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W Boson Production at NLO*

Michael Spira†

II. Institut für Theoretische Physik‡, Universität Hamburg, D-22761 Hamburg, Germany

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Abstract

We discuss W boson production at HERA including NLO QCD corrections. A detailed comparison with previous work is presented.

1 Introduction

The c.m. energy $\sqrt{s} \approx 300$ GeV of the ep collider HERA [1] is sufficiently large to produce on-shell W bosons. Since the production cross sections for the processes $e^{\pm}p \to e^{\pm}W + X$ reach values of about 1 pb at HERA, the number of W events allows to study the production mechanisms of W bosons in some detail and to probe the existence of anomalous $WW\gamma$ trilinear couplings [2–4]. Moreover, W boson production represents an important SM background to several new physics searches such as the measurement of isolated high energy muons [5]. In order to observe possible discrepancies between the observations and Standard Model (SM) predictions, the latter have to be sufficiently accurate and reliable. This is not guaranteed for the available leading order calculations [3, 4, 6] of W boson production. Clearly, for an unambiguous test of anomalous contributions, it is necessary to extend the previous analyses to NLO accuracy. A first step in this direction will be presented in this contribution.

2 QCD Corrections

2.1 Leading Order

The production of W bosons at ep colliders is mediated by photon, Z and W exchange between the electron/positron and the hadronic part of the process. It is useful to distinguish two regions, the DIS regime at large Q^2 and the photoproduction regime at small Q^2, Q^2 being the square of the transferred momentum. The photoproduction cross section

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†Heisenberg-Fellow
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can be calculated by convoluting the Weizsäcker-Williams photon spectrum,

$$f_{\gamma/e}(x) = \frac{\alpha}{2\pi}\left\{\frac{1+(1-x)^2}{x}\log\frac{Q^2_{max}}{Q^2_{min}} - 2m_e^2 x\left(\frac{1}{Q^2_{min}} - \frac{1}{Q^2_{max}}\right)\right\}, \tag{1}$$

with the cross section for $\gamma q \to q'W$:

$$\sigma(ep \to W + X) = \int_{M_W^2/s}^1 d\tau \sum_q \frac{d\mathcal{L}^{\gamma q}}{d\tau}\hat{\sigma}(\gamma q \to q'W; \hat{s} = \tau s) \tag{2}$$

where

$$\frac{d\mathcal{L}^{\gamma q}}{d\tau} = \int_\tau^1 \frac{dx}{x} f_{\gamma/e}(x)\; q_p\left(\frac{\tau}{x}, \mu_F^2\right) \tag{3}$$

is the photon-quark luminosity, α denotes the QED coupling, m_e the electron mass, and Q^2_{min}, Q^2_{max} the minimal and maximal values of the photon virtuality Q^2. The function $q_p(x, \mu_F^2)$ is the quark density of the proton at the momentum fraction x and the factorization scale μ_F. In order to separate photoproduction from the DIS region we impose an angular cut of $\theta_{cut} = 5^o$ on the outgoing lepton, which corresponds to an energy-dependent cut

$$Q^2_{max} = \frac{E_e^2(1-x)^2\theta^2_{cut} + m_e^2 x^2}{1-x}, \tag{4}$$

E_e being the initial lepton energy [7]. The minimal value of Q^2 is fixed by kinematics,

$$Q^2_{min} = m_e^2\frac{x^2}{1-x}, \tag{5}$$

where negligible higher order terms in the electron mass m_e have been omitted.

While the treatment of the DIS region is straightforward [a typical contribution is shown in the third diagram of Fig. 1], the small Q^2 region requires to include the contribution of the hadronic component of the photon giving rise to W production via the standard Drell-Yan mechanism [first diagram of Fig. 1]. In fact, this is the dominant production mechanism.

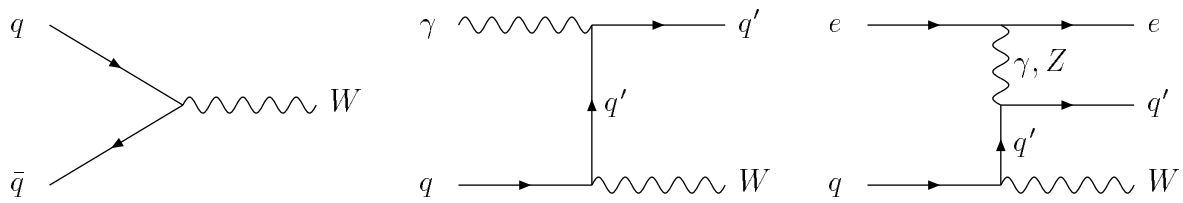

Figure 1: *Typical diagrams of W boson production at HERA: resolved, direct and DIS mechanism.*

The leading direct photon process $\gamma q \to q'W$ [a typical contribution is depicted by the second diagram of Fig. 1] develops a singularity when the final state quark q' becomes

collinear with the initial state photon. This singularity has to be subtracted and absorbed in the corresponding quark density of the photon. We have worked in the $\overline{\rm MS}$ scheme using dimensional regularization. The subtraction of the collinear pole introduces the factorization scale μ_F in the photonic quark density. The renormalized result for the direct contribution can be cast into the form [8]

$$\begin{aligned}\hat{\sigma}_{LO}^{dir} &= \frac{G_F M_W^2 \alpha}{2\sqrt{2}\hat{s}} \left\{ e_{q'}^2 \left[-2[z^2+(1-z)^2] \log\left(\frac{\mu_F^2 z}{M_W^2 (1-z)^2} \right) + 1 + 6z - 7z^2 \right] \right. \\ &\quad + 2 e_{q'} e_W \left[3(1-z^2) + 4(1+z^2)\log z \right] \\ &\quad \left. + e_W^2 \left[\frac{1-z}{z}(4+5z+7z^2) + (8+4z+4z^2)\log z \right] \right\} \end{aligned} \tag{6}$$

where G_F denotes the Fermi constant, M_W the W mass and $e_{q'}, e_W$ the electric charges of the scattered quark q' and W boson, i.e. $e_W = e_q - e_{q'} = \pm 1$. The variable z is defined to be $z = M_W^2/\hat{s}$. The subtracted direct component is accounted for by the resolved process $q'\bar{q} \to W$, where one initial quark comes from the proton and the other from the photon in the collinear regime. The corresponding production cross section is given by

$$\sigma^{res}(ep \to W + X) = \int_{M_W^2/s}^1 d\tau \sum_{q,q'} \frac{d\mathcal{L}^{q'\bar{q}}}{d\tau} \hat{\sigma}^{res}(q'\bar{q} \to W; \hat{s} = \tau s) \tag{7}$$

with the quark-antiquark luminosity

$$\frac{d\mathcal{L}^{q'\bar{q}}}{d\tau} = \int_\tau^1 \frac{dx}{x} \int_x^1 \frac{dy}{y} f_{\gamma/e}(y) \left[q'_\gamma\left(\frac{x}{y}, \mu_F^2\right) \bar{q}_p\left(\frac{\tau}{x}, \mu_F^2\right) + \bar{q}_\gamma\left(\frac{x}{y}, \mu_F^2\right) q'_p\left(\frac{\tau}{x}, \mu_F^2\right) \right] \tag{8}$$

and the partonic cross section at leading order [$z = M_W^2/\hat{s}$]

$$\hat{\sigma}_{LO}^{res}(q'\bar{q} \to W) = \frac{\sqrt{2} G_F \pi}{3} \delta(1-z). \tag{9}$$

The DIS, direct and resolved contributions add up to the total W production cross section. The consistency of the calculation requires that the dependence on the specific value of the cut Q^2_{max}, which separates the DIS and photoproduction regimes, should be small. This dependence is presented in Fig. 2 for the LO W^+ and W^- cross sections. It can be seen that the residual dependence is less than about 3% and thus indeed sufficiently small.

2.2 Next-to-leading Order

For the dominant resolved part we have evaluated the QCD corrections in the $\overline{\rm MS}$ scheme [8]. Since the resolved process coincides with the Drell-Yan production of W bosons, the

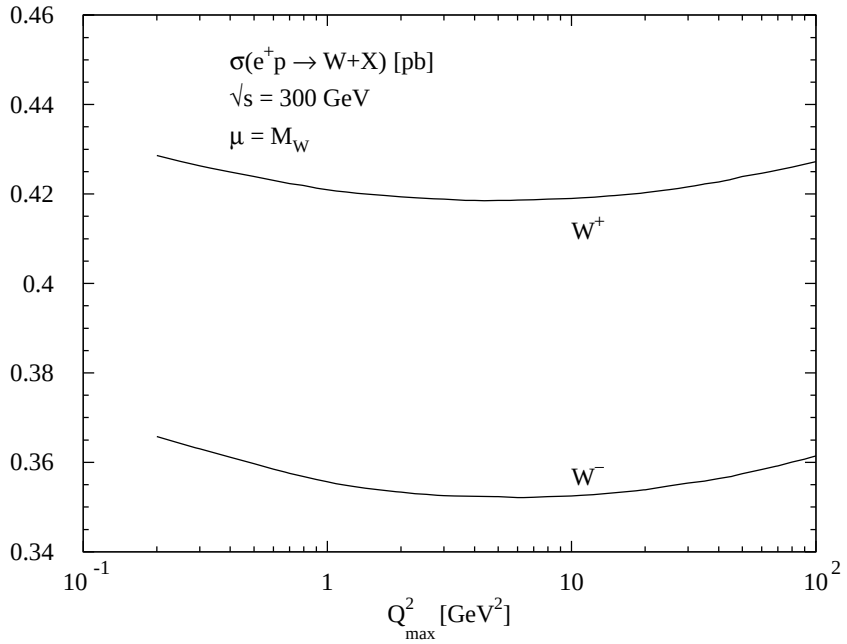

Figure 2: *Dependence of the total leading order $W^{\pm}$ production cross sections on the cut Q^2_{max} which separates the DIS and photoproduction regimes, after adding the DIS, direct and resolved contributions.*

final renormalized result for the total resolved cross section is given by [9]

$$\begin{aligned}
\sigma^{res}(ep \to W + X) &= \sigma^{res}_{LO} + \Delta\sigma^{res}_{q\bar{q}} + \Delta\sigma^{res}_{qg} \\
\Delta\sigma^{res}_{q\bar{q}} &= \frac{\sqrt{2}G_F\pi}{3}\,\frac{\alpha_s(\mu_R^2)}{\pi}\int_{M_W^2/s}^1 d\tau \sum_{q,q'}\frac{d\mathcal{L}^{q'\bar{q}}}{d\tau}\; z\;\omega_{q\bar{q}}(z) \\
\Delta\sigma^{res}_{qg} &= \frac{\sqrt{2}G_F\pi}{3}\,\frac{\alpha_s(\mu_R^2)}{\pi}\int_{M_W^2/s}^1 d\tau \sum_{q,\bar{q}}\frac{d\mathcal{L}^{qg}}{d\tau}\; z\;\omega_{qg}(z) \qquad (10)
\end{aligned}$$

with the coefficient functions $[z = M_W^2/(\tau s)]$

$$\begin{aligned}
\omega_{q\bar{q}}(z) &= -P_{qq}(z)\log\frac{\mu_F^2 z}{M_W^2} \\
&\quad +\frac{4}{3}\left\{2[\zeta_2 - 2]\delta(1-z) + 4\left(\frac{\log(1-z)}{1-z}\right)_+ - 2(1+z)\log(1-z)\right\} \\
\omega_{qg}(z) &= -\frac{1}{2}P_{qg}(z)\log\left(\frac{\mu_F^2 z}{(1-z)^2 M_W^2}\right) + \frac{1}{8}\left\{1 + 6z - 7z^2\right\}. \qquad (11)
\end{aligned}$$

Here, $P_{ij}(z)$ denote the usual Altarelli-Parisi splitting functions [10], and $\alpha_s(\mu_R^2)$ is the strong coupling at the renormalization scale μ_R. The quark-gluon luminosity is given by

$$\frac{d\mathcal{L}^{qg}}{d\tau} = \int_\tau^1 \frac{dx}{x}\int_x^1 \frac{dy}{y} f_{\gamma/e}(y)\;\left[q_\gamma\left(\frac{x}{y},\mu_F^2\right)\; g_p\left(\frac{\tau}{x},\mu_F^2\right)\; + g_\gamma\left(\frac{x}{y},\mu_F^2\right)\; q_p\left(\frac{\tau}{x},\mu_F^2\right)\right] \qquad (12)$$

with $g_{\gamma,p}(x,\mu_F^2)$ denoting the gluon densities of the photon and proton, respectively. Taking the cross section at the values $\mu_R = \mu_F = M_W$ for the renormalization and factorization scales, the QCD corrections enhance the resolved contribution by about 40% for W^+ and W^- production. In order to demonstrate the theoretical uncertainties, the renormalization/factorization scale dependence of the individual contributions to the processes $e^+p \to W^\pm + X \to \mu^\pm \overset{(-)}{\nu}_\mu + X$ are presented in Fig. 3 for HERA conditions [including the branching ratio $BR(W^\pm \to \mu^\pm \overset{(-)}{\nu}_\mu) = 10.84\%$]. One can clearly see that the scale dependence in the sum of direct and resolved contributions is significantly reduced, once the NLO corrections to the resolved part are included. The full curves show the total sum of NLO resolved, LO direct and LO DIS contribution, that is our prediction of the total $W^\pm$ production cross sections. The residual scale dependence is about 5%. Since the remaining dependence on Q^2_{max} is of similar size, the total theoretical uncertainty is estimated to be less than about 10%.

The radiation of an additional gluon also generates a finite transverse momentum p_T of the W bosons produced via the resolved Drell-Yan process. At sufficiently low p_T this may be expected to modify the total p_T distribution. As can be inferred from Fig 4, at p_T values below 20 GeV the resolved contribution amounts to about 5% and more of the total p_T distribution of the W bosons, while at larger values of p_T it falls off steeply[1]. There, the p_T spectrum is dominated by the direct photon mechanism and DIS which becomes more and more important as p_T increases. Moreover, for p_T values below about 15–20 GeV multi soft gluon radiation should become important. This would require resummation in order to obtain a finite result. The description of W production in this small p_T regime is beyond the scope of the present analysis.

3 Comparison with Earlier Results

Our approach differs from the analysis of Ref. [3] in several aspects:

(i) Whereas in Ref. [3] the DIS and photoproduction regimes are separated by a cut on the u-channel momentum transfer in the $\gamma^* q$ subprocess [see second diagram of Fig. 1], here these two regions are separated by a more conventional cut in the photon virtuality Q^2.

(ii) In Ref. [3] photoproduction is treated in the DIS_γ scheme, making use of the quark densities extracted from the structure function F_2^γ as measured in $\gamma^*\gamma \to q\bar{q}$. In contrast, our analysis is carried out in the conventional $\overline{\mathrm{MS}}$ scheme.

(iii) In Ref. [3] an approximation of the Weizsäcker-Williams spectrum of quasi-real photons is used which only includes the first logarithmic term of the curly bracket in eq.(1). Moreover, the input for $Q^2_{max/min}$ differs from our choice.

[1]The fraction of resolved W events in all events with $p_T > 15$ GeV is about 5%.

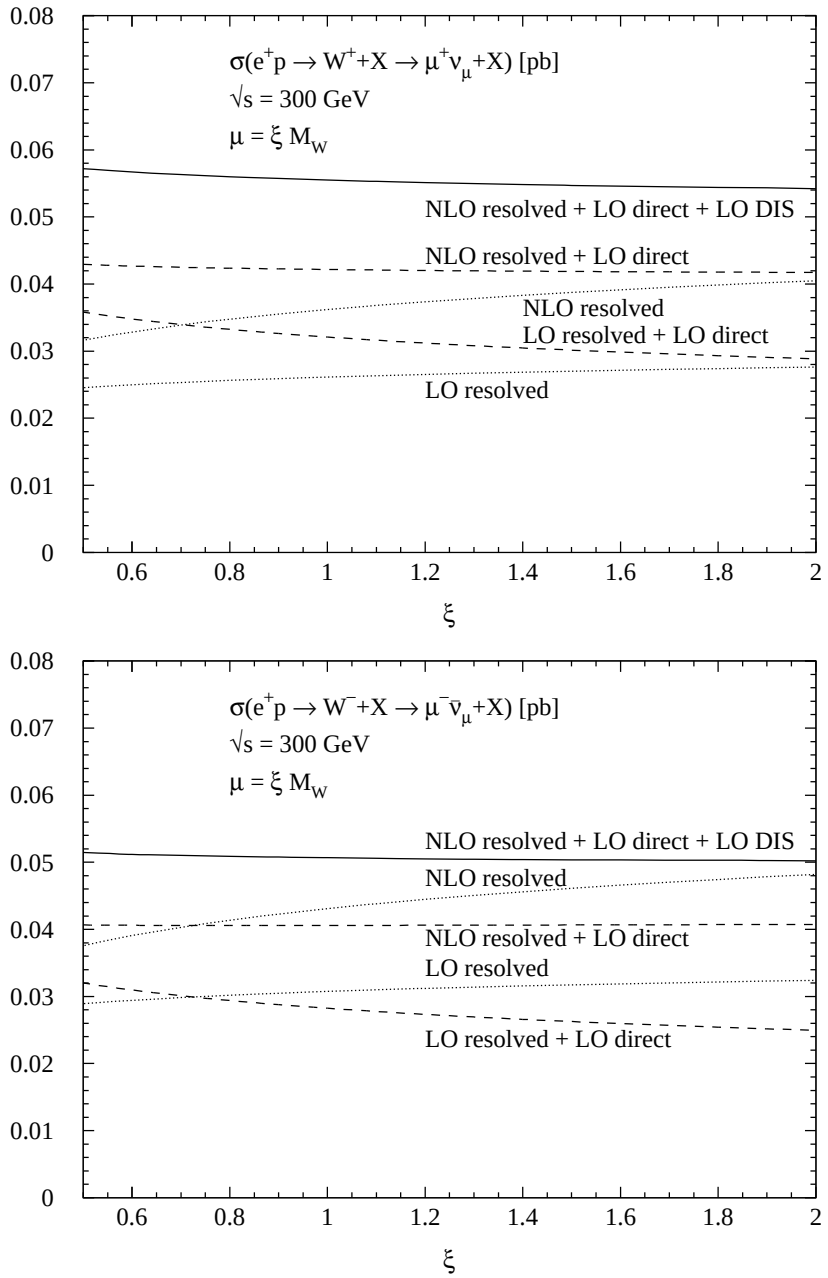

Figure 3: *Dependence of the individual contributions to W^+ (upper plot) and W^- (lower plot) production on the renormalization and factorization scale $\mu = \mu_F = \mu_R = \xi M_W$. The full curves represent the final predictions for the total cross section of $W^\pm$ production in e^+p collisions. We have chosen CTEQ4M [11] and ACFGP [12] parton densities for the proton and the photon, respectively. The strong coupling constant is taken at NLO with $\Lambda_5 = 202$ MeV. An angular cut of 5^o is introduced for the separation of photoproduction and deep inelastic scattering (DIS).*

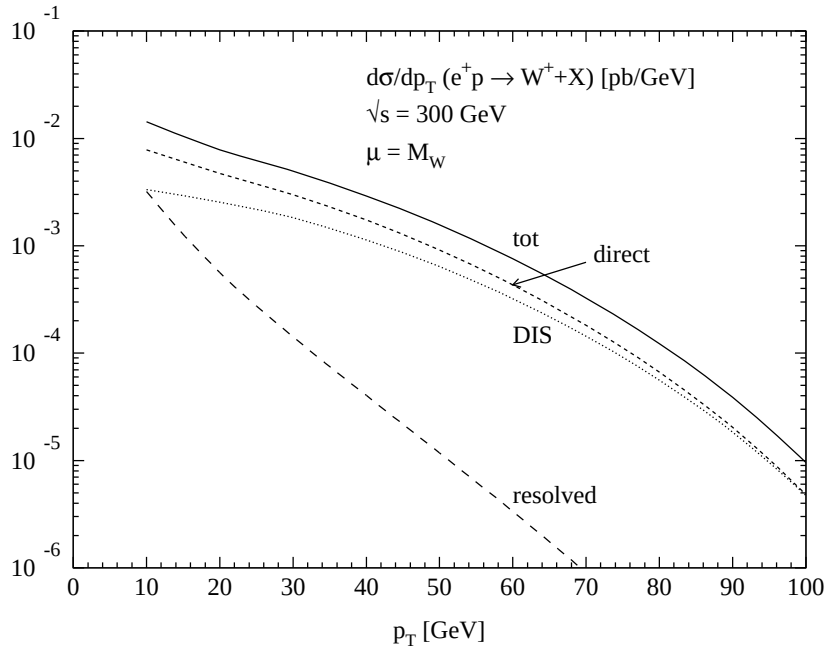

Figure 4: *Transverse momentum distribution of W^+ bosons at HERA. The full curve shows the total p_T distribution, while the broken lines exhibit the individual DIS, direct and resolved contributions.*

(iv) In Ref. [3], the full amplitude for off-shell W production is computed including the leading amplitudes for non-resonant 4-fermion final states. We have only considered on-shell W production.

If we make similar approximations for the Weizsäcker-Williams spectrum and work in the DIS_γ scheme, we are able to reproduce the results of Ref. [3] within less than 10%. The residual differences can be attributed to the different treatment of the DIS and photoproduction regimes and to non-resonant contributions.

4 Conclusions

We have presented predictions for W boson production at HERA including the QCD corrections to the dominant resolved photon mechanism. Working in the conventional $\overline{\mathrm{MS}}$ scheme we find that the QCD corrections enhance the resolved contributions by about 40% at the nominal renormalization/factorization scale $\mu_R = \mu_F = M_W$, and thus have a sizeable effect on the total W production rate. In addition, the NLO corrections reduce the residual scale dependence of the total cross section to a level of about 5%. Taking into account also the variation with the cut separating the DIS and photoproduction regimes, the total theoretical uncertainty is estimated to be smaller than about 10%. This does not yet include the uncertainties from the parton densities of the photon and proton. In spite of the dominance of the resolved photon mechanism in the total cross sections,

gluon radiation in the resolved photon process hardly affects the W transverse momentum spectrum at p_T values above about 15 GeV.

Our approach differs significantly from earlier analyses particularly in the treatment of the separation between the DIS and photoproduction regimes. Nevertheless, the resulting total W production cross sections differ by less than about 10%.

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References

[1] S. Adloff et al., H1 Collaboration, Report DESY 97-024;
J. Breitweg et al., ZEUS Collaboration, Report DESY 97-025.

[2] J. Breitweg et al., ZEUS Collaboration, Report DESY 99-054, in preparation.

[3] U. Baur, J.A.M. Vermaseren and D. Zeppenfeld, Nucl. Phys. **B375** (1992) 3.

[4] M.N. Dubinin and H.S. Song, Phys. Rev. **D57** (1998) 2927.

[5] C. Adloff et al., H1 Collaboration, Eur. Phys. J. **C5** (1998) 575.

[6] C.H. Llewellyn-Smith and B.H. Wiik, Report DESY 77/38;
P. Salati and J.C. Wallet, Z. Phys. **C16** (1982) 155;
K. Neufeld, Z. Phys. **C17** (1983) 145;
A.N. Kamal, J.N. Ng and H.C. Lee , Phys. Rev. **D24** (1984) 2842;
G. Altarelli, G. Martinelli, B. Mele and R. Rückl, Nucl. Phys. **B262** (1985) 204;
E. Gabrielli, Mod. Phys. Lett. **A1** (1986) 465;
M. Böhm and A. Rosado, Z. Phys. **C34** (1987) 117 and Z. Phys. **C39** (1988) 275;
U. Baur and D. Zeppenfeld, Nucl. Phys. **B325** (1989) 253;
J. Blümlein and G.A. Schuler, preprint PHE-90-21, in proceedings "Snowmass 1990, Research directions for the decade";
M. Janssen, Z. Phys. **C52** (1991) 165;
C.S. Kim and W.J. Stirling, Z. Phys. **C53** (1992) 601.

[7] S. Frixione, M.L. Mangano, P. Nason and G. Ridolfi, Phys. Lett. **B319** (1993) 339.

[8] P. Nason, R. Rückl and M. Spira, Proceedings of the 3rd UK Phenomenology Workshop on HERA Physics, Durham, 1998, hep-ph/9902296;
P. Nason, R. Rückl and M. Spira, in preparation.

[9] W. Furmanski and R. Petronzio, Z. Phys. **C11** (1982) 293.

[10] G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298.

[11] H.L. Lai, J. Huston, S. Kuhlmann, F. Olness, J. Owens, D. Soper, W.K. Tung and H. Weerts, Phys. Rev. **D55** (1997) 1280.

[12] P. Aurenche, P. Chiappetta, M. Fontannaz, J.P. Guillet and E. Pilon, Z. Phys. **C56** (1992) 589.