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BFKL pomeron in the next-to-leading approximation

V. S. Fadin* and L. N. Lipatov**

*Budker Nuclear Physics Institute and
Novosibirsk State University
Novosibirsk, 630090, Russia

**Petersburg Nuclear Physics Institute and
St.Petersburg State University
Gatchina, 188350, Russia

Abstract

We find one-loop correction to the integral kernel of the BFKL equation for the total cross section of the high energy scattering in QCD and calculate the next-to-leading contribution to anomalous dimensions of twist-2 operators near $j = 1$.

The BFKL equation is very important for the theory of the Regge processes at high energies $\sqrt{s}$ in the perturbative QCD[1]. In particular, it can be used together with the DGLAP evolution equation [2] for the description of structure functions for the deep inelastic ep scattering at small values of the Bjorken variable $x = -q^2/(2pq)$, where p and q are the momenta of the proton and the virtul photon correspondingly. But up to recent years the integral kernel for the BFKL equation was known only in the leading logarithmic approximation (LLA), which did not allow one to find its region of applicapability, including the scale in transverse momenta fixing the argument of the QCD coupling constant $\alpha(ck_\perp^2)$and the longitudinal scale $\sqrt{s_0}$ for the minimal initial energy. In this paper we calculate the QCD radiative corrections to this kernel.

In LLA the gluon is reggeized and the Pomeron is a compound state of two reggeized gluons. One can neglect multi-gluon components of the Pomeron wave function also in the next-to-leading logarithmic approximation (NLLA) and express the total cross-section $\sigma(s)$ for the high energy scattering of colourless particles A, B in terms of their impact factors $\Phi_i(\vec{q_i})$ and the t-channel partial wave $G_\omega(\vec{q}, \vec{q'})$ for the reggeized gluon scattering at $t = 0$:

$$\sigma(s) = \int \frac{d^2q}{2\pi q^2} \int \frac{d^2q'}{2\pi q'^2} \Phi_A(\vec{q}) \, \Phi_B(\vec{q'}) \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q\,q'}\right)^\omega G_\omega(\vec{q}, \vec{q'}) \,. \qquad (1)$$

Here $\vec{q}$ and $\vec{q'}$ are transverse momenta of gluons with the virtualities $-\vec{q}^{\,2} \equiv -q^2$ and $-\vec{q'}^{\,2} \equiv -q'^2$ correspondingly, $s = 2p_Ap_B$ is the squared invariant mass of the colliding particles with momenta p_A and p_B. Note, that the dependence of the Regge factor from

q and q' is natural from the point of view of the Watson-Sommerfeld representation for high energy scattering amplitudes. The change of the energy scale in this factor leads generally to the corresponding modification of the impact factors and the BFKL equation for G_ω but the physical results are not changed.

Using the dimensional regularization in the $\overline{MS}$-scheme to renormalize the QCD coupling constant and to remove infrared divergencies in the intermediate expressions, we write the generalized BFKL equation for $G_\omega(\vec{q}, \vec{q'})$ in the following form

$$\omega\, G_\omega(\vec{q}, \vec{q'}) = \delta^{D-2}(\vec{q} - \vec{q'}) + \int d^{D-2}\tilde{q}\;\; K(\vec{q}, \vec{\tilde{q}})\, G_\omega(\vec{\tilde{q}}, \vec{q'})\,. \tag{2}$$

Here

$$K(\vec{q_1}, \vec{q_2}) = 2\,\omega(q_1)\,\delta^{(D-2)}(\vec{q_1} - \vec{q_2}) + K_r(\vec{q_1}, \vec{q_2})\,, \tag{3}$$

and the space-time dimension is $D = 4 + 2\varepsilon$ for $\epsilon \to 0$. The gluon Regge trajectory $\omega(q)$ and the integral kernel $K_r(\vec{q_1}, \vec{q_2})$, related with the real particle production, are expanded in the series over the QCD coupling constant

$$\omega(q) = \omega_B(q) + \omega^{(2)}(q) + \ldots\,,\;\; K_r(\vec{q_1}, \vec{q_2}) = K_r^B(\vec{q_1}, \vec{q_2}) + K_r^{(1)}(\vec{q_1}, \vec{q_2}) + \ldots\,, \tag{4}$$

where the Born expressions, corresponding to LLA, are

$$\omega_B(q) = -\overline{g_\mu^2}(\frac{2}{\varepsilon} + 2\ln\frac{q^2}{\mu^2})\,,\quad K_r^B(\vec{q_1}, \vec{q_2}) = \frac{4\,\overline{g_\mu^2}\,\mu^{-2\varepsilon}}{\pi^{1+\varepsilon}\Gamma(1-\varepsilon)}\,\frac{1}{(\vec{q_1} - \vec{q_2})^2}\,. \tag{5}$$

Here

$$\overline{g_\mu^2} = \frac{g_\mu^2\, N_c\, \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}}$$

for the colour group $SU(N_c)$ and g_μ is the QCD coupling constant fixed at the normalization point μ in the $\overline{MS}$-scheme.

The program of calculating next-to-leading corrections to the BFKL equation was formulated several years ago [3]. It was shown, that the corrections can be expressed through the Born production amplitudes in the quasi-multi-regge kinematics with the use of the unitarity conditions in the s and t channels. These amplitudes were constructed in terms of various reggeon-particle vertices. The gauge-invariant action containing all such vertices was formulated recently [4].

The two-loop correction $\omega^{(2)}(q)$ to the gluon Regge trajectory is known [5] and for massless quarks can be written as follows

$$\omega^{(2)}(q) = -\bar{g}_\mu^4 \left[\left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right)\left(\frac{1}{\varepsilon^2} - \ln^2\left(\frac{\vec{q}^2}{\mu^2}\right)\right)\right.$$

$$\left. + \left(\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N_c}\right)\left(\frac{1}{\varepsilon} + 2\ln\left(\frac{\vec{q}^2}{\mu^2}\right)\right) - \frac{404}{27} + 2\zeta(3) + \frac{56}{27}\frac{n_f}{N_c}\right]\,. \tag{6}$$

where n_f is the number of light quarks. The poles in $\varepsilon = 0$ correspond to infrared divergencies cancelled in the total cross-section.

The one-loop correction to the integral kernel $K_r^{(1)}(\overrightarrow{q_1},\overrightarrow{q_2})$ is obtained as a sum of two contributions. The first one is related with the one-loop virtual correction to the one-gluon production cross-section [6] and the second one is determined by the Born cross-sections for production of two gluons [7] and quark-antiquark pair [8]. The final result for $K_r^{(1)}(\overrightarrow{q_1},\overrightarrow{q_2})$ can be written as follows

$$K_r^{(1)}(\overrightarrow{q_1},\overrightarrow{q_2}) = \frac{4\overline{g}_\mu^4\,\mu^{-2\varepsilon}}{\pi^{1+\varepsilon}\,\Gamma(1-\varepsilon)}\left\{\frac{1}{(\vec{q}_1-\vec{q}_2)^2}\left[\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\frac{1}{\varepsilon}\left(1-\left(\frac{(\overrightarrow{q_1}-\overrightarrow{q_2})^2}{\mu^2}\right)^{\varepsilon}(1-\varepsilon^2\frac{\pi^2}{6})\right)\right.\right.$$
$$\left.\left.+\left(\frac{(\overrightarrow{q_1}-\overrightarrow{q_2})^2}{\mu^2}\right)^{\varepsilon}\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N_c}+\varepsilon\left(-\frac{404}{27}+14\zeta(3)+\frac{56}{27}\frac{n_f}{N_c}\right)\right)\right]\right.$$
$$-\left(1+\frac{n_f}{N_c^3}\right)\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3(\vec{q}_1\vec{q}_2)^2}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left(\frac{2}{\vec{q}_2^{\,2}}+\frac{2}{\vec{q}_1^{\,2}}+(\frac{1}{\vec{q}_2^{\,2}}-\frac{1}{\vec{q}_1^{\,2}})\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{1}{(\vec{q}_1-\vec{q}_2)^2}\left(\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^2$$
$$+\frac{2(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}{(\vec{q}_1-\vec{q}_2)^2(\vec{q}_1+\vec{q}_2)^2}\left(\frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\vec{q}_1-\vec{q}_2)^4}{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})^4}\right)+L\left(-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-L\left(-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right)$$
$$-\left(3+(1+\frac{n_f}{N_c^3})\left(1-\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})^2}{8\vec{q}_1^{\,2}\vec{q}_2^{\,2}}-\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3\vec{q}_1^{\,4}-3\vec{q}_2^{\,4}}{16\vec{q}_1^{\,4}\vec{q}_2^{\,4}}(\vec{q}_1\vec{q}_2)^2\right)\right)\int_0^\infty\frac{dx\,\ln\left|\frac{1+x}{1-x}\right|}{\vec{q}_1^{\,2}+x^2\vec{q}_2^{\,2}}$$
$$\left.-\left(1-\frac{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}{(\vec{q}_1-\vec{q}_2)^2(\vec{q}_1+\vec{q}_2)^2}\right)\left(\int_0^1-\int_1^\infty\right)\frac{dz\,\ln\frac{(z\vec{q}_1)^2}{(\vec{q}_2)^2}}{(\vec{q}_2-z\vec{q}_1)^2}\right\}\,, \qquad (7)$$

where

$$L(z)=\int_0^z\frac{dt}{t}\ln(1-t)\,,\ \zeta(n)=\sum_{k=1}^{\infty}k^{-n}.$$

Note, that the gluonic part of the above expression for $K_r^{(1)}(\overrightarrow{q_1},\overrightarrow{q_2})$ is different from the corresponding piece of the so called "irreducible part" of the kernel, which was constructed by the authors of ref.[9] and it leads to different estimates of radiative corrections to the intercept of the BFKL Pomeron. This difference is due to the fact, that the result of Ref.[9]) is incomplete. So-called scale-dependent contributions were not calculated by these authors and in their opinion there is an ambiguity in fixing these contributions. We stress again, that physical results do not depend on the energy scale in the Regge factors in eq.(1) because its change is compensated by the corresponding modification of the impact factors and the kernel. Moreover, this modification does not have any influence on the correction to the Pomeron intercept.

In our approach the contribution to the total cross section from one gluon production in the central rapidity region can be presented as follows

$$\sigma_{2\to3}=\int\frac{d^2q}{2\pi q^2}\int\frac{d^2q'}{2\pi q'^2}\Phi_A(\overrightarrow{q})\,\Phi_B(\overrightarrow{q'})\int_{q'k_\perp/s}^{k_\perp/q}\frac{d\beta}{\beta}\left(\frac{k_\perp}{q\,\beta}\right)^{2\omega(q)}R^g(q,q')\left(\frac{s\,\beta}{q'k_\perp}\right)^{2\omega(q')}\,, \qquad (8)$$

where $\overrightarrow{k_\perp} = \overrightarrow{q} - \overrightarrow{q'}$ is the emitted gluon transverse momentum and β is its Sudakov variable, $\beta = (kp_B)/(p_A p_B)$. Note, that acting more accurately one should introduce in the impact factors and in the limits of integration over β some intermediate cut-offs in such way, that the physical results do not depend on them. The quantity $R^g(q,q')$ is the total contribution to the BFKL kernel from one gluon producton and it is expressed through the product of the Reggeon-Reggeon-gluon vertices Γ_μ (see [6]) with a subtraction of the terms proportional to the product of the gluon Regge trajectories ω_B and the vertices Γ^B_μ in the Born approximation

$$R^g(q,q') \sim \Gamma_\mu \Gamma_\mu - \left(\omega_B(q) \ln \frac{\mu^4}{q^2 k_\perp^2} + \omega_B(q') \ln \frac{\mu^4}{q'^2 k_\perp^2}\right) \Gamma^B_\mu \Gamma^B_\mu \, .$$

The neccesity of this subtraction is related with the renormalization of the Regge factors in eq. (8) in comparison with their definition in ref. [6].

The ultraviolet divergency in the integral over the relative gluon rapidity for the contribution to the BFKL kernel from the two gluon emission [7] is cancelled with the corresponding divergency, related with the infrared cut-off in the relative rapidity for the produced gluons in the multi-Regge kinematics. The analogous cancellation is implied in the loop corrections to the impact factors $\Phi_i(\overrightarrow{q})$.

The solution of the inhomogenious BFKL equation can be presented as a linear combination of a complete set of the solutions of a homogenious equation proportional to spherical harmonics in the $D-2$ dimensional space. The biggest eigen value ω_P, leading to a rapid increase of total cross-sections $\sigma(s) \sim s^{\omega_P}$ with energy, corresponds to a spherically symmetric eigen function. Therefore it is natural to average the BFKL kernel over the angle between the momenta $\overrightarrow{q_1}$ and $\overrightarrow{q_2}$:

$$\begin{aligned}
\overline{K_r^{(1)}(\overrightarrow{q_1},\overrightarrow{q_2})} = \frac{4\overline{g}_\mu^4 \, \mu^{-2\epsilon}}{\pi^{1+\epsilon}\,\Gamma(1-\epsilon)} \Bigg\{ \frac{1}{|q_2^2 - q_1^2|} \Bigg[\left(\frac{11}{3} - \frac{2n_f}{3N_c}\right) \frac{1}{\epsilon} \left(\left(\frac{|q_2^2 - q_1^2|}{(\max(q_1^2, q_2^2))} \right)^{2\varepsilon} (1 + \varepsilon^2 \frac{\pi^2}{3}) \right. \\
- \left(\frac{(q_2^2 - q_1^2)^4}{\mu^2 (\max(q_1^2, q_2^2))^3} \right)^{\varepsilon} \left(1 + \varepsilon^2 \frac{5\pi^2}{6} \right) \Bigg) + \left(\frac{|q_2^2 - q_1^2|^4}{\mu^2 (\max(q_1^2, q_2^2))^3} \right)^{\varepsilon} \left(\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9} \frac{n_f}{N_c} \right. \\
\left. + \varepsilon \left(-\frac{404}{27} + 14\zeta(3) + \frac{56}{27} \frac{n_f}{N_c} \right) \right) \Bigg] - \frac{1}{32} \left(1 + \frac{n_f}{N_c^3} \right) \left(\frac{2}{q_2^2} + \frac{2}{q_1^2} + (\frac{1}{q_2^2} - \frac{1}{q_1^2}) \ln \frac{q_1^2}{q_2^2} \right) \\
- \frac{1}{|q_1^2 - q_2^2|} \left(\ln \frac{q_1^2}{q_2^2} \right)^2 - \left(3 + (1 + \frac{n_f}{N^3}) \left(\frac{3}{4} - \frac{(q_1^2 + q_2^2)^2}{32 q_1^2 q_2^2} \right) \right) \int_0^\infty \frac{dx}{q_1^2 + x^2 q_2^2} \ln \left| \frac{1+x}{1-x} \right| \\
+ \frac{1}{q_2^2 + q_1^2} \left(\frac{\pi^2}{3} - 4L(\min(\frac{q_1^2}{q_2^2}, \frac{q_2^2}{q_1^2})) \right) \Bigg\} \, . \qquad (9)
\end{aligned}$$

Instead of the dimensional regularization we can remove the infrared divergencies in the kernel by introducing a fictious gluon mass λ. Using eqs. (4)-(6), it is possible to

verify that the averaged kernel (9) at $\varepsilon \to 0$ is equivalent to the expression

$$\overline{K(\overrightarrow{q_1},\overrightarrow{q_2})} = -2\frac{\alpha_s(\mu^2)N_c}{\pi^2}\left\{\ln\frac{q_1^2}{\lambda^2} + \frac{\alpha_s(\mu^2)N_c}{4\pi}\left[\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\left(\ln\frac{{q_1}^2}{\lambda^2}\ln\frac{\mu^2}{\lambda^2}+\frac{\pi^2}{12}\right)\right.\right.$$

$$\left.\left. + \left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N_c}\right)\ln\frac{{q_1}^2}{\lambda^2} - 3\zeta(3)\right]\right\}\delta(q_1^2-q_2^2) + \frac{\alpha_s(\mu^2)N_c}{\pi^2}\frac{\theta(|q_1^2-q_2^2|-\lambda^2)}{|q_1^2-q_2^2|}$$

$$\times\left\{1-\frac{\alpha_s(\mu^2)N_c}{4\pi}\left[\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\ln\left(\frac{|{q_1}^2-{q_2}^2|^2}{\max(q_1^2,q_2^2)\mu^2}\right)-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N_c}\right)\right]\right\}$$

$$-\frac{\alpha_s^2(\mu^2)N_c^2}{4\pi^3}\left\{\frac{1}{32}\left(1+\frac{n_f}{N_c^3}\right)\left(\frac{2}{{q_2}^2}+\frac{2}{{q_1}^2}+(\frac{1}{{q_2}^2}-\frac{1}{{q_1}^2})\ln\frac{{q_1}^2}{{q_2}^2}\right)\right.$$

$$+\frac{1}{|q_1^2-q_2^2|}\left(\ln\frac{{q_1}^2}{{q_2}^2}\right)^2 + \left(3+(1+\frac{n_f}{N^3})\left(\frac{3}{4}-\frac{(q_1^2+q_2^2)^2}{32q_1^2q_2^2}\right)\right)\int_0^\infty\frac{dx}{q_1^2+x^2q_2^2}\ln\left|\frac{1+x}{1-x}\right|$$

$$\left.-\frac{1}{q_2^2+q_1^2}\left(\frac{\pi^2}{3}-4L(\min(\frac{q_1^2}{q_2^2},\frac{q_2^2}{q_1^2}))\right)\right\}\,, \qquad (10)$$

defined in the two-dimensional transverse space, with $\lambda \to 0$. Of course, the dependence from λ disappears when the kernel acts on a function. Moreover, the representation (10) permits to find such form of the kernel for which this cancellation is evident:

$$\int d^{D-2}q_2\;\; K(\overrightarrow{q_1},\overrightarrow{q_2})\,f(q_2^2) = \frac{\alpha_s(\mu^2)N_c}{\pi}\int dq_2^2\left\{\frac{1}{|{q_1}^2-{q_2}^2|}\left(f(q_2^2)-2\frac{\min(q_1^2,q_2^2)}{(q_1^2+q_2^2)}f(q_1^2)\right)\right.$$

$$\times\left[1-\frac{\alpha_s(\mu^2)N_c}{4\pi}\left(\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\ln\left(\frac{|{q_1}^2-{q_2}^2|^2}{\max(q_1^2,q_2^2)\mu^2}\right)-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N_c}\right)\right)\right]$$

$$-f(q_2^2)\frac{\alpha_s(\mu^2)N_c}{4\pi}\left[\frac{1}{32}\left(1+\frac{n_f}{N_c^3}\right)\left(\frac{2}{{q_2}^2}+\frac{2}{{q_1}^2}+(\frac{1}{{q_2}^2}-\frac{1}{{q_1}^2})\ln\frac{{q_1}^2}{{q_2}^2}\right)\right.$$

$$+\frac{1}{|q_1^2-q_2^2|}\left(\ln\frac{{q_1}^2}{{q_2}^2}\right)^2 + \left(3+(1+\frac{n_f}{N^3})\left(\frac{3}{4}-\frac{(q_1^2+q_2^2)^2}{32q_1^2q_2^2}\right)\right)\int_0^\infty\frac{dx}{q_1^2+x^2q_2^2}\ln\left|\frac{1+x}{1-x}\right|$$

$$\left.\left.-\frac{1}{q_2^2+q_1^2}\left(\frac{\pi^2}{3}-4L(\min(\frac{q_1^2}{q_2^2},\frac{q_2^2}{q_1^2}))\right)\right]\right\} + \frac{\alpha_s^2(\mu^2)N_c^2}{4\pi^2}\left(6\zeta(3)-\frac{5\pi^2}{12}\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\right)f(q_1^2)\,. \qquad (11)$$

The μ - dependence in the right hand side of this equality leads to the violation of the scale invariance and is related with running the QCD coupling constant.

The form (11) is very convenient for finding the action of the kernel on the eigenfunctions $q_2^{2(\gamma-1)}$ of the Born kernel:

$$\int d^{D-2}q_2\;\; K(\overrightarrow{q_1},\overrightarrow{q_2})\left(\frac{q_2^2}{q_1^2}\right)^{\gamma-1} = \frac{\alpha_s(q_1^2)\,N_c}{\pi}\left(\chi(\gamma)+\delta(\gamma)\frac{\alpha_s(q_1^2)N_c}{4\,\pi}\right)\,, \qquad (12)$$

were within our accuracy we expressed the result in terms of the running coupling constant

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1+\frac{\alpha_s(\mu^2)N_c}{4\pi}\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\ln\left(\frac{q^2}{\mu^2}\right)} \simeq \alpha_s(\mu^2)\left(1-\frac{\alpha_s(\mu^2)N_c}{4\pi}\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\ln\left(\frac{q^2}{\mu^2}\right)\right) .$$

The introduced quantity $\chi(\gamma)$ is proportional to the eigenvalue of the Born kernel

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\,, \;\; \psi(\gamma) = \Gamma'(\gamma)/\Gamma(\gamma)\,, \tag{13}$$

and the correction $\delta(\gamma)$ is given below

$$\begin{aligned}\delta(\gamma) = -\Bigg[&\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\frac{1}{2}\left(\chi^2(\gamma)-\psi'(\gamma)+\psi'(1-\gamma)\right)-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N_c}\right)\chi(\gamma)\\ &-6\zeta(3)+\frac{\pi^2\cos(\pi\gamma)}{\sin^2(\pi\gamma)(1-2\gamma)}\left(3+\left(1+\frac{n_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right)\\ &-\psi''(\gamma)-\psi''(1-\gamma)-\frac{\pi^3}{\sin(\pi\gamma)}+4\phi(\gamma)\Bigg] .\end{aligned} \tag{14}$$

The function $\phi(\gamma)$ is

$$\begin{aligned}\phi(\gamma) &= -\int_0^1 \frac{dx}{1+x}\left(x^{\gamma-1}+x^{-\gamma}\right)\int_x^1\frac{dt}{t}\ln(1-t)\\ &= \sum_{n=0}^{\infty}(-1)^n\left[\frac{\psi(n+1+\gamma)-\psi(1)}{(n+\gamma)^2}+\frac{\psi(n+2-\gamma)-\psi(1)}{(n+1-\gamma)^2}\right] .\end{aligned} \tag{15}$$

Note, that the contribution in the last line of eq. (14) is different from the result of ref.[9], which leads to different estimates of the next-to-leading corrections to the intercept of the BFKL pomeron, for the reasons mentioned before.

Almost all terms in the right hand side of eq. (12) except the contribution

$$\Delta(\gamma) = \frac{\alpha_s^2(\mu^2)N_c^2}{4\pi^2}\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\frac{1}{2}\left(\psi'(\gamma)-\psi'(1-\gamma)\right)$$

are symmetric to the transformation $\gamma \leftrightarrow 1-\gamma$. Moreover, it is possible to cancel $\Delta(\gamma)$ if one would redefine the function $q^{2(\gamma-1)}$ by including in it the logarithmic factor $\left(\frac{\alpha_s(q^2)}{\alpha_s(\mu^2)}\right)^{-1/2}$.

Because the radiative correction to ω is negative, it is convenient to introduce the relative correction $c(\gamma)$ by the definition $\delta(\gamma) = -c(\gamma)\,\chi(\gamma)$. In particular, for the symmetric point $\gamma = 1/2$, corresponding to the rightmost singularity of the t-channel partial wave, we obtain

$$\begin{aligned}c\left(\frac{1}{2}\right) = 2\left(\frac{11}{3}-\frac{2n_f}{3N_c}\right)\ln 2 - \frac{67}{9}+\frac{\pi^2}{3}+\frac{10}{9}\frac{n_f}{N_c}+\frac{1}{4\ln 2}\Bigg[16\int_0^1\arctan(\sqrt{t})\ln(\frac{1}{1-t})\frac{dt}{t}\\ +22\zeta(3)+\frac{\pi^3}{2}\left(\frac{27}{16}+\frac{11}{16}\frac{n_f}{N_c^3}\right)\Bigg] = 25,8388+0.1869\frac{n_f}{N_c}+10.6584\frac{n_f}{N_c^3},\end{aligned} \tag{16}$$

which is almost two times larger, than its estimate in ref. [9]. For example, if $\alpha_s(q^2) = 0.15$, where the Born intercept is $\omega_P^B = 4N_c\alpha_s(q^2)/\pi \ \ln 2 = .39714$, the relative correction for $n_f = 0$ is very big:

$$\frac{\omega_P}{\omega_P^B} = 1 - c\left(\frac{1}{2}\right)\frac{\alpha_s(q^2)}{4\pi}N_c = 0.0747.$$

The maximal value of $\omega_P \simeq 0.1$ is obtained for $\alpha_s(q^2) \simeq 0.08$. These numerical estimates show, that in the kinematical region of HERA probably it is not enough to take into account only the next-to-leading correction. The value of this correction strongly depends on its representation. For example, if one takes into account the next-to-leading correction by the corresponding increase of the argument of the running QCD coupling constant, the intercept of the pomeron turns out to be only two times smaller, than its Born value.

Due to the effect of running the coupling constant the eigenfunctions of the NLLA kernel, which can be easily found, do not coincide with $q^{2(\gamma-1)}$; moreover, the position and the nature of the Pomeron singularity are strongly affected by the nonperturbative effects [10]. The BFKL equation with the next-to-leading corrections can be considered as the quantized version of the renormalization group equations. Expression (16) can be used only for a rough estimate of the power of the energy dependence of the total cross section.

The above results can be applied for the calculation of anomalous dimensions of the local operators in the vicinity of the point $\omega = J - 1 = 0$. To begin with, it is necessary to emphasize the difference between the t-channel partial wave $\mathcal{G}_\omega^{gB}(q^2)$ for the reggeized gluon scattering off the colourless particle B at $t = 0$

$$\mathcal{G}_\omega^{gB}(q^2) = \int \frac{d^2q'}{2\pi q'^2} G_\omega(\overrightarrow{q}, \overrightarrow{q'})\Phi_B(\overrightarrow{q'}) \tag{17}$$

and the deep-inelastic moments $\mathcal{F}_\omega^{gB}(q^2)$ defined as follows

$$\mathcal{F}_\omega^{gB}(q^2) = \int_{q^2}^{\infty} \frac{ds}{s}\left(\frac{s}{q^2}\right)^{-\omega} \sigma^{gB}(q^2, s)\,, \tag{18}$$

where

$$\sigma^{gB}(q^2, s) = \int \frac{d^2q'}{2\pi q'^2}\,\Phi_B(\overrightarrow{q'}) \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i}\left(\frac{s}{q\,q'}\right)^{\omega} G_\omega(\overrightarrow{q}, \overrightarrow{q'})\,. \tag{19}$$

This difference was not essential in LLA but becomes important in NLLA. Whereas the t-channel partial wave $\mathcal{G}_\omega^{gB}(q^2)$ obeys the integral equation of the type (2) with the same kernel (and the inhomogenious term equal to $\frac{\Phi_B(\overrightarrow{q})}{2\pi q^2}$), the kernel of the corresponding equation for the deep inelastic moments $\mathcal{F}_\omega^{gB}(q^2)$ in NLLA is

$$\widetilde{K}(\overrightarrow{q_1}, \overrightarrow{q_2}) = K(\overrightarrow{q_1}, \overrightarrow{q_2}) - \frac{1}{2}\int d^{D-2}q'\ \ K^B(\overrightarrow{q_1}, \overrightarrow{q'})\ \ln\frac{q'^2}{q_1^2}K^B(\overrightarrow{q'}, \overrightarrow{q_2})\,, \tag{20}$$

where $K^B(\overrightarrow{q'}, \overrightarrow{q_2})$ is the LLA kernel. The action of the modified kernel on the Born eigen functions $q_2^{2(\gamma-1)}$ can be calculated easily:

$$\int d^{D-2}q_2 \;\; \widetilde{K}(\overrightarrow{q_1},\overrightarrow{q_2}) \;\; \left(\frac{q_2^2}{q_1^2}\right)^{\gamma-1} = \frac{\alpha_s(q_1^2)\,N_c}{\pi}\left(\chi(\gamma)+\widetilde{\delta}(\gamma)\frac{\alpha_s(q_1^2)N_c}{4\,\pi}\right), \qquad (21)$$

where

$$\widetilde{\delta}(\gamma) = \delta(\gamma) - 2\chi(\gamma)\,\chi'(\gamma)\,. \qquad (22)$$

The anomalous dimensions

$$\gamma = \gamma_0(\alpha_s/\omega) + \alpha_s\gamma_1(\alpha_s/\omega)$$

of the twist-2 operators near point $\omega = 0$ are determined from the solution of the equation

$$\omega = \frac{\alpha_s N_c}{\pi}\left(\chi(\gamma)+\widetilde{\delta}(\gamma)\frac{\alpha_s N_c}{4\,\pi}\right) = \frac{\alpha_s N_c}{\pi}\left(\frac{1}{\gamma}+O(\gamma^2)\right) - \frac{\alpha_s^2 N_c^2}{4\,\pi^2}\left(\frac{11+2n_f/N_c^3}{3\,\gamma^2}+\right.$$

$$\left.\frac{n_f(10+13/N_c^2)}{9\gamma\,N_c}+\frac{395}{27}-2\zeta(3)-\frac{11}{3}\frac{\pi^2}{6}+\frac{n_f}{N_c^3}(\frac{71}{27}-\frac{\pi^2}{9})+O(\gamma)\right) \qquad (23)$$

for $\gamma \to 0$. The singularity in $\delta(\gamma) \simeq -2/\gamma^3$ for $\gamma \to 0$ is exactly cancelled in the transition to $\widetilde{\delta}(\gamma)$ because $2\chi(\gamma)\,\chi'(\gamma) \simeq -2/\gamma^3$ in the same limit. It gives a possibility to find the corrections to the anomalous dimensions. In particular for the low orders of the perturbation theory we reproduce the known results and predict the higher loop correction for $\omega \to 0$:

$$\gamma = \frac{\alpha_s\,N_c}{\pi}(\frac{1}{\omega}-\frac{11}{12}-\frac{n_f}{6N_c^3}+O(\omega)) - \left(\frac{\alpha_s}{\pi}\right)^2\frac{n_f\,N_c}{6\,\omega}(\frac{5}{3}+\frac{13}{6N_c^2}+O(\omega))$$

$$-\frac{1}{4\omega^2}\left(\frac{\alpha_s N_c}{\pi}\right)^3\left(\frac{395}{27}-2\zeta(3)-\frac{11}{3}\frac{\pi^2}{6}+\frac{n_f}{N_c^3}(\frac{71}{27}-\frac{\pi^2}{9})+O(\omega)\right)+O\left(\frac{\alpha_s^4}{\omega^4}\right)\;. \qquad (24)$$

Note, that our formulas for the anomalous dimensions are different from the expressions obtained by authors of ref.[9]. Their results are not complete because they did not calculate the contributions of so called energy-scale dependent parts of the kernel.

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References

[1] V. S. Fadin, E. A. Kuraev, L. N. Lipatov, Phys. Lett. **B60** (1975) 50;
Ya. Ya. Balitsky, L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.

[2] V. N. Gribov, L. N. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 438;
L. N. Lipatov, Sov. J. Nucl. Phys. **20** (1975) 94;
G. Altarelli, G. Parisi, Nucl. Phys. **B26** (1977) 298;
Yu. L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641.

[3] L. N. Lipatov, V. S. Fadin, Sov. J. Nucl. Phys. **50** (1989) 712.

[4] L. N. Lipatov, Nucl. Phys. **B452** (1995) 369; Physics Reports **286** (1997) 132.

[5] V. S. Fadin, R. Fiore, M. I. Kotsky, Phys. Lett. **B359** (1995) 181; **B387** (1996) 593.

[6] V. S. Fadin, L. N. Lipatov, Nucl. Phys. **B406** (1993) 259;
V. S. Fadin, R. Fiore and A. Quartarolo, Phys. Rev. **D50** (1994) 5893;
V. S. Fadin, R. Fiore, M. I. Kotsky, Phys. Lett. **B389** (1996) 737.

[7] V. S. Fadin, L. N. Lipatov, Nucl.Phys. **B477** (1996) 767;
V. S. Fadin, M. I. Kotsky, L. N. Lipatov, Phys. Lett. **B415** (1997) 97

[8] S. Catani, M. Ciafaloni and F.Hautman, Phys. Lett. **B242** (1990) 97; Nucl.Phys. **B366** (1991) 135;
G. Camici and M. Ciafaloni, Phys. Lett. **B386** (1996) 341; Nucl.Phys. **B496** (1997) 305;
V. S. Fadin, R. Fiore, A. Flashi, M. I. Kotsky, BUDKERINP-97-86, hep-ph 9711427.

[9] G. Camici, M. Ciafaloni, Phys. Lett. **B412** (1997) 396.

[10] L. N. Lipatov, JETP **63** (1986) 904;
G. Camici and M. Ciafaloni, Phys. Lett. **B395** (1997) 118.