x$. If the effect is real, it could mean that the experimental structure functions contain substantial higher-twist contributions. One would expect higher-twist contributions to show up predominantly at large $x$.

The lowest moment of the gluon distribution function, $\langle x \rangle^{(g)}$, turns out to be consistent with experiment. The relatively large error has to do with the fact that the calculation requires a delicate subtraction between two terms similar in magnitude [3]. Quark and gluon contributions to $\langle x \rangle$ must add up to one, as a result of energy-momentum conservation. We obtain

$$\langle x \rangle^{(u)} + \langle x \rangle^{(d)} + \langle x \rangle^{(g)} = 1.12(23). \tag{7}$$

Let us now turn to the polarized structure functions in table 2. Here we have a lot of experimental data to compare with, so that we do not have to rely only on the phenomenological distribution functions. A recent, and yet unpublished, piece of information comes from the *SMC* measurement of the charge asymmetry of fast pions in polarized muon-nucleon collisions [15]. In the asymmetry the effect of sea quarks drops out, and one is lead directly to the moments of the valence quark distribution functions

$$\Delta^{(1)}u = \frac{1}{2}a_1^{(u)}, \ \Delta^{(1)}d = \frac{1}{2}a_1^{(d)}. \tag{8}$$

The lattice calculation of $\Delta^{(1)}u$ and $\Delta^{(1)}d$ is reported here for the first time [19]. For the higher moments sea quark effects should not play any significant role any more because they are restricted to the region of small $x$.

With two exceptions, the moments of the polarized structure functions agree surprisingly well with experiment. One exception is the axial vector coupling constant $g_A = \Delta u - \Delta d$. Experimentally, this is one of the best known quantities, and the sea quark contribution is expected to cancel out. The lattice result turns out to be quite a bit lower than the experimental value. The other exception is the twist-three matrix element $d_2$, contributing to the second moment of the structure function $g_2$. Comparing existing data on $g_1$ and $g_2$, one is lead to the result $d_2 \approx 0$, both for the $u$ and the $d$ quark. We find that $d_2 \approx -a_2$ in the chiral limit. In the heavy quark limit, on the other hand, $d_2$

vanishes as one would expect. We have no explanation for this discrepancy. We do not think that it can be explained by finite cut-off effects and the effect of quenching.

## PION AND RHO STRUCTURE FUNCTIONS

The pion and rho structure functions have been computed from $O(500)$ gauge field configurations on the $16^3 \times 32$ lattice. As before, the coupling was taken to be $\beta = 6.0$.

For the pion we obtain in the chiral limit

$$\langle x \rangle = 0.274(13),\ \langle x^2 \rangle = 0.107(35),\ \langle x^3 \rangle = 0.048(20). \tag{9}$$

If we compare this result with indirect information from the Drell-Yan cross section [20], we find the same picture as for the nucleon: $\langle x \rangle$ comes out to be significantly larger than the phenomenological value, while $\langle x^2 \rangle$ and $\langle x^3 \rangle$ are consistent with the data.

The unpolarized rho structure function looks very similar to the pion structure function. For the moment $\Delta q$ (the counterpart of $\Delta^{(0)} q$ in eq. (4) with $m = 1$ instead of $1/2$) of the polarized structure function we find 0.59(5) indicating that the valence quarks carry 60% of the spin of the rho. This is about the same fraction as one finds for the nucleon. The lowest moment of the polarized structure function $b_1$ turns out to be positive and unexpectedly large, albeit with large statistical errors. A possible interpretation of this is that the valence quarks have a significant orbital angular momentum.

## RECENT DEVELOPMENTS

Wilson fermions (which we are using here) give rise to systematic errors of $O(a)$, while the standard gluonic action induces errors of $O(a^2)$ only. Therefore it is the fermionic action which is most in need of attention. From hadron mass calculations we know that discretization errors can have a significant effect on the results at present values of the coupling [21].

Since it is very expensive to reduce cut-off effects by reducing $a$, a better way of reducing them is by improving the action. A systematic improvement program reducing the cut-off errors order by order in $a$ has been proposed by Symanzik [22] and was further developed in [23]. By adding the (irrelevant) term

$$c \sum_x \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi \tag{10}$$

to the (Wilson) action [24] and choosing the coefficient $c$ appropriately [25], one can reduce the cut-off errors from $O(a)$ to $O(a^2)$, provided the operators are improved as well.

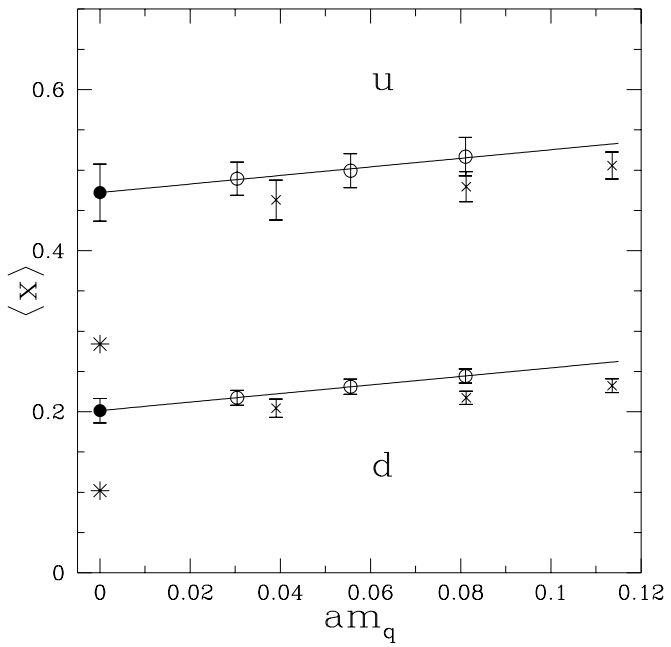


**FIGURE 2.** The moment $\langle x \rangle$ for $u$ and $d$ quark distributions as a function of the quark mass at $\beta = 6.0$. We compare results using the Wilson action ($\circ$) with results from the improved action ($\times$). The lines are a linear extrapolation of the Wilson action data to the chiral limit. The extrapolated values ($\bullet$) are compared with the $CTEQ$ results ($*$).

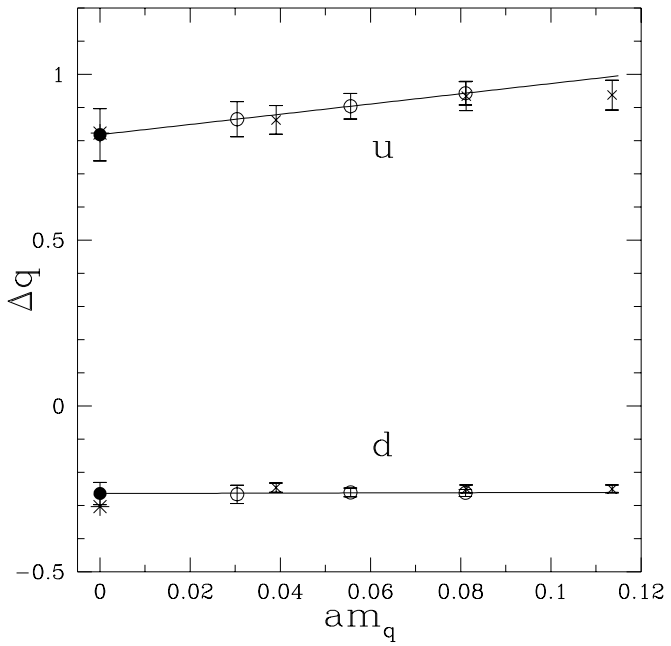


**FIGURE 3.** The same as fig. 1 but for $\Delta q$. Here the extrapolated values are compared with the phenomenological distribution $LO$ of ref. [16].

In figs. 2 and 3 we show first results of the $O(a)$ improved theory for the moments[3] $\langle x \rangle$ and $\Delta q$. The calculations are done on the $16^3 \times 32$ lattice at $\beta = 6.0$. We compare the improved results with the unimproved numbers. To facilitate the comparison, we have only looked at one operator on one lattice. In the case of $\Delta q$ we see no difference between the two results at any value of the (bare) quark mass. In the case of the moment $\langle x \rangle$ we see only a small effect, which is however not statistically significant. For both moments the quark mass dependence is rather weak and unspectacular, so that a linear extrapolation to the chiral limit is justified.

We may conclude that our numbers for $\langle x \rangle$ and $\Delta q$ are not far from the continuum values (of the quenched theory). This means that we have a real problem with the unpolarized structure functions. It is very likely that the solution lies in unexpectedly large higher-twist contributions at our values of $Q^2$. The situation of the axial vector coupling constant $g_A$ has also not changed. One should, however, keep in mind that the naive parton model identification of $g_A$ with $\Delta u - \Delta d$ might fail.

Whether discretization errors are also small for the other moments remains to be seen.

## CONCLUSIONS

We have reviewed the current status of lattice structure function calculations. The results so far are encouraging. But it is clear that much more remains to be done. We have only begun to improve the calculation by systematically removing discretization errors, and this will certainly occupy us throughout the next year.

A topic which is of upcoming interest [29] is power corrections (see also above). This includes higher twist effects on the one hand and renormalon contributions on the other. We hope to be able to present first results on this topic at next year's workshop.

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[3] For details see [26]. For the experts we like to mention that for $\Delta q$ the improved operators and the renormalization constants were taken from [25,27], while for $\langle x \rangle$ we used improved operators and renormalization constants suggested by tadpole improved perturbation theory [28].

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