Revised Version DESY 98-137

DESY 98-137
September 1998

hep-ph/9809411

hep-ph/9809411 v2 12 Nov 1998

Second order QCD corrections to the forward-backward asymmetry in e^+ e^--collisions

V. Ravindran and W.L. van Neerven [1]

DESY-Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany

Abstract

We will present the result of an analytical calculation of the second order contribution to the forward-backward asymmetry $A_{\rm FB}^{\rm H}$ and the shape constant $a^{\rm H}$ for heavy flavour production in e^+ e^--collisions. The calculation has been carried out by assuming that the quark mass is equal to zero. This is a reasonable approximation for the exact second order correction for charm and bottom quark production at LEP energies but not for top production at future linear colliders. Our result for $A_{\rm FB}^{\rm H}$ is a factor 2.6 (charm) and 4.7 (bottom) larger than obtained by a numerical calculation performed earlier in the literature. We study the effect of the second order corrections on the above parameters including their dependence on the renormalization scale. Further we make a comparison between the fixed pole mass and the running mass approach.

[1]On leave of absence from Instituut-Lorentz, University of Leiden,P.O. Box 9506, 2300 RA Leiden, The Netherlands .

Experiments carried out at electron positron colliders like LEP and LSD have provided us with a wealth of information about the constants [1] appearing in the standard model of the electroweak and strong interactions. One among them is given by the electroweak mixing angle defined by θ_W which can be very accurately extracted from the forward-backward asymmetry in heavy flavour production in particular when the flavour is represented by the bottom quark [2]. Recently this quantity is obtained for the charm quark [3] and in the future one also hopes to measure it for the top quark at the large linear $e^+\,e^-$-collider (see e.g. [4]). The forward-backward asymmetry is extracted from the differential cross section given by

$$\begin{aligned}\frac{d\sigma^{\rm H}(Q^2)}{d\cos\theta} &= \frac{3}{8}(1+\cos^2\theta)\left[\sigma_{\rm VV}(Q^2)f_T^v(\rho)+\sigma_{\rm AA}(Q^2)f_T^a(\rho)\right]\\ &\quad+\frac{3}{4}\sin^2\theta\left[\sigma_{\rm VV}(Q^2)f_L^v(\rho)+\sigma_{\rm AA}(Q^2)f_L^a(\rho)\right]+\frac{3}{4}\cos\theta\left[\sigma_{\rm VA}(Q^2)f_A^a(\rho)\right]\,, \qquad (1)\end{aligned}$$

where θ is the angle between the outgoing quark H and the incoming electron. The CM energy is denoted by Q and the scaling variable ρ is defined by $\rho = 4m^2/Q^2$ where m stands for the quark mass. Notice that the form of the cross section above is correct if only final state corrections are present which is the case for QCD investigated in this paper. For electroweak corrections (see [5], [6]) which occur in the initial as well as in the final state, including interference terms, the above formula has to be modified. The Born cross sections for quark final states appearing in Eq. (1) can be written as

$$\begin{aligned}\sigma_{VV}(Q^2) &= \frac{4\pi\alpha^2}{3Q^2}N\left[e_\ell^2e_q^2+\frac{2Q^2(Q^2-M_Z^2)}{|Z(Q^2)|^2}\,e_\ell e_q C_{V,\ell}C_{V,q}\right.\\ &\quad\left.+\frac{(Q^2)^2}{|Z(Q^2)|^2}\left(C_{V,\ell}^2+C_{A,\ell}^2\right)C_{V,q}^2\right]\,, \qquad (2)\\ \sigma_{AA}(Q^2) &= \frac{4\pi\alpha^2}{3Q^2}N\left[\frac{(Q^2)^2}{|Z(Q^2)|^2}\left(C_{V,\ell}^2+C_{A,\ell}^2\right)C_{A,q}^2\right]\,, \qquad (3)\\ \sigma_{VA}(Q^2) &= \frac{4\pi\alpha^2}{3Q^2}N\left[\frac{2Q^2(Q^2-M_Z^2)}{|Z(Q^2)|^2}\,e_\ell e_q C_{A,\ell}C_{A,q}+4\frac{(Q^2)^2}{|Z(Q^2)|^2}\,C_{A,\ell}C_{A,q}C_{V,\ell}C_{V,q}\right]\,, \qquad (4)\end{aligned}$$

where N denotes the number of colours in the case of the gauge group $SU(N)$ (in QCD one has $N=3$). Furthermore in the expressions above we adopt for the Z-propagator the energy independent width approximation

$$Z(Q^2) = Q^2 - M_Z^2 + iM_Z\Gamma_Z\,. \qquad (5)$$

The charges of the lepton and the quark are given by e_ℓ and e_q respectively and the angle θ_W defined at the beginning appears in the electroweak constants as follows.

$$\begin{aligned}&C_{A,\ell} = \frac{1}{2\sin 2\theta_W}, \qquad C_{V,\ell} = -C_{A,\ell}\,(1-4\sin^2\theta_W),\\ &C_{A,u} = -C_{A,d} = -C_{A,\ell}, \qquad (6)\\ &C_{V,u} = C_{A,\ell}\,(1-\frac{8}{3}\sin^2\theta_W), \quad C_{V,d} = -C_{A,\ell}\,(1-\frac{4}{3}\sin^2\theta_W)\,.\end{aligned}$$

The functions f_k^l $(k = T, L, A;\ l = v, a)$ in Eq. (1) can be computed order by order in perturbative QCD from the non-singlet quark coefficient function $\mathcal{C}_{k,q}^{l,\mathrm{NS}}$ as follows

$$f_k^l(\rho) = \int_{\sqrt{\rho}}^1 dx\, \mathcal{C}_{k,q}^{l,\mathrm{NS}}(x, \rho, \frac{Q^2}{\mu^2}) \quad \text{with} \quad \rho = \frac{4m^2}{Q^2}\,, \tag{7}$$

where μ stands for the factorization as well as the renormalization scale. The quark coefficient function appears in the fragmentation function $F_k(x, Q^2)$ with $x = 2p.q/Q^2$ where p is the momentum of the outgoing hadron which originates from the quark. These fragmentation functions describe the production of the quark and its subsequent decay into a hadron. The forward backward asymmetry, denoted by $A_{\mathrm{FB}}^{\mathrm{H}}$, appears when we divide the expression in Eq. (1) by the total cross section. The ratio can then be expressed in the following way

$$\frac{1}{\sigma_{\mathrm{tot}}^{\mathrm{H}}(Q^2)} \frac{d\sigma^{\mathrm{H}}(Q^2)}{d\cos\theta} = \frac{3}{8}\left(\frac{4}{3 + a^{\mathrm{H}}(Q^2)}\right)\left(1 + a^{\mathrm{H}}(Q^2)\cos^2\theta\right) + A_{\mathrm{FB}}^{\mathrm{H}}(Q^2)\cos\theta\,, \tag{8}$$

with

$$A_{\mathrm{FB}}^{\mathrm{H}}(Q^2) = \frac{3}{4}\frac{\sigma_{\mathrm{VA}}(Q^2)\, f_A^a(\rho)}{\sigma_{\mathrm{tot}}^{\mathrm{H}}(Q^2)}\,. \tag{9}$$

Further the shape coefficient a^{H} is defined by

$$a^{\mathrm{H}}(Q^2) = \frac{\sigma_{\mathrm{VV}}(Q^2)\Big[f_T^v(\rho) - 2f_L^v(\rho)\Big] + \sigma_{\mathrm{AA}}(Q^2)\Big[f_T^a(\rho) - 2f_L^a(\rho)\Big]}{\sigma_{\mathrm{VV}}(Q^2)\Big[f_T^v(\rho) + 2f_L^v(\rho)\Big] + \sigma_{\mathrm{AA}}(Q^2)\Big[f_T^a(\rho) + 2f_L^a(\rho)\Big]}\,, \tag{10}$$

and the total cross section for heavy flavour production is equal to

$$\sigma_{\mathrm{tot}}^{\mathrm{H}}(Q^2) = \sigma_{\mathrm{VV}}(Q^2)\Big[f_T^v(\rho) + f_L^v(\rho)\Big] + \sigma_{\mathrm{AA}}(Q^2)\Big[f_T^a(\rho) + f_L^a(\rho)\Big]\,. \tag{11}$$

The functions f_k^l can be expanded in the strong coupling constant α_s as follows

$$f_k^l(\rho) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n f_k^{l,(n)}(\rho)\,. \tag{12}$$

The lowest order contributions corresponding to the Born reaction

$$V \to \mathrm{H} + \overline{\mathrm{H}}\,, \tag{13}$$

with $V = \gamma, Z$ are given by

$$\begin{aligned}
f_T^{v,(0)}(\rho) &= \sqrt{1-\rho} \qquad f_L^{v,(0)}(\rho) = \frac{\rho}{2}\sqrt{1-\rho}\,,\\
f_T^{a,(0)}(\rho) &= (1-\rho)^{3/2} \qquad f_L^{a,(0)}(\rho) = 0\,,\\
f_A^{a,(0)}(\rho) &= 1-\rho\,,
\end{aligned} \tag{14}$$

where ρ is defined below Eq. (1). The next-to-leading order (NLO) contributions originate from the one-loop virtual corrections to the Born reaction (13) and the gluon bremsstrahlungs process

$$V \to \mathrm{H} + \overline{\mathrm{H}} + g\,. \tag{15}$$

The NLO contributions have been calculated by several groups in the literature for the case $m \neq 0$ (see [7]-[10]). However there were some discrepancies between the results so that we decided to calculate them again using a different method. Here they are derived from the order α_s contribution to the coefficient functions for heavy quarks which appears in the integrand of Eq. (7). Our computations lead to the same answers as given in appendix A of [11]. After performing the integral in Eq. (7) we obtain the following results for $k = T, L$

$$\begin{aligned}
f_T^{v,(1)}(\rho) &= C_F\Big[\frac{1}{2}\rho(1+\rho)F_1(t) + \sqrt{\rho}(1-3\rho)F_2(t) + (32 - \frac{39}{2}\rho - \frac{7}{2}\rho^2)\mathrm{Li}_2(t) \\
&\quad + (16 - 10\rho - 2\rho^2)F_3(t) + 2\sqrt{1-\rho}\,F_4(t) + (8 - 6\rho - 2\rho^2)\ln(t)\ln(1+t) \\
&\quad + (-12 + 9\rho - \frac{5}{4}\rho^2)\ln(t) + \sqrt{1-\rho}\,(1 + \frac{13}{2}\rho)\Big]\,, \qquad (16) \\
f_L^{v,(1)}(\rho) &= C_F\Big[-\frac{1}{2}\rho(1+\rho)F_1(t) - \sqrt{\rho}(1-3\rho)F_2(t) + (\frac{39}{2}\rho - \frac{9}{2}\rho^2)\mathrm{Li}_2(t) \\
&\quad + (10\rho - 2\rho^2)F_3(t) + \rho\sqrt{1-\rho}\,F_4(t) + 6\rho\ln(t)\ln(1+t) \\
&\quad + (-7\rho + 3\rho^2)\ln(t) + 2(1-\rho)^{3/2}\Big]\,, \qquad (17) \\
f_T^{a,(1)}(\rho) &= C_F\Big[\frac{1}{2}\rho(1+2\rho)F_1(t) + \sqrt{\rho}(1-4\rho)F_2(t) + (32 - \frac{103}{2}\rho + 9\rho^2)\mathrm{Li}_2(t) \\
&\quad + (16 - 26\rho + 4\rho^2)F_3(t) + 2(1-\rho)^{3/2}F_4(t) + (8 - 14\rho)\ln(t)\ln(1+t) \\
&\quad + (-12 + 15\rho - \frac{9}{4}\rho^2)\ln(t) + \sqrt{1-\rho}\,(1 + \frac{1}{2}\rho)\Big]\,, \qquad (18) \\
f_L^{a,(1)}(\rho) &= C_F\Big[-\frac{1}{2}\rho(1+2\rho)\Big(F_1(t) - 4F_3(t) - 7\mathrm{Li}_2(t) - 4\ln(t)\ln(1+t)\Big) \\
&\quad - \sqrt{\rho}(1-4\rho)F_2(t) + (-4\rho + \rho^2 - \frac{3}{8}\rho^3)\ln(t) + \sqrt{1-\rho}\,(2 - \frac{19}{2}\rho + \frac{3}{4}\rho^2)\Big]\,, \qquad (19)
\end{aligned}$$

with

$$t = \frac{1-\sqrt{1-\rho}}{1+\sqrt{1-\rho}}\,. \qquad (20)$$

Further the colour factor C_F is given by $C_F = (N^2-1)/2N$. The functions $F_i(t)$ appearing above are defined by

$$\begin{aligned}
F_1(t) &= \mathrm{Li}_2(t^3) + 4\zeta(2) + \frac{1}{2}\ln^2(t) + 3\ln(t)\ln(1+t+t^2)\,, \qquad (21) \\
F_2(t) &= \mathrm{Li}_2(-t^{3/2}) - \mathrm{Li}_2(t^{3/2}) + \mathrm{Li}_2(-t^{1/2}) - \mathrm{Li}_2(t^{1/2}) + 3\zeta(2) + 2\ln(t)\ln(1+\sqrt{t}) \\
&\quad -2\ln(t)\ln(1-\sqrt{t}) + \frac{3}{2}\ln(t)\ln(1+t-\sqrt{t}) - \frac{3}{2}\ln(t)\ln(1+t+\sqrt{t})\,, \qquad (22) \\
F_3(t) &= \mathrm{Li}_2(-t) + \ln(t)\ln(1-t)\,, \qquad (23) \\
F_4(t) &= 6\ln(t) - 8\ln(1-t) - 4\ln(1+t)\,, \qquad (24)
\end{aligned}$$

where $\zeta(n)$, which appears for $n = 2, 3$ in the formulae of this paper, represents the Riemann ζ-function and $\mathrm{Li}_2(x)$ denotes the dilogarithm. Using Eqs. (16)-(19) one can check that the order

α_s contribution to the total cross section in (11) is in agreement with the literature [12] (for the vector part see also [13]). Integration of the asymmetry quark coefficient function provides us with the result

$$\begin{aligned} f_A^{a,(1)}(\rho) &= C_F\Big[-4(2-\rho)\sqrt{1-\rho}\, G_1(t) + 2(4-5\rho)G_2(t) + 8\ln(1+t-\sqrt{t}) \\ &\quad -8(1-\rho)\Big(\ln(1+t) + 2\ln(1-\sqrt{t})\Big) + \Big(4(1-2\rho) + 2\sqrt{1-\rho}\,(-2+3\rho)\Big)\ln(t) \\ &\quad +4(\rho-\sqrt{\rho})\Big]\,, \end{aligned} \tag{25}$$

which involves the following functions

$$G_1(t) = \mathrm{Li}_2(-t^{3/2}) - 3\mathrm{Li}_2(-t^{1/2}) - 4\mathrm{Li}_2(t^{1/2}) - \mathrm{Li}_2(-t) - \frac{1}{2}\zeta(2) - \frac{1}{8}\ln^2(t)\,, \tag{26}$$

$$G_2(t) = \mathrm{Li}_2\left(\frac{\sqrt{t}}{1+t}\right) - \frac{1}{2}\zeta(2) - \frac{1}{2}\ln(t)\ln(1+t) + \frac{1}{2}\ln^2(1+t) - \frac{1}{8}\ln^2(t)\,. \tag{27}$$

The functions $f_k^{l,(1)}$ $(k = T, L;\ l = v, a)$ are related to the functions H_2, and H_6 presented in Eq. (15) and appendix A of [9] and we agree with their result. The same holds for $f_A^{a,(1)}$ which is proportional to H_5 in the reference above. There is also agreement with the calculation in [10]. The comparison is made by expanding the functions above and those in [9] up to seven powers in ρ. The next-to-next-to-leading order (NNLO) contributions come from the following processes. First one has to compute the two-loop vertex corrections to the Born process (13) and the one-loop corrections to (15). Second one has to add the radiative corrections due to the following reactions

$$V \to \mathrm{H} + \overline{\mathrm{H}} + g + g\,, \tag{28}$$

$$V \to \mathrm{H} + \overline{\mathrm{H}} + \mathrm{H} + \overline{\mathrm{H}}\,, \tag{29}$$

$$V \to \mathrm{H} + \overline{\mathrm{H}} + q + \overline{q}\,, \tag{30}$$

where q denotes the light quarks. The results for $f_k^{l,(2)}$ presented below are computed for the contributions where the vector boson V is always coupled to the heavy quark H so that the light quarks in Eq. (30) are only produced via fermion pair production emerging from gluon splitting. Besides these contributions there are other ones which have been treated in [14]. The latter consist of all one- and two-loop vertex corrections which contain the triangular quark-loop graphs. Following the notation in [14] their contribution to the forward-backward symmetry will be denoted by F_{QCD}^{3-jet} and F_{QCD}^{2-jet} respectively. They only show up if the quarks are massive and are coupled to the Z-boson via the axial-vector vertex. Notice that one has to sum over all members of one quark family in order to cancel the anomaly. Adopting the mass assignment in [14] we take the top to be massive and put the other quark masses, including that of the bottom, equal to zero. Further in [14] one has included all terms originating from reaction (30) where diagrams with light quarks attached to the vector boson V interfere with those describing the coupling of the heavy quarks to the vector boson. Notice that this contribution denoted in [14] by F_{QCD}^{F} vanishes if all quarks including H are taken to be massless provided one sums over all

members in one family. However we will omit that part of reaction (30) where the heavy quarks are produced via gluon splitting and the light quarks q are coupled to the vector boson V. This contribution denoted by $F_{QCD}^{\rm Branco}$ in [14] needs a cut on the invariant mass of the heavy flavour pair and it was computed for the first time in [15]). Finally notice that these additional contributions, denoted by F_{QCD} above, only show up in order α_s^2. Moreover if we put $m = 0$ they only appear in the forward-backward asymmetry (9) but cancel between numerator and denominator in the shape coefficient (10). The results for $f_k^{l,(2)}$ follow from the transverse and longitudinal coefficient functions calculated in [16] and the asymmetry coefficient function computed in [17]. Because of the complexity of the calculation of these functions the heavy quark mass was taken to be zero. This approximation is good for the charm and bottom quark but not for the top quark as we will see below. Substituting these coefficient functions in the integrand of Eq. (7) we obtain

$$\begin{aligned} f_T^{v,(2)} = f_T^{a,(2)} &= C_F^2 \left\{\frac{7}{2}\right\} + C_A C_F \left\{-\frac{11}{3}\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{347}{18} - 44\zeta(3)\right\} \\ &\quad + n_f C_F T_f \left\{\frac{4}{3}\ln\left(\frac{Q^2}{\mu^2}\right) - \frac{62}{9} + 16\zeta(3)\right\} , \qquad (31) \\ f_L^{v,(2)} = f_L^{a,(2)} &= C_F^2 \{-5\} + C_A C_F \left\{-\frac{22}{3}\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{380}{9}\right\} \\ &\quad + n_f C_F T_f \left\{\frac{8}{3}\ln\left(\frac{Q^2}{\mu^2}\right) - \frac{136}{9}\right\} , \qquad (32) \\ f_A^{a,(2)} &= C_A C_F \{-44\zeta(3)\} + n_f C_F T_f \{16\zeta(3)\} . \qquad (33) \end{aligned}$$

Here the colour factors are given by $C_A = N$ and $T_f = 1/2$ (for C_F see below Eq. (20)). Further n_f denotes the number of light flavours which originate from process (30). Finally μ appearing in the strong coupling constant α_s and the logarithms in Eqs.(31), (32) represents the renormalization scale. Notice that the coefficient of the logarithm is proportional to the lowest order coefficient of the β-function. The same holds for $\zeta(3)$ in Eqs. (31) and (33). The logarithm does not appear in $f_A^{a,(2)}$ because $f_A^{a,(1)} = 0$ at $m = 0$. Since the mass is equal to zero there is no distinction anymore between $f_k^{v,(2)}$ and $f_k^{a,(2)}$ $(k = T, L)$ unlike in the case for the first order corrections in Eqs. (16)-(19) where the heavy quark was taken to be massive. Furthermore one can check that substitution of Eqs. (31), (32) into (11) provides us with the order α_s^2 contribution to the total cross section which is in agreement with the results obtained in [18]. For zero mass quarks the forward-backward asymmetry becomes equal to

$$\begin{aligned} A_{\rm FB}^{\rm H}(Q^2) &= A_{\rm FB}^{\rm H,(0)}(Q^2)\left[1 - \frac{\alpha_s(\mu)}{4\pi}C_F\{3\} + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\left\{C_F^2\left(\frac{21}{2}\right)\right.\right. \\ &\quad \left.\left. + C_A C_F\left(11\ln\left(\frac{Q^2}{\mu^2}\right) - \frac{123}{2}\right) + n_f C_F T_F\left(-4\ln\left(\frac{Q^2}{\mu^2}\right) + 22\right)\right\}\right] . \qquad (34) \end{aligned}$$

For the discussion below and the notation often used in the literature (see [2]) it is convenient to write $A_{\rm FB}^{\rm H}$ in the following way

$$A_{\rm FB}^{\rm H}(Q^2) = A_{\rm FB}^{\rm H,(0)}(Q^2)\left[1 - \frac{\alpha_s(\mu)}{\pi}c_1 - \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 c_2\right] . \qquad (35)$$

The order α_s^2 contribution presented above has been also calculated in [14] but then in a numerical way. Unfortunately we get a different answer. First we disagree with the statement above Eq. (48) in [14] that reaction (29) does not contribute to $A_{\rm FB}^{\rm H}$. For the latter we obtain the following contribution to c_2, denoted by $\Delta^{(1)}c_2$, which equals

$$\Delta^{(1)}c_2 = -(C_F^2 - \frac{1}{2}C_A C_F)\frac{1}{16}\left[-\frac{19}{2} + 6\zeta(2) + 8\zeta(3)\right] = 0.14\,. \tag{36}$$

The result for reactions (28) and (30), including the virtual corrections, can be obtained by subtracting Eq. (36) from Eq. (34). Using the notation in Eq. (45) [14] and choosing $n_f = 5$ we find the following contributions

$$\begin{aligned} c_1 = \frac{3}{4}C_F \qquad & \Delta^{(2)}c_2 = -\frac{1}{4}C_F\left[\frac{9}{4}C_F + CC_F + NN_C + TT_R\right]\,, \\ \text{Eq. 46 [14]} \qquad & CC_F = 5.8 \quad NN_C = -31.0 \quad TT_R = 14.2\,, \\ \text{our result} \qquad & CC_F = -2.83 \quad NN_C = -42.38 \quad TT_R = 13.75\,. \end{aligned} \tag{37}$$

From the results above we infer that the discrepancies mainly occur in the C_F^2 and $C_A C_F$-terms of Eq. (34). Substituting $C_F = 4/3$ in the expression above we obtain for the coefficient of the $(\alpha_s(Q)/\pi)^2$-term the value -9.49 instead of -2.6 quoted in Eq. (4.7) of [14] which amounts to a discrepancy of a factor of about 3.7. Notice that the bulk of the second order correction to Eq (34) is coming from $f_A^{a,(2)}$ in Eq. (33) which amounts to 9.216. The remaining part can be traced back to the functions $f_k^{l,(2)}$ in Eqs. (31), (32) leading to the contribution 0.409.
Finally we are now also able to present the second order correction for the shape coefficient (10) in an analytical way

$$\begin{aligned} a^{\rm H}(Q^2) \;=\; & a^{\rm H,(0)}(Q^2)\left[1 - \frac{\alpha_s(\mu)}{4\pi}C_F\{8\} + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\left\{C_F^2\,(60)\right.\right. \\ & \left.\left. + C_A C_F\left(\frac{88}{3}\ln\left(\frac{Q^2}{\mu^2}\right) - \frac{1520}{9}\right) + n_f C_F T_F\left(-\frac{32}{3}\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{544}{9}\right)\right\}\right]\,, \end{aligned} \tag{38}$$

which can be written in the same way as has been done for $A_{\rm FB}^{\rm H}$ in Eq. (35) so that we get

$$a^{\rm H}(Q^2) = a^{\rm H,(0)}(Q^2)\left[1 - \frac{\alpha_s(\mu)}{\pi}d_1 - \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 d_2\right]\,. \tag{39}$$

We will now discuss the effect of the order α_s^2 corrections on the forward backward symmetry and the shape coefficient for the bottom and the charm quark. Further we omit any effect coming from the electroweak sector. Our results are obtained by choosing the following parameters (see [19]). The electroweak constants are: $M_Z = 91.187$ GeV/c^2, $\Gamma_Z = 2.490$ GeV/c^2 and $\sin^2\theta_W = 0.23116$. For the strong parameters we choose : $\Lambda_5^{\overline{MS}} = 237$ MeV/c ($n_f = 5$) which implies $\alpha_s(M_Z) = 0.119$ (two-loop corrected running coupling constant). Further we take for the renormalization scale $\mu = Q$ unless mentioned otherwise. Notice that we study $A_{\rm FB}^{\rm H}$ and $a^{\rm H}$ for H $= c, b$ at the CM energy $Q = M_Z$. For the heavy flavour masses the following values are adopted : $m_c = 1.50$ GeV/c^2, $m_b = 4.50$ GeV/c^2 and $m_t = 173.8$ GeV/c^2. The results for the bottom quark can be found in table 1. The values for c_2 are obtained by adding to Eq. (34) the

$A_{\rm FB}^b$		
	$m_b = 4.50$ GeV/c^2	$\overline{m}_b(M_Z) = 2.80$ GeV/c^2
c_1	0.789	0.858
c_2	8.89 (9.63)	8.89 (9.63)
$A_{\rm FB}^{(0)}$	0.1052	0.1052
$A_{\rm FB}^{(1)}$	0.1020 (- 3.04 %)	0.1017 (- 3.33 %)
$A_{\rm FB}^{(2)}$	0.1007 (- 4.28 %)	0.1004 (- 4.56 %)
a^b		
d_1	2.08	2.33
d_2	23.0	23.0
$a^{(0)}$	0.994	0.998
$a^{(1)}$	0.916 (- 7.85 %)	0.910 (- 8.82 %)
$a^{(2)}$	0.883 (- 11.2 %)	0.877 (- 12.1 %)

Table 1: The forward-backward asymmetry and the shape constant of the bottom quark.

contributions mentioned below Eq. (30). They were denoted by F_{QCD}^{2-jet}, F_{QCD}^{3-jet} and $F_{QCD}^{\rm F}$ in [14]. For the value of the top mass given above we obtain for bottom production $c_{2,QCD}^{2-jet} = -0.645$, $c_{2,QCD}^{3-jet} = -0.218$ and $c_{2,QCD}^{\rm F} = 0.123$. In the entry for c_2 we have also mentioned between the brackets the result obtained from the contribution of (34) which is equal to $c_2 = 77/8$. Notice that the second order contributions to the quantities in the tables are obtained by multiplying $A_{\rm FB}^{(0)}$ at $m = 0$ with c_2. Furthermore we have put in the tables the correction in percentages of the radiatively corrected quantities $A_{\rm FB}^H$, $a^{\rm H}$ with respect to their zeroth order result. From table 1 we infer that the order α_s as well as the order α_s^2 corrections to both $A_{\rm FB}^b$ and a^b are negative. In the case of the forward-backward asymmetry the QCD corrections are moderate in particular the second order ones. The latter would be even smaller if we had taken the value $c_2 = 1.9$ quoted in [14] which is a factor 4.7 less with respect to our result. In the case of the shape coefficient the QCD corrections are at least twice as large. The latter are reduced if for the reference axis the thrust axis is taken instead of the quark axis [2]. Notice that the quark axis has been chosen in our calculation. Further we want to mention that the zero mass approximation for the second order coefficient c_2 is quite reasonable. This is revealed by the first order coefficient when the quark mass is chosen to be zero which leads to the values $c_1 = 1$ Eq. (34) and $d_1 = 8/3$ Eq. (38). In the case of the bottom quark we observe a deviation of 11 % for c_1 whereas for d_1 it amounts to 22 % which is twice as large. For the charm quark (see table 2) these values become smaller i.e. about 7.5 % for both coefficients. If we expect that the same deviations occur for the coefficients c_2 Eq. (34)and d_2 Eq. (38) one gets a reasonable estimate of the theoretical error on the second order corrections. We also studied the effect of the running quark mass on the forward-backward asymmetry and the shape coefficient. For this purpose one has to change the on-mass shell scheme used in Eqs. (16)-(19), (25) into the $\overline{\rm MS}$-scheme. This can be done by substituting in all expressions the fixed pole mass m by the running mass $\overline{m}(\mu)$. Moreover one has to add to the first order contributions (16)-(19), (25) the finite counter term

$$\Delta f_k^{l,(1)} = \overline{m}(\mu) C_F \left[4 - 3\ln\left(\frac{\overline{m}^2(\mu)}{\mu^2}\right)\right] \left(\frac{d\ f_k^{l,(0)}(\rho)}{d\ m}\right)_{m=\overline{m}(\mu)}, \tag{40}$$

$A^c_{\rm FB}$		
	$m_c = 1.50$ GeV/c^2	$\overline{m}_c(M_Z) = 0.662$ GeV/c^2
c_1	0.924	0.962
c_2	11.5 (9.63)	11.5 (9.63)
$A^{(0)}_{\rm FB}$	0.0750	0.0750
$A^{(1)}_{\rm FB}$	0.0724 (- 3.47 %)	0.0723 (- 3.60 %)
$A^{(2)}_{\rm FB}$	0.0712 (- 5.07 %)	0.0710 (- 5.20 %)
a^c		
d_1	2.46	2.58
d_2	23.0	23.0
$a^{(0)}$	1.000	1.000
$a^{(1)}$	0.907 (- 9.30 %)	0.903 (- 9.70 %)
$a^{(2)}$	0.874 (- 12.6 %)	0.870 (- 13.0 %)

Table 2: The forward-backward asymmetry and the shape constant of the charm quark.

where μ stands for the mass renormalization scale for which we choose $\mu = Q$. Further we adopt the two-loop corrected running mass with the initial condition $\overline{m}(\mu_0) = \mu_0$. Using the relation between the $\overline{MS}$-mass and the fixed pole mass, as is indicated by the first factor on the right-hand side in Eq. (40), we have taken for bottom production $\mu_0 = 4.10$ GeV/c^2 which corresponds with a pole mass $m_b = 4.50$ GeV/c^2. This choice leads to $\overline{m}_b(M_Z) = 2.80$ GeV/c^2 which is 5 % above the experimental value 2.67 GeV/c^2 measured at LEP [20]. The results are presented in the second column of table 1. Comparing the latter with the first column we observe that the QCD corrections become a little bit more negative. The values of $A^{(i)}_{\rm FB}$ $(i = 0, 1, 2)$ decrease a little too. The same features are also shown by $a^{(i)}$ except for $a^{(0)}$ which slightly increases.

In table 2 we also present results for the charm quark. In the case of the running mass we have chosen $\mu_0 = 1.30$ GeV/c^2 so that the pole mass becomes $m_c = 1.50$ GeV/c^2. This leads to a value of $\overline{m}_c(M_Z) = 0.662$ GeV/c^2 which is rather low. Furthermore for charm quark production the additional contributions become $c^{2-jet}_{2,QCD} = 1.359$, $c^{3-jet}_{2,QCD} = 0.323$ and $c^{\rm F}_{2,QCD} = 0.211$. Our result for c_2 is about 2.6 times larger than the value 4.4 obtained in [14]. The features are the same as for the bottom quark. The differences between the numbers in the left-hand and right-hand column in table 2 become even less. The reason that the running of the mass in the case of the charm and the bottom quark hardly introduces any effect on the forward-backward asymmetry and the shape constant can be attributed to the fact that the mass of both quarks are small with respect to the CM energy so that the phase space is not much affected. Moreover, as far as the dynamics is concerned, these quantities are not proportional to the mass. This is also revealed by the constants c_1 and d_1 which do not deviate very much from their zero mass values. These arguments do not apply to the top quark except when $Q \gg m_t$. This is shown in table 3 where we have studied the above quantities at a CM energy $Q = 500$ GeV/c. For the running mass we have chosen $\mu_0 = 166.1$ GeV/c^2 so that the pole mass becomes equal to $m_t = 173.8$ GeV/c^2. This leads to a value $\overline{m}_t(Q) = 153.5$ GeV/c^2. The constants c_1 and d_1 in the perturbation series completely differ from the ones given at $m = 0$ at which they become 1 and 8/3 respectively. Therefore the running mass will have a large effects on these constants which is revealed by the switch of sign in table 3. Hence the Born approximations to $A^t_{\rm FB}$ and

$A_{\rm FB}^t$		
	$m_t = 173.8$ GeV/c^2	$\overline{m}_t(Q) = 153.5$ GeV/c^2
c_1	- 0.757	2.11
$A_{\rm FB}^{(0)}$	0.407	0.455
$A_{\rm FB}^{(1)}$	0.417 (2.46 %)	0.426 (- 6.37 %)
a^t		
d_1	- 0.911	5.13
$a^{(0)}$	0.406	0.514
$a^{(1)}$	0.417 (2.71 %)	0.435 (- 15.4 %)

Table 3: The forward-backward asymmetry and the shape constant of the top quark at $Q = 500$ GeV/c.

a^t will change while going from the fixed pole mass to the running mass approach. However the order α_s corrected quantities are less sensitive to the choice between the running or the fixed pole mass because of the compensating term in Eq. (40). From the above it is clear that the zero mass approximation to c_2 and d_2 makes no sense in case of the top quark and we have omitted these contributions to $A_{\rm FB}^t$ and a^t in table 3. Finally we want to comment on the renormalization scale dependencies of the forward backward asymmetry and the shape constant. If we vary the scale μ between $Q/2$ and 2 Q the changes in $A_{\rm FB}^{(2)}$ are small. It introduces an error of 0.002 for the bottom quark and 0.003 for the charm quark. For $a^{(2)}$ one can draw the same conclusion and the error becomes 0.005 and 0.007 respectively. In the case of the top quark a variation in the renormalization scale makes no sense because of the missing order α_s^2 correction. Its computation for massive quarks will be a enormous enterprise.

Summarizing our findings we have computed the order α_s^2 contributions to the forward-backward asymmetry and the shape constant in an analytical way provided the heavy flavour mass is chosen to be zero. Further we found a discrepancy with a numerical result calculated earlier in the literature for $A_{\rm FB}^{\rm H(2)}$. The second order corrections are noticeable. The transition from the fixed pole mass to the running mass approach does not introduce large changes in the values of $A_{\rm FB}^{\rm H}$ and $a^{\rm H}$ except for the first order constant in the perturbation series when $H = t$. This indicates that the zero mass approach breaks down unless $Q \gg m$. Also a variation of the renormalization scale does not lead to large effects. The latter are almost equal to the differences between the results obtained by the fixed pole mass and the running mass approach.

Acknowledgements

The authors would like to thank J. Blümlein for reading the manuscript and giving us some useful remarks. W.L. van Neerven would like to thank P.W. Zerwas for discussions concerning the importance of mass corrections to $A_{\rm FB}^b$ and a^b. This work is supported by the EC-network under contract FMRX-CT98-0194.

References

[1] G.Altarelli, R. Barbieri and F. Caravaglios, Int. J. Mod. Phys. **A13** (1998) 1031.

[2] A. Djouadi, J.H. Kühn and P.M. Zerwas, Z. Phys. **C46** (1990) 411;
A. Djouadi, B. Lampe and P.M. Zerwas, Z. Phys. **C67** (1995) 123;
A. Djouadi and P.M. Zerwas, Internal Report, DESY T-97-04;
D. Abbaneo et al., Eur.Phys.J. **C4** (1998) 185.

[3] R. Barate et al. (ALEPH-Collaboration), CERN-EP/98-101.

[4] V. Driesen, W. Hollik, A. Kraft, in "e^+ e^- collisions at TeV energies: the physics potential", Proceedings of the workshop - Annecy, Gran Sasso, Hamburg, February 1995 to September 1995, ed. P.M. Zerwas, DESY 96-123D, p. 33-40.

[5] D.Y. Bardin et al., Z. Phys. **C44** (1989) 493;
D.Y. Bardin et al., Nucl. Phys. **B351** (1991) 1.

[6] W. Beenakker, W. Hollik and S.C. van der Marck, Nucl. Phys. **B365** (1991) 24;
W. Beenakker and W. Hollik, Phys. Lett. **B269** (1991) 425;
W. Beenakker, A. Denner and A. Kraft, Nucl. Phys. **B410** (1993) 219.

[7] J. Jersak, E. Laermann and P.M. Zerwas, Phys. Lett. **B98** (1980) 363, Phys. Rev. **D25** (1980) 1218, Erratum ibid. **D36** (1987) 310.

[8] A. Djouadi, Z. Phys. **C39** (1988) 561.

[9] A. Arbuzov, D.Y. Bardin and A. Leike, Mod. Phys. Lett. **A7** (1992) 2029, Erratum ibid. **A9** (1994) 1515.

[10] J.B. Stav and H.A. Olsen, Phys. Rev. **D52** (1995) 1359.

[11] P. Nason and B.R. Webber, Nucl. Phys. **B421** (1994) 473.

[12] K. Schilcher, M.D. Tran and N.F. Nasrallah, Nucl. Phys. **B181** (1981) 91, Erratum ibid. **B187** (1981) 594;
J.L. Reinders, S. Yazaki and H.R. Rubinstein, Phys. Lett. **B103** (1981 63;
T.H. Chang, K.J.F. Gaemers, W.L. van Neerven, Phys. Lett. **B108** (1982) 222, Nucl. Phys. **B202** (1982) 407.

[13] G. Källen and A. Sabry, Dan. Mat. Fys. Medd. **29** (1955) 17;
J. Schwinger, in "Particles, Sources and Fields", Addison-Wesley, New York, 1973, vol II, ch. 5.

[14] G. Altarelli and B. Lampe, Nucl. Phys. **B391** (1993) 3.

[15] G.C. Branco, H.P. Nilles and K.H. Streng, Phys. Lett. **B85** (1979) 269.

[16] P.J. Rijken and W.L. van Neerven, Phys. Lett. **B386** (1996) 422, Nucl. Phys. **B487** (1997) 233.

[17] P.J. Rijken and W.L. van Neerven, Phys. Lett. **B392** (1997) 207.

[18] K.G. Chetyrkin, A.L. Kataev, F.V. Tkachov, Phys. Lett. **B85** (1979) 277;
M. Dine and J. Sapirstein, Phys. Rev. Lett. **43** (1979) 668;
W. Celmaster and R.J. Gonsalves, Phys. Rev. Lett. **44** (1980) 560.

[19] C. Caso et al., "Review of Particle Physics", Eur. Phys. J. **C3** (1998) 19, 24, 87.

[20] P. Abreu et al. (DELPHI-collaboration), Phys. Lett. **B418** (1998) 430.