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Scaling tests in O(a)–improved quenched lattice QCD*

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We present a scaling investigation of renormalized correlation functions in O(a)–improved quenched lattice QCD. As one observable the renormalized PCAC quark mass is considered, others are constructed such that they become the vector meson mass, and the pseudoscalar and vector meson decay constants in large volume. Presently, we remain in intermediate volume, $(0.75^3 \times 1.5)$ fm^4, and study the approach to the continuum limit.

1. Introduction

Discretization errors of O(a) in lattice QCD can be removed via a systematic approach based on the Symanzik improvement programme [1], which adds appropriate higher-dimensional operators to action and fields of interest [2]. Exploiting chiral symmetry restoration and certain current Ward identities on the lattice, a (mostly) non-perturbative O(a)–improvement for action and quark currents as well as their renormalization has been achieved in the quenched case within this framework [3,5,6].

Thus one is not only interested in the influence of improvement on agreement of lattice data with experiment at fixed β–values, but also in the quality of scaling and the size of its violation. In this context it was reported [9] that at $a \simeq 0.1$ fm the residual O(a^2) lattice artifacts may be fairly large e.g. for $f_K r_0$ (~ 10 %), while they are already very small for other quantities like $m_\rho/\sqrt{\sigma}$ (~ 2 %). Restricting to an intermediate volume, we therefore examined the impact of O(a)–improvement thoroughly and with high accuracy for different observables.

2. Fermionic correlation functions

Consider correlation functions in the Schrödinger functional (SF) [10] with all details found in [2,3]. We use $\zeta(\bar{\zeta})$ as $x_0 = 0$ boundary (anti-)quark fields in $\mathcal{O}^a = a^6 \sum_{\mathbf{u},\mathbf{v}} \bar{\zeta}(\mathbf{u}) \gamma_5 \frac{\tau^a}{2} \zeta(\mathbf{v})$ and $\mathcal{Q}_k^a = a^6 \sum_{\mathbf{u},\mathbf{v}} \bar{\zeta}(\mathbf{u}) \gamma_k \frac{\tau^a}{2} \zeta(\mathbf{v})$, and axial (vector) current $A_\mu^a (V_\mu^a)$ and pseudoscalar (tensor) density $P^a (T_{\mu\nu}^a)$ to form the expectation values

$$f_{\rm A}(x_0) = -\frac{1}{3} \langle A_0^a(x)\, \mathcal{O}^a \rangle \quad (1a)$$

$$f_{\rm P}(x_0) = -\frac{1}{3} \langle P^a(x)\, \mathcal{O}^a \rangle \quad (1b)$$

$$f_1 = -\frac{1}{3L^6} \langle \mathcal{O}'^a \mathcal{O}^a \rangle \quad (1c)$$

$$k_{\rm V}(x_0) = -\frac{1}{9} \langle V_k^a(x)\, \mathcal{Q}_k^a \rangle \quad (1d)$$

$$k_{\rm T}(x_0) = -\frac{1}{9} \langle T_{k0}^a(x)\, \mathcal{Q}_k^a \rangle \ . \quad (1e)$$

Given the improvement coefficients $c_{\rm A}$ and $c_{\rm V}$, the improved currents $(A_{\rm I})_\mu^a \equiv A_\mu^a + a c_{\rm A} \tilde{\partial}_\mu P^a$ and $(V_{\rm I})_\mu^a \equiv V_\mu^a + a c_{\rm V} \tilde{\partial}_\nu T_{\mu\nu}^a$ lead to define the corresponding fermionic correlation functions

$$f_{\rm A}^{\rm I}(x_0) = f_{\rm A}(x_0) + a c_{\rm A} \tilde{\partial}_0 f_{\rm P}(x_0) \quad (2a)$$

$$k_{\rm V}^{\rm I}(x_0) = k_{\rm V}(x_0) + a c_{\rm V} \tilde{\partial}_0 k_{\rm T}(x_0) \ , \quad (2b)$$

where the symmetrized lattice derivative acts as usual as $\tilde{\partial}_\mu f(x) = [f(x + a\hat{\mu}) - f(x - a\hat{\mu})]/2a$.

3. Observables under study and results

Employing a mass-independent renormalization respecting O(a)–improvement, the quantities

$$(A_{\rm R})_\mu^a = Z_{\rm A}(1 + b_{\rm A} a m_{\rm q})(A_{\rm I})_\mu^a \quad (3a)$$

$$(V_{\rm R})_\mu^a = Z_{\rm V}(1 + b_{\rm V} a m_{\rm q})(V_{\rm I})_\mu^a \quad (3b)$$

$$P_{\rm R}^a = Z_{\rm P}(1 + b_{\rm P} a m_{\rm q}) P^a \quad (3c)$$

induce the renormalized correlation functions

$$f_{\rm A}^{\rm R}(x_0) = Z_{\rm A}(1 + b_{\rm A} a m_{\rm q}) f_{\rm A}^{\rm I}(x_0) \quad (4a)$$

$$k_{\rm V}^{\rm R}(x_0) = Z_{\rm V}(1 + b_{\rm V} a m_{\rm q}) k_{\rm V}^{\rm I}(x_0) \quad (4b)$$

with $Z_{\rm X}$ and $b_{\rm X}$, X=A,V,P, which are *not* functions of the quark mass $a m_{\rm q} = (1/\kappa - 1/\kappa_{\rm c})/2$.

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Now we construct the following set of observables:

$$m(x_0) = \frac{\tilde{\partial}_0 f_{\rm A}(x_0) + a c_{\rm A} \tilde{\partial}_0^2 f_{\rm P}(x_0)}{2 f_{\rm P}(x_0)} \quad (5)$$

$$m_{\rm PS}(x_0) = \frac{\tilde{\partial}_0 f_{\rm P}(x_0)}{f_{\rm P}(x_0)}\,, \quad m_\pi = m_{\rm PS}(\tfrac{T}{2}) \quad (6)$$

$$m_{\rm V}(x_0) = \frac{\tilde{\partial}_0 k_{\rm V}^{\rm I}(x_0)}{k_{\rm V}^{\rm I}(x_0)}\,, \quad m_\rho = m_{\rm V}(\tfrac{T}{2}) \quad (7)$$

$$a f_{\rm PS}(x_0) \propto \frac{f_{\rm A}^{\rm R}(x_0)}{\sqrt{f_1}}\,, \quad f_\pi = f_{\rm PS}(\tfrac{T}{2}) \quad (8)$$

$$f_{\rm V}^{-1}(x_0) \propto \frac{k_{\rm V}^{\rm R}(x_0)}{\sqrt{f_1}}\,, \quad f_\rho^{-1} = f_{\rm V}^{-1}(\tfrac{T}{2})\,. \quad (9)$$

The division by $\sqrt{f_1}$ cancels the renormalizations of the boundary quark fields ensuring that f_π,f_ρ are scaling quantities, and the proportionality constants in eqs. (8),(9) are such that these ratios turn, as $T \to \infty$, into the familiar matrix elements, which define the π and ρ meson decay constants. The renormalized PCAC (current) quark mass in the SF scheme is obtained as [7]

$$\overline{m} = \frac{Z_{\rm A}}{Z_{\rm P}}\, m_{\rm PCAC}\,, \quad m_{\rm PCAC} = m(x_0)\Big|_{x_0=\frac{T}{2}}\,, \quad (10)$$

where $(b_{\rm A} - b_{\rm P}) a m_{\rm q}$ may be neglected [6].

For the analysis we use $c_{\rm sw}$, $c_{\rm X}$ and $Z_{\rm X}$ non-perturbatively determined in [3,5,8] for $\beta \geq 6.0$, while $b_{\rm A}$, $b_{\rm V}$ and $b_{\rm P}$, as well as the SF specific improvement coefficients of the boundary counterterms $c_{\rm t}$ and $\tilde{c}_{\rm t}$, are taken from 1–loop perturbation theory [2,4,10].

L/a	β	κ	L/r_0	$m_\pi L$
8	6.0	0.13458	1.490(6)	2.004(9)
10	6.14	0.13538	1.486(7)	1.946(14)
12	6.26	0.13546	1.495(7)	2.050(16)
16	6.48	0.13541	1.468(8)	1.991(15)

Table 1. *Simulation points for the LCP studied.*

The strategy was then to keep a finite physical volume and the quark mass fixed by prescribing the geometry $T/L = 2$, 'pion' mass $m_\pi L = 2.0$ and spatial lattice size $L/r_0 = 1.49$ using the recent results on the hadronic scale r_0/a [11], see table 1. This yields an intermediate volume of $(0.75^3 \times 1.5)$ fm^4, moving on a line of constant physics (LCP) in parameter space with lattice resolutions ranging from 0.1 to 0.05 fm. As an important prerequisite for the reliability of the scaling test we verified by variation of the 1–loop values as simulation input that the dependence on $c_{\rm t}, \tilde{c}_{\rm t}$ is small enough to be neglected. The leading scaling violations should therefore be $\mathrm{O}(a^2)$. Any small mismatch with the renormalization conditions on $m_\pi L$ and L/r_0 was corrected by an estimation of the corresponding slopes. Finally, we performed extrapolations to the continuum limit, assuming convergence with a rate $\propto a^2$.

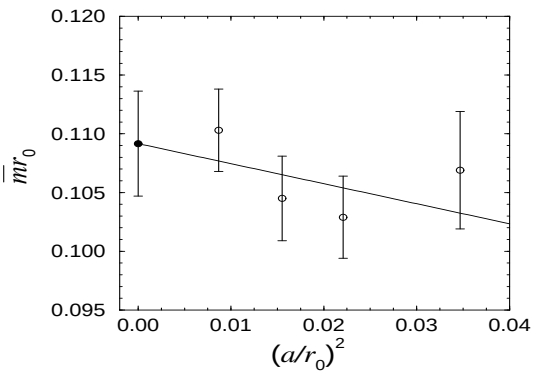

Figure 1. *Scaling of the PCAC quark mass in intermediate volume with SF boundary conditions.*

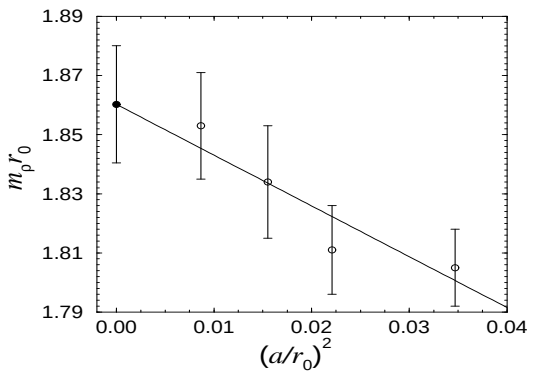

Figure 2. *Same as figure 1 but for the rho mass.*

$\overline{m}r_0$	$m_\rho r_0$	$f_\pi r_0$	$1/f_\rho$
0.1092(45)	1.860(20)	0.704(13)	0.826(22)
2.1 %	3.0 %	4.8 %	28 %

Table 2. *Continuum limits and their percentage deviations from $\beta = 6.0$ ($a \simeq 0.1$ fm).*

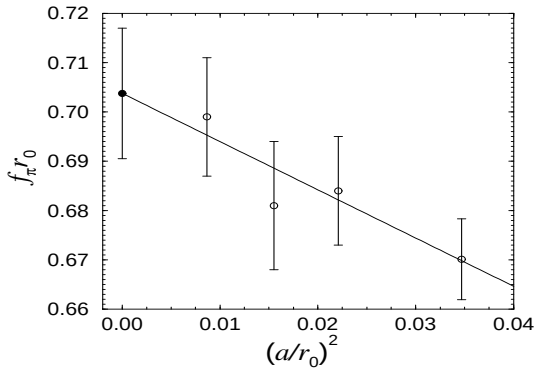

Figure 3. *As figure 1 but for the π decay constant.*

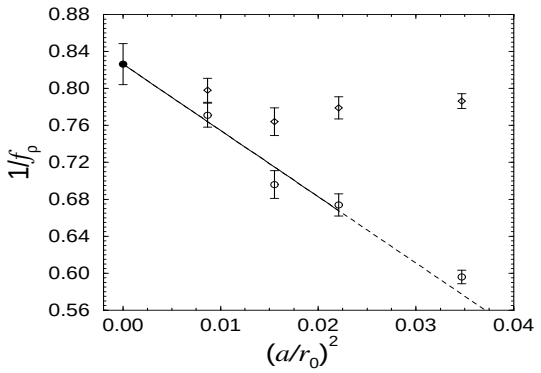

Figure 4. *As figure 1 but for the ρ decay constant. Diamonds refer to an analysis with $c_V = 0$.*

The fits are displayed in figures 1 – 4, where the total error is always dominated by the uncertainties of the renormalization factors Z_X. One observes the leading corrections to the continuum to be compatible with $O(a^2)$. Moreover, it can be inferred from table 2 that the difference of the continuum limits from the values at $\beta = 6.0$ ($a \simeq 0.1$ fm) is below 5 % in the improved theory. The only exception is the inverse 'rho' meson decay constant, whose slope is quite large; for that reason we discard the $\beta = 6.0$ point to extrapolate to the continuum limit.

4. Discussion and outlook

Our numerical simulations of renormalized and improved correlation functions show an overall behaviour completely consistent with being linear in a^2 at $\beta \geq 6.0$ for all quantities under consideration. Changing a by a factor 2 gives very stable continuum extrapolations.

But the residual $O(a^2)$–effect in $1/f_\rho$ is still large at $a \simeq 0.1$ fm ($\lesssim 30$ %). Here we find that artificially setting $c_V = 0$ results in data with an overall weaker dependence on a. However, as indicated in figure 4, for such a c_V–value the functional form of the a–effects seems no longer compatible with a^2. Additionally, a chiral Ward identity is badly violated for $c_V = 0$ at $O(a)$–level [5]. Thus there is no choice of c_V, which makes $O(a^2)$–effects small in both channels. This example clearly illustrates that even in the $O(a)$–improved theory the remaining $O(a^2)$–effects have to be assessed by varying the lattice spacing.

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