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VACUUM ENERGY: "IF NOT NOW, THEN WHEN?"

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Abstract

For a flat universe presently dominated by static or dynamic vacuum energy, cosmological constant (LCDM) or quintessence (QCDM), we calculate the asymptotic collapsed mass fraction as function of the present ratio of smooth energy to matter energy $\mathcal{R}_0 = (1-\Omega_{m0})/\Omega_{m0}$. Identifying these collapsed fractions as anthropic probabilities, we find the observed present ratio $\mathcal{R}_0 \sim 2$ to be likely in LCDM, but most likely in QCDM.

1 A Cosmological Constant or Quintessence?

Absent a known symmetry principle protecting its value, no theoretical reason for making the cosmological constant zero or small has been found. Inflation makes the universe flat, so that, at present, the vacuum or smooth energy density $\Omega_{Q0} = 1 - \Omega_{m0} < 1$, is 10^{120} times smaller than would be expected on current particle theories. To explain this small but non-vanishing present value, a dynamic vacuum energy, quintessence, has been invoked, which obeys the equation of state $w_Q \equiv P/\rho < 0$. (The limiting case, $w_Q = -1$, a static vacuum energy or Cosmological Constant, is homogeneous on all scales.)

The evidence for a flat low-density universe come from [1, 2]: (1) The location of the first Doppler peak in the CBR anisotroy at $l \sim 200$: $\Omega_m + \Omega_Q = 1 \pm 0.2$; (2) The slow evolution of rich clusters, the mass power spectrum, the CBR anisotropy, the cosmic flow:$\Omega_{m0} = 0.3 \pm 0.05$; (3) Curvature in the SNIa Hubble diagram, dynamic age, height of first Doppler peak, cluster evolution: $\Omega_{Q0} = 1 - \Omega_{m0} \sim 2/3$. Of these, the SNIa evidence is most subject to systematic errors due to precursor intrinsic evolution and the possibilty of grey dust extinction. The combined data nevertheless implies a flat, low-density universe with $\Omega_{m0} \sim 1/3$ and a smooth energy component with present energy density $\Omega_{Q0} \sim 2/3$ and negative pressure $-1 \leq w_Q \leq -1/2$.

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Accepting this small but non-vanishing value for static or dynamic vacuum energy, a flat Friedmann cosmology (CDM) is characterized by Ω_{m0}, $\Omega_{Q0} = 1 - \Omega_{m0}$ or the present ratio

$$\mathcal{R}_0 \equiv u_0^3 \equiv \Omega_{Q0}/\Omega_{m0} = (1 - \Omega_{m0})/\Omega_{m0} \ ,$$

and by the equation of state for the smooth energy. The *Cosmic Coincidence* problem now becomes pressing: Why do we live when the clustered matter density $\Omega(a)$, which is diluting as a^{-3} with cosmic scale a, is just now comparable to the static vacuum energy or present value of the smooth energy i.e. when the ratio $\mathcal{R}_0 \sim 2$?

In this paper, we distinguish the two limiting cases allowed [1, 2] for the smooth energy component: LCDM: Cosmological constant: $w_Q = -1$ and QCDM: Quintessence: $w_Q = -1/2$. In the next section, we compare the expansion of these two limiting low-density flat universes. In Section 3, we extend to QCDM the calculation of asymptotic mass fraction as function of a hypothetical continuous variable Ω_{m0} presented by Martel *et al* [6, 9] for QCDM. Finally, we statistically infer that, absent any prior information about Ω_{m0}, the observed present ratio $\mathcal{R}_0$ is reasonable for a LCDM universe, and most likely for a QCDM universe: "If not now, then when?" [3]

2 Expansion of a Low Density Flat Universe

The Friedmann equation in a flat universe with clustered matter and smooth energy density is

$$H^2(x) \equiv (\dot{a}/a)^2 = (8\pi G/3)(\rho_m + \rho_Q),$$

or, in units of $\rho_{cr}(x) = 3H^2(x)/8\pi G$, $1 = \Omega_m(x) + \Omega_Q(x)$, where the reciprocal scale factor $x \equiv a_0/a \equiv 1 + z \to \infty$ in the far past, $\to 0$ in the far future.

With the effective equation of state $w \equiv P/\rho =$ constant, different kinds of energy density dilute at different rates $\rho \sim a^{-n}$, $n \equiv 3(1 + w)$, and contribute to the deceleration at different rates $(1 + 3w)/2$ shown in the table:

substance	w	n	*(1+3w)/2*
radiation	1/3	4	1
NR matter	0	3	1/2
quintessence	-1/2	3/2	-1/4
cosmolconst	-1	0	-1

Table 1: Energy Dilution for Various Equations of State

The expansion rate in present Hubble units is

$$H(x)/H_0 = (\Omega_{m0}x^3 + (1 - \Omega_{m0})x_Q^n)^{1/2}.$$

The Friedmann equation has an unstable fixed point in the far past and a stable attractor in the far future. (Note the tacit application of the anthropic principle: Why does our universe expand, rather than contract?)

The second Friedmann equation is $-\ddot{a}a/\dot{a}^2 = (1+3w_Q\Omega_Q)/2$. The ratio of smooth energy to matter energy, $\mathcal{R}(a) = \mathcal{R}_0(a_0/a)^{3w_Q}$, increases as the cosmic expansion dilutes the matter density. A flat universe, characterized by R_0, w_Q, evolves out of an SCDM universe in the remote past towards a flat de Sitter universe in the future. As shown by the inflection points (O) on the middle curves of Figure 1, for fixed $\mathcal{R}_0$, QCDM expands faster than LCDM, but begins accelerating only at the present epoch. The top and bottom curves refer respectively to a de Sitter universe ($\Omega_m = 0$), which is always accelerating, and an SCDM universe ($\Omega_m = 1$), which is always decelerating.

The matter-smooth energy transition ("freeze-out") $\Omega_Q/\Omega_m = 1$ took place only recently at $x*^{-w_Q} = \mathcal{R}_0^{1/3} \equiv u_0$ or at $x* = 1+z* = u_0^2 = 1.59$ for QCDM and, even later, at $x* = 1+z* = u_0 = 1.26$ for LCDM. Because, for the same value of u_0, a matter-QCDM freeze-out would take place earlier and more slowly than a matter-LCDM freeze-out, it imposes a stronger constraint on structure evolution. As summarized in the table below, quintessence dominance begins 3.6 Gyr earlier and more gradually than cosmological constant dominance. (In this table, the deceleration $q(x) \equiv -\ddot{a}/aH_0^2$ is measured in *present* Hubble units.) The recent lookback time

$$H_0 t_L(z) = z - (1+q_0)z^2 + ..., \quad z < 1,$$

where $q_0 = 0$ for QCDM and $= -1/2$ for LCDM.

event	*LCDM*	*QCDM*
Onset of Vacuum Dominance		
reciprocal scale x*=$a_0/a = 1+z$	u_0=1.260	u_0^2=1.587
age $t(x*)/H_0^{-1}$	0.720	0.478
in units h_{65}^{-1}Gyr	10.8	7.2
horizon size in units cH_0^{-1}	2.39	1.58
in units h_{65}^{-1}Gpc	11.0	7.24
deceleration q(x*) at freeze-out	-0.333	0.333
Present Epoch		
age t_0/H_0^{-1}	0.936	0.845
h_{65}^{-1}Gyr	14.0	12.7
horizon in units cH_0^{-1}	3.26	2.96
in units h_{65}^{-1}Gpc	15.0	13.6
present deceleration q_0	-0.500	0

Table 2: Comparative Evolution of LCDM and QCDM Universes

3 Evolution of Large Scale Structure

In this section, we extend to QCDM earlier LCDM calculations [6, 7, 9] of the asymptotic mass fraction $f_{c,\infty}$ that ultimately collapses into evolved galaxies. This is presumably a measure of the number density of galaxies like our own,

that are potentially habitable by intelligent life. We then compare the QCDM and LCDM asymptotic mass fraction distribution functions, as function of an assumed Ω_{m0}.

The background density for large-scale structure formation is overwhelmingly Cold Dark Matter (CDM), consisting of clustered matter Ω_m and smooth energy or quintessence Ω_Q. Baryons, contributing only a fraction to Ω_m, collapse after the CDM and, particularly in small systems, produce the large overdensities that we see.

Structure formation begins and ends with matter dominance, and is characterized by two scales: The horizon scale at the first cross-over, from radiation to matter dominance, determines the power spectrum $P(k, a)$, which is presently characterized by a shape factor $\Gamma_0 = \Omega_{m0}h = 0.25 \pm 0.05$. The horizon scale at the second cross-over, from matter to smooth energy, determines a second scale factor, which for quintessence, is at ~ 130 Mpc, the scale of voids and superclusters. A cosmological constant is smooth at all scales.

Quasars formed as far back as $z \sim 5$, galaxies at $z \geq 6.7$, ionizing sources at $z = (10 - 30)$. The formation of *any* such structures, already sets an upper bound $x* < 30$ or $(\Omega_\Lambda/\Omega_{m0}) < 1000, \Omega_{Q0} < 30$, for *any* structure to have formed. A much stronger upper bound, $u_0 < 5$, is set by when *typical* galaxies form i.e. by estimating the *probability* of our observing $\mathcal{R}_0 = 2$ at the present epoch.

3.1 Asymptotic Collapsed Mass Parameter β

Martel *et al* [6] and Garriga *et al* [9] have already calculated the asymptotic mass fraction from the Press-Schechter formalism

$$f_{c,\infty} = \mathrm{erfc}(\sqrt{\beta}) = (2/\sqrt{\pi}) \int_{\sqrt{\beta}}^{\infty} \exp(-t^2)\, dt,$$

depending only on

$$\beta \equiv \delta_{i,c}^2/(2\sigma_i^2),$$

where σ_i^2 is the variance of the denssity field, smoothed on some scale R_G, and $\delta_{i,c}$ is the minimum density contrast at recombination which will ultimately make a bound structure. This minimum density contrast grows with scale factor a, and is, except for a numerical factor of order unity [9], $\delta_{i,c} \sim x*/(1 + z_i)$. Both numerator and denominator in β refer to the epoch of recombination, but this factor $(1 + z_i)$ cancels out in the quotient. (MSW and MS have improved on the Press-Schechter formalism by assuming spherical collapse of Gaussian fluctuations or linear fluctuations that are surrounded by equal volumes of compensating underdensity. Except in the limit $\beta \to 0$, the PS formula overestimates the collapsed mass by factor $\approx (1.70) * \beta^{0.085}$, or about 40% near $\Omega_{m0} = 1/3$. For simplicitly, this paper adheres to the PS formula with $R_G = 1$ Mpc. In a forthcoming paper, we will use the improved MSW formula for both $R_G = 1,\ 2$ Mpc.)

The variance of the mass power spectrum depends on the cosmological model (Ω_{m0}) and on the relevant co-moving galactic size scale R_G, but is insensitive to w_Q, for $w_Q < -1/3$ [8]. For the QCDM model we consider, $\sigma_i^2(\Omega_{m0}, R_G)$ is therefore the same as that already calculated [6, 9] for LCDM, for a scale-invariant

mass spectrum smoothed with a top-hat window function. For the observed ratio $\mathcal{R}_0 = 2, \Omega_{m0} = 1/3$, at recombination $1000\sigma = 3.5, 2.4$, for comoving galactic size scale $R_G = 1,\ 2$ Mpc.

The numerical factor in $\delta_{i,c}$ is $9/5(4)^{1/3} = 1.1339$ for both $w_Q = -1$ and $w_Q = -1/2$, so that $\delta_{i,c} = 1.1339x*/(1+z_i)$. Thus, the collapsed mass parameter $\sqrt{\beta} = 0.80x*/\sigma_i(R_G, u_0)$, depends explicitly on u_0 through $x* = u_0, u_0^2$ for LCDM, QCDM respectively. It also depends implicitly on u_0 through σ_i. Nevertheless, in going from LCDM the argument of $f_{c,\infty}$ scales simply as $\sqrt{\beta}_{QCDM} = \sqrt{\beta}_{LCDM} \cdot u_0$.

Both asymptotic mass fractions are practically unity for large Ω_{m0}, but fall off with increasing ratio $\mathcal{R}_0 > 1$. For any $\mathcal{R}_0 > 1$, QCDM always leads to a smaller asymptotic mass fraction than LCDM. For ratio $\mathcal{R}_0 < 1$, $f_{c,\infty}$ changes slowly and the differences between QCDM and LCDM are not large. At the observed ratio $R_0 = 2$, the Press-Schechter asymptotic mass fractions are 0.696, 0.623 for LCDM, QCDM respectively.

3.2 Asymptotic Collapsed Mass Fraction Distribution Function

As function of the ratio Ω_{m0}, the asymptotic mass fraction defines a distribution function

$$f_{c,\infty} = d\mathcal{P}/d\mathcal{R}_0.$$

In Figure 2, instead of $f_{c,\infty}$ we plot the logarithmic distribution function in the ratio R_0

$$F(\Omega_{m0}) = \mathcal{R}_0 \cdot f_{c,\infty} = d\mathcal{P}/d\log\mathcal{R}_0,$$

for LCDM and for QCDM and galactic size scale 1 Mpc. (Even for LCDM, this differs by a factor $\sigma_i^3(\Omega_{m0})$ from the logarithmic distribution in β, $d\mathcal{P}/d\log(\beta^{3/2})$ that is plotted by MSW and GLV.) $F(\Omega_{m0})$ may be thought of as the ratio R_0 weighted by the number density of galaxies $f_{c,\infty}$.

The figure shows broad peaks in the logarithmic distributions in Ω_{m0} at $((\Omega_{m0}, F) = (0.23, 1.27)$ for LCDM and at $(0.32, 1.78)$ for QCDM. At the observed $\Omega_{m0} = 1/3$, shown by circles (O), the asymptotic mass fraction logarithmic distribution in R_0 falls at 97% of the QCDM peak and at 78% of the LCDM peak.

4 $\Omega_Q \sim \Omega_m$ is Quite Likely for Our Universe

It is not surprising that our universe, containing at least one habitable galaxy, has $\mathcal{R}_0 = \mathcal{O}(1)$. What is impressive is that our observed low-density universe, is almost exactly that which will maximize the number of habitable galaxies. Our existence does not explain Ω_{m0}, but the observed value makes our existence (and that of other evolved galaxies) most likely.

What epistemological inference should we draw from this remarkable coincidence between our observed universe and the possible asymptotic mass fractions

in either LCDM or QCDM universes? What should we infer statistically about any fundamental theory determining the parameters of our universe?

An anthropic interpretation has already been given [4, 5, 6, 7] to "explain" a non-vanishing cosmological constant, in an assumed universe of subuniverses with all possible values for the vacuum energy $\Omega_\Lambda = 1 - \Omega_{m0}$. In each of these subuniverses, the probability for habitable galaxies to have emerged before the present epoch, is a function of Ω_Λ:

$$\mathcal{P}rob(\Omega_{m0}) \propto (\text{prior distribution in } \Omega_{m0}) \times F(\Omega_{m0}).$$

As always, the overall probability depends on the assumed prior. MSW, assuming nothing about initial conditions, take a prior flat in Ω_{m0}. GLV argue that the prior should be determined by a theory of initial conditions and is *not* flat for most theories.

We prefer not to assume a distribution of real subuniverses, but to inversely apply Bayes' Theorem to our own universe. In the absence of any *physical* explanation of the smooth energy, or until one is found, the partial information that intelligent astronomers exist tells us the observed smooth energy is just about what would be expected from equal *a priori* probabilities for Ω_{m0} in the very early universe. That our universe is realized at or near the maximum in the asymptotic mass distribution function confirms that the prior is flat or peaked at $\Omega_{m0} \sim 1/3$. Any phenomenologically viable fundamental theory must ultimately produce this value or be indifferent to the cosmological parameters.

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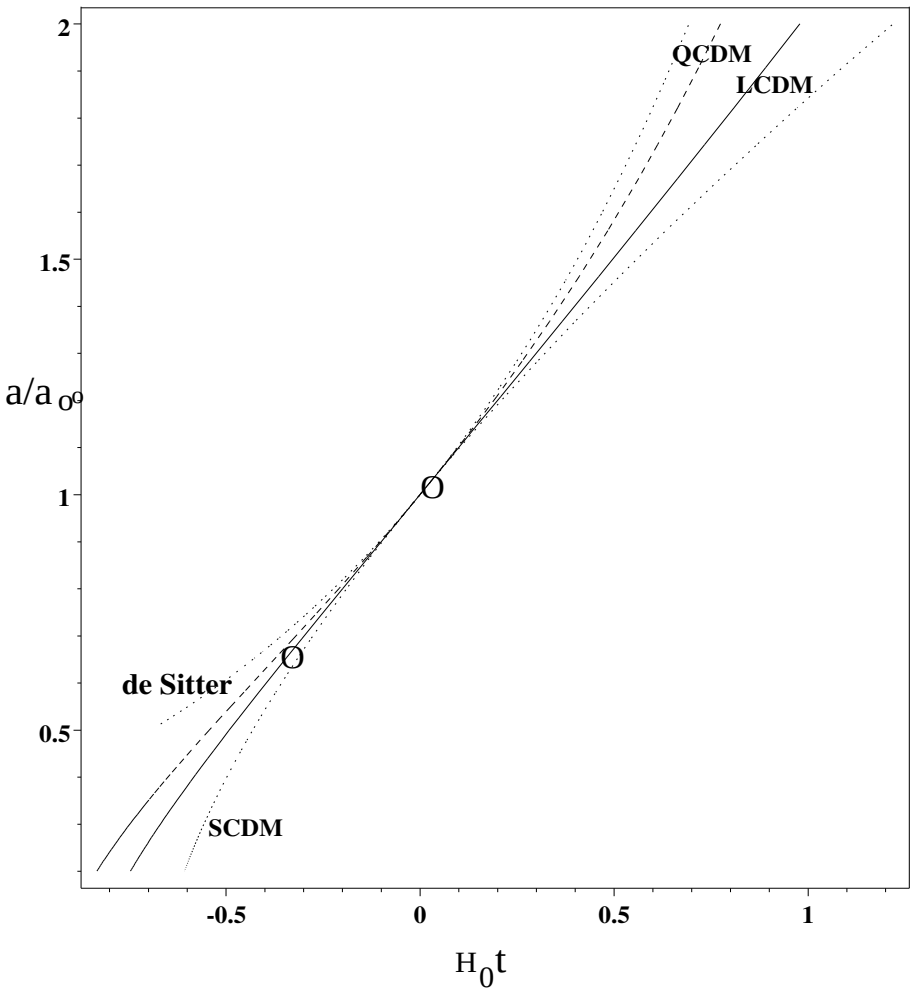

Figure 1: Scale evolution of LCDM and QCDM low-density flat universes in the recent past and near future. The lower curve shows the SCDM universe from which both LCDM and QCDM evolved in the far past. The upper curve shows the flat de Sitter universe towards which both LCDM and QCDM will evolve in the far past. The inflection points marked (O) show where first LCDM and later QCDM change over from decelerating to accelerating universes.

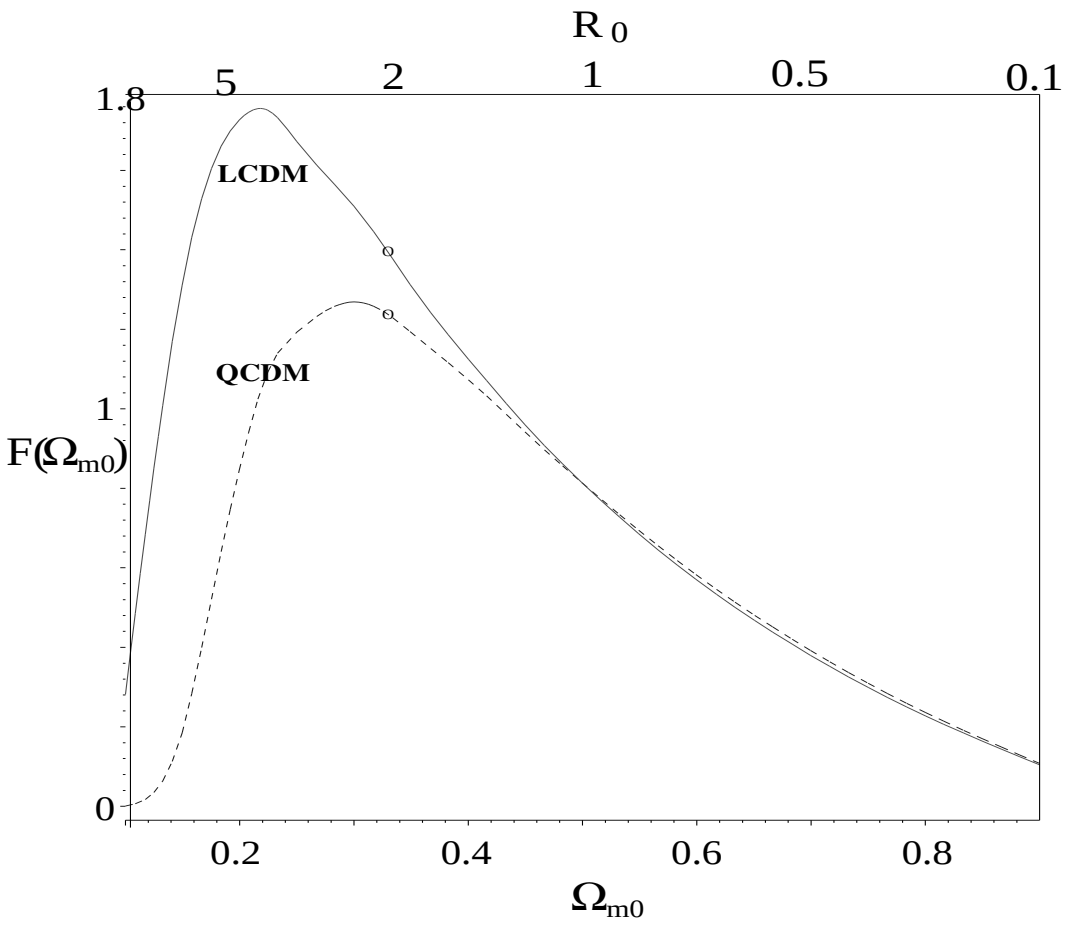

Figure 2: Logarithmic distribution function for the Press-Schecter asymptotic collapsed mass fraction as function of hypothetical present matter density Ω_{m0} (bottom scale) or smooth energy/matter ratio R_0 (top scale). Our observed universe (O) with $\Omega_{m0} \sim 1/3$, $R_0 \sim 2$ falls within the broad peak of the LCDM distribution and remarkably close to the peak of the QCDM distribution.