DESY 98-188

DESY 98-188

The Gauge-Higgs System in Three Dimensions to Two-loop Order

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hep-ph/9811513 30 Nov 1998

Abstract

The 3-dimensional gauge-Higgs system describes the non-perturbative infrared effects of the high-temperature phase of the Standard Model. We calculate the two-loop self-energies in the 3-dimensional $SU(2)$ Higgs model and in the corresponding gauged non-linear σ-model. As an application of the results, we estimate the dynamically generated vector boson mass in the symmetric phase of the Higgs model by means of gap equations.

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1 Introduction

The 3-dimensional gauge-Higgs system is interesting in many respects. The phase structure of the Abelian model describes type-I and type-II superconductors [1], whereas the non-abelian $SU(2)$ Higgs model serves as the high-temperature limit of the corresponding 4-dimensional theory and the electroweak Standard Model. It is important for the study of the electroweak phase transition and, since it contains all the infrared physics of the full theory, for investigating non-perturbative effects, which are expected in the high-temperature (symmetric) phase [2,3]. Already Feynman stressed the importance of examining the 3-dimensional theory as a first step to understand confinement in QCD [4]. Both the 3- and 4-dimensional theories confine with a linear potential.

Lattice simulations of the Higgs model in 3 dimensions have shown that a mass gap exists in the symmetric phase with a particle spectrum consisting of bound states of gluons with scalars [5] and of bound states of gluons only (glueballs) [6]. In order to illuminate the connection between Higgs phase and symmetric phase in 3 dimensions, it is important to understand the behaviour of the vector boson propagator. A propagator mass for the vector boson was studied on the lattice in [7]. In this paper, we concentrate on an analytical treatment of the model, in particular on propagator effects. We calculate the self-energy of the Higgs and vector boson to two-loop order. The one-loop self-energy for the vector boson turns out to be dominated by the diagrams obtained in the corresponding non-linear σ-model [8]. This is why we consider this simpler model first in our two-loop calculation.

In the symmetric phase of the 3-dimensional $SU(2)$ Higgs model, gap equations provide an analytical tool to calculate the vector boson propagator mass [8]. The gap equation is a self-consistent equation for the self-energy after resumming perturbation theory. We have recently extended this method to two-loop order in the non-linear σ-model [9]. Here, we solve the two-loop gap equation for the vector boson in the linear Higgs model and compare the result with the two-loop gap mass in the non-linear case. With the help of the two-loop self-energies evaluated here, we show that the non-linear σ-model constitutes a reasonable approximation for infrared effects of the linear Higgs model.

In section 2, the two-loop calculation of the vector boson self-energy in the non-linear σ-model is presented in unitary and in Feynman gauge. The gauge-independence of the pole of the propagator is verified. In section 3, the corresponding calculations are done for the Higgs and the vector field in the $SU(2)$ Higgs model.

In section 4, the two-loop gap equation is calculated in the $SU(2)$ Higgs model. The analysis of the two-loop gap equation for the vector boson mass in Feynman gauge suggests that the gap equation approach is a reliable method to calculate the transverse propagator mass of the vector boson in the symmetric phase.

Appendix A summarizes the basic two-loop integrals. In appendix B, the two-loop results of the non-linear σ-model are presented in more detail keeping the dimension arbitrary, and

finally in appendix C, the two-loop Higgs and vector boson self-energy in the $SU(2)$ Higgs model are given in unitary and in Feynman gauge.

2 The vector boson self-energy in the non-linear σ-model

2.1 The model

Our starting point is the action of the 3-dimensional $SU(2)$ Higgs model, which is given by

$$S = \int d^3x \, \mathrm{Tr} \left[\frac{1}{2} W_{\mu\nu} W_{\mu\nu} + (D_\mu \Phi)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger \Phi + 2\lambda \left(\Phi^\dagger \Phi \right)^2 \right] , \tag{1}$$

with

$$\Phi = \frac{1}{2} \left(\sigma + i \vec{\pi} \cdot \vec{\tau} \right) , D_\mu \Phi = \left(\partial - i g W_\mu \right) \Phi , W_\mu = \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \, . \tag{2}$$

Here $\vec{W}_\mu$ is the vector field, σ is the Higgs field, $\vec{\pi}$ is the Goldstone boson field and $\vec{\tau}$ the triplet of Pauli matrices. To obtain the non-linear σ-model, one eliminates one degree of freedom by the constraint

$$\sigma^2 = v^2 - \pi^2 \, , \tag{3}$$

and takes the limit $\lambda, \mu \to \infty$. Setting $m = \frac{g^2 v^2}{4}$, one then arrives at the following Lagrangian,

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{4} \vec{W}_{\mu\nu} \vec{W}_{\mu\nu} + \frac{1}{2\xi} \left(\partial_\mu \vec{W}_\mu \right)^2 + \frac{1}{2} m^2 \vec{W}_\mu^2 \\ & + \frac{1}{2} \left(\partial_\mu \vec{\pi} \right)^2 + \frac{\xi}{2} m^2 \vec{\pi}^2 + \frac{g}{2} \left(\vec{W}_\mu \times \vec{\pi} \right) \\ & + \partial_\mu \vec{c}^{\,*} \partial_\mu \vec{c} + \xi m^2 \vec{c}^{\,*} \vec{c} + g \partial_\mu \vec{c}^{\,*} \cdot \left(\vec{W}_\mu \times \vec{c} \right) \\ & + \xi \frac{g}{2} m \vec{c}^{\,*} \cdot \left(\vec{\pi} \times \vec{c} \right) - \xi \frac{g^2}{8} \vec{\pi}^2 \vec{c}^{\,*} \vec{c} \\ & + \frac{g^2}{8} \frac{\left(\vec{\pi} \partial_\mu \vec{\pi} \right)^2}{m^2} - \frac{g^2}{4} \vec{W}_\mu \cdot \vec{\pi} \frac{\vec{\pi} \cdot \partial_\mu \vec{\pi}}{m} + \frac{g^2}{8} \vec{W}_\mu \cdot \partial_\mu \vec{\pi} \frac{\vec{\pi}^2}{m} \, . \end{aligned} \tag{4}$$

Note that we have neglected all higher-dimensional operators which do not contribute to the two-loop self-energy. In unitary gauge the unphysical degrees of freedom decouple and one is left with a massive Yang-Mills theory,

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} m^2 W^a_\mu W^a_\mu \, , \tag{5}$$

with

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu \, . \tag{6}$$

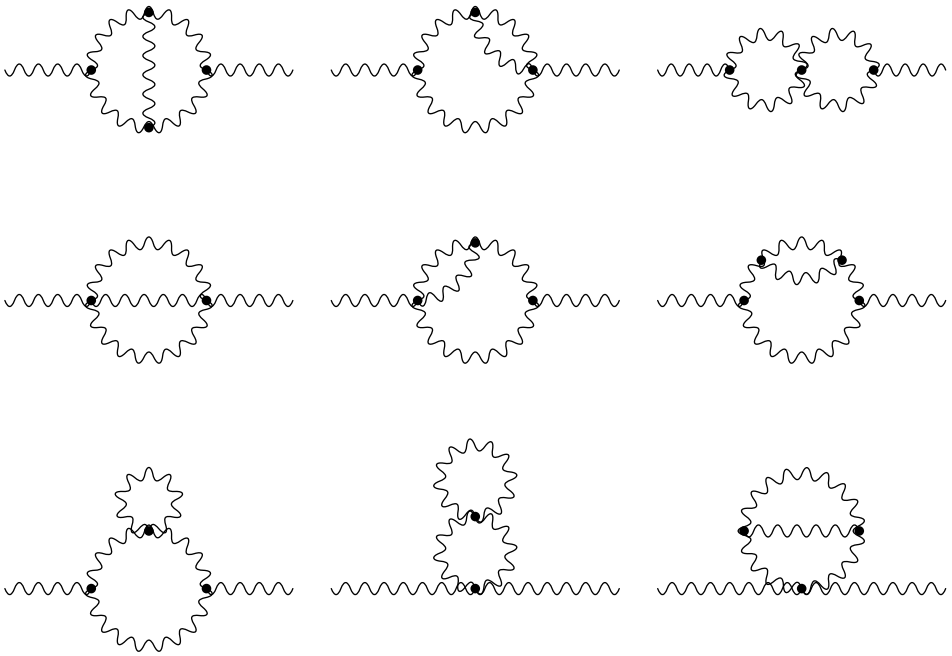


Figure 1: *Diagrams contributing to the two-loop gap equation in the unitary gauge*

The advantage of the unitary gauge is that only a minimal amount of diagrams has to be calculated. The one-loop self-energy in unitary and renormalizable gauges can be found in [8,10]. On mass-shell they coincide due to the BRS-invariance of the Lagrangian.

In the non-linear σ-model there are non-renormalizable vertices. At the one-loop level, no problem concerning renormalization arises, as the non-renormalizable couplings do not contribute to this loop order. Moreover, in dimensional regularization, all one-loop integrals are finite in 3 dimensions. To two loops the situation is more difficult.

2.2 Two-loop self-energy in massive Yang-Mills theory

In unitary gauge, only 9 two-loop diagrams have to be evaluated for the non-linear σ-model. They are depicted in fig. 1.

As all propagators are massive and the external momentum does not vanish, the reduction of the scalar integrals to basic integrals with no momenta in the numerators turns out to be the most difficult step in the calculation. For propagator type integrals this task has been achieved only recently by Tarasov [11]. Using his recurrence relations it is possible to reduce the self-energy integrals to a small set of linearly independent basic integrals. For the first time, this method achieves a complete reduction and stays on an algebraic level as far as possible. Since the recurrence relations are in some cases quite involved, they have to be implemented into a FORM package [12]. In unitary gauge, the situation is even more complex due to the high powers of momenta in the numerator. A peculiarity of the unitary gauge is that the limit $\xi \to \infty$ must be performed before divergent integrals are evaluated [13]. Otherwise, one would

get an infinite result for the self-energy.

The reduction program in FORM yields for the sum of the transverse parts

$$
\begin{aligned}
\frac{1}{g^4}\Pi_T^{2-loop}(p^2) &= \left(\frac{63}{4}m^4 - \frac{111}{8}p^2m^2 - \frac{67}{16}p^4 - \frac{33}{32}\frac{p^6}{m^2} + \frac{1}{16}\frac{p^8}{m^4}\right) F(m,m,m,m,m) \\
&+ \left(-\frac{63}{2}m^2 - \frac{113}{8}p^2 - \frac{27}{16}\frac{p^4}{m^2} + \frac{109}{64}\frac{p^6}{m^4}\right) V(m,m,m,m) \\
&+ \left(-\frac{189}{2}\frac{m^4}{p^2} + \frac{237}{4}m^2 + \frac{12257}{80}p^2 - \frac{21}{8}\frac{p^4}{m^2} - \frac{167}{80}\frac{p^6}{m^4}\right) I_{211}(m,m,m) \\
&+ \left(\frac{63}{4}\frac{m^2}{p^2} - \frac{111}{8} - \frac{159}{16}\frac{p^2}{m^2} + \frac{463}{192}\frac{p^4}{m^4} - \frac{1}{60}\frac{p^6}{m^6}\right. \\
&\quad \left. -\frac{387}{4}\frac{m^2}{p^2}\epsilon + \frac{903}{8}\epsilon + \frac{1597}{20}\frac{p^2}{m^2}\epsilon - \frac{149}{15}\frac{p^4}{m^4}\epsilon - \frac{1}{50}\frac{p^6}{m^6}\epsilon\right) I_{111}(p^2)(m,m,m) \\
&+ \left(-\frac{63}{4}\frac{m^2}{p^2} + \frac{111}{8} + \frac{159}{16}\frac{p^2}{m^2} - \frac{117}{64}\frac{p^4}{m^4}\right. \\
&\quad \left. +\frac{135}{4}\frac{m^2}{p^2}\epsilon - \frac{195}{8}\epsilon - \frac{207}{8}\frac{p^2}{m^2}\epsilon - \frac{3}{4}\frac{p^4}{m^4}\epsilon\right) I_{111}(0)(m,m,m) \\
&+ \left(\frac{37}{4}m^2 - \frac{5}{2}p^2 - \frac{387}{32}\frac{p^4}{m^2} - \frac{1}{32}\frac{p^6}{m^4} + \frac{35}{128}\frac{p^8}{m^6} - \frac{1}{64}\frac{p^{10}}{m^8}\right) B^2(p^2,m^2,m^2) \\
&+ \left(\frac{23}{2} - \frac{151}{8}\frac{p^2}{m^2} - \frac{57}{8}\frac{p^4}{m^4} + \frac{1}{4}\frac{p^6}{m^6} + \frac{1}{16}\frac{p^8}{m^8}\right) B(p^2,m^2,m^2)A(m^2) \\
&+ \left(-\frac{25}{8}\frac{1}{m^2} + \frac{7}{8}\frac{p^2}{m^4} + \frac{87}{160}\frac{p^4}{m^6} - \frac{1}{16}\frac{p^6}{m^8}\right) A(m^2)A(m^2)\,. \qquad (7)
\end{aligned}
$$

The sum of the longitudinal parts adds up to 0 for all external momenta p, which is a nice check of the calculation.

In 3 dimensions, $I(p^2)(m,m,m)$ and $I(0)(m,m,m)$ are logarithmically UV-divergent, whereas all other basic integrals are finite in dimensional regularization. In $d = 3 - 2\epsilon$, these two integrals exhibit the following behaviour for small ϵ,

$$
I(p^2)(m,m,m) = I(0)(m,m,m) = \frac{1}{64\pi^2\epsilon} + \text{finite}\,, \qquad (8)
$$

leading to poles in the self-energy,

$$
\Pi_T^{2-loop} = \left(\frac{7}{12}\frac{p^4}{m^4} - \frac{1}{60}\frac{p^6}{m^6}\right)\frac{1}{64\pi^2\epsilon} + \text{finite}\,, \qquad (9)
$$

which cannot be dealt with by a mass or wave function renormalization. As we will see in the next section, this is due to the bad high-energy behaviour of the propagator in unitary gauge.

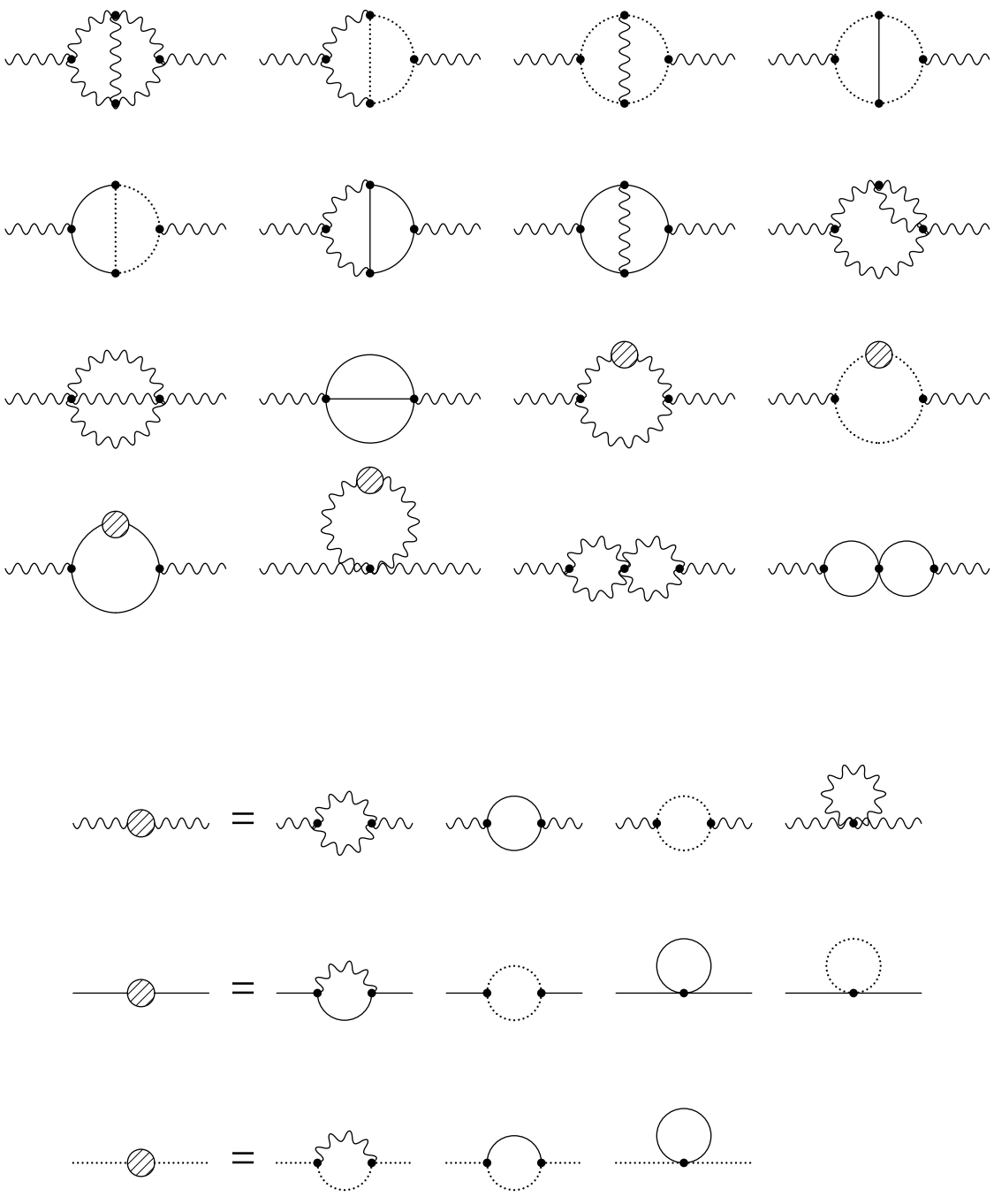


Figure 2: *Generic two-loop self-energy diagrams in the non-linear σ-model*

A similar problem arises already at one loop, if one uses cutoff-regularization. Calculations of counter-terms cannot be done in unitary gauge. However, if one is interested in finite parts of gauge-invariant quantities like poles in propagators, the unitary gauge provides a convenient short-cut of the calculation.

2.3 Two-loop calculation in Feynman gauge

The diagrams which have to be evaluated in Feynman gauge are depicted in figure 2. In the sum of the transverse parts of the generic two-loop diagrams Π_T^{2-loop}, the coefficients in front of the products of one-loop basic integrals do not coincide in unitary and in Feynman gauge.

However, this gauge-dependence combined with the gauge-dependence in $\frac{\partial}{\partial p^2}\Pi_T^{1-loop}$ leads to a gauge-invariant result for $\Pi_T(p^2=-m^2)\left(1+\frac{\partial}{\partial p^2}\Pi_T(p^2=-m^2)\right)$ at the two-loop level. This quantity is nothing but the two-loop pole of the propagator, which should be gauge-invariant in BRS-symmetric theories [14]. The preceding statements can be checked using the result of the last section and the sum of the generic two-loop diagrams in Feynman gauge written in appendix B. We obtain the same position for the pole of the propagator as in unitary gauge.

The Feynman gauge is a renormalizable gauge. Collecting the coefficients of $I(p^2)(m,m,m)$ and $I(0)(m,m,m)$, we get the following poles in ϵ for the self-energy

$$\frac{1}{g^4}\Pi_{T,\xi=1}^{2-loop}(p^2)=\left(\frac{7}{12}-\frac{1}{60}\frac{p^2}{m^2}\right)\frac{1}{64\pi^2\epsilon}+\text{finite}\,. \tag{10}$$

According to eq. (10), a mass and wave function renormalization removes the infinities in the two-loop self-energy. This is also suggested by naive power counting.

3 The Higgs and vector boson self-energy in the $SU(2)$ Higgs model

3.1 The model

Consider now the 3-dimensional $SU(2)$ Higgs model of eq. (1). Varying μ^2/g^4 one expects a phase transition which is of first order for sufficiently small values of λ/g^2.

We change parameters according to

$$\mu^2=\frac{1}{2}M^2,\lambda=\frac{g^2}{8}\frac{M^2}{m^2}\,, \tag{11}$$

shift the Higgs field σ around its classical minimum $\sigma=\frac{2m}{g}+\sigma'$, and add an R_ξ-gauge fixing term $\mathcal{L}_{GF}=\frac{1}{2\xi}\left(\partial_\mu W_\mu^a-\xi m\pi^a\right)$ and the corresponding ghost terms in the usual way. The resulting Lagrangian reads,

$$
\begin{aligned}
\mathcal{L}_R &= \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} + \frac{1}{2\xi}\left(\partial_\mu \vec{W}_\mu\right)^2 + \frac{1}{2}m^2\vec{W}_\mu^2 \\
&+\frac{1}{2}\left(\partial_\mu \sigma'\right)^2 + \frac{1}{2}M^2\sigma'^2 + \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 + \frac{\xi}{2}m^2\vec{\pi}^2 \\
&+\frac{g}{2}m\sigma'\vec{W}_\mu^2 + \frac{g}{2}\vec{W}_\mu \cdot \left(\vec{\pi}\partial_\mu\sigma' - \sigma'\partial_\mu\vec{\pi}\right) + \frac{g}{2}\left(\vec{W}_\mu \times \vec{\pi}\right)\cdot\partial_\mu\vec{\pi} \\
&+\frac{g^2}{8}\vec{W}_\mu^2\left(\sigma'^2 + \vec{\pi}^2\right) + \frac{g}{4}\frac{M^2}{m}\sigma'\left(\sigma'^2 + \vec{\pi}^2\right) + \frac{g^2}{32}\frac{M^2}{m^2}\left(\sigma'^2 + \vec{\pi}^2\right)^2 \\
&+\partial_\mu\vec{c^*}\partial_\mu\vec{c} + \xi m^2\vec{c^*}\vec{c} \\
&+g\partial_\mu\vec{c^*}\cdot\left(\vec{W}_\mu \times \vec{c}\right) + \xi\frac{g}{2}m\sigma'\vec{c^*}\vec{c} + \xi\frac{g}{2}m\vec{c^*}\cdot\left(\vec{\pi}\times\vec{c}\right) . \qquad (12)
\end{aligned}
$$

The one-loop results for the self-energies in R_ξ-gauge can be found in [8]. The on-shell self-energies coincide for all gauges.

3.2 Two-loop self-energy for the Higgs field

The generic two-loop Higgs self-energy diagrams are depicted in appendix C. The sum of these diagrams is evaluated on mass-shell, $p^2 = -M^2$. The lengthy expressions resulting from the reduction to basic integrals can also be found in appendix C in Feynman gauge and in unitary gauge. For the sum of the generic two-loop diagrams Σ^{2-loop}, the unitary gauge result differs from the result in Feynman gauge only by products of one-loop integrals. Quantitatively,

$$
\begin{aligned}
&\Sigma^{2-loop}_{\xi=1}(p^2 = -M^2) - \Sigma^{2-loop}_{\xi=\infty}(p^2 = -M^2) = \\
&\Sigma^{1-loop}_{\xi=1,\infty}(p^2 = -M^2)\left(\frac{\partial}{\partial p^2}\Sigma^{1-loop}_{\xi=1}(p^2 = -M^2) - \frac{\partial}{\partial p^2}\Sigma^{1-loop}_{\xi=\infty}(p^2 = -M^2)\right) . \qquad (13)
\end{aligned}
$$

This ensures that neglecting the resummation counter-terms, the pole of the Higgs boson propagator is gauge parameter independent to two loops. The underlying reason for this powerful check of the calculation is the BRS-invariance of the linear model. Eq. (13) can be verified using the expressions in appendix C and the one-loop results in [8] and section 3.1.

Concerning renormalization, eq. (29) leads to the following pole structure in ϵ for the Higgs self-energy (keeping the external momentum p^2 arbitrary),

$$
\Sigma^{2-loop}_{\xi<\infty}(p^2) = \left(\frac{51}{8} + \frac{9}{4}\frac{M^2}{m^2} - \frac{3}{8}\frac{M^4}{m^4}\right)\frac{1}{64\pi^2\epsilon} + \text{finite.} \qquad (14)
$$

As expected in the super-renormalizable 3-dimensional Higgs model, no wave function renormalization is necessary. For the Higgs field, we have to add only a mass renormalization counter-term.

3.3 Two-loop self-energy for the vector field

The analogous calculation is performed for the vector boson field. The sum of the generic two-loop diagrams shown in appendix C is evaluated in Feynman and unitary gauge in $d = 3 - 2\epsilon$ on mass-shell, $p^2 = -m^2$ (note, that we only draw those Feynman diagrams, whose transverse part is non-zero).

A relation similar to eq. (13) holds for the transverse two-loop vector field self-energy. As for the Higgs field, it can be verified, that the pole of the transverse part of the vector boson propagator is gauge-invariant to two-loop order. In dimensional regularization, the divergent terms in the vector self-energy (evaluated for arbitary external momentum p^2) can be obtained from eq. (30),

$$\Pi_T^{2-loop}(p^2) = \left(\frac{51}{8}\frac{m^2}{M^2} + \frac{9}{4} - \frac{3}{8}\frac{M^2}{m^2} \right) \frac{1}{64\pi^2\epsilon} + \text{finite}\,. \tag{15}$$

As for the Higgs field, no wave function renormalization is needed for the vector field. There is only a renormalization of the vacuum expectation value. Comparing eq. (15) with eq. (14), one can see a simple relation between the divergent terms: the coefficients of the divergent terms differ only by the factor $\frac{m^2}{M^2}$ (Ward-identity).

4 Two-loop gap equation in the $SU(2)$ Higgs model

In the high-temperature phase of the standard model a naive perturbative expansion with a vanishing vector boson mass leads to severe infrared divergences in the magnetic sector of the theory [3]. Introducing a non-vanishing mass which acts as an infrared cut-off can cure these problems. The symmetric phase is expected to be governed by non-perturbative effects whose size is determined by the magnetic screening length, which is the inverse of the magnetic mass. In an apparently massless 3-dimensional Yang-Mills theory, the gauge coupling g^2 carries the dimension of mass, thereby providing a natural mass scale.

The propagator mass of the vector boson can be calculated analytically using gap equations [8]. In this chapter, we investigate two-loop effects on the gap equation in the 3-dimensional Higgs model.

4.1 Gap equations and resummation

We again consider the model defined by eq. (1). We are interested in the Higgs and vector boson masses in both phases which determine the exponential fall-off of the corresponding two-point functions at large separation $|x - y|$,

$$\begin{aligned} \langle \sigma(x)\sigma(y) \rangle &\sim e^{-M|x-y|}\,, \\ \langle W_\mu(x) W_\mu(y) \rangle &\sim e^{-m|x-y|}\,. \end{aligned} \tag{16}$$

In the Higgs phase, these 2-point functions can be evaluated in perturbation theory. The masses m and M are given by the gauge-independent poles of the corresponding propagators in momentum space. In eq. (1) we shift the Higgs field σ around its vacuum expectation value v, $\sigma = v + \sigma'$, add an R_ξ-gauge fixing term and the corresponding ghost terms in the usual way. This yields the following masses for the vector boson and the Higgs field,

$$m_0^2 = \frac{g^2}{4} v^2 , \; M_0^2 = \mu^2 + 3\lambda v^2 . \tag{17}$$

The ghost and Goldstone boson mass is given by $\sqrt{\xi} m_0$.

In order to extract a non-vanishing mass in the symmetric phase, where in ordinary perturbation theory $v = 0$, we add and subtract a mass-term. The tree-level masses m_0^2 and M_0^2 are expressed as

$$m_0^2 = m^2 - \delta m^2 , \; M_0^2 = M^2 - \delta M^2 , \tag{18}$$

where m and M enter the propagators of the loop expansion, and δm^2 and δM^2 are treated perturbatively as counter-terms. For a gauge-invariant one-loop gap equation it is necessary and sufficient to have a BRS-invariant resummed tree-level action. This requires a suitable resummation of the ghost and Goldstone boson mass as well as of the following vertices,

$$\frac{g^2 v}{2} = gm - \delta V^g , \; \lambda v = \frac{gM}{4m} - \delta V^{\lambda v} , \; \lambda = \frac{g^2 M^2}{8m^2} - \delta V^\lambda . \tag{19}$$

The resulting Lagrangian reads [8],

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_R + \mathcal{L}_1 + \mathcal{L}_0 , \\
\mathcal{L}_1 &= -\delta m^2 \left(\frac{1}{2} \vec{W}_\mu^2 + \frac{\xi}{2} \vec{\pi}^2 + \xi \vec{c}^* \vec{c} \right) - \frac{1}{2} \delta M^2 \sigma'^2 + \frac{1}{2} \left(\mu^2 + \lambda v^2 \right) \vec{\pi}^2 \\
&\quad + v \left(\mu^2 + \lambda v^2 \right) \sigma' - \frac{1}{2} \delta V^g \left(\sigma' \vec{W}_\mu^2 + \xi \sigma' \vec{c}^* \vec{c} + \xi \vec{c}^* \cdot (\vec{\pi} \times \vec{c}) \right) \\
&\quad - \delta V^{\lambda v} \sigma' \left(\sigma'^2 + \vec{\pi}^2 \right) - \frac{1}{4} \delta V^\lambda \left(\sigma'^2 + \vec{\pi}^2 \right)^2 , \\
\mathcal{L}_0 &= \frac{1}{2} \mu^2 v^2 + \frac{1}{4} \lambda v^4 ,
\end{aligned} \tag{20}$$

where $\mathcal{L}_R$ equals the one in eq. (12),

In resummed perturbation theory, the vertices defined by $\mathcal{L}_1$ are treated as counter-terms. The coupled set of gap equations for the poles of the Higgs and vector propagator then reads,

$$\frac{\Pi_T(p^2 = -m^2)}{1 - \frac{\partial}{\partial p^2} \Pi_T(p^2 = -m^2)} = 0 ,$$

$$\begin{aligned} \frac{\Sigma(p^2=-M^2)}{1-\frac{\partial}{\partial p^2}\Sigma(p^2=-M^2)} &= 0\,, \\ \langle\sigma'\rangle &= 0\,, \end{aligned} \tag{21}$$

where Σ is the one-loop Higgs boson self-energy and Π_T is the transverse part of the vacuum polarization tensor. In resummed perturbation theory, one expands eq. (21) to the desired order and solves the set of gap equations for m. In theories with a BRS-symmetry the position of the pole of the propagator and therefore the first two eqs. in (21) are gauge-independent on mass-shell [14]. The self-energy itself is not gauge-invariant on mass-shell except at the one-loop level. Only to one-loop, the denominators in the LHS of eq. (21) can be neglected.

To one loop, the third equation of (21), which determines the vacuum expectation value v of the Higgs field self-consistently, is not gauge parameter independent since v is no physical observable. On the other hand, the masses obtained form the gap equations (21) must be gauge independent. The weak gauge dependence induced by the gauge dependence of v therefore has to be cancelled by higher order contributions.

Details of the one-loop calculation in renormalizable gauges and the solutions of the gap equations in the linear Higgs model in Landau gauge can be found in [8]. The main result is that deeply in the symmetric phase, the value for the gap mass is approximately the same as the one obtained in a non-linear σ model, which requires the evaluation of much less diagrams. The analytical result for the one-loop gap mass in the non-linear σ-model is,

$$m = \frac{1}{16\pi}(\frac{63}{4}\ln 3 - 3)\, g^2 \simeq 0.28\, g^2\,. \tag{22}$$

4.2 Two-loop gap equation in the non-linear σ-model

The two-loop calculation of the gap equation in the non-linear σ-model using the self-energies evaluated in chapter 2 can be found in [9]. It is a crucial test for the consistency of the whole approach since the loop expansion does not correspond to an expansion in a small parameter $\frac{g^2}{m}$. Nevertheless, this does not exclude that the one-loop result provides a reasonable approximation for the true mass gap. This is a question of numerical factors. The two-loop gap equation is quadratic in m, whereas at one loop it is linear. The existence of a positive solution is therefore a non-trivial check for the method. The results for the gap mass are listed in table 1.

The calculation in the non-linear model shows, that the two-loop correction to the one-loop gap mass is only $15-20\%$. The dependence on the gauge parameter ξ and the renormalization scale $\mu_{\overline{MS}}$ is very small numerically. For a more detailed discussion see [9]. The two-loop gap mass is in good agreement with the results form the other one-loop calculations [10,15,16] and in perfect aggrement with the lattice result in [7]. To judge the significance of the result in the non-linear case, it is crucial to perform the whole calculation in the Higgs model, which is super-renormalizable.

$\frac{\mu_{\overline{MS}}}{m}$	0.3	1	3
$\frac{m}{g^2}, \xi = 1, \infty$	0.343	0.335	0.327
$\frac{m}{g^2}, \xi = 2$	0.345	0.336	0.328
$\frac{m}{g^2}, \xi = 10$	0.350	0.342	0.334

Table 1: *Solutions of the two-loop gap equation*

4.3 Two-loop gap equation in the Higgs model

In the two-loop calculation, we will not solve the complete set of three gap equations, but restrict ourselves to the first of eqs. (21), the gap equation for the vector boson mass. For different values of $z = M/m$, we insert the corresponding μ and v from the one-loop solution and then solve the equation for m. We also investigate the dependence of m on varying μ and v around the one-loop value.

The two-loop gap equation for the vector boson is gauge parameter dependent. First, as in the one-loop case, this is caused by a ξ-dependent v. Second, as in the two-loop case in the non-linear σ-model, it is due to the one-loop (resummation) counter-term diagrams. We perform the two-loop calculation in the linear Higgs model in Feynman gauge, in contrast to the one-loop calculation in [8], where Landau gauge, $\xi = 0$, is used. For a suitable comparison of one- and two-loop results, we first solve the one-loop gap equations in Feynman gauge.

Table 2 shows the one-loop solutions in Feynman gauge for μ, v and m for different values of z, with $\frac{\lambda}{g^2} = \frac{1}{8}$. From the treatment in Landau gauge in [8] we see, that $1 \leq z \leq 2$ is a reasonable choice for the symmetric phase. $z \geq 2$ is forbidden, since in this case the Higgs boson can decay into two vector bosons. As a consequence of this, there will be poles in the two-loop result for the self-energy for $M = 2m$. The one-loop gap equation for M is complex for $z > 2$.

It can be seen that there is a constant value for the vector boson mass deeply in the symmetric phase.

We aim at a solution of eq. (21) to two-loop order. Since solving the complete set of three gap equations (21) would be unnecessarily complicated, we will use the following short-cut. We look at first equation in eq. (21), the gap equation for the vector boson, for different values of z, which are typical for the symmetric phase according to the one-loop calculations. For μ^2 and v we will insert the corresponding one-loop results from table 2. As already explained, the gap equation is gauge parameter dependent. We restrict the discussion to Feynman gauge.

In setting up the vector field gap equation we have to insert the third equation of (21).

z	1	1.2	1.4	1.6	1.8
$\frac{\mu^2}{g^4}$	0.111	0.153	0.208	0.277	0.367
$\frac{v}{g}$	0.159	0.162	0.160	0.155	0.148
$\frac{m}{g^2}$	0.226	0.231	0.234	0.236	0.237

Table 2: *Solutions of the one-loop gap equation in Feynman gauge*

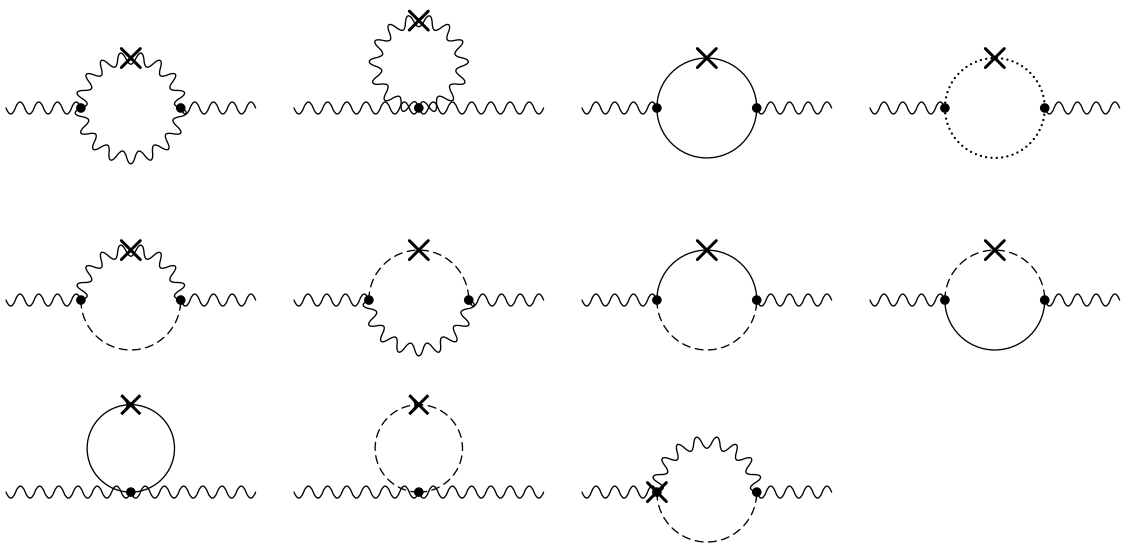


Figure 3: *One-loop CT diagrams contibuting to the gap equation*

This condition reduces the amount of one-loop counter-term and generic two-loop self-energy diagrams which contribute to the first equation of (21). We can leave out all the two-loop diagrams involving tadpoles as well as the one-loop counter-term diagrams which contain the scalar one-point function. The remaining one-loop diagrams with resummation counter-terms contributing to the first equation of (21) are depicted in fig. 3. Their on-shell value is given by

$$\Pi_T^{1-loop-CT}(p^2=-m^2) = \delta m^2 \left[-\frac{1}{64\pi m}\left(50 + 2\frac{M}{m} + \frac{M^2}{m^2}\right) \right.$$
$$\left. + \frac{1}{64\pi(2m+M)}\left(\frac{M^2}{m^2} + 8\frac{m^2}{M^2} - 4\right) + \frac{9}{16\pi m}\ln 3 + \frac{1}{32\pi}\frac{M^2}{m^3}\ln\left(1+2\frac{m}{M}\right)\right]$$
$$+\delta M^2 \left[\frac{1}{64\pi}\left(3\frac{M}{m^2} - \frac{2}{m} - \frac{2}{M}\right) + \frac{1}{64\pi(2m+M)}\left(\frac{M^3}{m^3} - 4\frac{M}{m} + 8\frac{m}{M}\right)\right.$$

z	1	1.2	1.4	1.6	1.8
$\frac{m}{g^2}$ (one-loop)	0.226	0.231	0.234	0.236	0.237
$\frac{m}{g^2}$(two-loop)	0.303	0.307	0.310	0.309	0.299

Table 3: *Solutions of the two-loop gap equation in Feynman gauge*

$$
\begin{aligned}
&+\frac{1}{32\pi m}\left(2-\frac{M^2}{m^2}\right)\ln\left(1+2\frac{m}{M}\right)\Bigg]+\frac{\delta V}{4\pi}\ln\left(1+2\frac{m}{M}\right)\\
&-\left(\mu^2+\lambda v^2\right)\left[\frac{1}{64\pi m}\left(2+2\frac{M}{m}+\frac{M^2}{m^2}\right)+\frac{1}{64\pi(2m+M)}\left(4-\frac{M^2}{m^2}\right)\right.\\
&\left.-\frac{1}{32\pi}\frac{M^2}{m^3}\ln\left(1+2\frac{m}{M}\right)-\frac{1}{16\pi m}\ln 3\right]\,. \qquad (23)
\end{aligned}
$$

With all the quantities evaluated above, we are now in the position to discuss the two-loop gap equation for the vector boson field,

$$
\begin{aligned}
&m^2-\frac{g^2}{4}v^2+\Pi_T^{1-loop}(p^2=-m^2)\\
&+\Pi_T^{2-loop}(p^2=-m^2)+\Pi_T^{1-loop-CT}(p^2=-m^2)\\
&+\frac{\partial}{\partial p^2}\Pi_T^{1-loop}(p^2=-m^2)\left(\Pi_T^{1-loop}(p^2=-m^2)+m^2-\frac{g^2}{4}v^2\right)=0\,. \qquad (24)
\end{aligned}
$$

The gap equation is investigated for different z, with $1 \leq z \leq 2$. We choose $z = 1, 1.2, 1.4, 1.6, 1.8$. As in the non-linear σ-model, we work in the $\overline{MS}$-scheme. It turns out, that the coefficient in front of the $\mu_{\overline{MS}}$-dependent terms is negligibly small. Therefore, we set $\mu_{\overline{MS}} = m$ in what follows.

The solutions of eq. (24) for different values of z are listed in table 3. They are compared with the one-loop result in Feynman gauge.

For the scalar coupling $\frac{\lambda}{g^2}$, we choose $\frac{1}{8}$. For this value, a crossover behaviour was found for the transition between the Higgs and the symmetric phase. As table 3 shows, the two-loop solutions exhibits a similar behaviour as the one-loop gap mass: it is numerically nearly independent of the value of the Higgs mass. Moreover, the two-loop correction is of the same sign and approximately of the same size as the correction in the non-linear σ-model. The $15-20\%$ difference between the numerical value of the one-loop gap mass in the non-linear and linear model in Feynman gauge still remains at two loops.

In solving eq. (24), the one-loop values for μ^2 and v are inserted for each value of z according to table 2. At two loops these values change, if one solves the set of three gap equations exactly.

$z = 1.6, \frac{\mu^2}{g^4} = 0.277, \frac{v}{g} =$	0.1	0.155	2
$\frac{m}{g^2}$	0.303	0.309	0.317

Table 4: *Solutions of the two-loop gap equation for different* $\frac{v}{g}$

$z = 1.6, \frac{v}{g} = 0.155, \frac{\mu^2}{g^4} =$	0.177	0.277	0.377
$\frac{m}{g^2}$	0.304	0.309	0.315

Table 5: *Solutions of the two-loop gap equation for different* $\frac{\mu^2}{g^4}$

To estimate this effect, we vary μ^2 and v around the one-loop solutions of table 2 for a fixed value of z ($z = 1.6$) and show that there is only a small numerical influence on the two-loop gap mass (see tables 4 and 5).

The small numerical difference between the gap mass in the linear and non-linear model as well as the approximate independence of the gap mass of the Higgs mass M shows that the non-linear σ-model describes the infrared limit of the linear Higgs model and of the electroweak Standard Model at finite temperature to a very good approximation.

5 Conclusions

We have investigated two-loop effects on the propagator in the following 3-dimensional theories: a resummed massive Yang-Mills theory, a resummed non-linear σ-model in arbitrary gauge and a resummed $SU(2)$ Higgs model in unitary and Feynman gauge.

The two-loop calculation of the transverse vector self-energy in the non-linear σ-model in unitary gauge shows divergences with high powers of the external momentum. They cannot be removed by a renormalization of the mass or the wave-function. This is because the unitary gauge is a non-renormalizable gauge. In renormalizable gauges, a mass and wave-function renormalization are sufficient to get rid of the infinities. To three-loop order, naive power counting suggests that a similar problem also arises in renormalizable gauges. This is due to the non-renormalizability of the non-linear σ-model. In the linear Higgs model, however, we have seen that at the two-loop level a mass renormalization is sufficient in Feynman gauge, as expected in a super-renormalizable theory in 3 dimensions. In unitary gauge of the linear model, however, the problematic situation remains.

The two-loop self-energies for the Higgs and the vector field have then been applied to set

up the gap equations for the vector boson mass in the symmetric phase of the Higgs model. The two-loop gap equation in a resummed non-linear σ-model was already discussed in our previous paper [9] and shows a real and positive solution for the vector boson mass $m \simeq 0.34\, g^2$. The corresponding calculation in the super-renormalizable Higgs model is crucial to judge the significance of a calculation in the non-renormalizable non-linear sigma model. We have solved the gap equation for the vector boson mass varying the Higgs mass.

In the symmetric phase, the result for the gap mass $m \simeq 0.31\, g^2$ is almost independent of the Higgs mass. This proves that the non-linear σ-model is a very good approximation for infrared phenomena of the linear Higgs model. Moreover, the two-loop correction in the linear model is of similar size as in the non-linear model.

A vector boson mass $\simeq 0.31 g^2$ or $\simeq 0.34 g^2$ is not in contradiction with confinement. It is of the same size as the confinement scale given by the string tension [17]. The connection of such a propagator mass to the heavier glueball masses $\sim \mathcal{O}(1) g^2$ [6] remains to be clarified.

The result of the two-loop calculation in the considered 3-dimensional models suggests that the gap equation approach is a reliable method to calculate the transverse propagator mass of the vector boson in the symmetric phase. The two-loop calculation is a crucial test for the consistency of the whole method. The physical interpretation and the connection to the masses of bound states studied on the lattice requires further investigations [20].

Acknowledgments

I would like to thank W. Buchmüller and O. Philipsen for highly instructive discussions and comments. I am also most grateful to O. Tarasov for an independent calculation of eq. (7) as well as for helpful discussions.

A Basic two-loop integrals

In 3 Euclidean dimensions, the two-loop basic integrals are defined as,

$$
\begin{aligned}
F(m_1,m_2,m_3,m_4,m_5) &= \int\int \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}\frac{1}{(k_1^2+m_1^2)\,(k_2^2+m_2^2)} \\
&\quad \frac{1}{((k_1-q)^2+m_3^2)\,((k_2-q)^2+m_4^2)\,((k_1-k_2)^2+m_5^2)}\,, \\
V(m_1,m_2,m_3,m_4) &= \int\int \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}\frac{1}{(k_2^2+m_1^2)\,((k_1-q)^2+m_2^2)} \\
&\quad \frac{1}{((k_2-q)^2+m_3^2)\,((k_1-k_2)^2+m_4^2)}\,, \\
I_{111}(q^2)(m_1,m_2,m_3) &= \int\int \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}\frac{1}{(k_1^2+m_1^2)\,((k_2-q)^2+m_2^2)}\,, \\
&\quad \frac{1}{((k_1-k_2)^2+m_3^2)}\,, \\
I_{211}(m_1,m_2,m_3) &= -\frac{\partial}{\partial m_1^2} I_{111}(q^2)(m_1,m_2,m_3)\,, \\
I_{121}(m_1,m_2,m_3) &= -\frac{\partial}{\partial m_2^2} I_{111}(q^2)(m_1,m_2,m_3)\,, \\
I_{112}(m_1,m_2,m_3) &= -\frac{\partial}{\partial m_3^2} I_{111}(q^2)(m_1,m_2,m_3)\,, \\
I_{111}(0)(m_1,m_2,m_3) &= \int\int \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}\frac{1}{(k_1^2+m_1^2)\,(k_2^2+m_2^2)\,((k_1-k_2)^2+m_3^2)}\,. \qquad (25)
\end{aligned}
$$

The one-loop integrals A_0 and B_0 are,

$$
\begin{aligned}
A_0(m^2) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^2+m^2} = -\frac{m}{4\pi}, \\
B_0(p^2,m_1^2,m_2^2) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+m_1^2)\,((k+p)^2+m_2^2)} = \frac{1}{4\pi p}\arctan\frac{p}{m_1+m_2}\,. \qquad (26)
\end{aligned}
$$

Apart from the master integral $F(m_1,m_2,m_3,m_4,m_5)$, which has to be evaluated numerically, there exist analytic expressions for the basic integrals in $d=3-2\epsilon$ dimensions [18]. For F a one-dimensional integral remains.

B Two-loop results in the non-linear σ-model

In section 2, the transverse two-loop vector boson self-energy is calculated in $d=3-2\epsilon$ dimensions on as well as off mass-shell in the non-linear σ-model. Here the on-shell result of the reduction is given in arbitrary dimension d. The basic integrals are defined in eq. (25).

$$
\begin{aligned}
&\frac{1}{g^4}\Pi_T^{2-loop}(p^2) = \\
&\frac{3}{16}m^4\frac{176d-245}{d-1}F(m,m,m,m,m) \\
&-\frac{3}{16}m^2\frac{144d^3-712d^2+1241d-760}{(d-1)^2}V(m,m,m,m) \\
&-\frac{1}{48}\frac{10800d^4-70632d^3+165227d^2-166654d+61752}{(d-1)^2(3d-4)}I(p^2=-m^2)(m,m,m) \\
&-\frac{3}{16}\frac{(d-2)(32d^3-312d^2+656d-405)}{(d-1)^2}I(0)(m,m,m) \\
&+\frac{3}{32}\frac{32d^2-148d+155}{(d-1)^2}B(p^2=-m^2,m^2,m^2)B(p^2=-m^2,m^2,m^2) \\
&-\frac{3}{4}\frac{16d^4-188d^3+668d^2-940d+465}{(d-1)^2}B(p^2=-m^2,m^2,m^2)A(m^2) \\
&-\frac{1}{8m^2}\frac{(2d-3)(24d^5-164d^4+452d^3-680d^2+597d-242)}{(d-1)^2(3d-4)}A(m^2)A(m^2)\,, \qquad (27)
\end{aligned}
$$

Switching back to $d = 3 - 2\epsilon$, we write down the result for the off-shell transverse two-loop self-energy of the vector field in Feynman gauge. It is

$$
\begin{aligned}
&\frac{1}{g^4}\Pi_T^{2-loop}(p^2) = \\
&\left(\frac{257}{16}m^4-\frac{351}{32}p^2m^2-\frac{1}{2}p^4\right)F(m,m,m,m,m) \\
&+\left(-\frac{259}{8}m^2-\frac{1265}{64}p^2-\frac{261}{32}\frac{p^4}{m^2}\right)V(m,m,m,m) \\
&+\left(\frac{8163}{20}m^8-\frac{4607}{80}p^2m^6-\frac{12183}{20}p^4m^4-\frac{12243}{80}p^6m^2-\frac{77}{8}p^8\right)\frac{I_{211}(m,m,m)}{m^2p^2(p^2+4m^2)} \\
&+\Big(-\frac{279}{4}m^6+\frac{1409}{48}p^2m^4+\frac{53279}{960}p^4m^2+\frac{3923}{480}p^6 \\
&+\frac{8717}{20}m^6\epsilon-\frac{14647}{60}p^2m^4\epsilon-\frac{225067}{600}p^4m^2\epsilon-\frac{5473}{100}p^6\epsilon\Big)\frac{I_{111}(p^2)(m,m,m)}{m^2p^2(p^2+4m^2)} \\
&+\Big(+\frac{279}{4}m^6-\frac{507}{16}p^2m^4-\frac{3585}{64}p^4m^2-\frac{261}{32}p^6 \\
&-\frac{655}{4}m^6\epsilon-\frac{51}{8}p^2m^4\epsilon+\frac{537}{8}p^4m^2\epsilon+\frac{35}{4}p^6\epsilon\Big)\frac{I_{111}(0)(m,m,m)}{m^2p^2(p^2+4m^2)} \\
&+\text{products of one-loop integrals}\,. \qquad (28)
\end{aligned}
$$

The coefficients in front of the generic basic two-loop integrals coincide in unitary and Feynman gauge on mass-shell, see eq. (7). More detailed results for single two-loop diagrams can be found in [19].

C Two-loop results in the $SU(2)$ Higgs model

The on-shell value of the Higgs boson self-energy and of the transverse vector boson self-energy to two loops is given in unitary and Feynman gauge in $3-2\epsilon$ dimensions, neglecting resummation counter-terms. The coefficients in front of the generic basic two-loop integrals are identical in both gauges. In the unitary gauge result, we therefore write only the part containing products of one-loop integrals. With the following formulae and the one-loop results in [8] and in sect. 3.1, the gauge-invariance of the pole of the Higgs and the vector propagator can easily be proved to two loops. The relevant diagrams are given in figs. 4, 5 and 6.

The two-loop self-energy for the Higgs field in Feynman gauge reads,

$$
\begin{aligned}
\Sigma^{\xi=1}(p^2=-M^2) &= \left(-\frac{189}{4}m^4+\frac{81}{4}M^2m^2-\frac{33}{16}M^4+\frac{3}{8}\frac{M^6}{m^2}\right)F(m,m,m,m,m)\\
&+\left(3m^4-3M^2m^2-\frac{3}{4}M^4+\frac{3}{4}\frac{M^6}{m^2}+\frac{3}{32}\frac{M^8}{m^4}\right)F(m,m,m,m,M)\\
&+\left(9M^2m^2-\frac{27}{4}M^4+\frac{9}{16}\frac{M^8}{m^4}\right)F(m,M,m,M,m)\\
&+\frac{81}{32}\frac{M^8}{m^4}F(M,M,M,M,M)\\
&+\left(\frac{189}{8}m^2+6M^2-\frac{213}{64}\frac{M^4}{m^2}\right)V(m,m,m,m)\\
&+\left(-3m^2+\frac{3}{2}M^2+3\frac{M^4}{m^2}-\frac{3}{2}\frac{M^6}{m^4}+\frac{15}{64}\frac{M^8}{m^6}\right)V(m,m,m,M)\\
&+\left(-\frac{9}{8}M^2+\frac{45}{16}\frac{M^4}{m^2}-\frac{9}{64}\frac{M^6}{m^4}\right)V(M,m,M,m)\\
&-\frac{27}{64}\frac{M^6}{m^4}V(M,M,M,M)\\
&+\left(-\frac{567}{2}m^8+\frac{2097}{4}M^2m^6-\frac{5061}{16}M^4m^4+\frac{651}{8}M^6m^2-\frac{93}{16}M^8\right)\frac{I_{211}(m,m,m)}{M^2m^2(M^2-4m^2)}\\
&+\left(24m^{10}-18M^2m^8-\frac{57}{4}M^6m^4+\frac{81}{8}M^8m^2-\frac{15}{8}M^{10}\right)\frac{I_{211}(M,m,m)}{M^2m^4(M^2-4m^2)}\\
&+\left(-\frac{189}{4}m^6+\frac{459}{8}M^2m^4-\frac{111}{4}M^4m^2+\frac{309}{64}M^6\right.
\end{aligned}
$$

$$+\frac{675}{2}m^6\epsilon-\frac{1449}{4}M^2m^4\epsilon+\frac{2727}{16}M^4m^2\epsilon-\frac{1953}{64}M^6\epsilon\Big)\frac{I_{111}(p^2=-M^2)(m,m,m)}{M^2m^2(M^2-4m^2)}$$

$$+\Big(6m^{10}-\frac{15}{2}M^2m^8+\frac{27}{8}M^4m^6-\frac{27}{8}M^6m^4+\frac{105}{64}M^8m^2$$

$$-\frac{15}{64}M^{10}-42m^{10}\epsilon+\frac{45}{2}M^2m^8\epsilon-\frac{87}{8}M^4m^6\epsilon+\frac{45}{4}M^6m^4\epsilon$$

$$-\frac{171}{32}M^8m^2\epsilon+\frac{51}{64}M^{10}\epsilon\Big)\frac{I_{111}(p^2=-M^2)(M,m,m)}{M^2m^6(M^2-4m^2)}$$

$$+\left(-\frac{21}{64}\frac{M^4}{m^4}+\frac{81}{32}\frac{M^4}{m^4}\epsilon\right)I_{111}(p^2=-M^2)(M,M,M)$$

$$+\Big(\frac{189}{4}m^6-\frac{711}{8}M^2m^4+\frac{261}{8}M^4m^2-\frac{261}{64}M^6$$

$$-\frac{297}{2}m^6\epsilon+\frac{909}{4}M^2m^4\epsilon-\frac{837}{16}M^4m^2\epsilon+\frac{225}{64}M^6\epsilon\Big)\frac{I_{111}(0)(m,m,m)}{M^2m^2(M^2-4m^2)}$$

$$+\Big(-6m^{10}+\frac{27}{2}M^2m^8-\frac{87}{8}M^4m^6+\frac{45}{8}M^6m^4-\frac{117}{64}M^8m^2$$

$$+\frac{15}{64}M^{10}+18m^{10}\epsilon-\frac{63}{2}M^2m^8\epsilon+\frac{111}{8}M^4m^6\epsilon-\frac{9}{8}M^6m^4\epsilon$$

$$-\frac{27}{32}M^8m^2\epsilon+\frac{9}{64}M^{10}\epsilon\Big)\frac{I_{111}(0)(M,m,m)}{M^2m^6(M^2-4m^2)}$$

$$+\left(\frac{9}{64}\frac{M^4}{m^4}-\frac{9}{32}\frac{M^4}{m^4}\epsilon\right)I_{111}(0)(M,M,M)$$

$$+\left(-\frac{3}{2}m^2-\frac{9}{16}M^2-\frac{9}{16}\frac{M^4}{m^2}-\frac{15}{64}\frac{M^6}{m^4}\right)B(p^2=-M^2)(m^2,m^2)B(p^2=-M^2)(m^2,m^2)$$

$$+\left(-\frac{9}{4}M^2+\frac{9}{16}\frac{M^4}{m^2}-\frac{9}{32}\frac{M^6}{m^4}\right)B(p^2=-M^2)(m^2,m^2)B(p^2=-M^2)(M^2,M^2)$$

$$-\frac{27}{64}\frac{M^6}{m^4}B(p^2=-M^2)(M^2,M^2)B(p^2=-M^2)(M^2,M^2)$$

$$+\Big(36m^{10}+60M^2m^8-\frac{99}{2}M^4m^6+\frac{75}{4}M^6m^4$$

$$-\frac{57}{32}M^8m^2-\frac{15}{64}M^{10}\Big)\frac{B(p^2=-M^2)(m^2,m^2)A(m^2)}{M^2m^6(M^2-4m^2)}$$

$$+\Big(-6m^{10}+18M^2m^8-\frac{39}{4}M^4m^6+\frac{45}{8}M^6m^4$$

$$-\frac{63}{32}M^8m^2+\frac{15}{64}M^{10}\Big)\frac{B(p^2=-M^2)(m^2,m^2)A(M^2)}{M^2m^6(M^2-4m^2)}$$

$$+\Big(\frac{27}{4}M^2m^4+\frac{27}{8}M^4m^2-\frac{27}{16}M^6\Big)\frac{B(p^2=-M^2)(M^2,M^2)A(m^2)}{m^4(M^2-4m^2)}$$

$$+\left(27m^8-\frac{39}{4}M^2m^6+\frac{33}{32}M^6m^2-\frac{15}{64}M^8\right)\frac{A(m^2)A(m^2)}{M^2m^6(M^2-4m^2)}$$
$$+\left(\frac{57}{4}m^6-\frac{51}{32}M^4m^2+\frac{15}{64}M^6\right)\frac{A(m^2)A(M^2)}{m^6(M^2-4m^2)}$$
$$-\frac{9}{16}\frac{M^2}{m^4}A(M^2)A(M^2)\,. \tag{29}$$

The vector boson self-energy in Feynman gauge is to two-loop order,

$$\Pi_T^{\xi=1}(p^2=-m^2)=\frac{849}{32}m^4F(m,m,m,m,m)$$
$$+\left(-\frac{63}{8}m^4+\frac{27}{8}M^2m^2-\frac{11}{32}M^4+\frac{1}{16}\frac{M^6}{m^2}\right)F(m,m,m,m,M)$$
$$+\left(-\frac{63}{2}m^4+\frac{27}{2}M^2m^2-\frac{11}{8}M^4+\frac{1}{4}\frac{M^6}{m^2}\right)F(M,m,m,m,m)$$
$$+\left(m^4-M^2m^2-\frac{1}{4}M^4+\frac{1}{4}\frac{M^6}{m^2}+\frac{1}{32}\frac{M^8}{m^4}\right)F(M,m,m,M,m)$$
$$+\left(\frac{3}{2}M^2m^2-\frac{9}{8}M^4+\frac{3}{32}\frac{M^8}{m^4}\right)F(m,m,M,M,M)$$
$$+\left(-\frac{2115}{64}m^2+M^2-\frac{1}{4}\frac{M^4}{m^2}\right)V(m,m,m,m)$$
$$+\left(\frac{63}{8}m^2+2M^2-\frac{71}{64}\frac{M^4}{m^2}\right)V(m,M,m,m)$$
$$+\left(\frac{3}{2}\frac{m^4}{M^2}+\frac{1}{2}m^2-\frac{9}{8}M^2-\frac{3}{16}\frac{M^4}{m^2}-\frac{17}{128}\frac{M^6}{m^4}\right)V(m,m,M,m)$$
$$+\left(\frac{63}{16}m^2+M^2-\frac{71}{128}\frac{M^4}{m^2}\right)V(M,m,m,m)$$
$$+\left(-\frac{1}{2}m^2+\frac{1}{4}M^2+\frac{1}{2}\frac{M^4}{m^2}-\frac{1}{4}\frac{M^6}{m^4}+\frac{5}{128}\frac{M^8}{m^6}\right)V(M,M,m,m)$$
$$+\left(\frac{3}{8}M^2+\frac{3}{4}\frac{M^4}{m^2}-\frac{15}{64}\frac{M^6}{m^4}\right)V(m,M,M,M)$$
$$+\left(\frac{189}{4}m^8-\frac{699}{8}M^2m^6+\frac{1687}{32}M^4m^4-\frac{217}{16}M^6m^2+\frac{31}{32}M^8\right)\frac{I_{211}(m,m,M)}{m^4(M^2-4m^2)}$$
$$+\left(-4m^{10}+\frac{9}{2}M^4m^6+\frac{1}{2}M^6m^4-\frac{21}{16}M^8m^2+\frac{5}{16}M^{10}\right)\frac{I_{211}(M,m,M)}{m^6(M^2-4m^2)}$$
$$+\left(\frac{3}{2}m^8+\frac{1021}{16}M^2m^6-\frac{997}{64}M^4m^4-\frac{1}{2}M^6m^2+\frac{13}{128}M^8\right.$$

$$-\frac{21}{2}m^8\epsilon-\frac{1881}{4}M^2m^6\epsilon+\frac{1851}{16}M^4m^4\epsilon+\frac{5}{8}M^6m^2\epsilon-\frac{5}{128}M^8\epsilon\Big)\frac{I_{111}(p^2=-m^2)(m,m,m)}{M^2m^4(M^2-4m^2)}$$

$$+\Big(-15m^6+\frac{799}{64}M^2m^4-\frac{559}{128}M^4m^2+\frac{31}{64}M^6$$

$$+\frac{1617}{16}m^6\epsilon-\frac{2251}{32}M^2m^4\epsilon+\frac{737}{32}M^4m^2\epsilon-\frac{41}{16}M^6\epsilon\Big)\frac{I_{111}(p^2=-m^2)(M,m,m)}{m^4(M^2-4m^2)}$$

$$+\Big(-\frac{1}{2}m^8+\frac{9}{8}M^2m^6-\frac{13}{16}M^4m^4+\frac{1}{16}M^6m^2+\frac{5}{128}M^8$$

$$-2m^8\epsilon-\frac{23}{4}M^2m^6\epsilon+\frac{9}{8}M^4m^4\epsilon+\frac{1}{64}M^6m^2\epsilon-\frac{3}{32}M^8\epsilon\Big)\frac{I_{111}(p^2=-m^2)(M,M,m)}{m^6(M^2-4m^2)}$$

$$+\Big(-\frac{63}{2}m^6-\frac{951}{16}M^2m^4+\frac{483}{32}M^4m^2+\frac{87}{128}M^6$$

$$99m^6\epsilon+\frac{1647}{16}M^2m^4\epsilon-\frac{1101}{32}M^4m^2\epsilon+\frac{3}{32}M^6\epsilon\Big)\frac{I_{111}(0)(m,m,m)}{M^2m^2(M^2-4m^2)}$$

$$+\Big(\frac{9}{2}m^6+\frac{17}{2}M^2m^4-9M^4m^2+\frac{7}{2}M^6-\frac{41}{128}\frac{M^8}{m^2}-\frac{5}{128}\frac{M^{10}}{m^4}$$

$$-\frac{27}{2}m^6\epsilon-\frac{29}{2}M^2m^4\epsilon+\frac{49}{4}M^4m^2\epsilon-\frac{21}{4}M^6\epsilon+\frac{149}{128}\frac{M^8}{m^2}\epsilon-\frac{1}{16}\frac{M^{10}}{m^4}\epsilon\Big)\frac{I_{111}(0)(M,m,m)}{M^2m^2(M^2-4m^2)}$$

$$+\Big(\frac{9}{8}M^2m^4+\frac{3}{4}M^4m^2-\frac{21}{64}M^6-\frac{3}{4}M^2m^4\epsilon+\frac{3}{16}M^4m^2\epsilon+\frac{9}{64}M^6\epsilon\Big)\frac{I_{111}(0)(M,M,M)}{m^4(M^2-4m^2)}$$

$$+\Big(\frac{45}{16}m^2+\frac{1}{4}M^2-\frac{1}{16}\frac{M^4}{m^2}\Big)B(p^2=-m^2)(m^2,m^2)B(p^2=-m^2)(m^2,m^2)$$

$$+\Big(\frac{9}{2}m^2-\frac{5}{4}M^2+\frac{1}{2}\frac{M^4}{m^2}\Big)B(p^2=-m^2)(m^2,m^2)B(p^2=-m^2)(M^2,m^2)$$

$$+\Big(-\frac{1}{2}m^2+\frac{5}{8}M^2-\frac{1}{8}\frac{M^4}{m^2}\Big)B(p^2=-m^2)(M^2,m^2)B(p^2=-m^2)(M^2,m^2)$$

$$+\Big(-54m^6-\frac{963}{8}M^2m^4+\frac{63}{2}M^4m^2+\frac{63}{64}M^6\Big)\frac{B(p^2=-m^2)(m^2,m^2)A(m^2)}{M^2m^2(M^2-4m^2)}$$

$$+\Big(\frac{63}{4}m^6-\frac{207}{8}M^2m^4+\frac{135}{16}M^4m^2-\frac{63}{64}M^6\Big)\frac{B(p^2=-m^2)(m^2,m^2)A(M^2)}{M^2m^2(M^2-4m^2)}$$

$$+\Big(9m^{10}+\frac{9}{2}M^2m^8-\frac{35}{4}M^4m^6+\frac{9}{2}M^6m^4$$

$$-\frac{35}{64}M^8m^2-\frac{5}{128}M^{10}\Big)\frac{B(p^2=-m^2)(M^2,m^2)A(m^2)}{M^2m^6(M^2-4m^2)}$$

$$+\Big(-m^{10}+M^2m^8-\frac{9}{8}M^4m^6+\frac{7}{8}M^6m^4$$

$$
-\frac{5}{16}M^8m^2+\frac{5}{128}M^{10}\Big)\,\frac{B(p^2=-m^2)(M^2,m^2)A(M^2)}{M^2m^6(M^2-4m^2)}
$$

$$
+\Big(18m^{10}-39M^2m^8+\frac{51}{4}M^4m^6+\frac{11}{32}M^6m^4
$$

$$
-\frac{11}{32}M^8m^2+\frac{5}{128}M^{10}\Big)\,\frac{A(m^2)A(M^2)}{M^4m^6(M^2-4m^2)}
$$

$$
+\Big(3m^{10}+M^2m^8-\frac{19}{8}M^4m^6-\frac{17}{16}M^6m^4
$$

$$
+\frac{29}{64}M^8m^2-\frac{5}{128}M^{10}\Big)\,\frac{A(m^2)A(M^2)}{M^4m^6(M^2-4m^2)}
$$

$$
+\Big(\frac{1}{2}m^6+\frac{1}{8}M^2m^4+\frac{11}{32}M^4m^2-\frac{7}{64}M^6\Big)\,\frac{A(M^2)A(M^2)}{M^2m^4(M^2-4m^2)}\,. \tag{30}
$$

In unitary gauge, the products of one-loop integrals in the two-loop self-energy of the Higgs field read,

$$
\Sigma^{\xi=\infty}(p^2=-M^2)=\ldots
$$

$$
+\left(-\frac{3}{2}m^2+\frac{27}{16}M^2-\frac{27}{16}\frac{M^4}{m^2}+\frac{3}{64}\frac{M^6}{m^4}\right)B(p^2=-M^2)(m^2,m^2)B(p^2=-M^2)(m^2,m^2)
$$

$$
+\left(-\frac{9}{4}M^2+\frac{9}{16}\frac{M^4}{m^2}+\frac{9}{16}\frac{M^6}{m^4}\right)B(p^2=-M^2)(m^2,m^2)B(p^2=-M^2)(M^2,M^2)
$$

$$
-\frac{27}{64}\frac{M^6}{m^4}B(p^2=-M^2)(M^2,M^2)B(p^2=-M^2)(M^2,M^2)
$$

$$
+\Big(36m^{10}+51M^2m^8-\frac{207}{4}M^4m^6+\frac{33}{2}M^6m^4
$$

$$
-\frac{15}{16}M^8m^2-\frac{15}{64}M^{10}\Big)\,\frac{B(p^2=-M^2)(m^2,m^2)A(m^2)}{M^2m^6(M^2-4m^2)}
$$

$$
+\Big(-6m^{10}+18M^2m^8-\frac{39}{4}M^4m^6+\frac{27}{8}M^6m^4
$$

$$
-\frac{45}{32}M^8m^2+\frac{15}{64}M^{10}\Big)\,\frac{B(p^2=-M^2)(m^2,m^2)A(M^2)}{M^2m^6(M^2-4m^2)}
$$

$$
+\Big(\frac{27}{4}M^2m^4-\frac{27}{32}M^6\Big)\,\frac{B(p^2=-M^2)(M^2,M^2)A(m^2)}{m^4(M^2-4m^2)}
$$

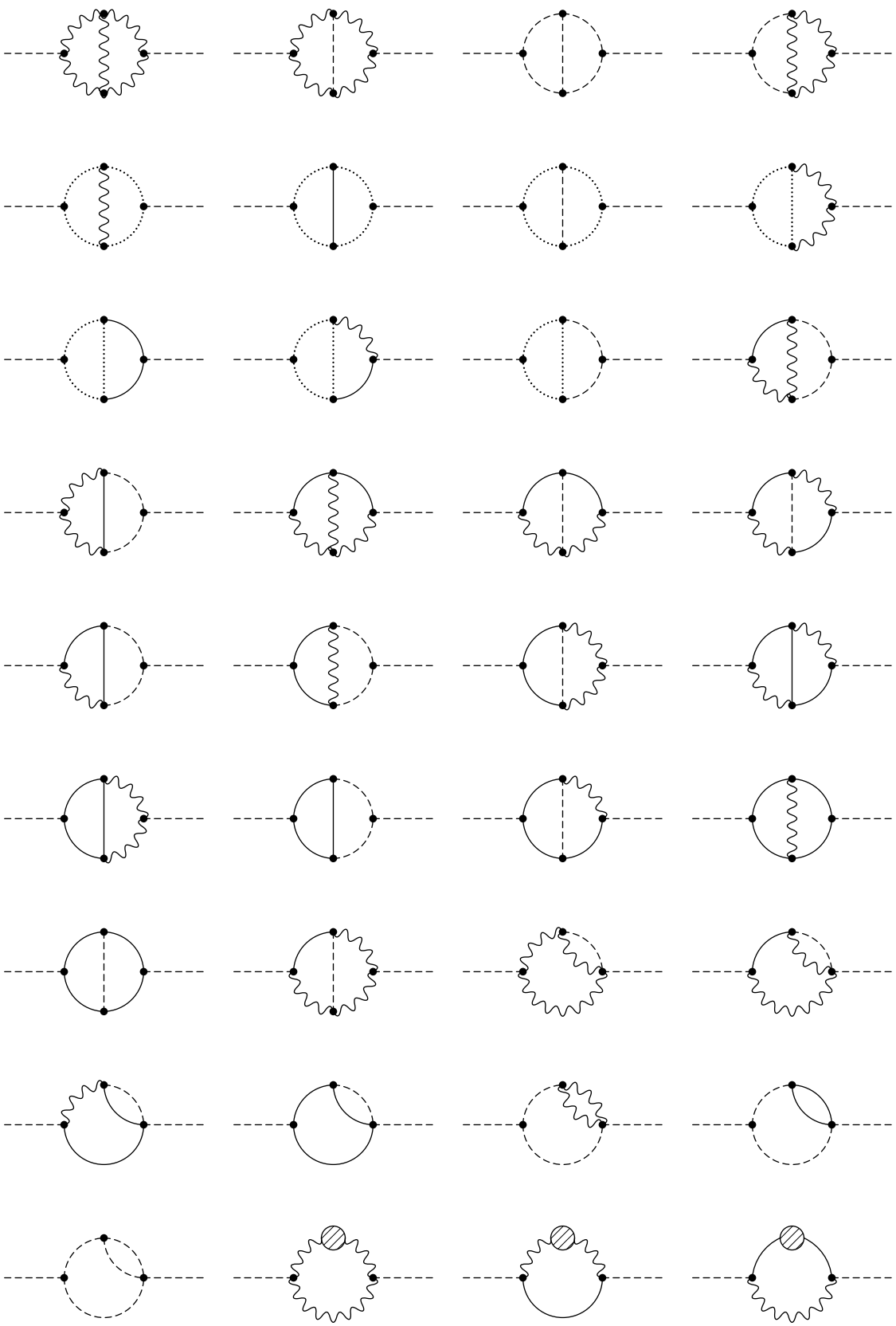
$$
+\Big(27m^8-\frac{75}{4}M^2m^6+\frac{51}{32}M^6m^2-\frac{15}{64}M^8\Big)\,\frac{A(m^2)A(M^2)}{M^2m^6(M^2-4m^2)}
$$

$$
+\Big(\frac{57}{4}m^6-\frac{9}{4}M^2m^4-\frac{33}{32}M^4m^2+\frac{15}{64}M^6\Big)\,\frac{A(m^2)A(M^2)}{m^6(M^2-4m^2)}
$$

$$-\frac{9}{16}\frac{M^2}{m^4}A(M^2)A(M^2)\,, \tag{31}$$

and for the vector field,

$$\begin{aligned}
&\Pi_T^{\xi=\infty}(p^2=-m^2) = \ \ldots \\
&+\left(-\frac{99}{32}m^2+\frac{1}{4}M^2-\frac{1}{16}\frac{M^4}{m^2}\right)B(p^2=-m^2)(m^2,m^2)B(p^2=-m^2)(m^2,m^2) \\
&+\left(\frac{21}{4}m^2-\frac{13}{8}M^2+\frac{19}{32}\frac{M^4}{m^2}\right)B(p^2=-m^2)(m^2,m^2)B(p^2=-m^2)(M^2,m^2) \\
&+\left(-\frac{1}{2}m^2+\frac{5}{8}M^2-\frac{1}{8}\frac{M^4}{m^2}\right)B(p^2=-m^2)(M^2,m^2)B(p^2=-m^2)(M^2,m^2) \\
&+\left(-63m^6-\frac{555}{8}M^2m^4+\frac{315}{16}M^4m^2+\frac{57}{64}M^6\right)\frac{B(p^2=-m^2)(m^2,m^2)A(m^2)}{M^2m^2(M^2-4m^2)} \\
&+\left(\frac{63}{4}m^6-\frac{219}{8}M^2m^4+\frac{135}{16}M^4m^2-\frac{57}{64}M^6\right)\frac{B(p^2=-m^2)(m^2,m^2)A(m^2)}{M^2m^2(M^2-4m^2)} \\
&+\left(9m^{10}-\frac{3}{2}M^2m^8-\frac{17}{4}M^4m^6+3M^6m^4\right. \\
&\left.-\frac{23}{64}M^8m^2-\frac{5}{128}M^{10}\right)\frac{B(p^2=-m^2)(M^2,m^2)A(m^2)}{M^2m^6(M^2-4m^2)} \\
&+\left(-m^{10}+M^2m^8-\frac{9}{8}M^4m^6+\frac{7}{8}M^6m^4\right. \\
&\left.-\frac{5}{16}M^8m^2+\frac{5}{128}M^{10}\right)\frac{B(p^2=-m^2)(M^2,m^2)A(M^2)}{M^2m^6(M^2-4m^2)} \\
&+\left(18m^{10}-57M^2m^8+\frac{81}{4}M^4m^6+\frac{11}{32}M^6m^4\right. \\
&\left.-\frac{17}{32}M^8m^2+\frac{5}{128}M^{10}\right)\frac{A(m^2)A(m^2)}{M^4m^6(M^2-4m^2)} \\
&+\left(3m^{10}+M^2m^8-\frac{43}{8}M^4m^6-\frac{17}{16}M^6m^4\right. \\
&\left.+\frac{41}{64}M^8m^2-\frac{5}{128}M^{10}\right)\frac{A(m^2)A(M^2)}{M^4m^6(M^2-4m^2)} \\
&+\left(\frac{1}{2}m^6+\frac{1}{8}M^2m^4+\frac{11}{32}M^4m^2-\frac{7}{64}M^6\right)\frac{A(M^2)A(M^2)}{M^2m^4(M^2-4m^2)}\,.
\end{aligned} \tag{32}$$



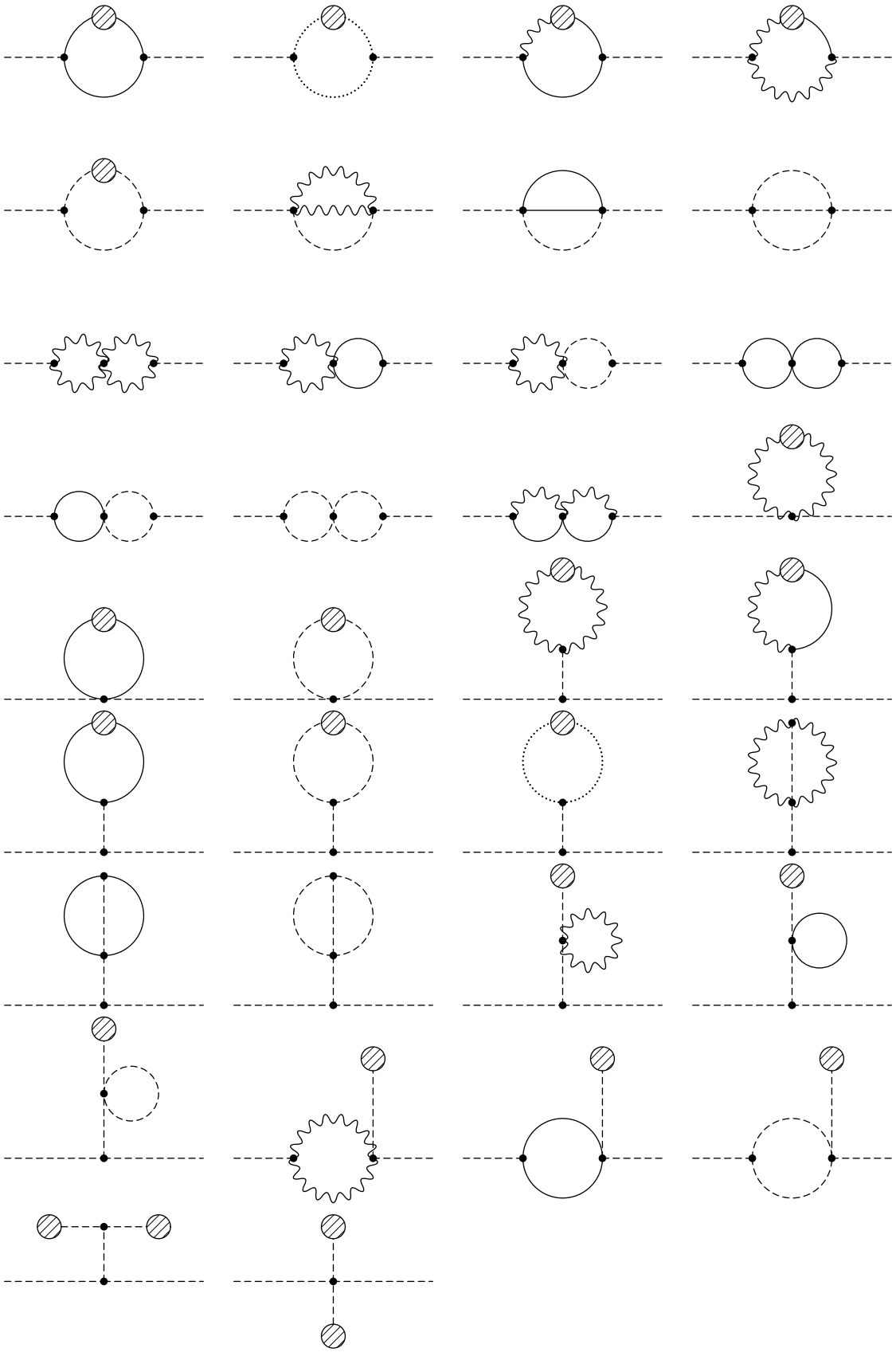
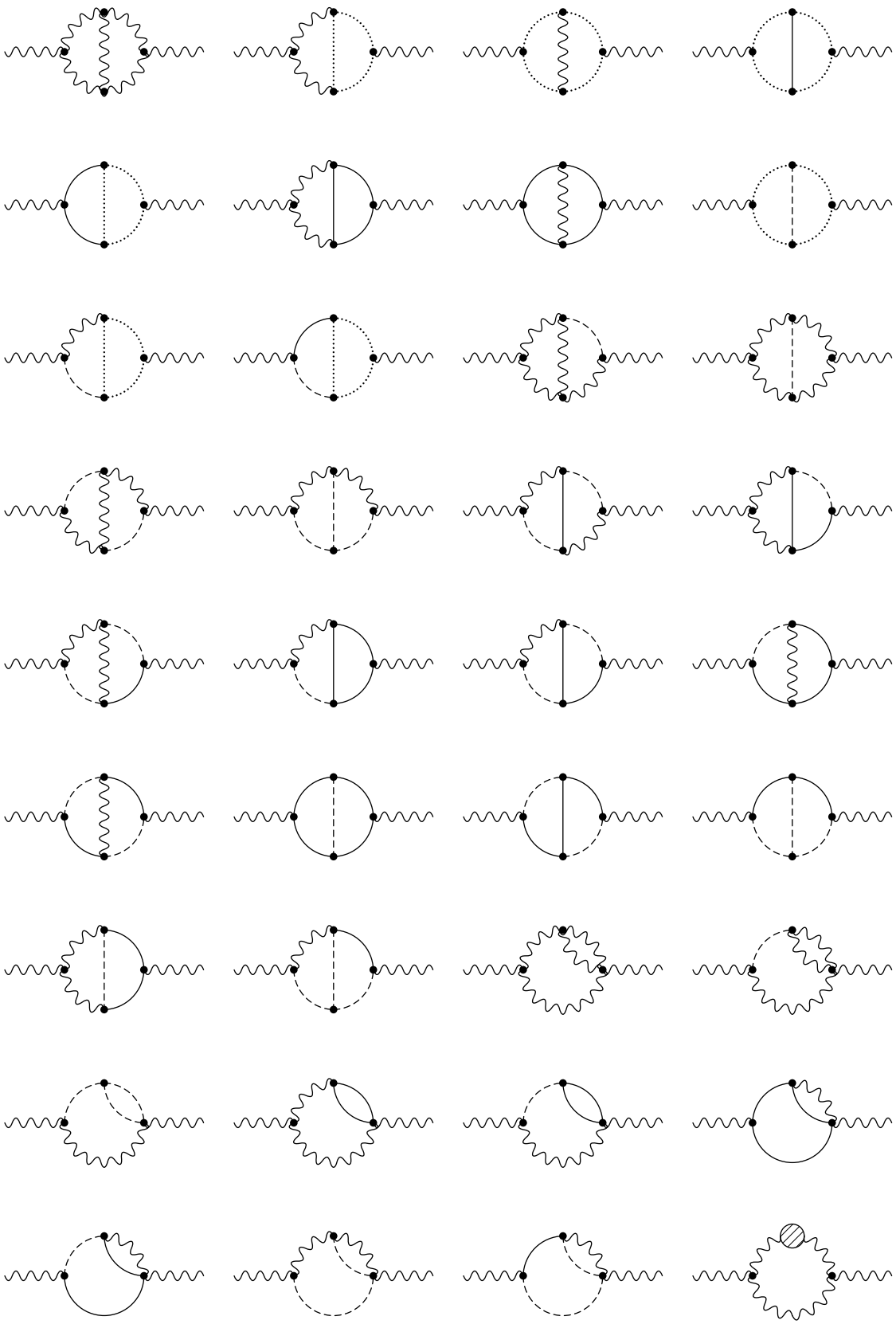


Figure 4: *Two-loop diagrams for the Higgs self-energy*



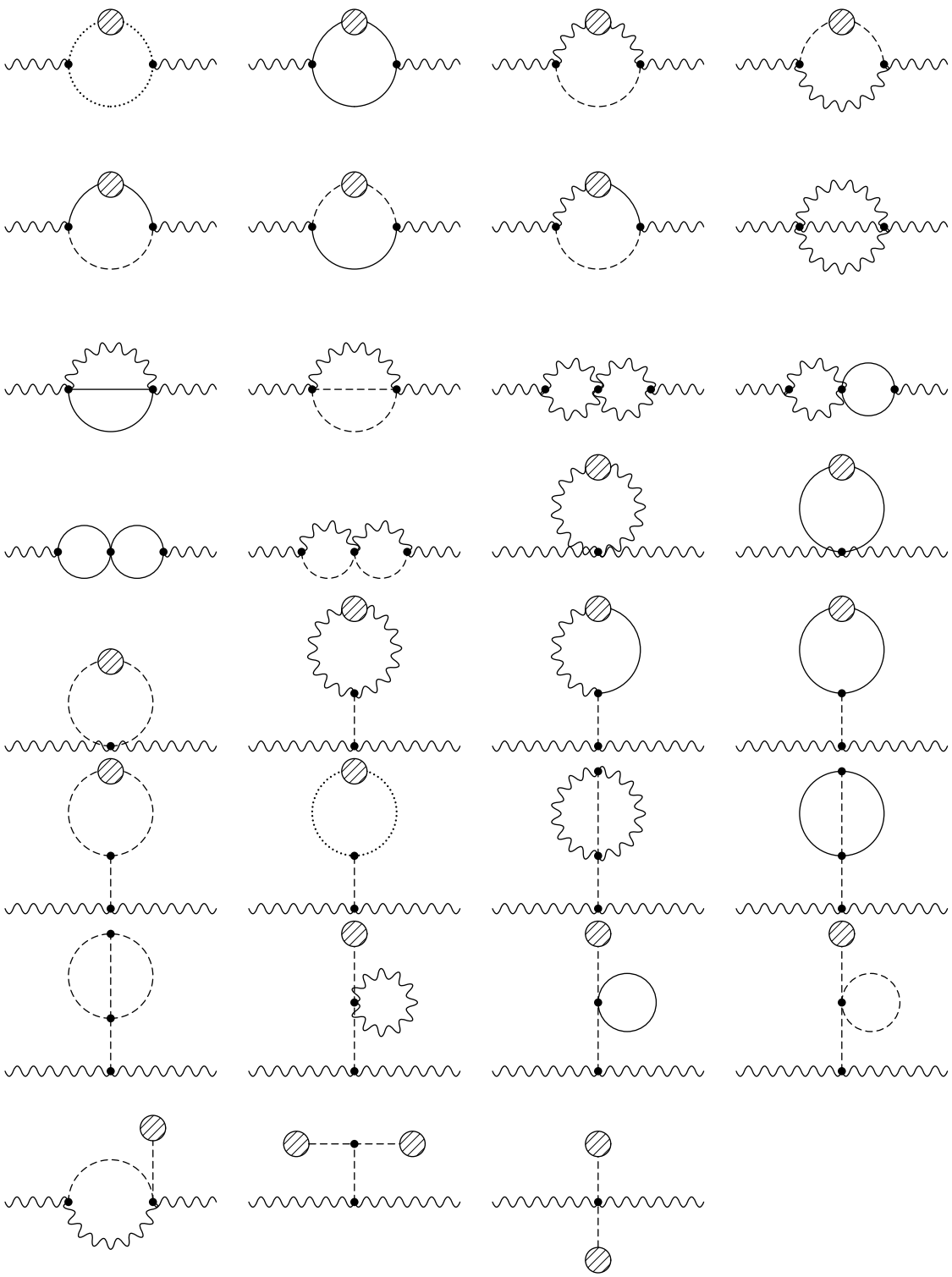


Figure 5: *Two-loop diagrams for the vector boson self-energy*

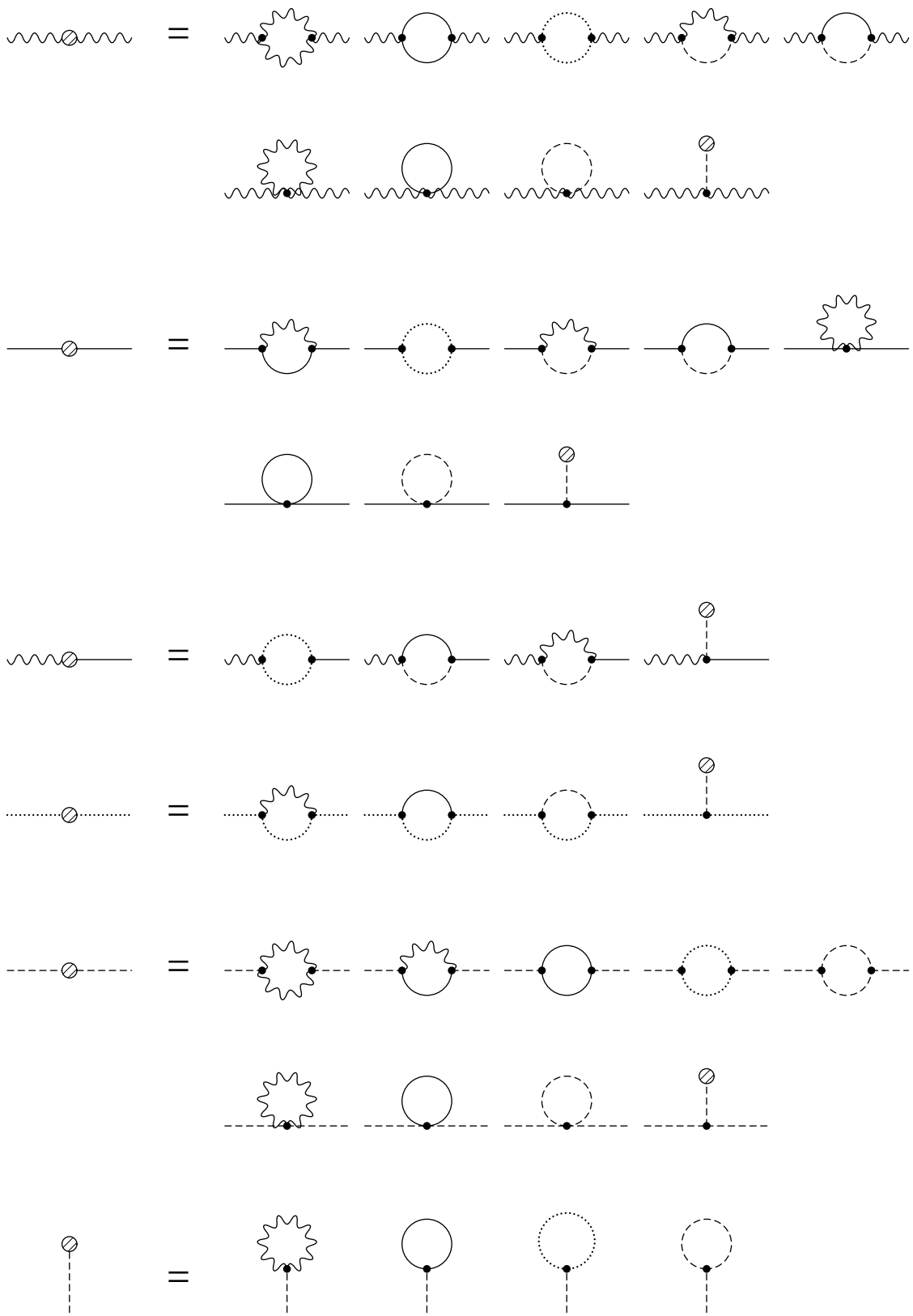


Figure 6: *One-loop self-energy insertions into two-loop diagrams*

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