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Scaling of Z' limits at the Linear Collider[1]

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Z' exclusion limits and errors of Z' model measurements are compared for different reactions at future linear colliders. The influence of the c.m. energy, integrated luminosity, beam polarization and systematic errors is discussed. The sensitivity to a Z' depends only on the product Ls and not on the integrated luminosity and the c.m. energy separately.

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1 Introduction

Extra neutral gauge bosons (Z') are predicted in grand unified theories (GUT's) with a unification group larger than $SU(5)$. They are candidates for particles, which could be discovered at the next linear collider.

In the previous years, extensive studies on the sensitivity of future e^+e^- colliders to a Z' have been completed [1]. See these references for more details and for related original references.

It is important to have a simple understanding how the sensitivity to new physics depends on the collider parameters, i.e. the c.m. energy $\sqrt{s}$ of the colliding particles, the integrated luminosity L, the beam polarization and the expected systematic errors of the different observables. We consider here the sensitivity of different reactions to a Z' with the help of simple analytical formulae predicting the results of time consuming exact analyses with an acceptable accuracy.

We assume that the interaction of neutral gauge bosons with Standard Model (SM) fermions is given by the lagrangian

$$\mathcal{L} = eA_\beta J^\beta_\gamma + g_1 Z_\beta J^\beta_Z + g_2 Z'_\beta J^\beta_{Z'}, \tag{1}$$

where the first two terms are SM contributions and the last term is due to the Z'. The gauge bosons couple through vector and axial vector couplings,

$$J^\beta_X = \sum_f \bar{u}_f \gamma^\beta \Big[\gamma_5 a_f(X) + v_f(X) \Big] u_f, \quad X = \gamma, Z, Z'. \tag{2}$$

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In general, we must admit that the symmetry eigenstates Z and Z' are different from the mass eigenstates Z_1 and Z_2. Mass and symmetry eigenstates are linked by a mixing matrix parametrized by the ZZ' mixing angle θ_M,

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}. \tag{3}$$

The masses and the widths of the mass and symmetry eigenstates are denoted as M_i, Γ_i, $i = 1, 2, Z, Z'$. The mixing must be small because the experiments at LEP 1 and SLC agree precisely with the SM.

We assume no mixing between the SM fermions and the new fermions present in a GUT. We assume that all new particles in the GUT are heavier than the Z'. See [2] for a review on Z' models and further references.

The experimental error of an observable O consists of a systematic $\Delta^{syst}O$ and a statistical $\Delta^{stat}O$ contribution. We add both contributions in quadrature,

$$\Delta O = \sqrt{(\Delta^{stat}O)^2 + (\Delta^{syst}O)^2} = \Delta^{stat}O \cdot \sqrt{1+r^2}, \quad r = \Delta^{syst}O/\Delta^{stat}O. \tag{4}$$

One has to distinguish between Z' *exclusion limits* and Z' *model measurements*. Exclusion limits will be obtained if there are no deviations from the SM. All present bounds on Z' theories are examples of exclusion limits. Model measurements will be possible if there are deviations from the SM predictions compatible with theories containing a Z'.

Lepton colliders have the advantage of a clean environment. Highly polarized electron beams are available. We have several reactions, which are sensitive to extra Z bosons.

Fermion pair production allows for a measurement of a large number of different observables. All couplings of the Z' to charged SM fermions can be constrained separately. This is a unique property of this reaction.

Bhabha and *Møller scattering* have large event rates. In addition, Møller scattering profits from two highly polarized electron beams. Of course, these reactions are sensitive to gauge boson couplings to electrons only.

W pair production is very sensitive to ZZ' mixing. This sensitivity is enhanced for large energies because additional amplitudes destroy the gauge cancellation present in the SM.

All *other reactions* in e^+e^- or e^-e^- collisions can not add useful information on extra neutral gauge bosons.

2 $e^+e^- \to f\bar{f}$

Off-resonance fermion pair production is almost insensitive to ZZ' mixing. We therefore set $\theta_M = 0$ in this section.

As has been proven by the LEP and SLC experiments, different fermions as e, μ, τ, c, b can be tagged in the final state. The tagging of top quarks is expected at the linear collider. The polarization of τ's and likely of top quarks can be measured.

Fermion pair production is sensitive to a Z' at energies far below the Z' peak. A Z' modifies cross sections and asymmetries due to interferences of the Z' amplitude with the SM amplitudes. With measurements below the Z' resonance, only the ratios $a_f(Z')/M_{Z'}$ and $v_f(Z')/M_{Z'}$ can be constrained and not the couplings and the mass separately, unless very high luminosities are available and the systematic errors are very small.

2.1 Exclusion Limits

An agreement of future measurements of an observable O with the SM prediction proves the Z' to be heavy,

$$M_{Z'} > M_{Z'}^{lim} \approx \frac{g_2}{g_1}\sqrt{s\frac{O}{\Delta O}} \approx 2.8\,TeV \cdot \frac{g_2}{g_1}\left[\frac{Ls}{1+r^2}\frac{fb}{TeV^2}\right]^{1/4}. \qquad (5)$$

The estimate (5) assumes that $\frac{g_2 \Delta O}{g_1 O} \gg 1$. We have $g_2/g_1 = \sqrt{\frac{5}{3}}\sin\theta_W \approx 0.62$ in the E_6 GUT. The scaling of the error with s and L , $\frac{\Delta O}{O} \approx \sqrt{(1+r^2)/N} \sim \sqrt{(1+r^2)s/L}$, is taken into account in the last step of the estimate (5). We further assume that all couplings of the Z' to fermions are fixed by the GUT. Then, many observables with leptons and hadrons in the final state contribute to $M_{Z'}^{lim}$. If there are no model assumptions on the Z' couplings to quarks, only observables with leptons in the final state will contribute. Then, the factor $2.8\,TeV$ has to be replaced by 1.9 TeV.

$M_{Z'}^{lim}$ depends on the fourth root of the experimental error. The dependence on the systematic error will be suppressed if it is not too large. Suppose that an analysis gives certain exclusion limits $M_{Z'}^{lim}$ without systematic errors. What changes are expected after the inclusion of systematic errors assuming $r = 1$? The estimate (5) predicts $M_{Z'}^{lim} \to M_{Z'}^{lim}/\sqrt[4]{2}$, which is a reduction by 16% only.

	χ	ψ	η	LR
$M_{Z'}^{lim}$ stat.	3.1	1.8	1.9	3.8
$M_{Z'}^{lim}$ +syst.	2.8	1.6	1.7	3.2
P_V^l stat.	2.00± 0.11	0.00± 0.064	$-3.00^{+0.53}_{-0.85}$	$-0.148^{+0.020}_{-0.024}$
P_V^l +syst.	2.00± 0.15	0.00± 0.13	$-3.00^{+0.73}_{-1.55}$	$-0.148^{+0.023}_{-0.026}$
P_L^b stat	-0.500± 0.018	0.500± 0.035	$2.00^{+0.33}_{-0.31}$	-0.143±0.033
P_L^b +syst.	-0.500± 0.070	0.500± 0.130	$2.00^{+0.64}_{-0.62}$	-0.143±0.066

Table 1 *The lower bound on Z' masses $M_{Z'}^{lim}$ in TeV excluded by $e^+e^- \to f\bar{f}$ at $\sqrt{s} = 0.5\,TeV$ and $L = 20\,fb^{-1}$ (first two rows). The Z' coupling combinations $P_V^l = v_l'/a_l'$ and $P_L^b = (v_b' + a_b')/(2a_b')$ and their 1-σ errors (last four rows) are derived under the same conditions as the exclusion limits but assuming $M_{Z'} = 1\,TeV$. The χ, ψ, η and LR models are the same as in the Particle Data Book. The numbers are given with and without systematic errors. They are taken from reference [3].*

Let us confront these findings with the numbers quoted in table 1. They include all SM corrections. The systematic errors included for observables with leptons in the final state are roughly as large as their statistical errors, i.e. $r \approx 1$. The systematic errors of observables with b quarks in the final state are roughly 4 times as large as the statistical errors, i.e. $r \approx 4$.

We see that the predicted reduction of $M_{Z'}^{lim}$ by 16% is reproduced by the numbers in the first two rows of table 1. Although $M_{Z'}^{lim}$ is defined by hadronic and leptonic observables, hadronic observables with large systematic errors don't spoil the estimate (5) because their contribution to $M_{Z'}^{lim}$ decreases in that case.

Polarized beams give almost no improvements to Z' exclusion limits [3].

2.2 Model Measurements

Suppose that there exists a Z' with $M_{Z'} < M_{Z'}^{lim}$. Then, a measurement of the ratios $a_f(Z')/M_{Z'}$ and $v_f(Z')/M_{Z'}$ is possible. Alternatively, one can measure the Z' mass for fixed couplings or the coupling strength for a fixed Z' mass, which could be known from hadron collisions. Considerations [4] similar to the previous section give an estimate of the *errors* of such measurements as

$$\frac{\Delta M_{Z'}}{M_{Z'}},\ \frac{\Delta g_2}{g_2} \approx \frac{1}{2}\left(\frac{M_{Z'}}{M_{Z'}^{lim}}\right)^2 \approx c_f \cdot \frac{g_1^2}{g_2^2}\frac{M_{Z'}^2}{TeV^2}\left[\frac{1+r^2}{Ls}\frac{TeV^2}{fb}\right]^{1/2} \sim \left[\frac{1+r^2}{Ls}\right]^{1/2} \tag{6}$$

with $c_f \approx 0.063$ for leptons in the final state. Model measurements depend on the square root of the experimental errors. Another important difference to the exclusion limit (5) is that the couplings of the Z' to leptons are measured by observables with leptons in the final state only, while the couplings of the Z' to b-quarks are measured by observables with b-quarks in the final state only.

In particular, the estimate (6) predicts (under the assumptions of the analysis [3]) that the errors of measurements of the Z' couplings to leptons (b-quarks) change as $\Delta P_V^l \to \Delta P_V^l\sqrt{2}$ ($\Delta P_L^b \to \Delta P_L^b\sqrt{17}$) after the inclusion of the systematic errors. Of course, these predictions are only rough approximations because they ignore details of the Z' models and differences of the ratio $\Delta^{syst}O/\Delta^{stat}O$ for the various observables entering the analysis. Nevertheless, they reproduce the main tendency of the last four rows in table 1. The estimate (6) explains why Z' model measurements are much more sensitive to systematic errors than Z' exclusion limits.

We remark that the scalings (5) and (6) depend on the product Ls only and not on s and L separately. The product Ls is the "currency", in which we have to pay for a Z' search. The estimate (6) relates the "price" $(Ls)_{det}$ one has to pay for the detection of a Z' of a certain model to the "price" $(Ls)_\varepsilon$ of a model measurement of the same model by the same observables at the same confidence level with the accuracy ε,

$$(Ls)_\varepsilon \approx \frac{1}{4\varepsilon^2} \cdot (Ls)_{det} \tag{7}$$

One can go one step further and try to determine the couplings and the mass of the Z' separately by a fit to the line shape below the Z' resonance. Such an measurement is proposed in reference [5]. Here is the list of "prices" for an investigation of $Z' = \chi,\ M_\chi = 1.6\,TeV,\ 95\%CL.,\ (s < M_2^2,)$:

Detection:	$Ls \approx 0.5 TeV^2/fb$
Measurement of $a_e(Z')/M_{Z'}$ and $v_e(Z')/M_{Z'}$ with 15% error:	$Ls \approx 8\,TeV^2/fb$
Measurement of $a_e(Z'), v_e(Z')$ and $M_{Z'}^2$ separately with 15% error:	$Ls \approx 260\,TeV^2/fb$

The first number is obtained from the first row of table 1 using the scaling (5). The second number is obtained from $M_{Z'}^{lim}$ calculated from leptonic observables only [3] and scaling (6). The third number is calculated from reference [5]. We see that the prices for the detection and the two proposed measurements are very different.

In contrast to exclusion limits, the electron polarization is very important for a Z' model measurement. It reduces the four-fold sign ambiguity of a Z' coupling measurement to a two-fold ambiguity [3]. This is a qualitative effect, which cannot by "bought" by an increase of Ls in an unpolarized measurement. Polarization of both beams would not give a further improvement.

3 $e^+e^- \to e^+e^-$ and $e^-e^- \to e^-e^-$

Bhabha and Møller scattering are as insensitive to ZZ' mixing as off-resonance fermion pair production. We therefore set the ZZ' mixing angle to zero in this section. Z' constraints arise by the same mechanisms as in fermion pair production. A difference occurs due to the gauge boson exchange in the t and u channels, which leads to a very singular angular distribution. Most of the scattered particles are near the beam pipe due to photon exchange. Therefore, the total cross sections are rather insensitive to Z' effects. Angular distributions are sensitive to Z' exchange.

3.1 Exclusion Limits

The exclusion limits from Bhabha and Møller scattering scale as in fermion pair production,

$$M_{Z'} > M_{Z'}^{lim} \approx 1.9\,TeV \cdot \frac{g_2}{g_1} \left[\frac{Ls}{1+r^2} \frac{fb}{TeV^2} \right]^{1/4} . \tag{8}$$

g_1 and g_2 are here the coupling strengths to electrons. The numerical factor $1.9\,TeV$ is obtained from reference [6]. The exclusion limit from Møller scattering is comparable to that from fermion pair production with leptons in the final state. This agrees with the results of reference [7].

In contrast to fermion pair production, the polarization of both beams is important for the exclusion limits in Møller scattering [8]. However, it improves the limits only quantitatively. An equivalent gain could be "bought" by unpolarized beams with approximately 6 times higher Ls.

As Møller scattering, Bhabha scattering has a definite final state. With an unpolarized positron beam, the constraints from Bhabha scattering are weaker than those from Møller scattering [9]. However, Bhabha scattering does not need a special option of the linear collider. A polarized positron beam is expected to improve the Z' constraints from Bhabha scattering.

3.2 Model Measurements

Bhabha and Møller scattering can only constrain the couplings of the Z' to electrons. Fermion pair production and Møller scattering are complementary in model measurements involving these couplings [6]. Fermion pair production reduces the sign ambiguities present in the measurements with Bhabha and Møller scattering. The estimate (6) for leptons in the final state applies.

4 $e^+e^- \to W^+W^-$

The individual interferences between the different amplitudes to W pair production rise like s. The two leading powers in s cancel due to gauge cancellations making the total cross section proportional to $\ln s/s$. Due to the $SU_L(2)$ gauge invariance, a Z' can couple to a W pair only in presence of a non-zero ZZ' mixing. These additional contributions to W pair production destroy the cancellation mechanism leading to a huge enhancement of the sensitivity to a Z'.

A Z' signal in W pair production can be absorbed [10] in (s dependent) anomalous couplings of the photon and the Z boson to the W pair. W pair production can constrain the combinations $a_e(Z')\theta_M$ and $v_e(Z')\theta_M$ and not the couplings of the Z' to electrons and the ZZ' mixing angle separately [11].

4.1 Exclusion Limits

Polarized electron beams are necessary to constrain *all* Z' models. Without polarized beams, an infinite band in the $a_e(Z')\theta_M, v_e(Z')\theta_M$ plane would be allowed [11]. Polarized positron beams would not give a further improvement. Assuming $v_e(Z') \approx v_e(Z)$ and $a_e(Z') \approx a_e(Z)$, one can estimate the sensitivity to θ_M as [4]

$$|\theta_M| < 3.4 \cdot \frac{\Delta\sigma_T}{\sigma_T} \cdot \frac{M_Z^2}{s} \frac{g_1}{g_2} \Re e \left| 1 - \frac{s - M_Z^2}{s - M_2^2 + i\Gamma_2 M_2} \right|^{-1} \sim \left[\frac{1 + r^2}{Ls} \right]^{1/2} , \qquad (9)$$

where σ_T is the total cross section. $\Delta\sigma_T$ in the estimate is dominated by systematic errors. Again, the best Z' constraints are obtained by fits to the angular distribution. The factor $\Re e(\ldots)$ is one for $s \ll M_2^2$ and approximately $2\Gamma_2/M_2$ for $\sqrt{s} = M_2 \pm \Gamma_2/2$. The exclusion limit is as sensitive to systematic errors as the model measurement in fermion pair production. Assuming that future colliders will measure W pair production with a certain relative error, we see that the sensitivity to θ_M is enhanced by the factor M_Z^2/s. It is enhanced by the additional factor $2\Gamma_2/M_2$ in measurements near the Z_2 peak [11]. These enhancement factors overcompensate the effects of the larger statistics in fermion pair production. As a result, the sensitivity of W pair production to θ_M is much larger than that of fermion pair production at the Z_2 peak [12].

4.2 Model Measurements

One could try a Z' model measurement in the case of a non-zero ZZ' mixing. The error of such a measurement is expected to scale with L, s and r as (9).

5 Conclusion

A linear collider allows to study Z' effects in different reactions. Off resonance fermion pair production, Bhabha and Møller scattering constrain ratios of Z' couplings and the Z' mass. They are insensitive to ZZ' mixing. For a fixed Z' model, the mass exclusion limits from fermion pair production are better than those from Bhabha or Møller scattering. If there are no model assumptions linking the couplings of the Z' to leptons and quarks, the exclusion limits from fermion pair production, Bhabha and Møller scattering will be comparable. Fermion pair production, Bhabha and Møller scattering are complementary in a model measurement involving the couplings of the Z' to electrons.

W pair production is very sensitive to ZZ' mixing. In a fixed model, the ZZ' mixing angle and the Z' mass are linked. Then, the Z' constraints from all considered reactions can be compared.

Polarized electron beams give important improvements to Z' exclusion limits except fermion pair production. They are very important in all reactions for model measurements. Polarization of the positron beam gives almost no improvement for a Z' search.

In all reactions, the sensitivity to a Z' depends on the combination $Ls/(1+r^2)$ only, and not on the integrated luminosity, the c.m. energy squared and the ratio of the systematic and statistical errors separately. Z' exclusion limits are always less sensitive to systematic errors than Z' model measurements.

References

[1] P. Chiappetta, C. Verzegnassi (conv.), Proc. of the LEP 2 workshop, CERN yellow report CERN 96-01, vol. 1, p. 577;
T.G. Rizzo, et al., SLAC-PUB-7365, hep-ph/9612440, to appear in the Proceedings of the 1996 DPF/DPB Summer Study on New Directions for High Energy Physics-Snowmass 96;
see P.M. Zerwas (ed.), Proc. of the Workshop on "e^+e^- collisions at TeV energies: The physics potential", DESY 96-123D, p. 335, p. 345, p. 353 and p. 363.

[2] For a review see e.g. J.L. Hewett, T.G. Rizzo, Phys. Rep. **183** (1989) 193;
P. Langacker, M. Luo, A.K. Mann, Rev. Mod. Phys. **64** (1992) 87;
M. Cvetič, S. Godfrey, in "Electroweak Symmetry Breaking and Beyond the SM", T. Barklow, S. Dawson, H. Haber, J. Siegrist (eds), World Scientific 1995.

[3] A. Leike, S. Riemann, Z. Phys. **C75** (1997) 341.

[4] A. Leike, habilitation thesis, in preparation.

[5] T.G. Rizzo, preprint SLAC-PUB-7151, hep-ph/9604420.

[6] F. Cuypers, Nucl. Phys.**B474** (1996) 72.

[7] T.G. Rizzo, SLAC-PUB-7279, hep-ph/9609248, to appear in the Proceedings of the 1996 DPF/DPB Summer Study on New Directions for High Energy Physics-Snowmass 96.

[8] D. Choudhury, F. Cuypers, A. Leike, Phys. Lett. **B333** (1994) 531.

[9] F. Cuypers, hep-ph/9611336, to appear in the Proceedings of the 1996 DPF/DPB Summer Study on New Directions for High Energy Physics-Snowmass 96.

[10] J.-M. Frère, M. Tytgat, J.M. Moreno, J. Orloff, Nucl. Phys. **B429** (1994) 3.

[11] A.A. Pankov, N. Paver, Phys. Lett. **B393** (1997) 437.

[12] A.A. Pankov, N. Paver, Phys. Rev. **D48** (1993) 63.