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Spin Structure Functions from Lattice QCD*

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Abstract

We report on new results of the spin dependent structure functions g_1 and h_1 of the nucleon. An attempt is made to convert the moments, which is what one computes on the lattice, to quark distribution functions.

1 Introduction

In the past polarization data have often been the graveyard of fashionable models. Measurements of the polarized deep-inelastic structure functions of the nucleon over

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the last decade have once again borne out this experience. While the naive quark parton model has been very successful in predicting the gross features of hadrons, it was a surprise to find that it fails to explain the spin properties of the nucleon.

There is significant interest in an *ab initio* calculation of the nucleon structure functions now. The theoretical framework of such a calculation is the operator product expansion. While the Wilson coefficients can be computed perturbatively, the hard part of the calculation is the determination of the forward nucleon matrix elements of the operators. This is a non-perturbative problem, and the technique to solve it is lattice QCD. For recent work on the subject see [1]. A confrontation of the experimental data with the lattice predictions will be a crucial test of our understanding of the structure of the nucleon.

At the twist two level the quark sector of the nucleon is completely specified by the spin averaged structure functions $F_1(x,Q^2)$, $F_2(x,Q^2)$ and the polarized structure functions $g_1(x,Q^2)$, $h_1(x,Q^2)$. In this talk we will report on new results of the polarized structure functions.

The talk is organized as follows. In sec. 2 we introduce the notation. In sec. 3 we give the results for the moments of g_1 and h_1. We furthermore give an extrapolation of the axial vector coupling of the nucleon, g_A, to the continuum limit. The moments can be converted into real structure functions by an inverse Mellin transform. We will present first results on $g_1(x,Q^2)$ in sec. 4. Finally, in sec. 5 we conclude.

2 Basics

The structure functions g_1 and h_1 have simple parton model interpretations. The structure function g_1 measures the quark helicity distribution in a longitudinally polarized nucleon, while h_1 measures the probability of finding a quark in a spin eigenstate of the operator $\not{s}_\perp\gamma_5$ in a transversely polarized nucleon.

In the following we shall consider only *non-singlet* structure functions and distributions. Only these distribution functions are accessible in quenched lattice QCD.

Let us first consider the structure function g_1. If the sea is assumed to be flavor symmetric, the leading twist contribution can be written

$$g_1(x,Q^2) = \frac{1}{2}\sum_f \int_x^1 \frac{\mathrm{d}y}{y} c_1^{(f)}\left(\frac{x}{y},\frac{Q^2}{\mu^2}\right)\Delta q^{(f)}(y,\mu), \tag{1}$$

where

$$\Delta q^{(f)}(x,\mu) = q_\uparrow^{(f)}(x,\mu) - q_\downarrow^{(f)}(x,\mu), \tag{2}$$

and $q_{\uparrow(\downarrow)}^{(f)}$ is the probability distribution of a quark of flavor f and spin parallel (anti-parallel) to the parent spin of the nucleon. The so-called splitting functions $c_1^{(f)}$ are

determined by the Wilson coefficients

$$c_{1,n}^{(f)}\left(\frac{Q^2}{\mu^2}\right) = \int_0^1 \mathrm{d}x x^n c_1^{(f)}\left(x, \frac{Q^2}{\mu^2}\right) \tag{3}$$

through an inverse Mellin transform. Similarly, the distribution functions can be derived from their moments,

$$\Delta^{(n)}q^{(f)}(\mu) = \int_0^1 \mathrm{d}x x^n \Delta q^{(f)}(x,\mu). \tag{4}$$

We denote the lowest moment by

$$\Delta q^{(f)} \equiv \Delta^{(0)}q^{(f)}. \tag{5}$$

According to the operator product expansion the moments are given by forward nucleon matrix elements of local operators. For $\Delta^{(n)}q^{(f)}$ we have

$$\langle \vec{p},\vec{s}|\mathcal{O}^{5(f)}_{\{\sigma\mu_1\cdots\mu_n\}}|\vec{p},\vec{s}\rangle = \frac{2}{n+1}\Delta^{(n)}q^{(f)}[s_\sigma p_{\mu_1}\cdots p_{\mu_n} + \cdots - \text{traces}], \tag{6}$$

where

$$\mathcal{O}^{5(f)}_{\{\sigma\mu_1\cdots\mu_n\}} = \left(\frac{\mathrm{i}}{2}\right)^n \bar{q}^{(f)}\gamma_\sigma\gamma_5 \overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} q^{(f)} - \text{traces}. \tag{7}$$

Here $\{\cdots\}$ means symmetrization. The lowest moment $\Delta q^{(f)}$ measures the *axial vector charge* of the nucleon.

Similar expressions can be derived for the structure function h_1 [2]. One simply has to replace $\Delta q^{(f)}(x,\mu)$ in eq. (1) by

$$\delta q^{(f)}(x,\mu) = q_\perp^{(f)}(x,\mu) - q_\top^{(f)}(x,\mu), \tag{8}$$

where $q_{\perp(\top)}^{(f)}$ is the probability distribution of a quark of flavor f and spin $\not{s}_\perp\gamma_5$ parallel (anti-parallel) to the spin of the nucleon, giving

$$\delta^{(n)}q^{(f)}(\mu) = \int_0^1 \mathrm{d}x x^n \delta q^{(f)}(x,\mu). \tag{9}$$

We denote the lowest moment by

$$\delta q^{(f)} \equiv \delta^{(0)}q^{(f)}. \tag{10}$$

The moments are given by the matrix elements

$$\langle \vec{p},\vec{s}|\mathcal{O}^{(f)}_{\sigma\{\mu_1\cdots\mu_n\}}|\vec{p},\vec{s}\rangle = \frac{2}{m_N}\delta^{(n-1)}q^{(f)}[(s_\sigma p_{\mu_1} - s_{\mu_1}p_\sigma)p_2\cdots p_{\mu_n} + \cdots - \text{traces}], \tag{11}$$

where

$$\mathcal{O}^{(f)}_{\sigma\{\mu_1\cdots\mu_n\}} = \left(\frac{\mathrm{i}}{2}\right)^{n-1} \bar{q}^{(f)}\sigma_{\sigma\mu_1}\gamma_5 \overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} q^{(f)} - \text{traces}. \tag{12}$$

The lattice operators, which are computed at the scale $1/a$ (a: lattice constant), must be renormalized and brought into the same scheme in which the Wilson coefficients have been calculated. Generically we can write

$$\mathcal{O}_i(\mu) = Z_{ij}\mathcal{O}_j(a), \tag{13}$$

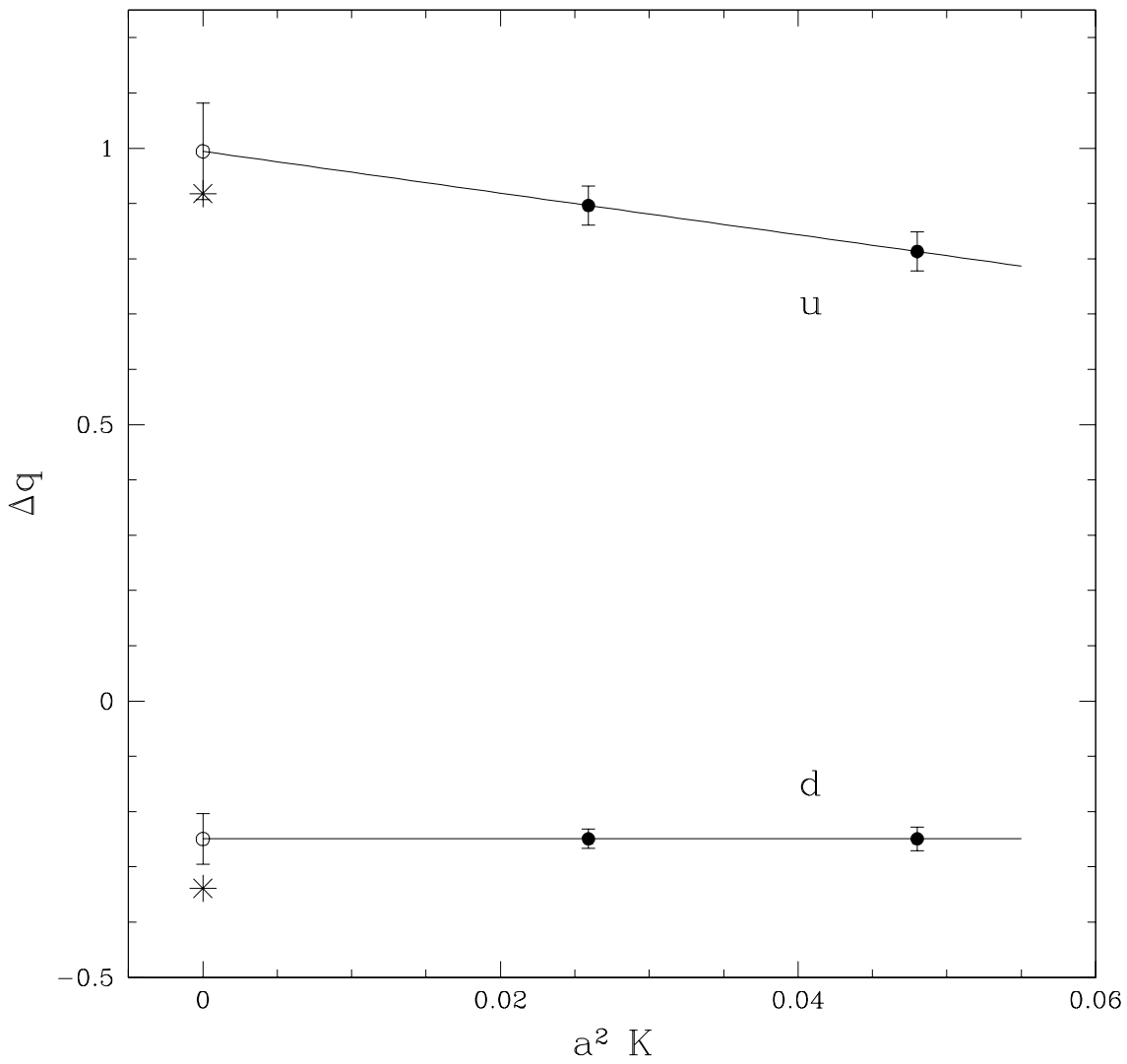

Figure 1: The quenched moments Δu and Δd plotted as a function of a^2. The lattice spacing is given in units of the string tension, K. The lattice data are denoted by $\bullet$, the extrapolated values by $\circ$. The phenomenological values [8] are denoted by $\ast$.

where the indices distinguish between the various lattice operators which can (and do) mix under renormalization. It is a task of its own to compute the renormalization constants accurately. For details of the calculation we refer the reader to the literature [3] and the talk of Schiller [4].

The lowest moment $\delta q^{(f)}$ measures the *tensor charge* of the nucleon. In the non-relativistic quark model axial vector and tensor charges are equal, i.e. $\delta q^{(f)} = \Delta q^{(f)}$. The structure function h_1 has opposite chiral properties to g_1. It can be measured in polarized Drell-Yan processes, but not in deep-inelastic lepton-hadron scattering. For other ideas see [5].

3 Moments

The quantities to be computed on the lattice are the moments (4) and (9). We have new results on $\Delta q^{(f)}$ and g_A, on $\Delta^{(1)} q^{(f)}$ and on $\delta q^{(f)}$. In the quenched approximation, which we are using, $f = u, d$, and we write $q^{(u)} = u$ and $q^{(d)} = d$. In the following we drop the argument μ from the quark distributions.

Δq

In lattice calculations one imagines the high frequency modes higher than the cut-off being integrated out. The logarithmically singular contributions of these modes are absorbed into the bare parameters, such as the coupling constant, while the power-behaved contributions are usually left unaccounted for. For Wilson fermions, which are widely used, the power corrections are of $O(a)$. By improving the action, these corrections can be reduced to $O(a^2)$ [6], thus giving results which are closer to the continuum limit $a = 0$.

Removing $O(a)$ effects from the matrix elements requires improving the operators as well. For the axial vector current the improved operator, and its renormalization constant, have been derived recently [7]. We will make use of these results here.

We have done calculations at two values of the coupling, $\beta = 6.0$ and 6.2, corresponding to lattice spacings of $a \approx 0.1$ and 0.07 fm, respectively. The results for Δu and Δd are plotted in Fig. 1. The values given refer to the chiral limit, i.e. zero quark mass, which are obtained from the lattice data by a suitable extrapolation.

As the remaining errors are of $O(a^2)$, we may fit the cut-off dependence by a formula of the form $c_0 + c_2 a^2$, and use this formula to extrapolate the result to the continuum ($a = 0$) limit. The outcome is shown by the solid lines. Clearly, we would have liked to have at least one more data point at another value of the coupling. We are working on that. We compare our results with the phenomenological valence quark distribution functions [8]. The agreement is good, considering that the errors on the phenomenological values are of the order of 10%.

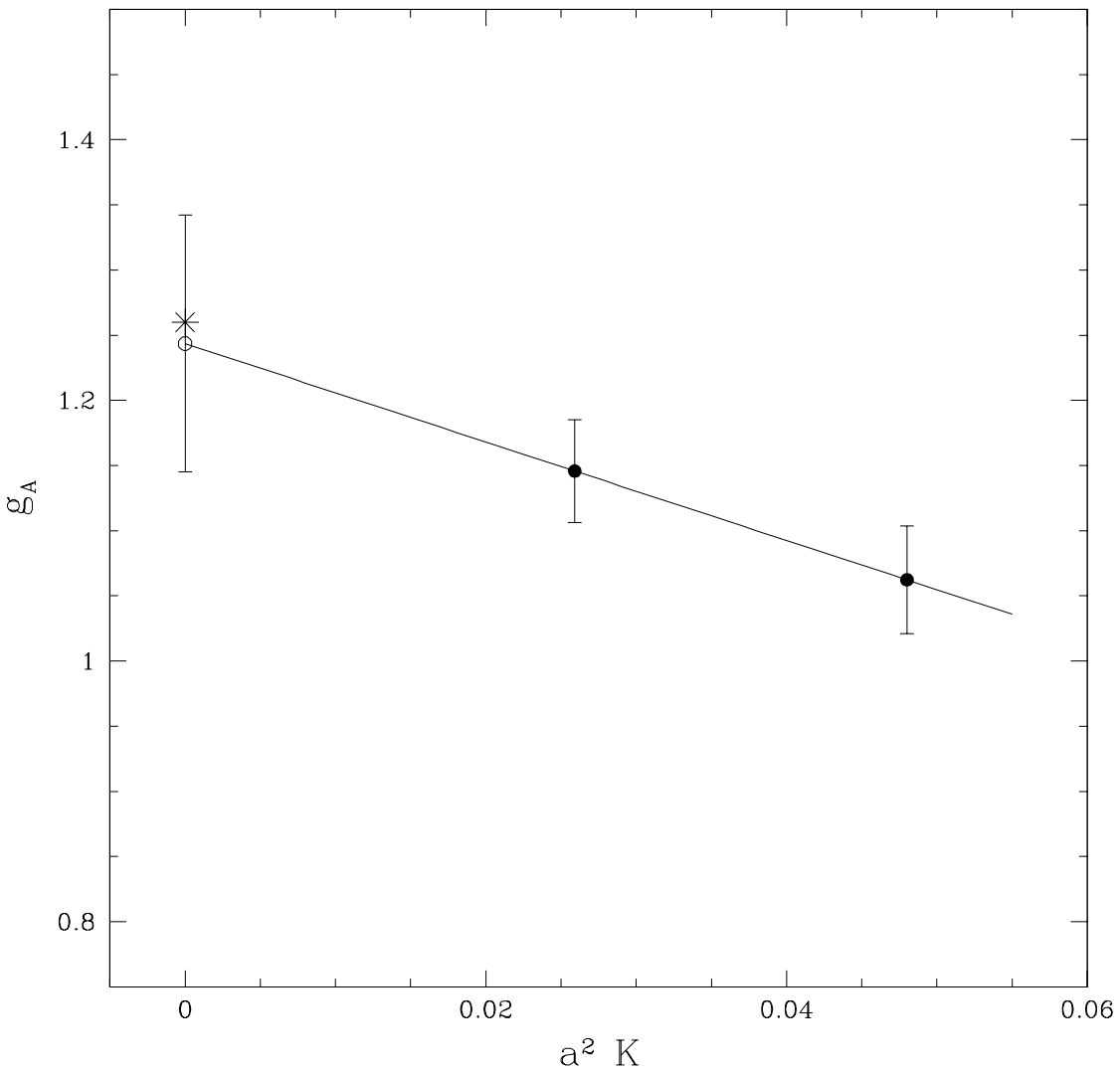

Figure 2: The axial vector coupling of the nucleon g_A as a function of a^2. The lattice spacing is given in units of the string tension, K. The lattice results are denoted by ●, the extrapolated values by ○. The experimental value is denoted by ∗.

g_A

A quantity, which is known very precisely experimentally, is the axial vector coupling of the nucleon,

$$g_A = \Delta u - \Delta d = 1.26. \tag{14}$$

In Fig. 2 we show our results for g_A, together with the extrapolation of the lattice data to the continuum limit. We find the continuum result to be in good agreement with the experimental value.

This figure indicates quite clearly how important it is to correct for finite cut-off effects. Even after improving the action and the operators, cut-off corrections can be quite substantial still. We see that the lattice result increases by approximately 20% going from our coarsest lattice at $\beta = 6.0$ to the continuum limit.

δq

The moments δu, δd have been computed using the improved action, but so far the operator has not been improved. We are currently working on this problem. For the renormalization constant we have taken the tadpole improved [9] one-loop perturbative result [10]. This means that the $O(a)$ corrections have not completely been removed in this case, as opposed to the previous case. The result of the calculation is shown in Fig. 3. This work adds another value of β to our previous calculation [11].

Before we discuss the result, let us find out what the tensor charge actually tells us about the structure of the nucleon. For a stationary nucleon the operator (12) differs from (7) by a factor of γ_0. In the non-relativistic limit fermions are in eigenstates of γ_0, and so δq and Δq are equal. By comparing δq and Δq for a real nucleon, we can gain insight into how relativistic the constituents are.

Comparing δq and Δq in Figs. 1, 3 now, we see that they are equal within the error bars. This shows that a non-relativistic description of the spin structure of the nucleon is quite adequate. Does this mean that the quarks are in a relative s-wave and the missing spin is coming from the gluons and the sea quarks? Further investigations will have to show.

Because the operator (12) with $n = 1$ is odd under charge conjugation, sea quarks do not contribute to δq. This means that we might hope that the quenched calculation is giving an answer close to the true value.

$\Delta^{(1)} q$

Another quantity which receives contributions from the valence quarks only is the second moment $\Delta^{(1)} q$. This moment is found from the $n = 1$ case of eqs. (6), (7). Again, the reason is that the operator is odd under charge conjugation.

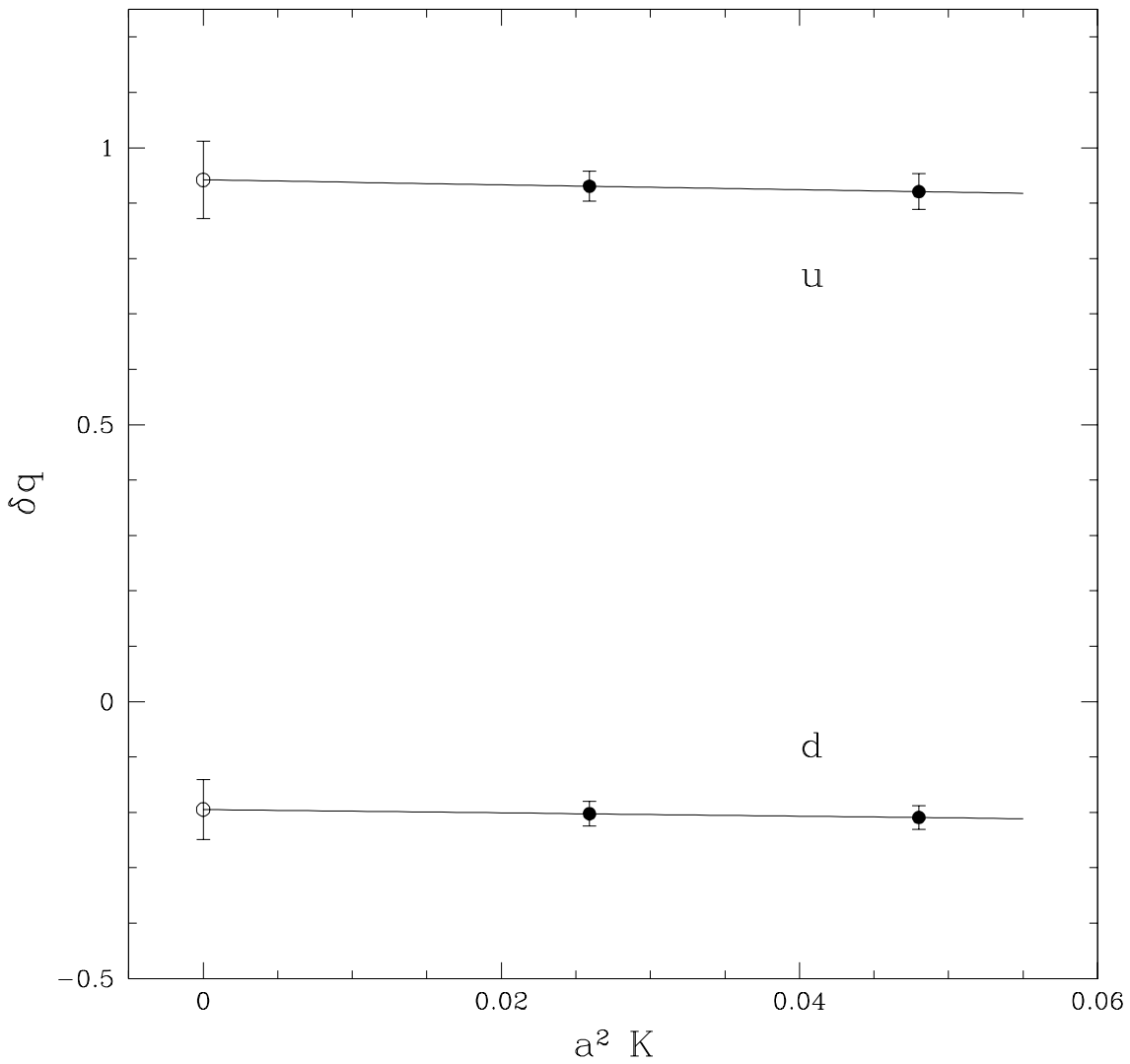

Figure 3: The quenched moments δu, δd as a function of a^2. The lattice spacing is given in units of the string tension, K. The lattice results are denoted by ●, the extrapolated values by ○. The numbers are renormalized at the scale $\mu^2 = 4\ \mathrm{GeV}^2$.

The special feature of this moment is that it is directly accessible experimentally, which allows us to test the valence quark distribution without further phenomenological analysis.

For the polarization asymmetry [12] of π^+ minus π^- inclusive cross sections one finds to lowest order in α_s

$$A_p^{\pi^+-\pi^-} = \frac{4\Delta u^{\rm val}(x) - \Delta d^{\rm val}(x)}{4u^{\rm val}(x) - d^{\rm val}(x)} \tag{15}$$

for a proton target, and

$$A_d^{\pi^+-\pi^-} = \frac{\Delta u^{\rm val}(x) + \Delta d^{\rm val}(x)}{u^{\rm val}(x) + d^{\rm val}(x)} \tag{16}$$

for a deuteron target. The fragmentation functions, as well as the sea quark contributions, drop out because of isospin invariance relating the various fragmentation functions with each other.

The polarization asymmetries have been measured by the SMC-Collaboration. For the lowest moment of the valence quark distribution they found [13] $\Delta u^{\rm val} = 1.01 \pm 0.19 \pm 0.14$ and $\Delta d^{\rm val} = -0.57 \pm 0.22 \pm 0.11$. The experimental errors are still a little too large to make a quantitative comparison with the lattice results. The SMC-Collaboration has recently extended their analysis to the second moment [14]. At $\mu^2 = 10\ \mathrm{GeV}^2$ they obtain the result

$$\Delta^{(1)}u^{\rm val} = 0.169 \pm 0.018 \pm 0.012, \tag{17}$$

$$\Delta^{(1)}d^{\rm val} = -0.055 \pm 0.027 \pm 0.011. \tag{18}$$

A recent lattice calculation, renormalized at the same scale, gives [15]

$$\Delta^{(1)}u = 0.189 \pm 0.08, \tag{19}$$

$$\Delta^{(1)}d = -0.0455 \pm 0.0032. \tag{20}$$

The lattice and experimental results agree within their respective errors.

4 Distribution Functions

The x-dependence of the structure functions carries valuable information about the dynamics of quarks and gluons which is not immediately available from the moments. Furthermore, because of limited experimental data, moments are sometimes hard to compare with experiment. This is, in particular, the case for the higher moments. Theoretically, the structure functions can be obtained from the moments by an inverse Mellin transform.

A first attempt of constructing nucleon structure functions from a few lower moments was reported in [16] for the unpolarized case. In this talk we shall consider a

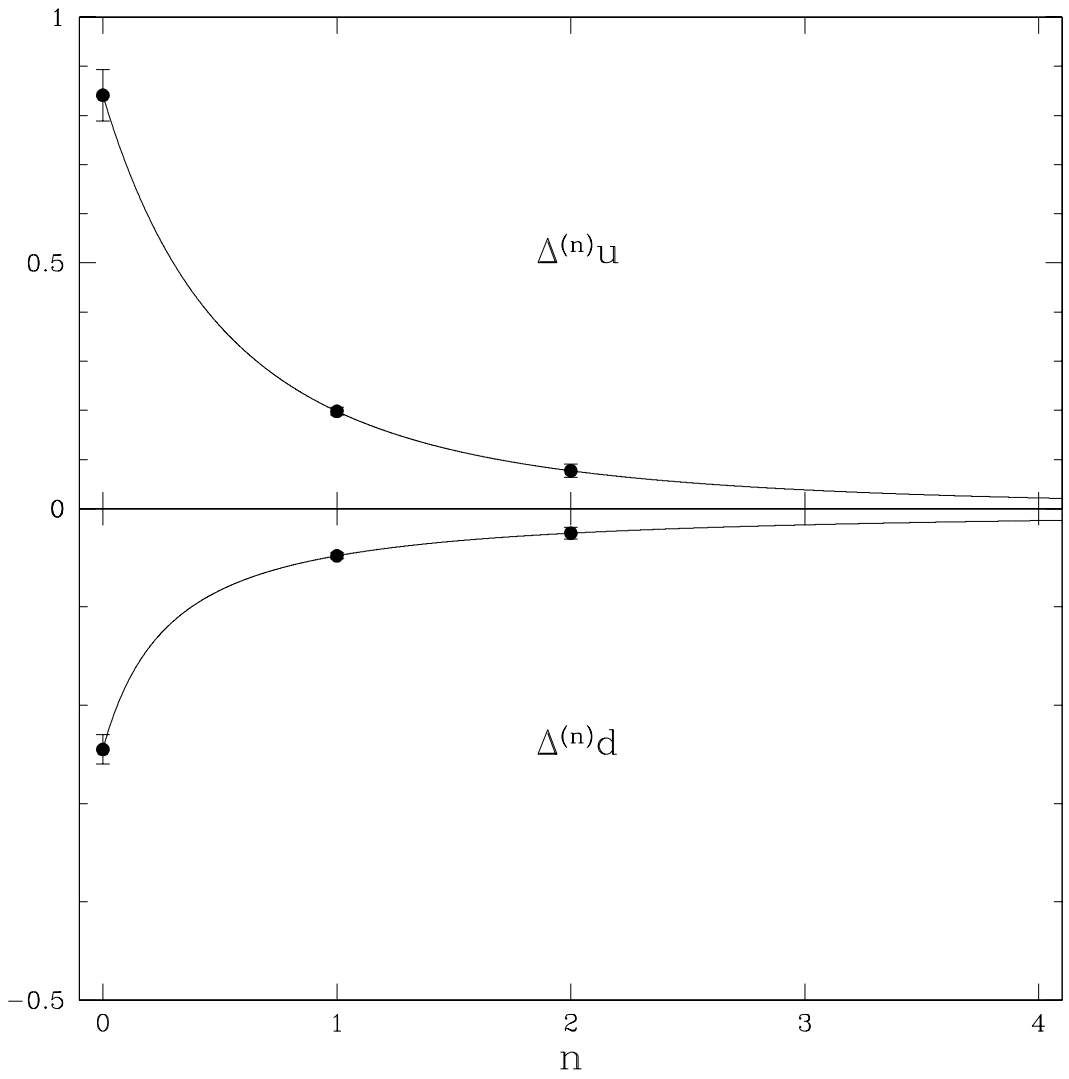

Figure 4: The lattice results for the first three moments of $\Delta u(x)$ and $\Delta d(x)$ at $\beta = 6.0$ and $\mu^2 = 4$ GeV^2. The curves are fits to the lattice data of the form (21). The parameters of the fit are $\alpha = 0.04(6)$ and $\beta = 2.21(9)$.

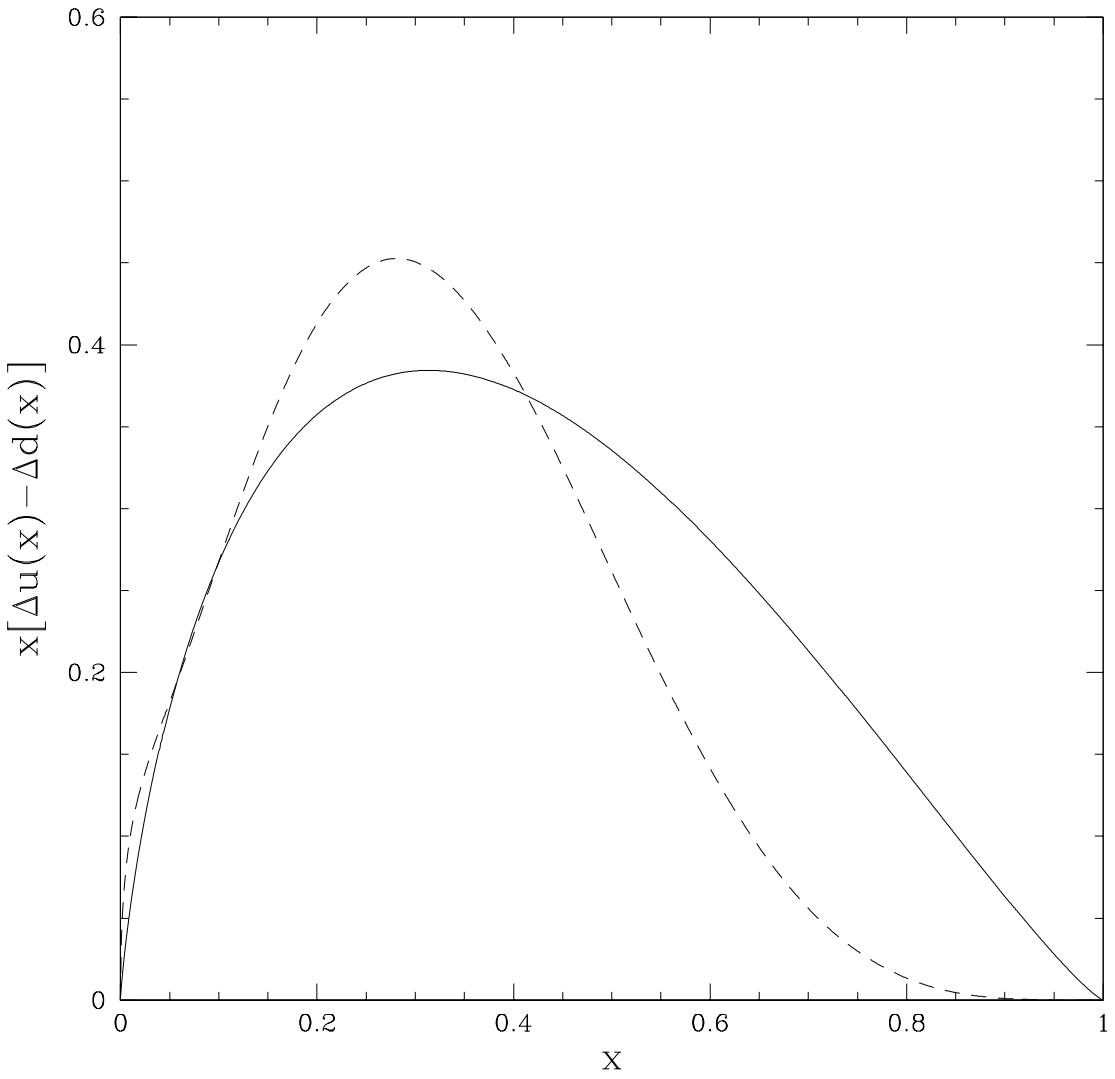

Figure 5: The distribution $x[\Delta u(x) - \Delta d(x)]$, together with the phenomenological valence quark distribution. The solid line is the result of the fit, the dashed line is taken from ref. [8].

different method and apply it to g_1.

We restrict ourselves to the coarser lattice at $\beta = 6.0$. We have computed the three lowest moments. Calculation of a fourth moment is possible. The moments can be well described by the formula

$$\Delta^{(n)}q = c\,\Gamma(\beta)(n+1+\alpha)^{-\beta}. \tag{21}$$

This formula seems also to describe the moments of the unpolarized structure functions well. A fit is shown in Fig. 4. An inverse Mellin transform of (21) gives

$$\Delta q(x) = c\,x^{\alpha}(-\ln x)^{\beta-1}. \tag{22}$$

In Fig. 5 we compare the result for $\Delta u(x) - \Delta d(x)$ with the phenomenological distributions. The outcome is encouraging. We have hope that with one more moment and precise lattice data we are able to derive phenomenologically useful quark distribution functions.

It must be said that one can only combine even and odd moments to make a single structure function if u and d sea quark contributions are assumed to be equal, as is commonly done [8]. However, this may not be a good approximation [17].

5 Conclusions

Lattice calculations of nucleon structure functions have improved in many respects. The calculations are now done with improved actions and using improved operators, so as to reduce cut-off effects. Furthermore, the renormalization constants of the lattice operators, which are another source of errors, are gradually being computed non-perturbatively [7, 18]. On top of that, we have seen that it is important to do the calculation at several values of the coupling and do an extrapolation to $a = 0$. By adding one or two more data points, and with increased statistics, we will soon be ready to report reliable continuum results, at least in the quenched approximation.

Where we can compare the lattice results with experiment or the phenomenological analysis, we find good agreement. Our efforts over the last year have, in particular, paid off for g_A, the axial vector coupling of the nucleon. Two years ago this quantity was considered a problem for quenched lattice QCD [19].

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