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LIMITATIONS OF A STANDARD MODEL HIGGS BOSON *

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ABSTRACT

This contribution reviews the latest results of the perturbative calculations of heavy-Higgs amplitudes. A comparison of perturbative results with nonperturbative lattice calculations is made, and the theoretical uncertainties of the lower and upper bound on the Standard Model Higgs mass are presented.

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1. Introduction

The simplest model of breaking the electroweak gauge symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ spontaneously is the standard Higgs model. It consists of the spin-zero Higgs boson and three massless Goldstone bosons, the latter ultimately being absorbed by the weak gauge bosons. The standard Higgs model is regarded to be an effective theory, only valid up to a cutoff energy Λ. The maximal allowed value of Λ depends on the value of the Higgs mass M_H, and it is connected to the renormalization group (RG) behaviour of the Higgs sector.

In the absence of a more complete theory, it is important to understand the perturbative limitations of the standard Higgs model: if M_H is too large, perturbation theory ceases to be a useful tool for calculating physical observables of the Higgs sector, such as cross sections and Higgs decay width. This yields perturbative upper bounds on M_H. At the same time lattice calculations provide nonperturbative upper bounds on M_H.

Here we review these upper bounds, eventually comparing them with results for the lower bounds on M_H as obtained from vacuum stability requirements.

In Sect. 2 we briefly present the framework of the Higgs sector, introducing the Lagrangian and the Higgs running coupling. Sect. 3 presents the perturbative upper limits from decay amplitudes, while Sect. 4 is devoted to bounds from scattering processes. The latter can be more stringent since the running of the Higgs quartic coupling is involved. Sect. 5 summarizes the present-day status of both upper and lower Higgs mass constraints from theory as a function of the cutoff-scale Λ.

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2. The Lagrangian for heavy-Higgs radiative corrections

Neglecting gauge and Yukawa couplings, the Lagrangian of the standard model Higgs sector reduces to

$$\mathcal{L}_H = \tfrac{1}{2}\left(\partial_\mu \Phi\right)^\dagger \left(\partial^\mu \Phi\right) - \tfrac{1}{4}\lambda\left(\Phi^\dagger\Phi\right)^2 + \tfrac{1}{2}\mu^2\Phi^\dagger\Phi\,, \tag{1}$$

where

$$\Phi = \begin{pmatrix} w_1 + i w_2 \\ h + i z \end{pmatrix} = \begin{pmatrix} \sqrt{2}w^+ \\ h + i z \end{pmatrix}. \tag{2}$$

The doublet Φ has a nonzero expectation value v in the physical vacuum. To facilitate perturbative calculations, the field h is expanded around the physical vacuum, absorbing the vacuum expectation value by the shift $h \to H + v$. Hence the field H has zero vacuum expectation value. Rewriting Eq. (1), $\mathcal{L}_H$ takes the form

$$\mathcal{L}_H \;=\; \tfrac{1}{2}\partial_\mu \mathbf{w}\cdot\partial^\mu \mathbf{w} + \tfrac{1}{2}\partial_\mu H\,\partial^\mu H - \tfrac{1}{2}M_H^2 H^2 + \mathcal{L}_{3pt} + \mathcal{L}_{4pt}\,, \tag{3}$$

with the three-point and four-point interactions of the fields given by

$$\mathcal{L}_{3pt} \;=\; -\lambda v\left(\mathbf{w}^2 H + H^3\right), \tag{4}$$

$$\mathcal{L}_{4pt} \;=\; -\tfrac{1}{4}\lambda\left(\mathbf{w}^4 + 2\mathbf{w}^2 H^2 + H^4\right). \tag{5}$$

Here $\mathbf{w}$ is the SO(3) vector of Goldstone scalars, (w_1, w_2, w_3), with $w_3 = z$. The tadpole term and an additive constant have been dropped. The $w^\pm$ and z bosons are massless, in agreement with the Goldstone theorem. The Higgs mass M_H and the Higgs quartic coupling λ are related by

$$\lambda = M_H^2/2v^2 = G_F M_H^2/\sqrt{2}, \tag{6}$$

where G_F is the Fermi constant, and $v = 2^{-1/4}G_F^{-1/2} = 246$ GeV.

The Lagrangian $\mathcal{L}_H$ is the starting point for carrying out calculations using the equivalence theorem [1,2,3]. Using power-counting arguments it has been shown that radiative corrections to $O((G_F M_H^2)^n) = O(\lambda^n)$ can also be calculated with the aid of $\mathcal{L}_H$, that is, without having to use the full SM Lagrangian. [2] The implementation of proper renormalization conditions is however crucial.

Including the Yukawa couplings by adding the fermionic Lagrangian $\mathcal{L}_F$ to $\mathcal{L}_H$, the basic Lagrangian for the calculation of $O((G_F M_H^2)^n (G_F m_t^2)^m)$ corrections is obtained. For a heavy Higgs particle these are the leading *and* subleading electroweak corrections, and they can be calculated using *massless* Goldstone bosons, hence simplifying their calculation greatly. For a Higgs mass of less than approximately 300 GeV those

corrections are not leading anymore: [3] The contributions from gauge couplings need to be taken into account using the full SM Lagrangian.

3. The decay $H \to W^+W^-$, ZZ: radiative corrections in powers of $G_F M_H^2$

Using the limit $M_H \gg M_W$ the leading corrections to the bosonic decay of the Higgs have been calculated to two loops. [4,5] *A priori* it is unknown for which value of M_H the two-loop correction term $O(\lambda^2)$ will dominate over the one-loop term of $O(\lambda)$. Since $\lambda \propto M_H^2/(246\ \mathrm{GeV})^2$ the breakdown of perturbation theory might occur for values of M_H less than 1 TeV.

The calculations of two-loop corrections to $H \to W^+W^-$, ZZ [4,5] are pioneering work with regard to the use of numerical methods in the evaluation of Feynman diagrams of three-point functions. The work by Ghinculov [4] uses analytic cancellation of all ultraviolet divergencies using dimensional regularization. Infrared singularities are regularized using a small mass for the Goldstone bosons which is taking to zero in the final result. The finite contributions of the Feynman diagrams are obtained by numerical integration of Feynman-parameter integrals. The calculation by Frink *et al.* [5] features massless Goldstone bosons. Both UV and IR divergent Feynman diagrams are calculated analytically, including their finite contributions. The sum of these diagrams leads to the explicit cancellation of both types of divergencies. The non-divergent Feynman diagrams are calculated using numerical integration in orthogonal momentum space components. The two results [4,5] for the two-loop coefficients agree to 1.7×10^{-3}, indicating the reliability at which these numerical methods operate. In particular: [5]

$$\Gamma(H \to ZZ,\ W^+W^-) \propto \lambda(M_H) \left(1 + 2.800\ \ldots \frac{\lambda}{16\pi^2} + 62.030\,8(86) \frac{\lambda^2}{(16\pi^2)^2} \right) . \quad (7)$$

It is interesting to compare the size of the coefficients with the leading heavy-Higgs corrections calculated in the case of fermionic Higgs decay. Using numerical or analytical methods one obtains [6,7,8]

$$\Gamma(H \to f\bar{f}) \propto g_f^2 \left(1 + 2.117\ \ldots \frac{\lambda}{16\pi^2} - 32.656\ \ldots \frac{\lambda^2}{(16\pi^2)^2} \right) . \quad (8)$$

Comparing the last two equations we see that the coefficients of the perturbative series are of similar size. This is not the case for scattering processes; see below.

The K-factors of both bosonic and fermionic decay width are given by the expressions in the large brackets of Eq. (7) and (8), respectively. They are plotted in Fig. 1 as a function of M_H. The corrections are less than 10% for $M_H < 670$ GeV, and for $M_H = 980$ the corrections are less than 30%. Yet perturbation theory is not meaningful for large Higgs masses. At $M_H = 930$ GeV the two-loop bosonic correction term is as large as the one-loop term. In the fermionic case the two-loop correction term

compensates the one-loop term if $M_H \approx 1100$ GeV. A different criterion for judging the breakdown of perturbation theory is the investigation of scale and scheme dependence. [9,10] For the Higgs decay processes this leads to a perturbative bound on M_H of about 700 GeV. [10]

In the case of bosonic Higgs decay, we are also able to compare the perturbative results with nonperturbative computations carried out using lattice techniques. [11,12] The lattice result obtained [12] for $M_H = 727$ GeV appears to be consistent with the perturbative results for $H \to W^+W^-$; see Fig. 1. The difference between the two-loop perturbative and the nonperturbative result can probably be contributed to the missing higher-order perturbative correction terms and the use of massive instead of massless Goldstone bosons (pions) in the lattice calculation. A detailed comparison of these results is in preparation.

For completeness we also mention that the two-loop heavy-Higgs correction to the loop-induced process $H \to \gamma\gamma$ has also been calculated. [13]

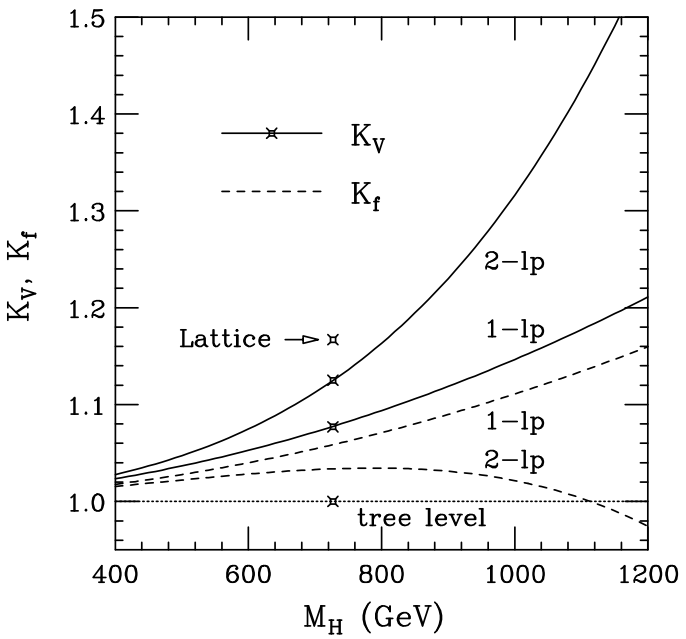

Fig. 1. The K-factors for $H \to W^+W^-$, ZZ and $H \to f\bar{f}$ at various orders in perturbation theory. For $M_H = 727$ GeV the bosonic perturbative K-factor is compared with a result from lattice calculations. [12]

4. Radiative corrections to scattering processes

Subprocesses such as $W^+W^- \to W^+W^-$ are important to extract experimental information on the Higgs resonance. The tree-level $O(\lambda \propto G_F M_H^2)$ contribution to this cross section is entirely due to the scattering of longitudinally polarized bosons,

$W_L^+W_L^- \to W_L^+W_L^-$. In the case of a heavy Higgs, this channel gives the dominant contribution to the cross section. The transverse polarizations only couple via gauge couplings and are suppressed as $g^2/\lambda \propto M_W^2/M_H^2$. In the case of $ZZ \to ZZ$, however, it has been shown that radiative gauge corrections can enhance the transverse channels significantly. [14] This is also expected to happen for $W^+W^- \to W^+W^-$.

The dominant heavy-Higgs corrections to longitudinal scattering amplitudes involving Z_L, W_L or H are known up to two loops. [15,16] In contrast to the Higgs decay amplitudes which only depend on one parameter (the coupling λ, or equivalently, the Higgs mass M_H), the $2 \to 2$ boson scattering amplitudes also depend on the center-of-mass energy $\sqrt{s}$ of the scattering process. In the high-energy limit terms of order M_H^2/s can be neglected, and the scattering amplitude exhibits a purely logarithmic energy dependence. The cross section for $W_L^+W_L^- \to W_L^+W_L^-$ in on-mass-shell (OMS) renormalization is [15,16]

$$\begin{aligned} \sigma(s) \;=\; \frac{1}{\pi s}\lambda^2 \Bigg[1 \,+\, \left(24 \ln \frac{s}{M_H^2} - \,48.64 \right) \frac{\lambda}{16\pi^2} \\ + \left(432 \ln^2 \frac{s}{M_H^2} - 2039.3 \ln \frac{s}{M_H^2} + \,3321.7 \right) \frac{\lambda^2}{(16\pi^2)^2} \,. \\ + \;\mathrm{O}\left(\lambda^3\right) + \;\mathrm{O}\left(\frac{M_H^2}{s}\right) \Bigg] + \;\mathrm{O}\left(g^2\right) \,. \end{aligned} \tag{9}$$

The coefficients found here are more than a factor 10 larger than the coefficients for the decay widths given in Eqs. (7) and (8).

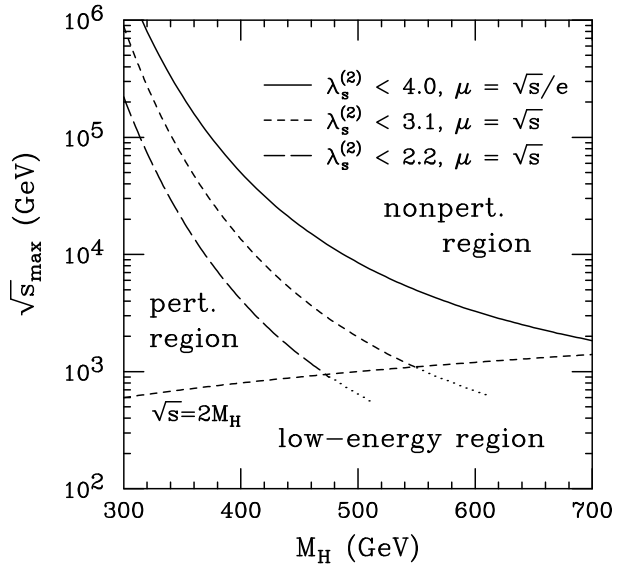

Fig. 2. The perturbative limits on $(M_H, \sqrt{s})$ found in high-energy scattering processes. The limits on the running coupling are derived with [20] (solid line) and without [17,18,16] (dashed lines) summation.

Applying renormalization-group methods, the logarithmic energy dependence can be absorbed into a running Higgs coupling, which at one loop is given by

$$\lambda(\mu) \;=\; \lambda(M_H)\left[1 - 12\,\frac{\lambda(M_H)}{16\pi^2}\ln\left(\frac{\mu^2}{M_H^2}\right)\right]^{-1} , \tag{10}$$

and $\lambda(M_H)$ is fixed by Eq. (6). Using the corresponding three-loop running coupling, [10] Eq. (9) can be rewritten as a next-to-next-to-leading-logarithmic (NNLL) cross section,

$$\sigma \;\propto\; \frac{1}{s}\lambda^2(\sqrt{s})\left(1 - 48.64\frac{\lambda(\sqrt{s})}{16\pi^2} + 3321.7\frac{\lambda^2(\sqrt{s})}{(16\pi^2)^2}\right) . \tag{11}$$

Here the natural choice $\mu = \sqrt{s}$ has been made, and a prefactor with a small s-dependence due to non-zero anomalous dimensions has been neglected. [16]

It is striking that the one-loop cross section is *negative* for the relatively low value of $\lambda(\sqrt{s}) \approx 3.2$. Using various criteria, the perturbative limit on λ can be given [10,16] as $\lambda(\sqrt{s}) \approx 2.2$.[a] Similar bounds on the running coupling are obtained using arguments concerning unitarity violations. [17,18] Tree-level unitarity bounds without the use of the running coupling are independent of $\sqrt{s}$ and less stringent: [19] They require $\lambda = \lambda(M_H) < 4\pi/3 \approx 4.2$.

Recently it has been discovered that the approximate summation of a subset of Feynman diagrams extends the range of validity of the perturbative results. [20] This summation corresponds to taking $\mu = \sqrt{s}/e \approx \sqrt{s}/2.7$ as the appropriate choice of scale in the running coupling.[b] The high-energy NNLL cross section then reads

$$\sigma \;\propto\; \frac{1}{s}\lambda^2(\sqrt{s}/e)\left(1 - 0.64\frac{\lambda(\sqrt{s}/e)}{16\pi^2} + 923.1\frac{\lambda^2(\sqrt{s}/e)}{(16\pi^2)^2}\right) . \tag{12}$$

The summed cross section is perturbative for much larger values of the running coupling. Using various criteria it has been concluded [20] that perturbative calculations in the Higgs sector are reliable for a *running* Higgs coupling up to $\lambda(\mu) \approx 4$, and perturbative unitarity is restored up to this value. This significantly extends the range in M_H and $\sqrt{s}$ for which high-energy calculations are reliable; see Fig. 2.

5. Renormalization-group behaviour of λ including all SM couplings

The one-loop running coupling introduced in the previous section, Eq. (10), is valid only if M_H is large. Increasing the scale μ, the coupling increases monotonically,

[a] Some authors use a different normalization of the Higgs potential, leading to a numerically different bound on the coupling. The bounds on the Higgs mass are unaffected by this redefinition.
[b] Starting from the $\overline{\rm MS}$ scheme, this choice of μ leads to the G-scheme. [21]

eventually approaching the Landau singularity. For small values of M_H the behaviour is different. In this case the contributions from gauge and Yukawa couplings need to be included. In particular, the presence of the top-quark Yukawa coupling g_t can cause the Higgs running coupling to decrease as μ increases, possibly leading to an unphysical negative Higgs coupling. This is due to the negative contribution of the top quark to the one-loop beta function of the Higgs coupling:

$$\beta_\lambda = 24\lambda^2 + 12\lambda g_t^2 - 6g_t^4 + \text{gauge contributions}, \tag{13}$$

where all couplings must be taken to be running couplings.

Requiring the Higgs coupling to remain finite and positive up to an energy scale Λ, constraints can be derived on the Higgs mass M_H. [22] Such analyses exist at the two-loop level for both lower [23,24] and upper [25,26] Higgs mass bounds. Since all Standard Model parameters are experimentally known except for the Higgs mass, the bound on M_H can be plotted as a function of the cutoff energy Λ. Taking the top quark mass to be 175 GeV and a QCD coupling $\alpha_s(M_Z) = 0.118$ the result is shown in Fig. 3.

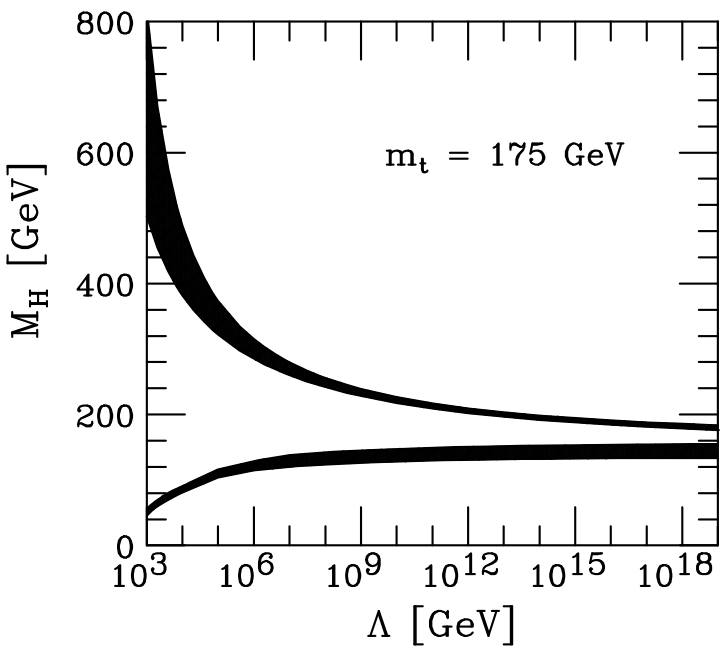

Fig. 3. The present-day theoretical uncertainties on the lower [23,24] and upper [26] M_H bounds when taking $m_t = 175$ GeV and $\alpha_s(M_Z) = 0.118$.

The bands shown in Fig. 3 indicate the theoretical uncertainties due to various cutoff criteria, the inclusion of matching conditions, and the choice of the matching scale. [26] If the Higgs mass is 160 to 170 GeV then the renormalization-group behaviour of the Standard Model is perturbative and well-behaved up to the Planck scale $\Lambda_{Pl} \approx 10^{19}$ GeV. For smaller or larger values of M_H new physics must set in below Λ_{Pl}.

6. Concluding remarks

The phenomenological aspects of a fundamental Higgs particle are well understood, and it seems a matter of time and money to prove (or disprove) its existence. Finding such a Higgs particle, however, would be just a first step. The multi-loop calculations presented in this and other talks of the workshop will enable us to check many properties of the Higgs boson, and comparison with experimental data hopefully provides us with new insight for the development of a more complete particle theory up to the Planck scale.

I would like to thank the organizers for giving me the opportunity to present a talk at this school. In particular, it was a great honor to give a talk on this topic while Professor Higgs himself was present.

Since the experimental discovery of the Higgs boson at a collider experiment may still take a long time, I would like to conclude with my amusing discovery of a different kind of Higgs Boson: http://homepages.enterprise.net/hboson/homefiles/higgsb.html.

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