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Radiative Leptonic Decays of B_c Meson

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Abstract

To the leading order, the radiative leptonic decays $B_c \to \gamma\ell\bar{\nu}$ ($\ell = e, \mu$) are studied carefully. In the study, a non-relativistic constituent quark model and the effective Lagrangian for the heavy flavour decays are used. As a result, the branching ratios turn out to be of the orders of 10^{-5} for $B_c \to \gamma\mu\bar{\nu}$ or for $B_c \to \gamma e\bar{\nu}$. Based on the study, we point out the decays being accessible experimentally at the future LHC, and the possibility to determine the decay constant f_{B_c} through the radiative decays.

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1 Introduction

The pure-leptonic decays of heavy mesons are very interesting from the point of not only theoretical but also experimental view [1, 2]. In principle, the pure-leptonic decays $\bar{B}_c \to \ell\bar{\nu}$ can be used to determine the decay constant f_{B_c}. It is also discussed that the pure-leptonic decays of B_c meson may be sensitive to new physics beyond the Standard Model (SM) at tree level [2]. Based on the estimates in [3, 4], except at Tevatron, numerous B_c mesons are hard to be produced in the current colliders, whereas at LHC a great number of B_c mesons, e.g. 2×10^8, may be produced. Thus we may expect that careful experimental studies on the B_c meson will be able to be accessible in the foreseeable future. In addition, the B_c meson decay channels can also contribute some background for probing $B^{\pm}$ decays with the same final states [5], thus, it makes an 'extra' reason to study these decay channels of the B_c meson precisely.

The decays of pseudoscalar mesons into light lepton pairs are helicity suppressed, i.e. their decay widths are suppressed by m_ℓ^2/m_B^2:

$$\Gamma(B_c \to \ell\bar{\nu}) = \frac{G_F^2}{8\pi^2}|V_{cb}|^2 f_{B_c}^2 m_{B_c}^3 \frac{m_\ell^2}{m_{B_c}^2}\left(1 - \frac{m_\ell^2}{m_{B_c}^2}\right)^2 . \tag{1}$$

Thus it is hard to collect enough events of the pure-leptonic decays i.e. it is very difficult to obtain very good statistics for the decays, so that it makes very difficult to determine the decay constant f_{B_c} from these processes. In SM, only the decay $B_c \to \tau\bar{\nu}_\tau$ does not suffer so much from this suppression and the branching ratio can be about 1.5% [1]. However the produced τ decays promptly and at least one more neutrino is generated when the cascade decay is taken into account, thus this decay channel is difficult to be identified. The experimental efficiency should be discussed in connection with a specific detector if one insists on using the τ leptonic decay for the purpose to determine the decay constant f_{B_c}.

In the present work, we will study the processes $B_c \to \gamma\ell\bar{\nu}$ within SM and with the effective Lagrangian for the heavy flavour decays. In the following section we will analyze $B_c \to \gamma\ell\bar{\nu}$ in the framework of a constituent quark model. Finally we will discuss the

obtained results briefly in the last section.

2 Model calculations

Because of the lightness of the leptons e and μ, the processes $B_c \to \ell\bar{\nu}$ are suppressed much by the helicity factor $m_l^2/m_{B_c}^2$ as in eqn.(1). If an additional photon line is attached to any of the charged lines of the Feynman diagrams for the pure-leptonic decays as done in Fig.1, i.e. the pure-leptonic processes change into the corresponding radiative ones. The situation will be different: now no helicity suppression exists any more, but there will be an additional α (the electro-magnetic coupling constant) suppression instead. To the radiative decays there are four tree diagrams to contribute, as shown in Fig. 1. It is easy to see that the fourth diagram (Fig.1d), in which the photon is emitted from the W boson, is suppressed further by a factor of m_b^2/m_W^2, if comparing it with the other three diagrams. Thus we neglect it for simplicity. To be consistent, in the following calculations we will neglect all the terms suppressed by this factor m_b^2/m_W^2. It is easy to check that the total amplitudes is gauge invariant at this accuracy. Hence to the accuracy, the amplitudes corresponding to the other three diagrams turn out to be

$$\begin{aligned}
\mathcal{H}_{a+b} &= -i\sqrt{2}G_F e V_{cb}\bar{c}\left[Q_c \not{\epsilon}_\gamma \frac{\not{p}_\gamma - \not{p}_c + m_c}{(p_c\cdot p_\gamma)}\gamma_\mu P_L + Q_b P_R \gamma_\mu \frac{\not{p}_b - \not{p}_\gamma + m_b}{(p_b\cdot p_\gamma)} \not{\epsilon}_\gamma\right] b\ (\bar{\ell}\gamma^\mu P_L \nu), \\
\mathcal{H}_c &= -i\sqrt{2}G_F e V_{cb}(\bar{c}\gamma^\mu P_L b)\ \left[\bar{\ell}\ \not{\epsilon}_\gamma \frac{\not{p}_\gamma + \not{p}_\ell + m_\ell}{(p_\ell\cdot p_\gamma)}\gamma_\mu P_L \nu\right]. \qquad (2)
\end{aligned}$$

To be at the "quark level", the amplitudes given in eqn.(2) are not sufficient enough to analyze the processes $B_c \to \gamma\ell\bar{\nu}$, indeed at least a model is needed to 'turn' the amplitudes into the 'hadronic level'. Here for the aim we adopt a simple constituent quark model (see, for example [6]). In this model both of the quark and anti-quark inside the meson are treated non-relativistically moving with the same velocity and the quark masses are the constituent

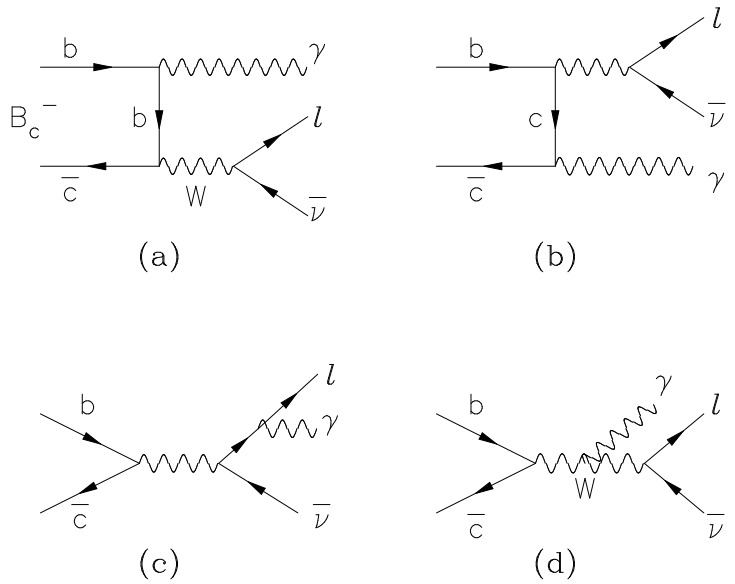

Figure 1: Feynman diagrams in standard model for $B_c \to \gamma \ell \bar{\nu}$

masses. In the constituent quark model, we have

$$p_c^\mu = (m_c/m_{B_c}) p_{B_c}^\mu, \qquad p_b^\mu = (m_b/m_{B_c}) p_{B_c}^\mu. \tag{3}$$

We use further the interpolating field technique [7] which relates the hadronic matrix elements to the decay constants of the mesons in the present case. With the decay constant definition:

$$< 0|\bar{q}\gamma^\mu\gamma_5 b|B_c >= i f_{B_c} p_B^\mu, \tag{4}$$

the whole amplitude for $B_c \to \gamma \ell \bar{\nu}$ decay is derived from eqn. (2,3):

$$\begin{aligned} \mathcal{A} &= \frac{\sqrt{2} e G_F V_{cb}}{6(p_{B_c} \cdot p_\gamma)} f_{B_c} \left[\left(\frac{m_{B_c}}{m_b} - 2\frac{m_{B_c}}{m_c} \right) i \epsilon_{\mu\nu\alpha\beta} p_{B_c}^\nu p_\gamma^\alpha \epsilon_\gamma^\beta \right. \\ &\left. + \left(6 - \frac{m_{B_c}}{m_b} - 2\frac{m_{B_c}}{m_c} \right) (p_{\gamma\nu}\epsilon_{\gamma\mu} - p_{\gamma\mu}\epsilon_{\gamma\nu}) p_{B_c}^\nu \right] (\bar{\ell}\gamma^\mu P_L \nu). \end{aligned} \tag{5}$$

Note once again that in the above calculations, all the terms suppressed by the factor of m_ℓ/m_b have been neglected. It is easily seen that the eqn.(5) is explicitly gauge invariant. Neglecting the mass of the lepton, we get the differential decay width:

$$\frac{d\Gamma}{d\hat{s}d\hat{t}} = \frac{\alpha G_F^2 |V_{cb}|^2}{144\ pi^2} f_{B_c}^2 m_{B_c}^3 \frac{\hat{s}}{(1-\hat{s})^2} \left[x_b(1-\hat{s}-\hat{t})^2 + x_c \hat{t}^2 \right], \tag{6}$$

where

$$x_b = \left(3 - \frac{m_{B_c}}{m_b}\right)^2, \qquad x_c = \left(3 - 2\frac{m_{B_c}}{m_c}\right)^2.$$

The $\hat{s}$, $\hat{t}$ are defined as $\hat{s} = (p_\ell + p_\nu)^2/m_{B_c}^2$, $\hat{t} = (p_\ell + p_\gamma)^2/m_{B_c}^2$. Hence the decay width is:

$$\Gamma = \frac{\alpha G_F^2 |V_{cb}|^2}{2592\pi^2} f_{B_c}^2 m_{B_c}^3 \left[x_b + x_c\right]. \tag{7}$$

Using $\alpha = 1/132$, $|V_{cb}| = 0.04$ [8], $m_c = 1.5$ GeV, $m_{B_c} = 6.258$ GeV [9], we obtain

$$\Gamma(B_c \to \gamma\ell\bar{\nu}) = 6.2 \times 10^{-17} \times \left(\frac{f_{B_c}}{360MeV}\right)^2 \text{ GeV}. \tag{8}$$

If the lifetime is taken as $\tau(B_c) = 0.52 \times 10^{-12} s$ [10], and the decay constant is used as $f_{B_c} = 360$MeV [11], the branching ratio is found to be 4.9×10^{-5}.

Since it is helpful for experiments to detect this decay channel, it is also useful to consider the differential spectra. The photon energy spectrum is easily derived from eqn.(6):

$$\frac{m_B}{\Gamma}\frac{d\Gamma}{dE_\gamma} = \frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\gamma} = 24\lambda_\gamma(1 - 2\lambda_\gamma), \tag{9}$$

with $\lambda_\gamma = E_\gamma/m_B$. We show the photon energy spectrum in Fig.2 as the solid line. This is clearly different from the bremsstrahlung photon spectrum, but the same with $B^\pm \to \gamma\ell\nu$ decay photon spectrum [12].

The invariant mass t of the charged lepton and photon combination is directly related to the energy carried by the neutrino, i.e. the missing energy in the process, so it is also measurable. Thus to present the charged lepton energy distribution and as well as the neutrino one are interesting. They are

$$\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\nu} &= \frac{36}{x_b + x_c}\{x_c(1 - 2\lambda_\nu)\left[2\lambda_\nu + (1 - 2\lambda_\nu)\ln(1 - 2\lambda_\nu)\right] \\ &\quad + x_b\left[2\lambda_\nu(3 - 5\lambda_\nu) + (1 - 2\lambda_\nu)(3 - 2\lambda_\nu)\ln(1 - 2\lambda_\nu)\right]\},\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\ell} &= \frac{36}{x_b + x_c}\{x_b(1 - 2\lambda_\ell)\left[2\lambda_\ell + (1 - 2\lambda_\ell)\ln(1 - 2\lambda_\ell)\right] \\ &\quad + x_c\left[2\lambda_\ell(3 - 5\lambda_\ell) + (1 - 2\lambda_\ell)(3 - 2\lambda_\ell)\ln(1 - 2\lambda_\ell)\right]\},\end{aligned} \tag{11}$$

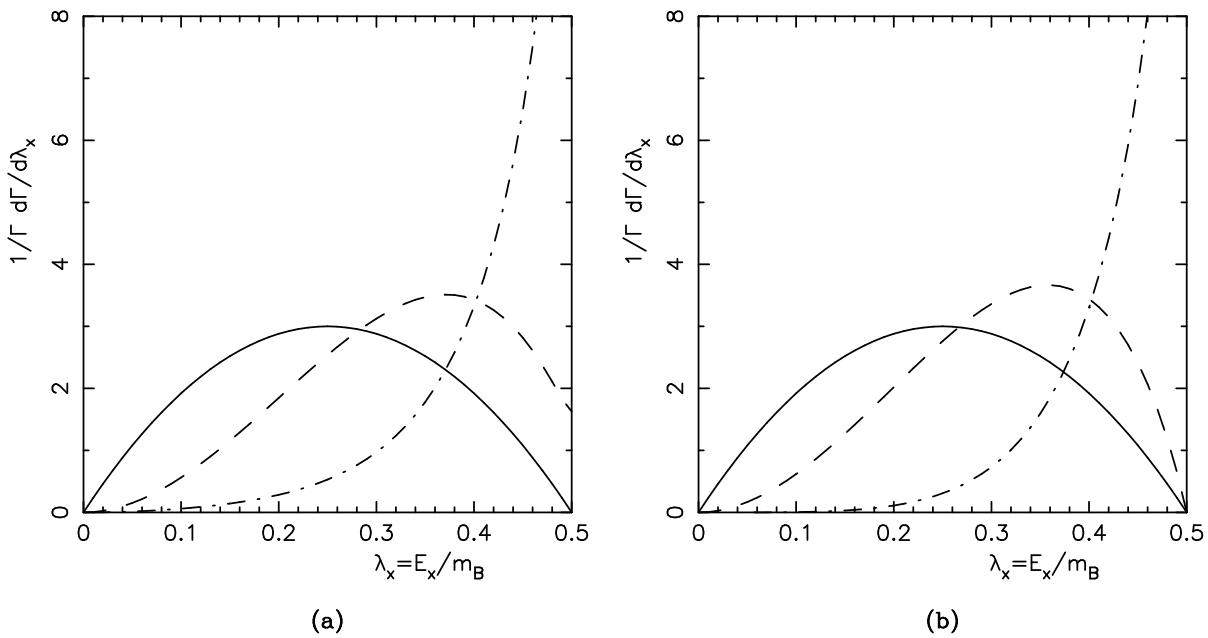

Figure 2: Normalized energy spectra of the decay $B_c \to \gamma\ell\bar{\nu}$ (a) and $B_u \to \gamma\ell\bar{\nu}$ (b). The solid line is for the photon energy spectrum, the dashed line is for the neutrino energy and the dash-dotted line is for the lepton energy spectrum, respectively.

where $\lambda_\nu = E_\nu/m_B$, $\lambda_\ell = E_\ell/m_B$. Since both c quark and b quark inside B_c meson are heavy, the contributions from the terms proportional to x_c, x_b, both are important.

To show the distributions more clearly, we present $\frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_x}, (x = \gamma, \nu, \ell)$ in Fig.2(a) as solid, dashed and dash-dotted lines. In $B^\pm \to \gamma\ell\nu$ decay, since the lightness of the u quark, $x_u >> x_b$, the energy spectra for charged lepton and neutrino are kept only the x_c terms in (10,11). They are shown in Fig.2(b) correspondingly. The behavior is similar, but the endpoint at $\lambda_i = 0.5$ is different 2E for $B_c \to \gamma\ell\bar{\nu}$,

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\nu}\bigg|_{\lambda_\nu=0.5} = 1.6, \qquad \frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\ell}\bigg|_{\lambda_\ell=0.5} = 16;$$

while for $B^\pm \to \gamma\ell\nu$,

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\nu}\bigg|_{\lambda_\nu=0.5} = 0, \qquad \frac{1}{\Gamma}\frac{d\Gamma}{d\lambda_\ell}\bigg|_{\lambda_\ell=0.5} = 18.$$

For comparison, the pure-leptonic decay branching ratios are also given with the same parameters as the radiative ones:

$$B(B_c \to \mu\bar{\nu}_\mu) \;\; = \;\; 6.2 \times 10^{-5},$$

$$B(B_c \to e\bar{\nu}_e) \;=\; 1.4 \times 10^{-9}. \tag{12}$$

It is easy to see that the branching ratios of the radiative leptonic decay and the pure-leptonic decay for the muon are at the same order.

3 Discussions

One may see from eqn.(7) that the decay rate of $B_c \to \gamma\ell\bar{\nu}$ is proportional to $f_{B_c}^2$, so one may use it to measure the decay constant $|f_{B_c}|$. We should note here that the decay rate eqn.(7) is model-dependent, because the 'hadronic piece' in the calculations is model dependent i.e. the non-relativistic constituent quark model is used. However, unlike the $B^{\pm} \to \gamma\ell\nu$ decays, here b and c quarks are both heavy. The long distance contributions can be controlled by the factor of Λ_{QCD}/m_c or Λ_{QCD}/m_b thus they will be able to estimate theoretically [13]. Therefore to determine the decay constance f_{B_c} from the measurements with our formula is still promised.

The similar decays, $B^{\pm} \to \gamma\ell\nu$, have been calculated by many authors [12]. The decay width has been shown in constituent quark model to be

$$\Gamma = \frac{\alpha G_F^2 |V_{ub}|^2}{648\pi^2} f_{B_u}^2 m_{B_u}^5 / m_u^2, \tag{13}$$

where $m_u = 350$MeV is the constituent quark mass of u-quark. To compare the importance of the $B_c \to \gamma\ell\bar{\nu}$ and that of the decay $B^{\pm} \to \gamma\ell\nu$, let us present the relative fraction of $\gamma\ell\nu$ final states coming from different sources B_c and B_u in a high energy production process:

$$\frac{N_{B_c}}{N_{B_u}} = \frac{1}{4}\frac{f(b \to B_c)}{f(b \to B_u)}\frac{|V_{cb}|^2}{|V_{ub}|^2}\frac{f_{B_c}^2}{f_{B_u}^2}\left(\frac{m_{B_c}}{m_{B_u}}\right)^3 [x_b + x_c]\,\frac{m_u^2}{m_{B_u}^2}, \tag{14}$$

where the factor $f(b \to B_c)$, $f(b \to B_u)$ are the inclusive probability that a b-quark hadronizes into a B_c meson and a B_u meson respectively. The $f(b \to B_c)$ is estimated to be a small number of the order 0.1%, and the probability $f(b \to B_u)$ is known at LEP with good accuracy [8]:

$$f(b \to B_u) = 0.378 \pm 0.022.$$

Using the central values of all parameters, one obtains

$$N_{B_c}/N_{B_u} = 0.8.$$

This means that the $\gamma\ell\nu$ final states coming from B_c and B_u are at the same order. We expect a similar fraction will also be obtained at LHC. If one would like to use the radiative decays to determine the decay constant f_{B_u} (or f_{B_c}), the background from the corresponding decays of the meson B_c (or B_u) should be considered very carefully. In B factories at KEK and SLAC, there will be no such problem, since B_c cannot be produced there. The energy spectra of the lepton and neutrino for B_c decays in Fig.2 have been emphasized to be different from that for the B_u decays [12]. This will be a good feature for experiments to distinguish the radiative decays of B_c from that of B_u meson.

In conclusion, we predict the branching ratios for $B_c \to \gamma\ell\bar{\nu}$ in SM at the order of 10^{-5}. With this branching ratio, they are hopeful detectable at Tevatron and LHC. When enough B_c meson events are collected, the radiative decays will be able to provide alternate channels for measuring and/or 'cross-checking' the decay constant f_{B_c} independently with certain accuracy, provided that the background from B_u decays is treated well. To enhance the accuracy of the theoretical predictions, or to estimate theoretical uncertainty of the calculations, various calculations on the B_c radiative decay with different models are needed [13].

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References

[1] C.-H. Chang and Y.-Q. Chen, Phys. Rev. D49 (1994) 3399.

[2] D.-S. Du, H.-Y. Jin and Y.-D. Yang, preprint BIHEP-Th/97-007, hep-ph/9705261.

[3] C.-H. Chang and Y.-Q. Chen, Phys. Rev. D48 (1993) 4086;
C.-H. Chang, Y.-Q. Chen, G.-P. Han and H.-T. Jiang, Phys. Lett. B364 (1995) 78;
C.-H. Chang, Y.-Q. Chen and R.J. Oakes Phys. Rev. D54 (1996) 4344.

[4] K. Cheung, Phys. Rev. Lett. 71 (1993) 3413.

[5] M.L. Mangano and S.R. Slabospitsky, preprint CERN-TH/97-150, hep-ph/9707248.

[6] H.Y. Cheng, et al. Phys. Rev. D51, 1199 (1995).

[7] H. Georgi, Phys. Lett. B240, 447 (1990);
M.B. Wise, Phys. Rev. D45, 2188 (1992).

[8] Particle Data Group, Phys. Rev. D54, Part II (1996) 1.

[9] E. Eichten, C. Quigg, Phys. Rev. D49 (1994) 5845.

[10] M. Beneke and G. Buchalla, Phys. Rev. D53 (1996) 4991.

[11] P. Colangelo, G. Nardulli, N. Paver, Z. Phys. C57 (1993) 43.

[12] G. Burdman, T. Goldman, D. Wyler, Phys. Rev. D51 (1995) 111;
D. Atwood, G. Eilam, A. Soni, Mod. Phys. Lett. A11 (1996) 1061;
G. Eilam, I. Halperin, R.R. Mendel, Phys. Lett. B361 (1995) 137;
P. Colangelo, F. De Fazio, G. Nardulli, Phys. Lett. B386 (1996) 328;
Phys. Lett. B372 (1996) 331.

[13] C.H. Chang et al., in progress.