hep-lat/9709037 15 Sep 1997

DESY 97–178
September 1997

Lattice Chiral Schwinger Model in the Continuum Formulation*

V. Bornyakov[a], G. Schierholz[b] and A. Thimm[c]

[a]Institute for High Energy Physics, RU-142284 Protvino, Russia

[b]Deutsches Elektronen-Synchrotron DESY, D-22603 Hamburg and D-15735 Zeuthen, Germany

[c]Institut für Theoretische Physik, Freie Universität Berlin, D-14195 Berlin, Germany

We pursue further an approach to lattice chiral fermions in which the fermions are treated in the continuum. To render the effective action gauge invariant, counterterms have to be introduced. We determine the counterterms for smooth gauge fields, both analytically and numerically. The final result is that the imaginary part of the effective action can be computed analytically from the lattice gauge field, while the real part is given by one half of the action of the corresponding vector model.

1. INTRODUCTION

In formulating chiral gauge theories on the lattice, it has been suggested [1] to discretize only the gauge fields and treat the fermions in the continuum. One starts from a lattice with spacing a. This is the lattice on which the simulations will be done. Then one constructs a finer lattice with spacing a_f. On this lattice one puts the fermions. Before one can do this, one has to extrapolate the gauge fields to the interior of the original lattice. This was done in [2]. The method makes use of Wilson fermions to remove the doublers. One then computes the effective fermionic action in the limit $a_f \to 0$, while keeping a fixed. This action will in general not be invariant under chiral gauge transformations, but it will already be close. So close that chiral gauge invariance can be restored by simply adding a few local counterterms to the action. For similar ideas see [3].

In this talk we shall restrict ourselves to gauge fields with zero topological charge. We start from the fermionic action

$$S_\pm = \frac{1}{2a}\sum_{x,\mu}\{\bar\psi(x)\gamma_\mu[(1+P_\pm U_\mu(x))\psi(x+\mu) - (1+P_\pm U_\mu^\dagger(x-\mu))\psi(x-\mu)]\} + S_W,$$

$$S_W = \frac{1}{2a}\sum_{x,\mu}\bar\psi(x)[2\psi(x) - U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu)]$$

with $P_\pm = (1\pm\gamma_5)/2$. Later on we will also consider an ungauged Wilson term, S_W, with $U_\mu \equiv 1$. The effective action is given by

$$\exp(-W_\pm) = \int \mathcal{D}\bar\psi\mathcal{D}\psi\,\exp(-S_\pm).$$

Let us denote the anomaly free effective action generically by the subscript a. We are now looking for an action of the form

$$W_\pm^\Sigma = W_\pm + \text{counterterms},$$

so that

$$\widehat{W}_a = \lim_{a_f\to 0} W_a^\Sigma$$

is invariant under chiral gauge transformations. It will then turn out that

$$\mathrm{Re}\widehat{W}_\pm = \frac{1}{2}(W+W_0),\ \mathrm{Im}\widehat{W}_a = \lim_{a_f\to 0}\mathrm{Im}W_a,$$

where W is the effective action of the corresponding vector model, and W_0 is the free action, both taken at $a_f \to 0$. The imaginary part of $W_\pm$ has been computed analytically [4]. It depends only on the zero gauge field modes (torons) of the background field (see below).

2. EFFECTIVE ACTION

For the extrapolation of the gauge fields we use [2]. The effective action is computed by

*Talk given by V. Bornyakov

means of the Lanczos method. The Lanczos vectors are re-orthogonalized after every iteration.

The gauge field in its most general form can be written

$$A_\mu(x) = \frac{2\pi}{L} t_\mu + \varepsilon_{\mu\nu}\partial_\nu \alpha(x) + \mathrm{ig}^{-1}(\mathrm{x})\partial_\mu \mathrm{g}(\mathrm{x}), \quad (1)$$

where it is assumed that $A_\mu(x) \in [-\pi, \pi)$, and where t_μ are the zero momentum modes (torons), $\partial^2\alpha(x) = F_{12}(x)$ and $g(x) \in U(1)$ is a gauge transformation. We assume periodic boundary conditions for the gauge fields and antiperiodic boundary conditions for the fermions.

Toron Field

Let us first consider the case

$$A_\mu(x) = \frac{2\pi}{L} t_\mu + \mathrm{ig}^{-1}(\mathrm{x})\partial_\mu \mathrm{g}(\mathrm{x}),$$

where $g(x)$ is a small gauge transformation. Under a small gauge transformation we understand a transformation that does not change t_μ. The effective action $\lim_{a_f \to 0} W_\pm$ is not gauge invariant. It is easy to identify the appropriate counterterm. It is

$$c \sum_x A_\mu^2(x). \quad (2)$$

The coefficient c can be computed analytically. We find $c = -0.0202$. We will use this counterterm throughout the paper. In Fig. 1 we plot the real and imaginary part of W_-^Σ as a function of $(a_f/a)^2$ for a particular toron field and $g(x) = 0$. We do not expect anomalous contributions in this case, so it is legitimate to consider one species of fermions only. These results are compared with $(W + W_0)/2$ and the analytical result for $\mathrm{Im}\widehat{W}_-$ [4]. We see that the real part converges rapidly to $(W+W_0)/2$, while the imaginary part is practically equal to its analytic value for all a_f. We have also considered an ungauged Wilson term. In this case the imaginary part converges less rapidly to its continuum value. We find that real and imaginary part of the effective action are gauge invariant in the limit $a_f \to 0$. We will show a picture in the next section.

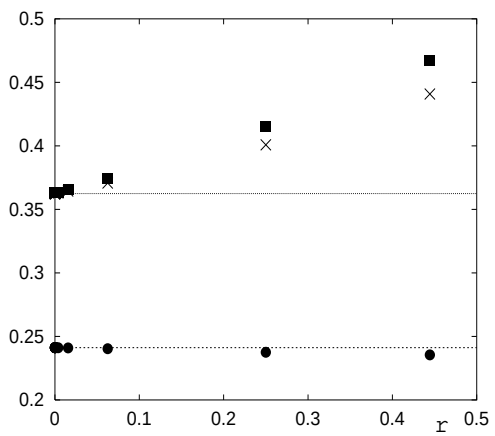

Figure 1. $\mathrm{Re}W_-^\Sigma - W_0$ (■), $\mathrm{Im}W_-$ (●) as well as $(W - W_0)/2$ (×) as a function of $r = (a_f/a)^2$. The lines are the continuum results.

Fluctuating Fields

Next we consider a general gauge field as given by (1) with $F_{12} \neq 0$. This configuration has been generated by a Monte Carlo method at $\beta = 6.0$ on a lattice small enough to avoid singular plaquettes. The average plaquette value at this coupling was ≈ 0.9. With the counterterm (2) we find similar results as in Fig. 1. In particular, we find that $\mathrm{Im}\widehat{W}_a$ is in complete agreement with the analytic result, meaning that the imaginary part depends alone on the magnitude of the toron field t_μ.

To test for gauge invariance we applied small random gauge transformations to the gauge field. To monitor the variation of the effective action under such gauge transformations we introduce the measure

$$\Delta X = \frac{1}{N} \sum_{\{g\}} |X^g - X|, \; X = \mathrm{Re}W_\pm, \mathrm{Im}W_{\pm,a},$$

where the sum is over a set of N gauge transformations, X is the initial result, and X^g is the result after the gauge transformation g. The effect of these gauge transformations is shown in Fig. 2. We see that the real part of W_-^Σ becomes gauge invariant in the limit $a_f \to 0$. For the imaginary part we have to distinguish between the anomalous and the anomaly free model. In the anomalous case the imaginary part is not gauge invari-

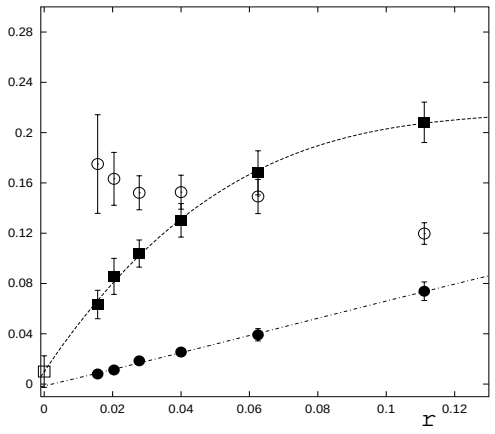

Figure 2. $\Delta\mathrm{Im}W_-$ (○), $\Delta\mathrm{Im}W_a$ (●) and $\Delta(\mathrm{Re}W_-^\Sigma - W_0)$ (■) as a function of $r = (a_f/a)^2$. The lines are polynomial fits.

ant, and that was also not expected. For the anomaly free model we take $e_- = (1,1,1,1)$ and $e_+ = 2$, $e_\pm$ being the charges of the right and left handed fermions, respectively. In this case we find that the imaginary part of the effective action is gauge invariant in the limit $a_f \to 0$.

Toron Field in Singular Gauge

Let us now go back to the toron field and allow for a gauge transformation which transforms $A_\mu(x)$ to

$$A_\mu(x) = 2\pi\delta_{x_\mu,1} t_\mu \ [\mathrm{mod}\, 2\pi].$$

The reason for considering such a transformation was to test our result for the imaginary part of the effective action under different conditions. Suppose that $|t_\mu| > 1/2$. For charge 2 it is sufficient that $|t_\mu| > 1/4$. For the anomalous model we then find that the analytic result changes under this transformation, unlike in the previous case, due to the compactness of the gauge field. Our lattice results show exactly this behavior. The real part of the effective action was found to be gauge invariant.

Vortex-Antivortex Configuration

Another large gauge transformation which changes t_μ is $g(x) = \exp(\mathrm{i}h(x))$ with

$$h(x) = 2\pi[\frac{x_2 - v_2 + 1}{L} + \frac{1}{2}][\theta(x_1 - v_1 - 1) - \theta(x_1 - \bar{v}_1 - 2)] - 2\pi[\frac{x_1 - \bar{v}_1 - 1}{L} + \frac{1}{2}] \times[\theta(x_2 - v_2) - \theta(x_2 - \bar{v}_2 - 1)],\ [\mathrm{mod}\, 2\pi]. \quad (3)$$

This transformation creates a vortex-antivortex pair at $x = v$ and $\bar{v}$, respectively. With the counterterm (2) it turns out that under this gauge transformation neither the real part, nor the imaginary part of the effective action are invariant in the limit $a_f \to 0$. This holds for the anomalous as well as for the anomaly free model. The good news is however that the imaginary part agrees with the analytic result which changes by exactly the same amount. As far as the real part is concerned, this indicates that further counterterms containing derivatives of the gauge field are needed to restore gauge invariance. It should be noted that this problem does not exist in the case of non-compact gauge field action because there (3) is not a gauge transformation.

3. CONCLUSIONS

For smooth gauge fields we have found an action which is ready to use in numerical simulations. The real part of the effective action can be expressed in terms of the action of the corresponding vector model, while the imaginary part can be computed analytically from the lattice gauge field.

REFERENCES

1. M. Göckeler and G. Schierholz, Nucl. Phys. B (Proc. Suppl.) 29B,C (1992) 114, 30 (1992) 609; G. 't Hooft, Phys. Lett. B349 (1995) 491.
2. M. Göckeler, A.S. Kronfeld, G. Schierholz and U.-J. Wiese, Nucl. Phys. B404 (1993) 839.
3. P. Hernandez and R. Sundrum, Nucl. Phys. B455 (1995) 287; G. T. Bodwin, Phys. Rev. D54 (1996) 6497.
4. L. Alvarez-Gaumé, G. Moore and C. Vafa, Commun. Math. Phys. 106 (1986) 1; M. Göckeler and G. Schierholz, unpublished (1994); R. Narayanan and H. Neuberger, Phys. Lett. B348 (1995) 549.