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Next-to-Leading Order QCD Corrections to the Lifetime Difference of B_s Mesons

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Abstract

We compute the QCD corrections to the decay rate difference in the B_s–$\bar{B}_s$ system, $\Delta\Gamma_{B_s}$, in the next-to-leading logarithmic approximation using the heavy quark expansion approach. Going beyond leading order in QCD is essential to obtain a proper matching of the Wilson coefficients to the matrix elements of local operators from lattice gauge theory. The lifetime difference is reduced considerably at next-to-leading order. We find $(\Delta\Gamma/\Gamma)_{B_s} = (f_{B_s}/210\,\mathrm{MeV})^2[0.006B(m_b) + 0.150B_S(m_b) - 0.063]$ in terms of the bag parameters B, B_S in the NDR scheme. As a further application of our analysis we also derive the next-to-leading order result for the mixing-induced CP asymmetry in inclusive $b \to u\bar{u}d$ decays, which measures $\sin 2\alpha$.

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1. Introduction. The width difference $(\Delta\Gamma/\Gamma)_{B_s}$ of the B_s meson CP eigenstates [1] is expected to be about $10-20\%$, among the largest rate differences in the b-hadron sector [2], and might be measured in the near future. A measurement of a sizeable $(\Delta\Gamma/\Gamma)_{B_s}$ would open up the possibility of novel CP violation studies with B_s mesons [3, 4]. In principle, a measured value for $\Delta\Gamma_{B_s}$ could also give some information on the mass difference ΔM_{B_s} [5], if the theoretical prediction for the ratio $(\Delta\Gamma/\Delta M)_{B_s}$ can be sufficiently well controlled [6]. Furthermore, as pointed out in [7], if non-standard-model sources of CP violation are present in the B_s system, $\Delta\Gamma_{B_s}$ can be smaller (but not larger) than expected in the standard model. For this reason a lower bound on the standard model prediction is of special interest.

The calculation of inclusive non-leptonic b-hadron decay observables, such as $\Delta\Gamma_{B_s}$, uses the heavy quark expansion (HQE). The decay width difference is expanded in powers of $\Lambda_{\rm QCD}/m_b$, each term being multiplied by a series of radiative corrections in $\alpha_s(m_b)$. In the case of $(\Delta\Gamma/\Gamma)_{B_s}$, the leading contribution is parametrically of order $16\pi^2(\Lambda_{\rm QCD}/m_b)^3$. In the framework of the HQE the main ingredients for a reliable prediction (less than 10% uncertainty) are a) subleading corrections in the $1/m_b$ expansion, b) the non-perturbative matrix elements of local four-quark operators between B-meson states and c) $\mathcal{O}(\alpha_s)$ radiative corrections to the Wilson coefficients of these operators. The first issue has been addressed in [6]. The hadronic matrix elements can be studied using numerical simulations in lattice QCD. In this letter, we present the next-to-leading order QCD radiative corrections to the Wilson coefficient functions for $\Delta\Gamma_{B_s}$. In addition to removing another item from the above list and reducing certain renormalization scale ambiguities of the leading order prediction, the inclusion of $\mathcal{O}(\alpha_s)$ corrections is necessary for a satisfactory matching of the Wilson coefficients to the matrix elements to be obtained from lattice calculations.

Our results provide the first calculation of perturbative QCD effects beyond the leading logarithmic approximation to spectator effects in the HQE for heavy hadron decays. The consideration of subleading QCD radiative effects has implications of conceptual interest for the construction of the HQE. Soft gluon emission from the spectator s quark in the B_s meson leads to power-like IR singularities in individual contributions, which would apparently impede the HQE construction, because they cannot be absorbed into matrix elements of local operators. It has already been explained in [8], how these severe IR divergences cancel in the sum over all cuts of a given diagram, so that the Wilson coefficients of four-quark operators relevant to lifetime differences, such as $\Delta\Gamma_{B_s}$, are free of infrared singularities. This infrared cancellation is confirmed by the result of our explicit calculation.

Using the HQE to finite order in Λ_{QCD}/m_b rests on the assumption of local quark-hadron duality. Little is known in QCD about the actual numerical size of duality-violating effects. (See, e.g., [9] for a recent discussion of the issue.) Experimentally no violation of local quark-hadron duality in inclusive observables of the B-meson sector has been established so far. In [10] it has been shown that for $\Delta\Gamma_{B_s}$ local duality holds exactly in the simultaneous limits of small velocity ($\Lambda_{QCD} \ll m_b - 2m_c \ll m_b$) and large

number of colours ($N_c \to \infty$). In this case

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \frac{G_F^2 m_b^3 f_{B_s}^2}{4\pi} |V_{cs}V_{cb}|^2 \sqrt{2 - 4\frac{m_c}{m_b}}\, \tau_{B_s} \approx 0.18. \tag{1}$$

It is interesting that the numerical value implied by the limiting formula (1) appears to be quite realistic and is in fact consistent with the results of more complete analyses [6, 10]. The duality assumption can in principle be tested by a confrontation of theoretical predictions, based on the HQE, with experiment. This aspect is another major motivation for computing $\Delta\Gamma_{B_s}$ accurately.

It is clear from these remarks that a detailed theoretical analysis of $\Delta\Gamma_{B_s}$, and particularly of $\mathcal{O}(\alpha_s)$ corrections, is very desirable, both for phenomenological and for conceptual reasons. In this letter we shall concentrate on the presentation of our results and a brief discussion of their main aspects. Details and an extension of our analysis to other b-hadron lifetime differences will be given in a forthcoming publication [11].

2. Formalism and next-to-leading order results. In the limit of CP conservation the mass eigenstates of the B_s–$\bar{B}_s$ system are $|B_{H/L}\rangle = (|B_s\rangle \pm |\bar{B}_s\rangle)/\sqrt{2}$, using the convention $CP|B_s\rangle = -|\bar{B}_s\rangle$. The width difference between mass eigenstates is then given by

$$\Delta\Gamma_{B_s} \equiv \Gamma_L - \Gamma_H = -2\,\Gamma_{12} = -2\,\Gamma_{21}, \tag{2}$$

where Γ_{ij} are the elements of the decay-width matrix, $i,j = 1,2$ ($|1\rangle = |B_s\rangle$, $|2\rangle = |\bar{B}_s\rangle$). In writing (2) we assumed standard CKM phase conventions [12]. For more information about the basic formulas see for instance [6] (and references therein).

The decay width is related to the absorptive part of the forward scattering amplitude via the optical theorem [2]. The off-diagonal element of the decay-width matrix may thus be written as

$$\Gamma_{21} = \frac{1}{2M_{B_s}} \langle \bar{B}_s | \mathcal{T} | B_s \rangle. \tag{3}$$

The normalization of states is $\langle B_s | B_s \rangle = 2EV$ (conventional relativistic normalization) and the transition operator $\mathcal{T}$ is defined by [2]

$$\mathcal{T} = \mathrm{Im}\, i \int d^4x\; T\, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0). \tag{4}$$

Here $\mathcal{H}_{eff}$ is the low energy effective weak Hamiltonian mediating bottom quark decay. The component that is relevant for Γ_{21} reads explicitly

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left(\sum_{r=1}^{6} C_r Q_r + C_8 Q_8 \right), \tag{5}$$

with the operators

$$Q_1 = (\bar{b}_i c_j)_{V-A} (\bar{c}_j s_i)_{V-A} \qquad Q_2 = (\bar{b}_i c_i)_{V-A} (\bar{c}_j s_j)_{V-A}, \tag{6}$$

$$Q_3 = (\bar{b}_i s_i)_{V-A} (\bar{q}_j q_j)_{V-A} \qquad Q_4 = (\bar{b}_i s_j)_{V-A} (\bar{q}_j q_i)_{V-A}, \tag{7}$$

$$Q_5 = (\bar{b}_i s_i)_{V-A}(\bar{q}_j q_j)_{V+A} \qquad Q_6 = (\bar{b}_i s_j)_{V-A}(\bar{q}_j q_i)_{V+A}, \tag{8}$$

$$Q_8 = \frac{g}{8\pi^2} m_b\, \bar{b}_i \sigma^{\mu\nu}(1-\gamma_5) T^a_{ij} s_j\, G^a_{\mu\nu}. \tag{9}$$

Here the i, j are colour indices and a summation over $q = u$, d, s, c, b is implied. $V \pm A$ refers to $\gamma^\mu(1 \pm \gamma_5)$ and $S - P$ (which we need below) to $(1 - \gamma_5)$. $C_1, \ldots, C_6$ are the corresponding Wilson coefficient functions, which are known at next-to-leading order. We have also included the chromomagnetic operator Q_8, contributing to $\mathcal{T}$ at $\mathcal{O}(\alpha_s)$. (Note that for a negative C_8, as conventionally used in the literature, the Feynman rule for the quark-gluon vertex is $-ig\gamma_\mu T^a$.) A detailed review and explicit expressions may be found in [13]. Cabibbo suppressed channels have been neglected in (5).

Expanding the operator product (4) for small $x \sim 1/m_b$, the transition operator $\mathcal{T}$ can be written, to leading order in the $1/m_b$ expansion, as

$$\mathcal{T} = -\frac{G_F^2 m_b^2}{12\pi}(V_{cb}^* V_{cs})^2 \; [F(z)Q(\mu_2) + F_S(z)Q_S(\mu_2)] \tag{10}$$

with $z = m_c^2/m_b^2$ and the basis of $\Delta B = 2$ operators

$$Q = (\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}, \qquad Q_S = (\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}\ . \tag{11}$$

In writing (10) we have used Fierz identities and the equations of motion to eliminate the colour re-arranged operators

$$\tilde{Q} = (\bar{b}_i s_j)_{V-A}(\bar{b}_j s_i)_{V-A}, \qquad \tilde{Q}_S = (\bar{b}_i s_j)_{S-P}(\bar{b}_j s_i)_{S-P}\ , \tag{12}$$

always working to leading order in $1/m_b$ (see below). The Wilson coefficients F and F_S can be extracted by computing the matrix elements between quark states of $\mathcal{T}$ in (4) (Fig. 1), as well as those of Q and Q_S, to a given order in QCD perturbation theory, and comparing them with (10). (The external b quarks are taken to be on-shell.) This matching procedure factorizes the perturbatively calculable short-distance contributions (Wilson coefficients) from the long-distance dynamics, parametrized by the non-perturbative matrix elements of local $\Delta B = 2$ operators. We do not use heavy quark effective theory (HQET) to expand these matrix elements explicitly in $1/m_b$. They are to be understood in full QCD, in accordance with the treatment of $1/m_b$ effects in [6].

In our discussion we shall first concentrate on the contribution to $\mathcal{T}$ from the operators Q_1 and Q_2 in (5). The penguin operators have small coefficients and are numerically less important. Their effect will be included later on. Thus for the time being only the diagrams D_1 – D_{10} in Fig. 1 are considered, while D_{11} and D_{12} belong to the penguin sector to be discussed below.

Working in leading order, the matching calculation for (10) has to be performed to zeroth order in α_s. At next-to-leading order the coefficients $C_{1,2}$ have to be taken in next-to-leading logarithmic approximation [14, 15] and the $\mathcal{O}(\alpha_s)$ matching corrections to the coefficients in (10) have to be computed. At this order the coefficients in (10) depend on the renormalization scheme chosen for their evaluation. This scheme dependence is cancelled by the matrix elements of the operators in (10), which have to be determined accordingly.

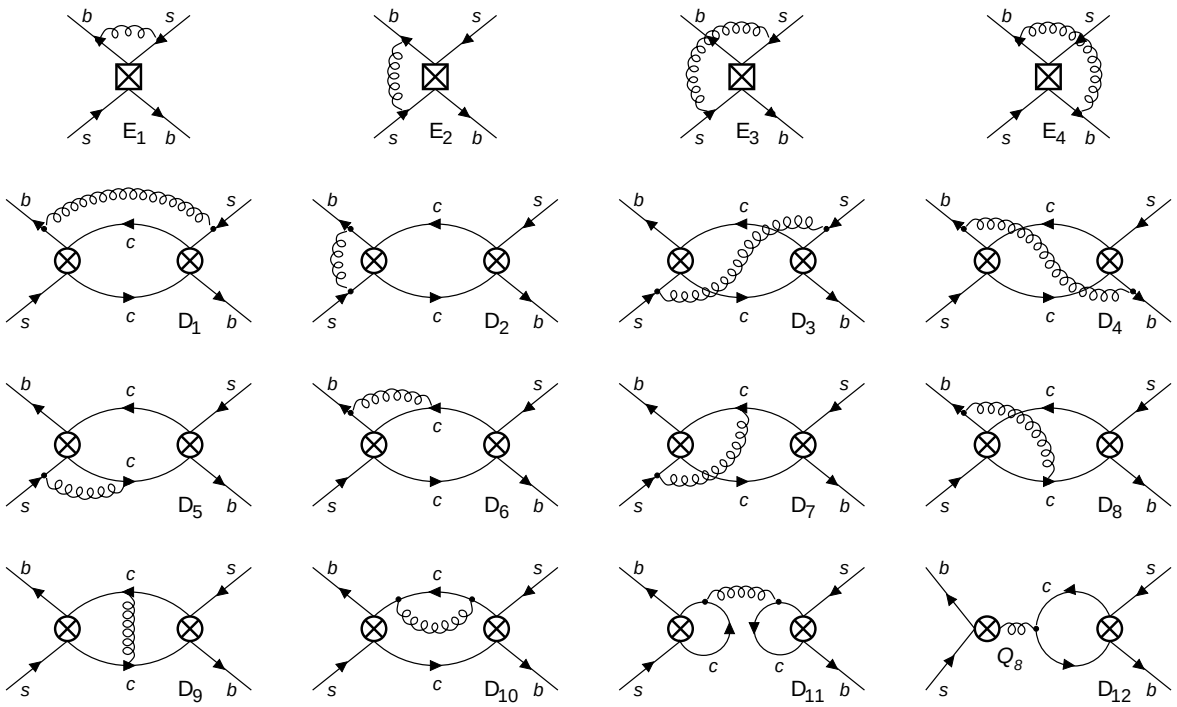

Figure 1: $\mathcal{O}(\alpha_s)$ corrections to the B_s–$\bar{B}_s$ transition operator, D_1 – D_{12}. Also shown are the corrections to the matrix elements of local $\Delta B = 2$ operators, E_1 – E_4, required for a proper factorization of short-distance and long-distance contributions. Not displayed explicitly are E_1', E_2', D_1', D_2', D_5', D_6', D_7', D_8', D_{10}' and D_{12}', which are obtained by rotating the corresponding diagrams by 180°.

The scheme we employ for our result is specified as follows. We use dimensional regularization with anti-commuting γ_5 and modified minimal ($\overline{MS}$) subtraction of ultraviolet singularities (NDR scheme). In addition we project D-dimensional Dirac-structures, arising at intermediate stages of the calculation, according to the prescriptions ($D = 4-2\varepsilon$)

$$[\gamma^\mu\gamma^\alpha\gamma^\nu(1-\gamma_5)]_{ij}\,[\gamma_\mu\gamma_\alpha\gamma_\nu(1-\gamma_5)]_{kl} \to (16-4\varepsilon)[\gamma^\mu(1-\gamma_5)]_{ij}\,[\gamma_\mu(1-\gamma_5)]_{kl}, \qquad (13)$$

$$[\gamma^\mu\gamma^\alpha\gamma^\nu(1-\gamma_5)]_{ij}\,[\gamma_\nu\gamma_\alpha\gamma_\mu(1-\gamma_5)]_{kl} \to (4-8\varepsilon)[\gamma^\mu(1-\gamma_5)]_{ij}\,[\gamma_\mu(1-\gamma_5)]_{kl}, \qquad (14)$$

$$[\gamma^\mu\gamma^\nu(1-\gamma_5)]_{ij}\,[\gamma_\mu\gamma_\nu(1-\gamma_5)]_{kl} \to (8-4\varepsilon)[1-\gamma_5]_{ij}\,[1-\gamma_5]_{kl} - (8-8\varepsilon)[1-\gamma_5]_{il}\,[1-\gamma_5]_{kj}. \qquad (15)$$

The projections are equivalent to the subtraction of evanescent operators, defined by the difference of the left- and right-hand sides of (13–15). The definition of evanescent operators is discussed in great detail in [15, 16, 17] and we use the basis of [17] for the projection. The prescriptions (13–15) complete the definition of our renormalization scheme. They preserve Fierz symmetry, i.e. the one-loop matrix elements of Q, Q_S and $\tilde{Q}_S$ equal the matrix elements of the operators that one obtains from Q, Q_S, $\tilde{Q}_S$ by (4-dimensional) Fierz transformations. The projections (13–15) are sufficient if we use the Fierz form of Q_1, Q_2 in (6) that corresponds to closed charm-quark loops in Fig. 1.

Since the renormalization of Q_1, Q_2 respects Fierz symmetry, this choice can always be made.

We now give the result for the transition operator (10) at next-to-leading order, still neglecting the penguin sector. The coefficients in (10) can be written as

$$F(z) = F_{11}(z)C_1^2(\mu_1) + F_{12}(z)C_1(\mu_1)C_2(\mu_1) + F_{22}(z)C_2^2(\mu_1), \tag{16}$$

$$F_{ij}(z) = F_{ij}^{(0)}(z) + \frac{\alpha_s(\mu_1)}{4\pi}F_{ij}^{(1)}(z) \tag{17}$$

and similarly for $F_S(z)$. The leading order functions $F_{ij}^{(0)}$, $F_{S,ij}^{(0)}$ read explicitly

$$F_{11}^{(0)}(z) = 3\sqrt{1-4z}\,(1-z) \qquad F_{S,11}^{(0)}(z) = 3\sqrt{1-4z}\,(1+2z), \tag{18}$$

$$F_{12}^{(0)}(z) = 2\sqrt{1-4z}\,(1-z) \qquad F_{S,12}^{(0)}(z) = 2\sqrt{1-4z}\,(1+2z), \tag{19}$$

$$F_{22}^{(0)}(z) = \frac{1}{2}(1-4z)^{3/2} \qquad F_{S,22}^{(0)}(z) = -\sqrt{1-4z}\,(1+2z). \tag{20}$$

The next-to-leading order expressions $F_{ij}^{(1)}$, $F_{S,ij}^{(1)}$ are

$$\begin{aligned}
F_{11}^{(1)}(z) &= 32(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
&+ 64(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
&- 4(13-26z-4z^2+14z^3)\ln\sigma \\
&+ \sqrt{1-4z}\Big[4(13-10z)\ln z - 12(3-2z)\ln(1-4z) \\
&\quad + \frac{1}{6}(109-226z+168z^2)\Big] + 2\sqrt{1-4z}(5-8z)\ln\frac{\mu_2}{m_b},
\end{aligned} \tag{21}$$

$$\begin{aligned}
F_{S,11}^{(1)}(z) &= 32(1-4z^2)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
&+ 64(1-4z^2)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
&- 16(4-2z-7z^2+14z^3)\ln\sigma \\
&+ \sqrt{1-4z}\Big[64(1+2z)\ln z - 48(1+2z)\ln(1-4z) \\
&\quad - \frac{8}{3}(1-6z)(5+7z)\Big] - 32\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b},
\end{aligned} \tag{22}$$

$$\begin{aligned}
F_{12}^{(1)}(z) &= \frac{64}{3}(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
&+ \frac{128}{3}(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right)
\end{aligned}$$

$$
\begin{aligned}
&+ (2 - 259z + 662z^2 - 76z^3 - 200z^4)\frac{\ln\sigma}{6z} \\
&- \sqrt{1-4z}\left[(2 - 255z + 316z^2)\frac{\ln z}{6z} + 8(3-2z)\ln(1-4z)\right. \\
&\quad \left. + \frac{2}{9}(127 - 199z - 75z^2)\right] \\
&- 2\sqrt{1-4z}(17-26z)\ln\frac{\mu_1}{m_b} + \frac{4}{3}\sqrt{1-4z}(5-8z)\ln\frac{\mu_2}{m_b}, \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
F^{(1)}_{S,12}(z) = {}& \frac{64}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
&+ \frac{128}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
&+ (1 - 35z + 4z^2 + 76z^3 - 100z^4)\frac{4\ln\sigma}{3z} \\
&- \sqrt{1-4z}\left[(1 - 33z - 76z^2)\frac{4\ln z}{3z} + 32(1+2z)\ln(1-4z)\right. \\
&\quad \left. + \frac{4}{9}(68 + 49z - 150z^2)\right] \\
&- 16\sqrt{1-4z}(1+2z)\ln\frac{\mu_1}{m_b} - \frac{64}{3}\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b}, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
F^{(1)}_{22}(z) = {}& \frac{4}{3}(4 - 21z + 2z^2)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
&+ \frac{4}{3}(1-2z)(5-2z)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
&- (7 + 13z - 194z^2 + 304z^3 - 64z^4)\frac{\ln\sigma}{6z} - \frac{\pi^2}{3}(1-10z) \\
&+ \sqrt{1-4z}\left[(7 + 27z - 250z^2)\frac{\ln z}{6z} - 4(1-6z)\ln(1-4z)\right. \\
&\quad \left. - \frac{1}{18}(115 + 632z + 96z^2)\right] \\
&- 2\sqrt{1-4z}(5-2z)\ln\frac{\mu_1}{m_b} + \frac{4}{3}\sqrt{1-4z}(2-5z)\ln\frac{\mu_2}{m_b}, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
F^{(1)}_{S,22}(z) = {}& -\frac{32}{3}(1+z)(1+2z)\left(\mathrm{Li}_2(\sigma^2) + \ln^2\sigma + \frac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\right) \\
&+ \frac{32}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma) + \frac{1}{2}\ln(1-\sigma)\ln\sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
&+ (1+7z+10z^2-68z^3+32z^4)\frac{4\ln\sigma}{3z}+\frac{8\pi^2}{3}(1+2z)\\
&- \sqrt{1-4z}\left[(1+9z+26z^2)\frac{4\ln z}{3z}-16(1+2z)\ln(1-4z)\right.\\
&\quad\left.+\frac{8}{9}(19+53z+24z^2)\right]\\
&- 16\sqrt{1-4z}(1+2z)\ln\frac{\mu_1}{m_b}+\frac{32}{3}\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b}. \qquad (26)
\end{aligned}
$$

In these equations we have set $N_c = 3$ and used

$$
\mathrm{Li}_2(x) = -\int_0^x dt\frac{\ln(1-t)}{t}, \qquad \sigma = \frac{1-\sqrt{1-4z}}{1+\sqrt{1-4z}}, \qquad z = \frac{m_c^2}{m_b^2}. \qquad (27)
$$

The dependence on the renormalization scale μ_1 in (21–26) cancels against the scale dependence of the $\Delta B = 1$ Wilson coefficients $C_i(\mu_1)$ in (16) to the considered order in α_s. Likewise, the dependence on μ_2 is cancelled by the matrix elements of the $\Delta B = 2$ operators Q and Q_S. To check this, we note that the scale dependence of matrix elements $\langle\vec{Q}\rangle = (\langle Q\rangle,\ \langle Q_S\rangle,\ \langle\tilde{Q}_S\rangle)^T$ is given by

$$
\frac{d}{d\ln\mu_2}\langle\vec{Q}\rangle = -\frac{\alpha_s}{4\pi}\gamma_2^{(0)}\langle\vec{Q}\rangle, \qquad (28)
$$

where

$$
\gamma_2^{(0)} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -28/3 & 4/3 \\ 0 & 16/3 & 32/3 \end{pmatrix}. \qquad (29)
$$

The operator $\tilde{Q}_S$, which is redundant at leading power in $1/m_b$, can then be eliminated using (35) below and the μ_2-independence can be verified.

Our results in (21-26) correspond to the use of the one-loop pole mass for the b quark in (10). In $z = m_c^2/m_b^2$ there is no difference between the ratios of pole masses and $\overline{MS}$ masses at next-to-leading order in α_s.

We next present the contribution from QCD penguins to the transition operator $\mathcal{T}$. This sector has been treated in [6] in the leading logarithmic approximation. At next-to-leading order the penguin coefficients $C_3,\ldots,C_6$ have to be computed with next-to-leading logarithmic accuracy [18, 19] and the diagram D_{11} in Fig. 1 must be evaluated. Because the coefficients $C_3,\ldots,C_6$ are small ($\sim$ few per cent), contributions of second order in these coefficients are safely negligible and it is sufficient to calculate only the interference of $C_3,\ldots,C_6$ with C_1 and C_2. A consistent way to implement this approximation at NLO is to treat $C_3,\ldots,C_6$ as formally of $\mathcal{O}(\alpha_s)$. The standard NLO formula (as reviewed in [13]) can be used for the penguin coefficients, except that terms of order $\alpha_s C_3,\ldots,\alpha_s C_6$ have to be dropped. Accordingly, only current-current operators Q_1, Q_2 are inserted into the diagrams D_1 – D_{11}. The NLO result thus obtained is manifestly scheme independent and formally of order $\mathcal{O}(C_{3,\ldots,6}) = \mathcal{O}(\alpha_s)$. A further contribution,

absent in leading logarithmic approximation, comes from the chromomagnetic operator Q_8 in (5) and is shown as diagram D_{12} of Fig. 1. Since this contribution arises first at $\mathcal{O}(\alpha_s)$, the lowest order expression is sufficient for C_8. For the NLO penguin contribution we then find

$$\mathcal{T}_p = -\frac{G_F^2 m_b^2}{12\pi}(V_{cb}^* V_{cs})^2 \left[P(z)Q + P_S(z)Q_S\right], \tag{30}$$

$$P(z) = \sqrt{1-4z}\left((1-z)K_1'(\mu_1) + \frac{1}{2}(1-4z)K_2'(\mu_1) + 3z\,K_3'(\mu_1)\right) + \frac{\alpha_s(\mu_1)}{4\pi}F_p(z)C_2^2(\mu_1), \tag{31}$$

$$P_S(z) = \sqrt{1-4z}(1+2z)\left(K_1'(\mu_1) - K_2'(\mu_1)\right) - \frac{\alpha_s(\mu_1)}{4\pi}8F_p(z)C_2^2(\mu_1), \tag{32}$$

$$F_p(z) = -\frac{1}{9}\sqrt{1-4z}(1+2z)\cdot \tag{33}$$
$$\cdot\left[2\ln\frac{\mu_1}{m_b} + \frac{2}{3} + 4z - \ln z + \sqrt{1-4z}(1+2z)\ln\sigma + \frac{3C_8(\mu_1)}{C_2(\mu_1)}\right],$$

where we defined the combinations $K_1' = 2(3C_1C_3 + C_1C_4 + C_2C_3)$, $K_2' = 2C_2C_4$ and $K_3' = 2(3C_1C_5 + C_1C_6 + C_2C_5 + C_2C_6)$. The explicit μ_1-dependence in $F_p(z)$ is cancelled by the μ_1-dependence of $C_3, \ldots, C_6$. Note that for the penguin sector the scale and scheme dependence of the matrix elements of Q and Q_S are effects beyond the considered order and numerically negligible.

Beyond leading order in the $1/m_b$ expansion, several further operators contribute to $\Delta\Gamma$, denoted by R_i in [6]. Inspection of the factorized matrix elements of these operators given in [6] shows that these superficially power-suppressed operators contribute at leading power beyond tree level. This is due to the fact that the operators are defined in QCD (rather than in HQET) and hence the one-loop matrix elements contain m_b explicitly. The leading power piece arises only from loop momenta of order m_b and can therefore be subtracted perturbatively. This subtraction is necessary for a complete calculation of the α_s correction at leading power and is taken into account in the above result. To be more explicit, consider as an example the operator

$$R_0 \equiv Q_S + \tilde{Q}_S + \frac{1}{2}Q, \tag{34}$$

which can be reduced to an explicitly power-suppressed operator using Fierz transformations and the equations of motion. (We have used this in our calculation to eliminate $\tilde{Q}_S$.) At order α_s, we find the leading power contribution

$$\langle R_0\rangle = \frac{\alpha_s}{4\pi}\left[\left((N_c+1)\ln\frac{\mu}{m_b} + \frac{2-N_c}{N_c}\right)\langle Q\rangle\right. \tag{35}$$
$$\left. + \left(4(N_c+1)\ln\frac{\mu}{m_b} + 2(N_c+1)\right)\langle Q_S\rangle\right] + \mathcal{O}(1/m_b).$$

It is crucial that this relation holds independent of the external state, so that power counting is again manifest after subtracting the right hand side of (35) from the matrix

element of R_0. (The procedure discussed here bears some similarity with mixing of higher dimension operators into lower dimension operators in cut-off or lattice regularizations.) These subtractions must be kept in mind when a non-perturbative evaluation of the matrix elements of the R_i is combined with the present NLO results. In the factorization approximation of [6] these subtractions correspond to using the pole b quark mass in the expressions for the factorized matrix elements. Indeed, in the $N_c \to \infty$ limit, we find [6]

$$\langle \bar{B}_s|R_0|B_s\rangle = f_{B_s}^2 M_{B_s}^2 \left(1 - \frac{M_{B_s}^2}{(\bar{m}_b+\bar{m}_s)^2}\right) \simeq f_{B_s}^2 M_{B_s}^2 \frac{\alpha_s N_c}{4\pi}\left(6\ln\frac{m_b}{\mu} - 4\right), \tag{36}$$

up to corrections of order $(N_c\alpha_s)^2$ and $\Lambda_{\rm QCD}/m_b$. The b-quark mass $\bar{m}_b$ in the second expression of (36) is the $\overline{MS}$ mass at the scale μ, which corresponds to our renormalization of the scalar operators Q_S, $\tilde{Q}_S$. To obtain the third expression we used the 1-loop relation between the pole and $\overline{MS}$ mass in the large-N_c limit and the fact that $M_{B_s} - m_{b,pole} = \mathcal{O}(\Lambda_{\rm QCD})$. The same result as (36) is obtained from the large-N_c limit of (35). (In deriving (36) we have used the Fierz transform of $\tilde{Q}_S$. For the coincidence of (36) with (35) it is crucial that the choice in (15) maintains the Fierz symmetry in the one-loop matrix elements entering (34).)

3. Discussion. The complete expression for $\Delta\Gamma_{B_s}$ with short-distance coefficients at NLO in QCD is given by

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \frac{16\pi^2 B(B_s\to Xe\nu)}{g(z)\tilde{\eta}_{QCD}} \frac{f_{B_s}^2 M_{B_s}}{m_b^3}|V_{cs}|^2 \cdot \tag{37}$$

$$\cdot\left(G(z)\,\frac{8}{3}\,B + G_S(z)\,\frac{M_{B_s}^2}{(\bar{m}_b+\bar{m}_s)^2}\,\frac{5}{3}\,B_S + \sqrt{1-4z}\;\delta_{1/m}\right),$$

where

$$G(z) = F(z) + P(z) \quad \text{and} \quad G_S(z) = -(F_S(z) + P_S(z)). \tag{38}$$

We eliminated the total decay rate Γ_{B_s} in favour of the semileptonic branching ratio $B(B_s \to Xe\nu)$, as done in [6]. This cancels the dependence of $(\Delta\Gamma/\Gamma)$ on V_{cb} and introduces the phase space function

$$g(z) = 1 - 8z + 8z^3 - z^4 - 12z^2\ln z, \tag{39}$$

as well as the QCD correction factor [20]

$$\tilde{\eta}_{QCD} = 1 - \frac{2\alpha_s(m_b)}{3\pi}\left[\left(\pi^2 - \frac{31}{4}\right)(1-\sqrt{z})^2 + \frac{3}{2}\right]. \tag{40}$$

The latter is written here in the approximate form of [21]. The bag factors B and B_S parametrize the matrix elements of Q and Q_S,

$$\langle \bar{B}_s|Q|B_s\rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B, \tag{41}$$

$$\langle \bar{B}_s|Q_S|B_s\rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b+\bar{m}_s)^2} B_S. \tag{42}$$

μ_1	$m_b/2$	m_b	$2m_b$
$G_S = -(F_S + P_S)$	0.743	0.937	1.018
$G_S^{(0)} = -(F_S^{(0)} + P_S^{(0)})$	1.622	1.440	1.292
$G = F + P$	0.023	0.030	0.036
$G^{(0)} = F^{(0)} + P^{(0)}$	0.013	0.047	0.097
$-F_S$	0.867	1.045	1.111
$-F_S^{(0)}$	1.729	1.513	1.341
F	0.042	0.045	0.049
$F^{(0)}$	0.030	0.057	0.103

Table 1: Numerical values of the Wilson coefficients G, G_S, F, F_S, at next-to-leading order in the NDR-scheme with evanescent operators subtracted as described in the text (for $\mu_2 = m_b$). Leading order results (superscript '(0)') are also shown for comparison.

The masses $\bar{m}_b \equiv \bar{m}_b(\mu)$, $\bar{m}_s$ refer to the $\overline{MS}$ definition. The relation of $\bar{m}_b$ to the pole mass m_b is

$$\bar{m}_b(\mu) = m_b \left[1 - \frac{\alpha_s}{\pi}\left(\ln\frac{\mu^2}{m_b^2} + \frac{4}{3}\right)\right]. \tag{43}$$

Finally, $\delta_{1/m}$ describes $1/m_b$ corrections. The explicit expression for $\delta_{1/m}$ in the factorization approximation can be found in [6]. In the present NLO approximation the b-quark mass that appears in $\delta_{1/m}$ is the pole mass as mentioned above.

For the numerical evaluation we use the following input parameters (central values):

$$m_b = 4.8 \text{ GeV} \quad \bar{m}_b(m_b) = 4.4 \text{ GeV} \quad z = 0.085 \quad \bar{m}_s = 0.2 \text{ GeV}, \tag{44}$$

$$M_{B_s} = 5.37 \text{ GeV} \quad f_{B_s} = 0.21 \text{ GeV} \quad B(B_s \to Xe\nu) = 0.104. \tag{45}$$

The two-loop expression is used throughout for the QCD coupling α_s in the form given in [13] with $\Lambda^{(5)}_{\overline{MS}} = 0.225$ GeV. The NLO coefficients $F(z)$, $F_S(z)$ in (10) and $P(z)$, $P_S(z)$ in (30) are consistently expanded to first order in α_s. We take $\mu_1 = \mu_2 = m_b$ as central values for the renormalization scales. The dependence of $\Delta\Gamma_{B_s}$ on $\bar{m}_s$ is marginal and its dependence on z stems almost totally from $g(z)$ in (39).

The results for the Wilson coefficients are displayed in Table 1. The contribution of Q_S dominates $\Delta\Gamma_{B_s}$ since the coefficient of Q_S is numerically much larger than the one of Q. These coefficients are independent of the scale μ_1, related to the $\Delta B = 1$ operators, up to terms of next-to-next-to-leading order. As can be seen from Table 1, the residual μ_1 dependence is indeed substantially decreased for the coefficient G of Q. The reduction of scale dependence is less pronounced for the coefficient G_S of Q_S. On the other hand, in our renormalization scheme, the central value at NLO is considerably smaller (by about 30%) than the leading order result. However, due to the scheme dependence of the Wilson coefficient, it is premature to draw definitive conclusions on

the size of $\Delta\Gamma_{B_s}$ without combining the coefficient functions with B and B_S, computed in the same scheme. For quick reference we rewrite (37) as

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\ \text{MeV}}\right)^2 \left[0.006\, B(m_b) + 0.150\, B_S(m_b) - 0.063\right], \tag{46}$$

where the numbers are obtained with our central parameter set.

A preliminary lattice evaluation of the relevant bag parameters can be found in [22], together with a complete 1-loop lattice-to-continuum matching. We can use their result to obtain a (conceptually) complete (but numerically preliminary) next-to-leading-order result for $(\Delta\Gamma/\Gamma)_{B_s}$. In [22] a continuum renormalization scheme different from ours for the operators Q_S and $\tilde{Q}_S$ is chosen. We computed the relation between the two schemes to $\mathcal{O}(\alpha_s)$ and found

$$B_S = B_4^+ + \frac{\alpha_s}{4\pi}\left(B_4^+ + \frac{1}{15}B_5^+\right), \tag{47}$$

$$\tilde{B}_S = B_5^+ + \frac{\alpha_s}{4\pi}\left(-\frac{35}{6}B_4^+ + \frac{1}{2}B_5^+\right). \tag{48}$$

Here $\tilde{B}_S$ is the bag parameter related to $\tilde{Q}_S$, defined in analogy with (42) with $Q_S \to \tilde{Q}_S$, $B_S \to \tilde{B}_S$, and the numerical factor $(-5/3) \to (+1/3)$. B_4^+ and B_5^+ are the bag parameters B_S and $\tilde{B}_S$, respectively, but in the scheme of [22]. From the quoted estimates $B_4^+(\mu_0) = 0.80 \pm 0.01$, $B_5^+(\mu_0) = 0.94 \pm 0.01$ [22] (where the errors are statistical only and $\mu_0 = 2.33$ GeV) we infer $B_S(\mu_0) = 0.81$, $\tilde{B}_S(\mu_0) = 0.87$. Using (29), and taking the running of $\bar{m}_b$ (43) into account, the bag parameter B_S at the scale m_b is given by

$$B_S(m_b) = B_S(\mu_0) + \frac{\alpha_s}{4\pi}\ln\frac{m_b}{\mu_0}\left(-\frac{20}{3}B_S(\mu_0) + \frac{4}{15}\tilde{B}_S(\mu_0)\right) = 0.75. \tag{49}$$

We also take $B(m_b) = 0.9$ from the compilation [23]. Without the $1/m_b$ corrections, (46) then becomes $(\Delta\Gamma/\Gamma)_{B_s} = 0.118\ (f_{B_s}/210\ \text{MeV})^2$. There is a $\pm 15\%$ error from (continuum) perturbation theory, obtained from varying μ_1 between $m_b/2$ and $2m_b$, and a negligible $\pm 1\%$ statistical error from the lattice simulation. In addition the sizeable $1/m_b$ correction, computed in [6], has to be included. The estimate -0.063 in (46) is obtained using factorization of hadronic matrix elements and has a relative error of at least $\pm 20\%$. As a preliminary result we may therefore write

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\ \text{MeV}}\right)^2 \left(0.054^{+0.016}_{-0.032}\ (\mu_1\text{-dep.}) \pm ???\ (\text{latt. syst.})\right). \tag{50}$$

We emphasize the preliminary nature of the central value which depends crucially on the estimate $B_4^+(\mu_0) = 0.80$ taken from [22]. There is an unspecified systematic error (indicated by the question marks) attached to this number, related to the fact that the lattice calculation has been performed in the quenched approximation, at finite lattice spacing and with a "b-quark" mass in the charm quark mass region, without extrapolation to the continuum limit and to realistic b quark masses, respectively. Clearly, for

further progress improved lattice determinations of bag parameters, most importantly of B_S, are mandatory, and the numerical value for $(\Delta\Gamma/\Gamma)_{B_s}$ above has to be regarded in this context.

The rather low number for the central value, compared to the leading order analysis of [6], is a consequence of the fact that $1/m_b$ effects and penguin contributions [6], as well as NLO QCD corrections, all lead to a reduction of $\Delta\Gamma_{B_S}$. This is further reinforced by the small value $B_S(m_b) = 0.75$ in our example.

It is interesting to consider the ratio of $\Delta\Gamma_{B_s}$ to the mass difference ΔM_{B_s} [5, 6], in which the dependence on the decay constant f_{B_s} cancels out and the sensitivity to V_{cb} is considerably reduced. In addition $(\Delta\Gamma/\Delta M)_{B_s}$ only depends on the ratio of bag parameters. Generalizing the results given in [6] to include the next-to-leading order QCD corrections we can write

$$\left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} = \frac{\pi}{2}\frac{m_b^2}{M_W^2}\left|\frac{V_{cb}V_{cs}}{V_{ts}V_{tb}}\right|^2 \frac{1}{\eta_B S_0(x_t)} \cdot \tag{51}$$
$$\cdot\left(\frac{8}{3}\,G(z) + \frac{5}{3}\frac{M_{B_s}^2}{(\bar{m}_b+\bar{m}_s)^2}\,G_S(z)\,\frac{B_S}{B} + \sqrt{1-4z}\;\delta_{1/m}\right),$$

where η_B is the (scheme-dependent) next-to-leading order QCD factor entering ΔM_{B_s} [24]. In the usual NDR scheme $\eta_B(m_b) = 0.846$. The top-quark mass dependent function $S_0(x_t) = S_0((\bar{m}_t/M_W)^2) = 2.41$ for $\bar{m}_t = 167\,\text{GeV}$. In analogy with (50) we then have

$$\left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} = \left(2.63^{+0.67}_{-1.36}\;(\mu_1\text{-dep.}) \pm ???\;(\text{latt. syst.})\right)\cdot 10^{-3}. \tag{52}$$

The next-to-leading order calculation presented in this article can also be applied, by taking the limit $z \to 0$, to the mixing-induced CP asymmetry in inclusive $B_d(\bar{B}_d) \to \bar{u}u\bar{d}d$ decays [25]. The time-dependent asymmetry is given by $\mathcal{A}(t) = \text{Im}\xi \sin\Delta M t$, where the coefficient $\text{Im}\xi$ is a measure of the CKM parameter $\sin 2\alpha$. With next-to-leading order QCD corrections included, the expression for $\text{Im}\xi$ in Eq. (25) of [25] is modified to read

$$\text{Im}\xi = -0.12\sin 2\alpha\left(\frac{f_B}{180\;\text{MeV}}\right)^2\left[0.14B + 0.64B_S - 0.07 - 0.06\,\frac{\sin\alpha\sin(\alpha+\beta)}{\sin\beta\sin 2\alpha}\right]. \tag{53}$$

In this equation the bag factors B and B_S (taken at $\mu = m_b$) are the B_d analogues of those defined in (41) and (42) for B_s. A detailed discussion of CP asymmetries will be presented in [11].

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