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On the Gluon Regge Trajectory in $O(\alpha_s^2)$ [1]

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Abstract

We recalculate the gluon Regge trajectory in next-to-leading order to clarify a discrepancy between two results in the literature on the constant part. We confirm the result obtained by Fadin et al. [1]. The effects on the anomalous dimension and on the s^ω behavior of inclusive cross sections are also discussed.

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1 Introduction

The Regge trajectory $\omega(L)$ forms an essential ingredient for the kernel of the Bethe–Salpeter equation [2, 3] to resum the small x contributions to the gluon anomalous dimension γ_{gg} in next-to-leading order in $\ln(1/x)$ and to all orders in the coupling constant. $\omega(L)$ is supposed to be a process–independent quantity, which, beyond one–loop order becomes renormalization–scheme dependent. For definiteness we will refer to the $\overline{\rm MS}$ scheme. There $\omega(L)$ is given by [1]

$$\omega(L) = \omega_q(L) + \omega_g(L) = -\overline{g}^2 \left[\frac{2}{\varepsilon} + 2L\right] - \overline{g}^4 \left\{c_0 \left[\frac{1}{\varepsilon^2} - L^2\right] + c_1 \left[\frac{1}{\varepsilon} + 2L\right] + c_2\right\} \tag{1}$$

with $\overline{g}^2 = g^2 N_c \Gamma(1-\varepsilon)/(4\pi)^{2+\varepsilon}$, $L = \log(q^2/\mu^2)$, N_c the number of colors, $\varepsilon = D/2 - 2$, and D the dimension of space-time. Previous studies [1, 4] agree on the the value of the coefficients

$$c_0 = \frac{\beta_0}{3} = \frac{11}{3} - \frac{2}{3}\frac{N_f}{N_c} \tag{2}$$

$$c_1 = \frac{67}{9} - 2\zeta(2) - \frac{10}{9}\frac{N_f}{N_c}, \tag{3}$$

but partly different results were reported on the coefficient c_2,

$$c_2 = -\frac{440}{27} + 2\zeta(3) + \frac{56}{27}\frac{N_f}{N_c}, \qquad [1] \tag{4}$$

$$c_2 = -\frac{440}{27} + 14\zeta(3) + \frac{56}{27}\frac{N_f}{N_c}, \qquad [4]\ . \tag{5}$$

Here N_f denotes the number of quark flavors.

The quarkonic contributions ($\omega_q(L)$) to the coefficients $c_i \propto N_f$ in eq. (1) were derived in ref. [5]. The gluonic contribution, $\omega_g(L)$, is represented by [1]

$$\omega_g(L) = \left(q^2\right)^{2\varepsilon} \left[A_0(I_1 - 2J_1) - 2(J_2 - J_3) - I_2\right] \tag{6}$$

with

$$A_0 = \psi(1) + 2\psi(\varepsilon) - \psi(1-\varepsilon) - 2\psi(1+2\varepsilon) + \frac{1}{2}\frac{(2+\varepsilon)(3+\varepsilon)}{\varepsilon(1+2\varepsilon)(3+2\varepsilon)} \tag{7}$$

and the integrals $I_{1,2}$ and $J_{1,2,3}$ are

$$I_i = \overline{g}^4 \frac{1}{\pi^{2(1+\varepsilon)}} \frac{1}{\Gamma^2(1-\varepsilon)} \int \frac{d^{2(1+\varepsilon)}q_1 d^{2(1+\varepsilon)}q_2 \left(q^2\right)^{2(1-\varepsilon)}}{q_1^2 q_2^2 (q_1-q)^2 (q_2-q)^2} a_i \tag{8}$$

$$J_i = \overline{g}^4 \frac{1}{\pi^{2(1+\varepsilon)}} \frac{1}{\Gamma^2(1-\varepsilon)} \int \frac{d^{2(1+\varepsilon)}q_1 d^{2(1+\varepsilon)}q_2 \left(q^2\right)^{(1-2\varepsilon)}}{q_1^2 q_2^2 (q-q_1-q_2)^2} b_i\ , \tag{9}$$

where $a_1 = b_1 = 1, a_2 = \log[(q_1-q_2)^2/q^2], b_2 = \log[(q-q_2)^2/q^2]$ and $b_3 = \log(q_2^2/q^2)$. Standard techniques allow to calculate the integrals I_1, J_1, J_2 and J_3 straightforwardly and the results given in ref. [1] are confirmed. The calculation of the integral I_2, to which the term $\propto \zeta(3)$ in eqs. (4,5) can be attributed, is much more difficult, enforcing the use of certain approximation methods in the previous calculation [1].

It is the aim of the present paper to recalculate this integral by a method different from that being used in [1] to obtain a thorough and independent check. Here we aim at using as general as possible representations in all steps.

As was outlined in refs. [6, 7] the term $\propto \zeta(3)$ in c_2 contributes to γ_{gg} at the 3–loop level and is essential for the small-x behavior of this quantity. Since the the gluonic anomalous dimension has not yet been calculated up to 3–loop order, this coefficient is an important *prediction* and its value has to be determined unambiguously.

2 The integral I_2

The integral I_2 is given by

$$I_2 = -\frac{\partial}{\partial\nu}\hat{I}_2(\nu)\Big|_{\nu=0} \tag{10}$$

with[3]

$$\hat{I}_2 = \overline{g}^4 \frac{1}{\pi^{2(1+\varepsilon)}} \frac{1}{\Gamma^2(1-\varepsilon)} \int \frac{d^{2(1+\varepsilon)}q_1 d^{2(1+\varepsilon)}q_2 \left(q^2\right)^{2(1+\varepsilon)+\nu}}{q_1^2 q_2^2 (q_1-q)^2 (q_2-q)^2 \left[(q_1-q_2)^2\right]^\nu} \ . \tag{11}$$

A general class of integrals, to which (11) belongs in principle, was studied in ref. [9] recently. As will be shown below the integral I_2 cannot be directly obtained from the solution given in [9], but appropriate limits are required in a series of parameters to obtain the final result. These may be performed without leaving general representations.

In the notation of [9] $\hat{I}_2$ reads

$$\hat{I}_2 = \frac{\overline{g}^4}{\Gamma^2(1-\varepsilon)} I_4(1,1,\nu,\delta) \tag{12}$$

where I_4 can be found in [9] and $\delta = \nu + 1 - \varepsilon$. Therefore the integral above is represented by

$$\hat{I}_2 = \overline{g}^4 \frac{2}{2\varepsilon-1} \frac{\nu(\nu+1-\varepsilon)}{\Gamma^2(1-\varepsilon)} G_2(1,2+\nu-\varepsilon) G_2(1,\nu+1) \lim_{b\to 0} S(\varepsilon-1,b,2\varepsilon-\nu-1,\nu-\varepsilon) \ . \tag{13}$$

Here the function G_2 is given by

$$G_2(\alpha_1,\alpha_2) = G_1(\alpha_1) G_1(\alpha_2) G_1(2-2\varepsilon-\alpha_1-\alpha_2) \tag{14}$$

and

$$G_1(\alpha) = \frac{\Gamma(1+\varepsilon-\alpha)}{\Gamma(\alpha)}. \tag{15}$$

The function $S(a,b,c,d)$ in eq. (13) consists of two terms

$$S(a,b,c,d) = S_1(a,b,c,d) + S_2(a,b,c,d) \tag{16}$$

with

$$S_1(a,b,c,d) = \frac{\pi\cot(\pi c)}{H(a,b,c,d)} - \frac{1}{c} \tag{17}$$

$$S_2(a,b,c,d) = -\frac{b+c}{bc} F(a+c,-b,-c,b+d), \tag{18}$$

[3] Already a long time ago integrals of the type (11) were studied, representing them by doubly infinite series [8]. We refer, complementary to this, to integral representations in the present treatment, since at the parameters relevant for the present problem we found it easier to derive the singularity structure in ε.

where

$$H(a,b,c,d) = \frac{\Gamma(1+a)\Gamma(1+b)\Gamma(1+c)\Gamma(1+d)\Gamma(1+a+b+c+d)}{\Gamma(1+a+c)\Gamma(1+a+d)\Gamma(1+b+c)\Gamma(1+b+d)} \ . \tag{19}$$

Note that the limit $\lim_{b\to 0} S_1(a,b,c,d)$ is regular. Due to the factor $1/\Gamma^2(1-\varepsilon)$ in eq. (11) all Γ-functions combine into Beta-functions and rational functions in ε and ν. This is also true for the Γ-functions in (19), guaranteeing the absence of the Mascheroni number γ_E in the subsequent evaluation.

The function $F(\hat{a},\hat{b},\hat{c},\hat{d})$ has the serial representation

$$F(\hat{a},\hat{b},\hat{c},\hat{d}) = \sum_{n=1}^{\infty} \frac{(-\hat{a})_n(-\hat{b})_n}{(1+\hat{c})_n(1+\hat{d})_n} \tag{20}$$

with $(r)_n = \prod_{k=1}^{n}(r+k-1)$ the Pochhammer–Barnes symbol. Since $F(\hat{a},b,\hat{c},\hat{d})$ emerges only in the limit

$$-\lim_{b\to 0}\frac{1}{b}F(\hat{a},-b,\hat{c},\hat{d})$$

we may represent S_2 directly by

$$\lim_{b\to 0} S_2(\hat{a},b,\hat{c},\hat{d}) = \frac{\nu+2-3\varepsilon}{(2+\nu-2\varepsilon)(1+\nu-\varepsilon)}{}_3F_2\left[\begin{array}{lll} 3(1-\varepsilon)+\nu, & 1, & 1 \\ 3+\nu-2\varepsilon, & 2+\nu-\varepsilon & \end{array}\ ;\ 1\right] \ . \tag{21}$$

The generalized hypergeometric function ${}_3F_2$ of unity argument in (21) obeys the integral representation, cf. [10],

$$\begin{aligned} &{}_3F_2\left[\begin{array}{lll} 3(1-\varepsilon)+\nu, & 1, & 1 \\ 3+\nu-2\varepsilon, & 2+\nu-\varepsilon & \end{array}\ ;\ 1\right] \\ &\qquad = \frac{1}{B(1,1+\nu-\varepsilon)}\int_0^1 dx(1-x)^{\nu-\varepsilon}{}_2F_1[3(1-\varepsilon)+\nu,1,3+\nu-2\varepsilon;x] \end{aligned} \tag{22}$$

and the hypergeometric function ${}_2F_1$ is given by

$${}_2F_1[3(1-\varepsilon)+\nu,1,3+\nu-2\varepsilon;x] = \frac{1}{B(1,2+\nu-2\varepsilon)}\int_0^1 dt(1-t)^{1+\nu-2\varepsilon}(1-tx)^{-3(1-\varepsilon)-\nu} \ . \tag{23}$$

Here $B(\alpha_1,\alpha_2)$ denotes Euler's Beta-function.

Therefore (21) obeys the representation

$$\begin{aligned} \lim_{b\to 0}\nu S_2 &= (\nu+2-3\varepsilon)\nu\int_0^1 dx\int_0^1 dt(1-x)^{\nu-\varepsilon}(1-t)^{1+\nu-2\varepsilon}(1-tx)^{-3(1-\varepsilon)-\nu} \\ &= -(2+\nu-3\varepsilon)\int_0^1 dx\int_0^1 dz\left[\frac{d}{dx}(1-x)^{\nu}\right]z^{1+\nu-2\varepsilon}(1-zx)^{-\varepsilon} \ . \end{aligned} \tag{24}$$

The factor ν in the r.h.s. of (24) has to be absorbed into the integral to obtain finite expressions in the subsequent calculation. All the above relations are exact transformations of $\hat{I}_2(\nu)$.

We now determine its expansion coefficient $\propto \nu$. For this purpose $\hat{I}_2$ is represented by

$$\hat{I}_2(\nu) = \overline{g}^4 C(\nu)\left[\nu S_2(\nu)\right] = \overline{g}^4\left[C_0+\nu C_1\right]\left[(S_{10}+S_{20})+\nu(S_{11}+S_{21})\right]+O(\nu^2) \ . \tag{25}$$

The coefficients C_0, C_1, S_{10} and S_{11} are obtained from a serial expansion of eqs. (13, 14) and (17). Also the coefficient S_{20} can be given in a closed form by partial integration of eq. (24). To determine S_{21} we apply a Taylor expansion of the latter integral with respect to ν and ε. The individual, partly transcendental integrals T_i contributing to this expression are listed in the appendix.

The different expansion coefficients read :

$$
\begin{aligned}
C_0 &= -\frac{6}{\varepsilon^2} + \frac{6}{\varepsilon} + 12\left[1+\zeta(2)\right] + 12\left[2-\zeta(2)-5\zeta(3)\right]\varepsilon + O(\varepsilon^2) && (26)\\
C_1 &= -\frac{4}{\varepsilon^3} - \frac{2}{\varepsilon^2} - \frac{4}{\varepsilon}\left[1-2\zeta(2)\right] - 4\left[2-\zeta(2)-8\zeta(3)\right] + O(\varepsilon) && (27)\\
S_{10} &= \frac{1}{3} + \frac{1}{3}\varepsilon + \left[1-2\zeta(2)\right]\varepsilon^2 + \left[3-2\zeta(2)-2\zeta(3)\right]\varepsilon^3 + O(\varepsilon^4) && (28)\\
S_{11} &= \frac{5}{18}\frac{1}{\varepsilon} + \frac{17}{18} + \frac{3}{2}\varepsilon + \left[\frac{3}{2} + 2\zeta(2) + \frac{4}{3}\zeta(3)\right]\varepsilon^2 + O(\varepsilon^3) && (29)\\
S_{20} &= -(2-3\varepsilon)B(2-2\varepsilon, 1-\varepsilon)\\
&= -1 - \varepsilon - \left[3-2\zeta(2)\right]\varepsilon^2 - \left[9-2\zeta(2)-6\zeta(3)\right]\varepsilon^3 + O(\varepsilon^4) && (30)\\
S_{21} &= \frac{\varepsilon}{4} + \frac{\varepsilon^2}{2} - \left[T_1 + 2T_2 + 2T_3\right]\varepsilon + \left[3T_2 + 5T_3 + T_4 + 4T_5 + 4T_6 + 2T_7 + 2T_8\right]\varepsilon^2\\
&\quad + O(\varepsilon^3)\\
&= \frac{5}{2}\varepsilon + \left[12 - 2\zeta(2) - 6\zeta(3)\right]\varepsilon^2 + O(\varepsilon^3) && (31)
\end{aligned}
$$

leading to

$$
I_2 = -\overline{g}^4\left[\frac{1}{\varepsilon^3} - \frac{2}{\varepsilon}\zeta(2) - 26\zeta(3)\right] \qquad (32)
$$

using eqs. (10,25), which confirms the result derived by Fadin et al. in ref. [1] before. Here the coefficients of $O(1/\varepsilon^3)$ and $O(1/\varepsilon^2)$ are independent of the value of S_{21}. For the constant term $O(\varepsilon^0)$ all rationals and the $\zeta(2)$-terms are canceling and only $\zeta(3)$-terms contribute. Finally c_2 is given by eq. (4).

3 Consequences for the small-x resummed anomalous dimension γ_{gg}^{NLO}

It is convenient to study Γ_k^n using DIS-schemes, cf. [11, 6]. The general expression for the resummed gluonic small-x anomalous dimension was derived up to next-to-leading order in ref. [11] and may be written as

$$
\begin{aligned}
\gamma_{gg}^{\rm DIS} &= \gamma_{gg}^{(0)} - \alpha_s\frac{\chi_1(\gamma) - 2\chi_0(\gamma)\chi_0'(\gamma)}{\chi_0'(\gamma)}\\
&\quad + \frac{\beta_0}{4\pi}\alpha_s^2\frac{d\log R(\gamma)}{d\alpha_s} + \frac{4}{9}\left[1-R(\gamma)\right]\gamma_{qg}^{Q_0}(\gamma) + \frac{\beta_0}{4\pi}\alpha_s^2\frac{d\log\left[\gamma\sqrt{-\chi_0'(\gamma)}\right]}{d\alpha_s}\ . \qquad (33)
\end{aligned}
$$

Here

$$
\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\ , \qquad (34)
$$

cf. [12], and $\gamma_{gg}^{(0)}$ is the solution of the equation

$$1 = \frac{N_c \alpha_s}{\pi} \frac{1}{N-1} \chi_0(\gamma_{gg}^{(0)}) \qquad \text{with} \qquad \gamma_{gg}^{(0)} \to \frac{N_c \alpha_s}{\pi} \frac{1}{N-1} \quad \text{for} \quad N \to \infty \; . \tag{35}$$

The function $R(\gamma)$ is specific for the $\overline{\rm MS}-$DIS scheme [13] and $\gamma_{qg}^{(Q_0)}$ denotes the quark-gluon anomalous dimension in the Q_0 scheme, see ref. [11]. The latter quantities contribute only beyond 3–loop order. For later use we decompose $\chi_1(\gamma)$ ([2, 3]) into the parts

$$\chi_1(\gamma) = \chi_1^{\rm scal}(\gamma) + \chi_1^{\rm conf}(\gamma) \tag{36}$$

where

$$\chi_1^{\rm scal}(\gamma) \;\; = \;\; \left[\frac{\beta_0}{6} + \frac{d}{d\gamma}\right] \left[\chi_0^2(\gamma) + \chi_0'(\gamma)\right] \; . \tag{37}$$

$\chi_0(\gamma) \equiv \chi_0^{\rm conf}(\gamma)$ and $\chi_1^{\rm conf}(\gamma)$ are symmetric under the interchange $\gamma \leftrightarrow 1-\gamma$.

Both terms are called conformal since they do not depend on the choice of scales, cf. also [3], and asymptotic scale invariance implies asymptotic conformal invariance [14]. Retaining only these parts of the kernel of the Bethe–Salpeter equation in refs. [12, 2] one obtains the solution

$$E_k^n(\mu^2) = E_k^n(\mu_0^2) \left(\frac{\mu^2}{\mu_0^2}\right)^{\Gamma_k^n} \tag{38}$$

which is *also* the solution of the renormalization group equation

$$\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \gamma_m \frac{\partial}{\partial m} + \gamma_{O_k} - n \gamma_\Phi\right] E_k^n = 0 \tag{39}$$

for $m = 0$ and $\beta = 0$ if one identifies

$$\Gamma_k^n = \gamma_{O_k} - n \gamma_\Phi \equiv \sum_{l=1}^{\infty} \alpha_s^l \gamma_l^{k,n} \; . \tag{40}$$

The coefficients $\gamma_l^{k,n}$ are then the expansion coefficients of the conformal part of (33).

At the one and two-loop level the contributions to Γ_k^n result from either $\chi_0(\gamma)$ or $\chi_1^{\rm conf}(\gamma)$ only and agree with the corresponding fixed order results. Moreover the type of DIS-scheme is unimportant at that level.

Beginning with the 3–loop order the first term in eq. (36), containing the scale dependent contributions, starts to determine the small x part of the anomalous dimension. At that level still the specific way in which the DIS–scheme was introduced is unimportant, cf. [6, 7]. Only beginning with the 4–loop level the small-x part of the anomalous dimension starts to depend on the specific type of DIS-schemes. Further running coupling effects beyond those considered in [2] are important, cf. [11, 6]. The difference between eq. (4) and (5) effects the gluon anomalous dimension at the 3–loop level. We note that the expansion coefficient in $O(\alpha_s^3)$ of the combination $\gamma_+ = \gamma_{gg} + (4/9)\gamma_{qg}$ would have been larger by a factor of 2.123 for $N_f = 4$ if one would have used eq. (5) instead of eq. (4), having thus significant impact. This factor is widely independent of N_f.

We finally would like to comment upon the intercept ω describing the s^ω–behavior of inclusive cross sections. In the saddle-point approximation [2] one obtains

$$\omega_{(1)} = \omega_{(0)} \left\{ 1 + \frac{\alpha_s^2}{\omega_{(0)}} \left[\chi_1^{\rm scal}(1/2) + \chi_1^{\rm conf}(1/2) \right] \right\}, \qquad \text{with} \qquad \omega_{(0)} = 12 N_c \log(2) \frac{\alpha_s}{3\pi} \tag{41}$$

and (cf. [2])

$$\chi_1^{\rm scal}(1/2) = -\omega_{(0)} \left[2 \left(\frac{11}{3} - \frac{2N_f}{3N_c} \right) \log(2) + 7 \frac{\zeta(3)}{\log(2)} \right] \tag{42}$$

$$\begin{aligned} \chi_1^{\rm conf}(1/2) &= -\omega_{(0)} \left\{ -\frac{67}{9} + 2\zeta(2) + \frac{10}{9}\frac{N_f}{N_c} \right. \\ &+ \left. \frac{1}{4\log(2)} \left[16 \int_0^1 \frac{dx}{x} \arctan(\sqrt{x}) \log(1/(1-x)) - 6\zeta(3) + \frac{\pi^3}{2} \left(\frac{27}{16} + \frac{11}{16}\frac{N_f}{N_c^3} \right) \right] \right\} \end{aligned} \tag{43}$$

corresponding to the decomposition in eq. (36). Using the full expression for $\chi_1(\gamma = 1/2)$ in the calculation of ω one obtains [4]

$$\omega = 2.64762 \alpha_s (1 - 6.36402 \alpha_s), \tag{44}$$

with a maximal value of 0.1040 at $Q^2 = 8.7 \cdot 10^6\,\mathrm{GeV}^2$. This value is only little above the soft pomeron's intercept 0.0808 and, moreover, *negative* for $Q^2 < 588\,\mathrm{GeV}^2$. Contrary to this, the conformal part of $\chi_1(\gamma)$ implies

$$\omega_{\rm conf} = 2.64762 \alpha_s (1 - 2.54664 \alpha_s), \tag{45}$$

with a maximal value of 0.259915 at $Q^2 = 87\,\mathrm{GeV}^2$. Unlike the value of ω in (44) $\omega_{\rm conf}$ is positive for $Q^2 > 1.86\,\mathrm{GeV}^2$. Thus the conformal part shows a reasonable behavior, while the scale–dependent effects have still to be understood. If instead of eq. (5), eq. (4) would have been used in eqs. (44, 45) the coefficient inside the brackets would be smaller by 0.6210 and lead to $\omega_{\rm max} = 0.09476$ and 0.20896 respectively. This difference, particularly in view of the last number, is not negligible.

In summary, we confirm the results of Fadin et al. given in ref. [1]. The difference between a result reported in ref. [4] and [1] was found to have significant impact both on the value of γ_+ in $O(\alpha_s^3)$, widely independent of the number of flavors, and on the conformal part of the intercept $\omega_{\rm conf}$ in the s^ω behaviour in next-to-leading order.

4 Appendix

The coefficients T_i in eq. (31) are given by

$$T_1 = \int_0^1 dx \int_0^1 dz \frac{z^2}{1-zx} = \frac{3}{4} \tag{46}$$

$$T_2 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log(1-x)}{1-zx} = -\frac{1}{2} - \frac{1}{2}\zeta(2) \tag{47}$$

[4] Note a numerical error in eq. (16) of ref. [2].

$$T_3 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log(z)}{1-zx} = -1 + \frac{1}{2}\zeta(2) \tag{48}$$

$$T_4 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log(1-zx)}{1-zx} = -\frac{7}{8} \tag{49}$$

$$T_5 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log(z)\log(1-x)}{1-zx} = \frac{5}{4} - \frac{1}{4}\zeta(2) - \frac{1}{2}\zeta(3) \tag{50}$$

$$T_6 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log^2(z)}{1-zx} = \frac{17}{8} - \frac{1}{2}\zeta(2) - \zeta(3) \tag{51}$$

$$T_7 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log(z)\log(1-zx)}{1-zx} = \frac{31}{16} - \frac{3}{4}\zeta(2) - \frac{1}{2}\zeta(3) \tag{52}$$

$$T_8 = \int_0^1 dx \int_0^1 dz \frac{z^2 \log(1-x)\log(1-zx)}{1-zx} = \frac{3}{4} + \frac{3}{4}\zeta(2) + \frac{1}{2}\zeta(3)\ . \tag{53}$$

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