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Momentum Resolution Studies for a TPC-Based Central Tracker

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Abstract

This note describes a study on the momentum resolution of the TPC-based central tracker design which will be included in the ECFA/DESY conceptual design report (CDR) for the proposed DESY 500 GeV electron-positron linear collider. The study found it to be likely that the central tracker could achieve a momentum resolution of approximately 10^{-4} $(\mathrm{GeV/c})^{-1}$, particularly if a layer of silicon microstrip detectors were to be added just inside the TPC.

1 Introduction

This note describes a study on the momentum resolution of the TPC-based central tracker design (CT) which will be included in the ECFA/DESY conceptual design report (CDR) for the proposed DESY 500 GeV electron-positron linear collider.

The study had two goals:

- To estimate the CT momentum resolution as a function of the Time Projection Chamber (TPC) coordinate resolution and level of relative misalignment between the TPC and and vertex detector (VD).
- To try to characterise the improvements which would be expected from adding a layer of silicon microstrip detectors (SI) outside the vertex detector and just inside the TPC.

Only the physically important high transverse momentum limit for the CT momentum resolution is considered:

$$p_t = \infty \;\Leftrightarrow\; \frac{1}{p_t} = 0, \tag{1}$$

and only for the barrel region of the CT.

Section 2 of this paper describes the CT model used for these studies, which is roughly the model which emerged from the discussions at the Munich ECFA/DESY workshop [1]. For this reason, the model will be referred to as the "Munich CT".

In section 3, details are given on the method used to determine the momentum resolution of the CT. Section 4 describes how the coordinate resolution of the TPC was estimated, based on both the discussion at the Munich

workshop and the known performance of the ALEPH TPC [2]. The performance of the ALEPH central tracker was also used to obtain an estimate for a reasonable level of TPC misalignment, as discussed in section 5.

The results of this study are presented in section 6, consisting of the momentum resolutions obtained for different alignment and resolution assumptions, and using different combinations of the central tracking subdetectors, and conclusions for the study are given in the final section of the report.

2 Central Tracker Model

Based on the discussions at the Munich ECFA/DESY workshop [1], the central tracker is assumed to consist of 4 layers of vertex detector, at radii of 2.5, 3.5, 7.5 and 10 cm, and a TPC with an active region between 40 cm and 150 cm radius. It is expected that the general conclusions of this study will apply for other similar configurations.

Optionally, a layer of silicon microstrips (SI) is added at a radius of 32 cm. This is 8 cm inside the inner active radius of the TPC, with the 8 cm gap corresponding roughly to the size of the inactive region on the inside of the ALEPH TPC. One part of the study investigates the effect of increasing the SI radius to 42 or 52 cm, with the inner radius of the TPC increased to 50 cm or 60 cm, respectively, to retain the 8 cm gap.

The study considers only the barrel region of the CT, which is defined by the outer corner of the TPC, at $r_{corner} = 150$ cm and $z_{corner} = 250$ cm. The corresponding polar angle is $\arctan(0.6) = 31$ degrees, which subtends 85.8 percent of the solid angle. The point resolutions of the vertex detector and silicon layer, in the coordinate for the bend plane, should be

relatively constant over this range. This should also be fairly true for the TPC, although the details depend on which effect limits the TPC resolution – as discussed in section 4.

It is easily seen that the area of Si needed to cover the barrel region is given by:

$$\text{Si area} = 2 \times \pi \times r \times (2 \times Z_{end}), \tag{2}$$

with

$$Z_{end} = z_{corner}/r_{corner} \times r = 1.67 \times r \tag{3}$$

giving

$$\text{Si area} = 20.9 \times r^2 \tag{4}$$

For $r = 32$ cm, 2.14 square metres of silicon microstrips are required. This would cost about 1.3 million CHF at a conservative total cost per unit area – detectors plus readout – of 60 CHF/cm^2 [3].

The vertexing layers and the Si were both assumed to have point resolutions of 8 microns. The TPC readout is assumed to be radially segmented into N_{TPC} equal-length pads, each with position resolution σ_{pad}. The choice of values for N_{TPC} and σ_{pad} is discussed in section 4.

In the interests of simplicity, the form of the TPC misalignment was assumed to be a simple transverse displacement relative to the SI layer, with a gaussian uncertainty. The magnitude chosen for this uncertainty is discussed in section 5. Much smaller transverse displacements of 5 microns were included to describe the misalignment of SI relative to VD, and VD relative to the beam spot.

3 Method for Determining Momentum Resolutions

This section describes the method used for determining the various momentum resolutions for the CT, which involves the Monte Carlo-based generation of large numbers of simulated infinite-momentum tracks. It begins by giving a derivation of the method for measuring high momentum tracks using a quadratic fit, followed by a description of the Monte Carlo-based method for determining the tracker's momentum resolution.

In a quadratic approximation, the measured coordinates in the bend plane of a high momentum track, $f_i(r_i)$, at the radial positions, r_i, $i = 1, N$, of the N detector elements, can be fitted to a "Chebychev-type" quadratic equation:

$$f(r) = a + b(r - m) + 0.25 \times c \times (2(r - m)^2 - m^2), \tag{5}$$

where m is the half-radius of the TPC and the fit is for the 3 parameters, a, b and c.

The relevant parameter for momentum resolution is the coefficient, c, of the quadratic term. For each detector configuration, the momentum resolution is obtained from the r.m.s. uncertainty, σ_c, of a Gaussian fit to the distribution of c values for 1000 infinite momentum tracks, as follows.

Using the Lorentz force equation, it is easy to translate the fitted value of c into a measurement of the transverse momentum, p_t [5]:

$$c = \frac{d^2 f}{dr^2} = \frac{B}{0.3 \times p_t}, \tag{6}$$

where f and r are in metres, B is in Tesla, p_t is in GeV/c and the factor of 0.3 accounts for the momentum units and the speed of light: $0.3 = 3.10^8$ m/s $\times$ 10^{-9}GeV/eV.

As a clarifying remark about the high momentum limit, equation 6 shows that the directly measured quantity is actually the inverse of the track momentum, $\frac{1}{p_t}$, and it is this quantity which has an approximately gaussian resolution in the infinite transverse momentum limit. In practice, the momentum itself will also have an approximately gaussian distribution for all the tracks encountered in an experiment, since these tracks will always have a low enough momentum that the inverse momentum can be measured with a fractional uncertainty much less than one. In this case, the sigmas of the gaussian widths are related by:

$$\sigma(\frac{1}{p_t}) = \frac{\sigma_{p_t}}{p_t^2}. \tag{7}$$

Using this relation, equation 6 can be converted to an equation for the transverse momentum resolution, σ_{p_t}, in terms of the width, σ_c of c:

$$\frac{\sigma_{p_t}}{p_t^2} = \frac{0.3 \times \sigma_c}{B}, \tag{8}$$

in units of $(\mathrm{GeV/c})^{-1}$, where σ_c is in units of $1/\mathrm{m}$ and B is in Tesla. This is the basic equation used in the procedure for determining the tracker resolution, which will now be described.

The procedure simulates the measurement of infinite-momentum tracks, which will have zero displacement, f, from a straight line at all radii (r):

$$f(r) \equiv 0, \text{ at all } r. \tag{9}$$

However, the measured points, at $f(r_i),\ r_i = 1, N$, will differ from zero due to point resolutions and misalignments.

For each of 1000 simulated tracks, the measured displacements at each of the N measurement positions were generated randomly in two stages:

(i) independent displacements were generated corresponding to the point resolutions, and (ii) the misalignments of the subdetectors were generated and then added to each measurement point in the subdetector.

Next, each of the tracks was fitted, using MINUIT, to the parameterization of equation 5. As expected, the fitted values of the coefficient of the quadratic term, c, formed a gaussian distribution centred on zero. The resolution parameter, σ_c, was obtained from a gaussian fit to this distribution, and converted to a momentum resolution, $\frac{\sigma_{p_t}}{p_t^2}$, using equation 8.

As a detail regarding the fitting method, the misalignments will introduce correlations between the coordinate measurements, f_i, so the optimal fit procedure would require the use of an $N \times N$ covariance matrix. This was regarded as being unnecessarily complicated, given that the assumed form of the misalignments is, anyway, unrealistically simple. Instead, correlations between the measurements were ignored in the fit, but the TPC point resolutions assumed for the fit were adjusted to compensate for this. The fit used:

$$\sigma_{fit}^2 = \sigma_{point}^2 + k.\sigma_{align}^2 \tag{10}$$

where it can be seen that the misalignment variance is added to the point variance weighted by an adjustable parameter, k. Several fits were performed on the ensemble of 1000 tracks – one each for different values of k – and the most precise value for the momentum resolution was chosen.

4 Estimation of TPC Point Resolutions

The TPC point resolution was estimated from the TPC-alone momentum resolution, 2.3×10^{-4} $(\mathrm{GeV/c})^{-1}$, that was quoted [4] at the Munich ECFA/DESY

woorkshop, for a TPC with an inner (outer) radius of 40 cm (150 cm) and a 3 Tesla magnetic field. The performance of the ALEPH TPC [2] was also considered. Before determining this point resolution, this section first discusses how the point resolution is parameterized, and then discusses the physical effects which determine the point resolution of a TPC.

For the purposes of this study, it is reasonable to assume that the measurements from the TPC pads are statistically independent and that σ_{pad} scales statistically according to the number of primary ionization electrons producing the pad signal. That is, it reduces as the inverse square root of the pad's radial length. In this approximation, it is possible to define a TPC position resolution:

$$\sigma_{TPC} \equiv \sigma_{pad} \times L_{pad}, \tag{11}$$

for L_{pad} the pad length and with units of μm.cm$^{1/2}$, which is independent of the radial segmentation of the pads.

Further, when equation 11 holds, the momentum resolution for high momentum tracks is also almost independent of the radial segmentation. The well-known Glueckstein formula [5] predicts the momentum resolution for a singly charged particle:

$$\sigma(\frac{1}{p_t}) = \frac{1}{0.3 \times B(\text{Tesla})} \times \frac{\sigma_{pad}}{L^2} \times \sqrt{\frac{720}{N_{TPC}+4}}, \tag{12}$$

with L the radial extent of the TPC, and all units of length in metres. If it is assumed that the pads occupy the entire radius of the TPC,

$$L = N_{TPC} \times L_{pad}, \tag{13}$$

and

$$N_{TPC} \gg 4, \tag{14}$$

then equations 11 and 12 can be combined into:

$$\sigma(\frac{1}{p_t}) \simeq \frac{\sqrt{720}}{0.3 \times B(\text{Tesla})} \times \frac{\sigma_{TPC}}{L^{2.5}}, \tag{15}$$

which is seen to be independent of the pad segmentation, N_{pad}.

It should also be noted that this argument would break down for tracks crossing the pads at large angles, for which the finite radial segmentation would contribute to the pad measurement uncertainty, σ_{pad}. This is not a problem for the studies in this report: in the high momentum limit that we are considering the track directions are always nearly radial.

As an aside, a more realistic model in the case of TPC pads with fine segmentation would include the correlations between track position measurements in nearby pads, which are mainly due to knock-on electrons (i.e. delta-rays) spanning more than one pad. However, because the momentum resolution is almost independent of the pad segementation it will be assumed that a finely segmented configuration can be considered to be equivalent to a configuration with coarse enough sampling that the statistical independence represented by equation 11 is still obeyed yet equation 14 is also still valid.

The statistical precision of a TPC's momentum resolution is limited by the diffusion of the electrons as they drift through the TPC gas. The diffusion limit can be calculated from the ALEPH parameters [2]:

- the specific ionization for a minimum ionising particle is 90 electrons/cm
- the magnetic field is 1.5 Tesla
- the transverse diffusion is 800 μm.m$^{-1/2}$ per electron for a magnetic field of 1.5 Tesla

For magnetic fields (B) of order a Tesla or larger, the diffusion scales approximately as 1/B, so the limiting precision on the track position, σ_{limit} is:

$$\begin{aligned}\sigma_{limit} &= 800\ \mu m \times (1.5\ \text{Tesla/B}) \times (\text{drift length in metres})^{1/2}/(\text{spec. ion.})^{1/2}, \\ &= 67 \mu\text{m.cm}^{1/2}\end{aligned}$$

for a magnetic field of 3 Tesla and a drift length of 2.5 m.

As well as the statistical uncertainty due to electronic diffusion, the TPC's momentum resolution will be degraded by systematic terms in the point resolution. Examples of such effects are: imperfect read-out (e.g. the "angular pad effect"), space-charge effects and possible systematics from the B field, E field and nearby tracks.

A potentially large contribution to the resolution smearing comes from the "angular pad effect": the avalanche electrons which are created at the sense wires are displaced sideways due to the Lorentz effect. From the information in the ALEPH references [2] it follows easily that this gives an uncertainty of $\sigma_{TPC} \simeq 70\ \mu\text{m.cm}^{1/2}$ for a 1.5 Tesla magnetic field and the ALEPH geometry. Unfortunately, this term would be expected to scale approximately in proportion to the magnetic field. For the ALEPH read-out geometry it would be twice as large for a 3 Tesla field – about $\sigma_{TPC} \simeq 140\ \mu\text{m.cm}^{1/2}$. However, one might assume that improvements can be made over the ALEPH read-out, either within the sense-wire concept or by using a new type of read-out such as microstrip gas chamber (MSGC) read-out.

Solving equation 15 for σ_{TPC} gives:

$$\sigma_{TPC} \simeq \frac{\sigma(\frac{1}{p_t}) \times L^{2.5} \times 0.3 \times B(\text{Tesla})}{\sqrt{720}}. \qquad (16)$$

Substituting in the parameters assumed at the Munich workshop, $B = 3$ Tesla, $L = 1.1$ m and $\sigma(\frac{1}{p_t}) = 2.3 \times 10^{-4}$, gives

$$\begin{aligned} \sigma_{TPC} &= 9.8 \times 10^{-6} \text{ m}^{3/2} \\ &= 98 \ \mu\text{m.cm}^{1/2} \quad \text{(Munich TPC)}. \end{aligned}$$

For comparision, the ALEPH parameters, $B = 1.5$ Tesla, $L = 1.32$ m and $\sigma(\frac{1}{p_t}) = 12 \times 10^{-4}$, correspond to:

$$\begin{aligned} \sigma_{TPC} &= 4.03 \times 10^{-5} \text{ m}^{3/2} \\ &= 403 \ \mu\text{m.cm}^{1/2} \quad \text{(ALEPH TPC)}, \end{aligned}$$

i.e. a factor of four worse precision. However, it should be noted that only 48 percent of the radial coordinate of the ALEPH TPC is instrumented for measuring the track position (21 pads of 3 cm length, in a total radial span of 132 cm). Since the average radial span of each of the ALEPH pads is 6.3 cm, the predicted point resolution corresponds to a pad resolution of $403/\sqrt{6.3} = 160$ μm, which agrees with the quoted ALEPH pad resolution of 170 microns.

Very similar predictions for the point resolutions of both TPC's were obtained using the fitting procedure of section 3: 95 and 395 μm.cm$^{1/2}$ for the Munich and ALEPH TPC's, respectively. (These values were obtained assuming $N_{TPC} = 40$; it was checked that $N_{TPC} = 20$ gave similar values.)

Based on these results, it was considered reasonable to perform the resolution studies on the CT for three different TPC point resolutions:

- $\sigma_{TPC} = 95 \ \mu\text{m.cm}^{1/2}$

- $\sigma_{TPC} = 195$ μm.cm$^{1/2}$
- $\sigma_{TPC} = 390$ μm.cm$^{1/2}$.

The first and last of these values correspond roughly to the momentum resolution assumed at Munich and to the ALEPH resolution, respectively, and the middle value corresponds to an intermediate resolution.

5 Estimation of Subdetector Misalignments

For those momentum calculations which take misalignments into account, the following gaussian misalignment uncertainties were assumed:

- 5 microns for the vertex detector relative to beam spot
- 5 microns for the Si layer relative to vertex detector
- 80 microns for the TPC relative to Si layer.

The 80 micron misalignment of the TPC was obtained using the measured [2] ALEPH transverse momentum (p_t) resolutions for the TPC alone, and with the inner tracking chamber (ITC) and vertex detector (VD):

$$\begin{aligned}
\sigma_{p_t}/p_t^2 &= 12.0 \times 10^{-4}\ (\mathrm{GeV/c})^{-1}: \quad \text{TPC alone} \\
\sigma_{p_t}/p_t^2 &= 8.0 \times 10^{-4}\ (\mathrm{GeV/c})^{-1}: \quad \text{TPC + ITC} \\
\sigma_{p_t}/p_t^2 &= 6.0 \times 10^{-4}\ (\mathrm{GeV/c})^{-1}: \quad \text{TPC + ITC + VD},
\end{aligned}$$

as follows.

As a very crude simulation of the ALEPH geometry, momentum resolutions were obtained, using the method of section 3, with the correct ALEPH

TPC parameters, $B = 1.5$ Tesla, $L = 1.32$ m and $\sigma(\frac{1}{p_t}) = 12 \times 10^{-4}$, and with a vertex detector layer at a radius of 6.3 cm with a point resolution of 8 microns. This configuration gives the correct resolution, $\sigma_{TPC} = 6.0 \times 10^{-4}$ (GeV/c)$^{-1}$, for a misalignment of 80 microns, while the resolution is too good if the misalignment is left out: $\sigma_{TPC} = 4.6 \times 10^{-4}$ (GeV/c)$^{-1}$.

This determination of the ALEPH misalignment is unrealistically simple because the ALEPH vertex detector actually contains two layers of microstrips with point resolutions of 12 cm, and the ITC has been ignored. However, the misalignment is much bigger than the point resolution of the vertex detector, and it was also checked that adding a second vertex layer didn't change the amount of misalignment needed.

6 Results

Tables 1 and 2 summarize the estimates for the CT momentum resolution for different combinations of the following model variations:

- the three different TPC coordinate resolutions listed in the preceding section.
- With and without some misalignment between the TPC and VD.
- With and without a vertex constraint.
- Momentum resolutions for the TPC alone, TPC plus VD, and the entire CT: TPC, VD and SI.
- Three different choices for the radius of the SI layer and inner radius of the CT.

TPC Align./ res.	TPC	TPC+VD	TPC+VD+SI
0 / 95	2.28	0.84	0.71
0 / 195	4.63	1.48	0.92
0 / 390	9.30	2.61	1.43
80 / 95	2.28	1.40	0.89
80 / 195	4.63	1.71	1.21
80 / 390	9.30	2.65	1.84

Table 1: **Fitted momentum resolutions for different alignment and resolution assumptions, and using different combinations of the central tracking subdetectors. The TPC misalignment is in units of microns and the resolution in units of μm.cm$^{1/2}$. The momentum resolutions are in units of $10^{-4} \times (\mathrm{GeV/c})^{-1}$. Values are given for the TPC alone, TPC plus vertex detector and TPC plus vertex detector plus a silicon microstrip layer at a radius of 32 cm. The momentum resolutions are in units of $10^{-4} \times (\mathrm{GeV/c})^{-1}$. The values have statistical uncertainties of approximately 5 percent. No vertex constraint is used.**

As a further study, it was found that increasing the radius of the SI layer generally gave little improvement to the momentum resolution. For example, the momentum resolution for perfect alignment, no vertex constraint and a TPC resolution of 195 μm.cm$^{1/2}$ improves from 0.93 (in units of 10^{-4} (GeV/c)$^{-1}$) to 0.85 when the SI radius is increased from 32 cm to 42 cm, and remains at 0.85 when the SI radius is further increased to 52 cm.

TPC Align./ res.	TPC	TPC+VD	TPC+VD+SI
0 / 95	0.79	0.70	0.55
0 / 195	1.57	1.05	0.74
0 / 390	3.03	1.39	1.11
80 / 95	2.04	1.16	0.68
80 / 195	2.47	1.30	1.24
80 / 390	3.60	1.74	1.78

Table 2: **Momentum resolutions as in table 1, except that a vertex constraint is used.**

7 Summary and Conclusions

By comparing the various resolution estimates in tables 1 and 2, the reader should be able to form some sort of idea of the expected momentum resolution of the CT under both optimistic and conservative assumptions about the subdetector performances, and to assess the expected level of improvement from adding the SI layer.

In section 3, the TPC-alone momentum resolution that was estimated at the Munich workshop – 2.3×10^{-4} $(\mathrm{GeV/c})^{-1}$ – was found to correspond to approximately a factor of 4 improvement in the TPC point resolution over the ALEPH performance. It is not yet clear whether or not such a level of improvement is achievable. However, the central tracker resolution seems to be rather robust against degradations in the TPC point resolutions or, in a simple model, misalignments of the TPC with respect to the vertex detector.

The addition of a silicon microstrip layer just inside the TPC would give increased robustness to the momentum resolution by providing a stable and

precise coordinate measurement at a radius approaching the mid-radius of the central tracker. This should improve the CT performance in two ways:

- The precise coordinate information near the mid-point should improve the statistical precision of the momentum fit.
- The alignment of the TPC with respect to the vertex detector should improve with the addition of a stable time-independent coordinate measurement near the TPC inner radius.

The first of these items can be seen from the tabulated results, while the second is more difficult to quantify.

Perhaps surprisingly, increasing the SI radius seems to give little improvement to the momentum resolution. It seems that the improved lever-arm of the Si is almost compensated for by the reduced resolution of the TPC.

In conclusion, it appears likely that the TPC-based central tracker design which was discussed at the ECFA/DESY Munich workshop, including a layer of silicon microstrips, could achieve a momentum resolution of approximately 10^{-4} $(\mathrm{GeV/c})^{-1}$.

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