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Smooth Energy: Cosmological Constant or Quintessence?

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ABSTRACT

For a flat universe presently dominated by smooth energy, either cosmological constant (LCDM) or quintessence (QCDM), we calculate the asymptotic collapsed mass fraction as function of the present ratio of smooth energy to matter energy $\mathcal{R}_0$. Identifying the normalized collapsed fraction as a conditional probability for habitable galaxies, we observe that the observed present ratio $\mathcal{R}_0 \sim 2$ is likely in LCDM, but more likely in QCDM. Inverse application of Bayes' Theorem makes the Anthropic Principle a predictive scientific principle: the data implies that the prior probability for $\mathcal{R}_0$ must be essentially flat over the anthropically allowed range. Interpreting this prior as a distribution over *theories* lets us predict that any future theory of initial conditions must be indifferent to $\mathcal{R}_0$. This application of the Anthropic Principle does not demand the existence of other universes.

1. A Flat Low-Density Universe

In the absence of a recognized symmetry principle protecting its value, no theoretical reason for making the cosmological constant zero or small has been found. Inflation makes the universe appear flat, so that, at present, the vacuum or smooth energy density $\Omega_{Q0} = 1 - \Omega_{m0} < 1$, is 10^{120} times smaller than would be expected on current particle theories. To explain this small but non-vanishing present value, a dynamic vacuum energy, quintessence, has been invoked: a background scalar field whose potential energy dominates its kinetic energy, so that the fluid pressure P and its ratio to energy density $w_Q \equiv P/\rho < 0$. (When we speak of a static vacuum energy or cosmological constant, we mean the limiting

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case, $w_Q = -1$, which is homogeneous on all scales.) With any positive cosmological constant or quintessence, an expanding universe starts out radiation or matter dominated, but ultimately becomes dominated by smooth energy and goes into exponential expansion (Fig. 1).

The best evidence for a flat low-density universe comes from the location (at $l \sim 200$) of the first Doppler peak in the CBR anisotroy in the combined BOOMERANG-98, MAXIMA-1 and COBE-DMR measurements: $\Omega_{m0} + \Omega_{Q0} = 1.11 \pm 0.07\ ^{+0.13}_{-0.12}$ (Jaffe 2000). Supporting evidence (Wang 1999; Roos 2000) comes from the slow evolution of rich clusters, the mass power spectrum, the curvature in the SNIa Hubble diagram, and the dynamic age. The cosmic flow implies $\Omega_{m0} = 0.3 \pm 0.05$. The height of the first Doppler peak, and gravitational lensing imply $\Omega_{Q0} = 1 - \Omega_{m0} \sim 2/3$. Of these, the SNIa evidence is most subject to systematic errors due to precursor intrinsic evolution.

A large set of such observational data have been combined (Roos 2000) in a two-step constrained fit. Firstly, ten independent constraints in the $(\Omega_{m0}, \Omega_\Lambda)$-plane yielded the result $\Omega_{m0} + \Omega_\Lambda = 0.99 \pm 0.14$, which clearly supports the view of a flat universe. Secondly, assuming exact flatness, five more constraints were included in the fit with the result $\Omega_{m0} = 1 - \Omega_\Lambda = 0.33 \pm 0.04$, or equivalently, $\mathcal{R}_0 = \Omega_{Q0}/\Omega_{m0} = 2.03 \pm 0.25$. We can interpret this as evidence that we live in a low-density universe with a smooth energy component with present density $\Omega_{Q0} \sim 2/3$ and negative pressure $-1 \leq w_Q < -1/3$.

Accepting this small but non-vanishing value for static or dynamic vacuum energy, a flat Friedmann cosmology (CDM) is characterized by Ω_{m0}, $\Omega_{Q0} = 1 - \Omega_{m0}$ or the present ratio

$$\mathcal{R}_0 \equiv \Omega_{Q0}/\Omega_{m0} = (1 - \Omega_{m0})/\Omega_{m0} \ ,$$

and by the equation of state for the smooth energy component. The *Cosmic Coincidence* problem now becomes pressing: Why do we live when the clustered matter density $\Omega(a)$,

which is diluting as a^{-3} with cosmic scale a, is just now comparable to the static vacuum energy or present value of the smooth energy i.e. when the ratio $\mathcal{R}_0 \sim 2$?

In this paper, we study the quintessence range $-1 \leq w_Q < -1/3$ for the smooth energy component, distinguishing in particular the two cases
LCDM: cosmological constant with $w_Q = -1$, and
QCDM: quintessence with the specific choice $w_Q = -1/2$.

The next section compares the cosmic expansion and the freeze-out of structure formation, in these two models for smooth energy. Section 3 extends to QCDM the calculation of asymptotic mass fraction as function of a hypothetical continuous variable Ω_{m0} for LCDM, presented by Martel *et al* (Martel 1998, 1999). In Section 4, identifying these collapsed mass fractions with anthropic probabilities for $\mathcal{R}_0$, we show that the presently observed ratio, while reasonable in an LCDM universe, is more likely in a QCDM universe. This confirms empirically that the prior probability for our universe is flat in Ω_{m0}, as is expected in a large class of fundamental theories (Weinberg 2000).

The anthropic answer to the cosmic coincidence problem is: "If not now, then when?" (Hillel 30)

2. Expansion of a Low Density Flat Universe

The Friedmann equation in a flat universe with clustered matter and smooth energy density is

$$H^2(x) \equiv (\dot{a}/a)^2 = (8\pi G/3)(\rho_m + \rho_Q),$$

or, in units of $\rho_{cr}(x) = 3H^2(x)/8\pi G$,

$$1 = \Omega_m(x) + \Omega_Q(x),$$

where the reciprocal scale factor $x \equiv a_0/a \equiv 1+z \to \infty$ in the far past, $\to 0$ in the far future.

With the effective equation of state $w \equiv P/\rho =$ constant, different kinds of energy density dilute at different rates $\rho \sim a^{-n},\ n \equiv 3(1+w)$, and contribute to the deceleration at different rates $(1+3w)/2$ shown in the table:

The expansion rate in present Hubble units is

$$H(x)/H_0 = (\Omega_{m0}x^3 + (1-\Omega_{m0})x_Q^n)^{1/2}.$$

The Friedmann equation has an unstable fixed point in the far past and a stable attractor in the far future. (Note the tacit application of the anthropic principle: Why does our universe expand, rather than contract?)

The second Friedmann equation is

$$q(a) \equiv -\ddot{a}a/\dot{a}^2 = \sum_i \Omega_i(1+3w_i)/2 = (1+\Omega_Q(a))/2.$$

The ratio of smooth energy to matter energy, $\mathcal{R}(a) = \mathcal{R}_0(a_0/a)^{3w_Q}$, increases as the cosmic expansion dilutes the matter density. A flat universe, characterized by $\mathcal{R}_0,\ w_Q$, evolves out of an SCDM universe in the remote past towards a flat de Sitter universe in the future. As shown by the inflection points (O) on the middle curves of Figure 1, for fixed $\mathcal{R}_0$, QCDM

substance	*w*	*n*	*(1+3w)/2*
radiation	1/3	4	1
NR matter	0	3	1/2
quintessence	-1/2	3/2	-1/4
cosmolconst	-1	0	-1

Table 1: Energy Dilution for Various Equations of State

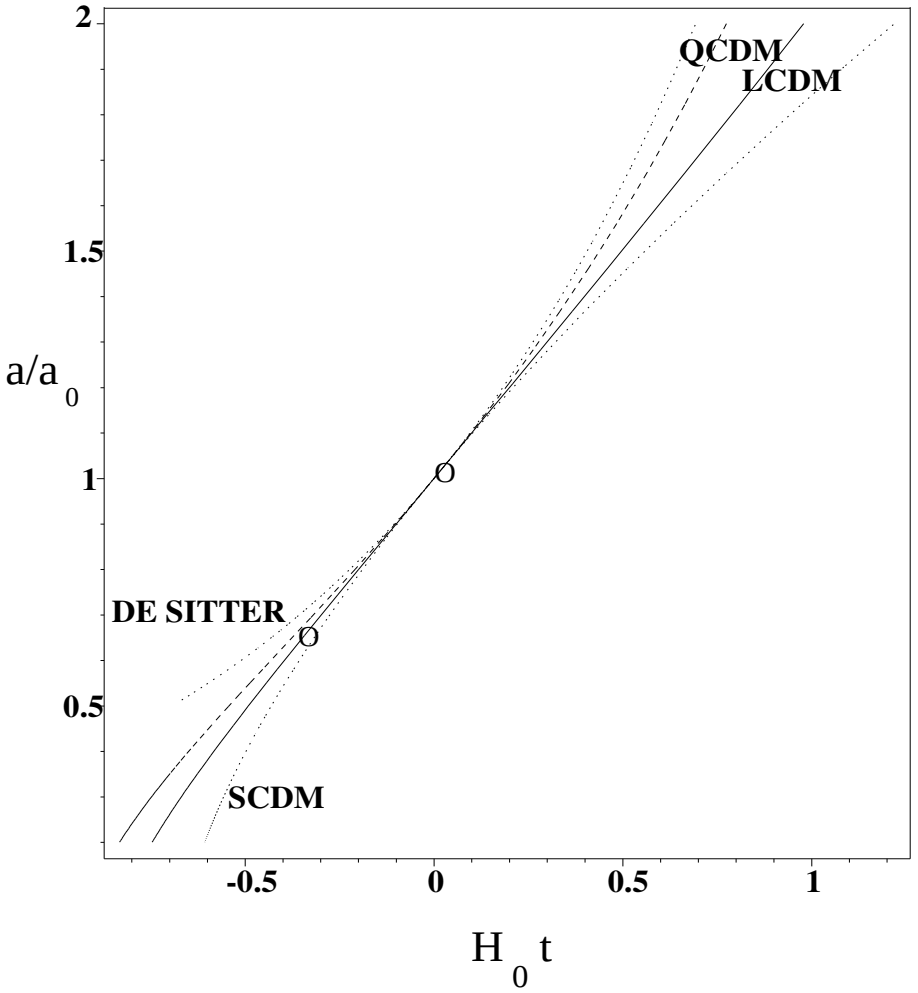

Fig. 1.— Scale evolution of LCDM and QCDM low-density flat universes in the recent past and near future. The lower curve shows the SCDM universe from which both LCDM and QCDM evolved in the far past. The upper curve shows the flat de Sitter universe towards which both LCDM and QCDM will evolve in the future. The inflection points marked (O) show where first LCDM and later QCDM change over from decelerating to accelerating universes.

expands faster than LCDM, but begins accelerating only at the present epoch. The top and bottom curves refer respectively to a de Sitter universe ($\Omega_m = 0$), which is always accelerating, and an SCDM universe ($\Omega_m = 1$), which is always decelerating.

The matter-smooth energy transition ("freeze-out") $\Omega_Q/\Omega_m = 1$ took place only recently at $(x^*)^{-w_Q} = \mathcal{R}_0^{-1/3}$ or at $x^* = \mathcal{R}_0^{2/3} = 1.59$ for QCDM and, even later, at $x^* = 1.26$ for LCDM. Because, for the same value of $\mathcal{R}_0$, a matter-QCDM transition would take place earlier and more slowly than a matter-LCDM transition, it imposes a stronger constraint on structure evolution. As summarized in the table below, quintessence dominance begins 3.6 Gyr earlier and more gradually than cosmological constant dominance. (In this table, the deceleration $q(x) \equiv -\ddot{a}/aH_0^2$ is measured in *present* Hubble units.) The recent lookback time is

$$H_0 t_L(z) = z - (1 + q_0)z^2 + ..., \quad z < 1,$$

where $q_0 = 0$ for QCDM and $q_0 = -1/2$ for LCDM.

3. Evolution of Large Scale Structure

In this section, we extend to QCDM earlier LCDM calculations (Martel 1998; Garriga 1999; Martel 1999) of the asymptotic mass fraction $f_{c,\infty}$ that ultimately collapses into evolved galaxies. This is presumably a measure of the number density of galaxies like our own, that are potentially habitable by intelligent life. We then compare the QCDM and LCDM asymptotic mass fraction distribution functions, as function of an assumed Ω_{m0}.

The background density for large-scale structure formation is overwhelmingly Cold Dark Matter (CDM), consisting of clustered matter Ω_m and smooth energy or quintessence Ω_Q. Baryons, contributing only a fraction to Ω_m, collapse after the CDM and, particularly in small systems, produce the large overdensities that we see.

event	*LCDM*	*QCDM*
Cross-Over to Smooth Energy Dominance		
reciprocal scale $x^* = a_0/a = 1+z$	$\mathcal{R}_0^{1/3}$=1.260	$\mathcal{R}_0^{2/3}$=1.587
age $t(x^*)/H_0^{-1}$	0.720	0.478
in units h_{65}^{-1}Gyr	10.8	7.2
horizon size in units cH_0^{-1}	2.39	1.58
in units h_{65}^{-1}Gpc	11.0	7.24
deceleration $q(x^*)$ at freeze-out	-0.333	0.333
Present Epoch		
age t_0/H_0^{-1}	0.936	0.845
h_{65}^{-1}Gyr	14.0	12.7
horizon in units cH_0^{-1}	3.26	2.96
in units h_{65}^{-1}Gpc	15.0	13.6
present deceleration q_0	-0.500	0

Table 2: Comparative Evolution of LCDM and QCDM Universes

Structure formation begins and ends with matter dominance, and is characterized by two scales: The horizon scale at the first cross-over, from radiation to matter dominance, determines the power spectrum $P(k,a)$, which is presently characterized by a shape factor $\Gamma_0 = \Omega_{m0}h = 0.25 \pm 0.05$. The horizon scale at the second cross-over, from matter to smooth energy, determines a second scale factor, which for $w_Q = -1/2$ quintessence, is at ~ 130 Mpc, the scale of voids and superclusters. A cosmological constant is smooth at all scales.

Quasars formed as far back as $z \sim 5$, galaxies at $z \geq 6.7$, ionizing sources at $z = (10 - 30)$. The formation of *any* such structures, already sets a large upper bound $x^* < 30$ or $(\Omega_\Lambda/\Omega_{m0}) < 1000, \Omega_{Q0} < 30$, for *any* structure to have formed. A much stronger upper bound, $u_0 < 5$, is set by when *typical* galaxies form i.e. by estimating the *probability* of our observing $\mathcal{R}_0 = 2$ at the present epoch.

3.1. Asymptotic Collapsed Mass Parameter β

Garriga *et al* (Garriga 1999) and Bludman (Bludman 2000) have already calculated the asymptotic mass fraction from the Press-Schechter formalism

$$f_{c,\infty}(\beta) = \mathrm{erfc}(\sqrt{\beta}) = (2/\sqrt{\pi}) \int_{\sqrt{\beta}}^{\infty} \exp(-t^2)\, dt,$$

depending only on

$$\beta \equiv \delta_{i,c}^2/(2\sigma_i^2),$$

where σ_i^2 is the variance of the density field, smoothed on some scale R_G, and $\delta_{i,c}$ is the minimum density contrast at recombination which will ultimately make a bound structure. This minimum density contrast grows with scale factor a, and is, except for a numerical factor of order unity (Eq.(2(Martel 1999)), $\delta_{i,c} \sim x^*/(1 + z_i)$. Both numerator and denominator in β refer to the epoch of recombination, but this factor $(1 + z_i)$ cancels out in

the quotient.

MSW (Martel 1998) and MS (Martel 1999) have improved on the Press-Schechter formalism by assuming spherical collapse of Gaussian fluctuations or linear fluctuations that are surrounded by equal volumes of compensating underdensity. Except in the limit $\beta \to 0$, the PS formula overestimates the collapsed mass by factor $\approx (1.70) \cdot \beta^{0.085}$, or about 50% near $\Omega_{m0} = 1/3$ (Bludman 2000). Here we will use the improved MSW formula for both $R_G = 1,\ 2$ Mpc,

$$f_{c,\infty}(\beta) = \frac{1}{\sqrt{\pi}} \int_\beta^\infty \frac{\exp(-x)dx}{\sqrt{x} + \sqrt{\beta}} \ . \tag{1}$$

The variance of the mass power spectrum depends on the cosmological model (Ω_{m0}) and on the relevant co-moving galactic size scale R_G, but is insensitive to w_Q, for $w_Q < -1/3$ (Wang 1998). For the QCDM model we consider, $\sigma_i^2(\Omega_{m0}, R_G)$ is therefore the same as that already calculated (Martel 1998, 1999) for LCDM, for a scale-invariant mass spectrum smoothed with a top-hat window function. For the observed ratio $\mathcal{R}_0 = 2,\ \Omega_{m0} = 1/3$, the value of $\sigma_i \cdot 10^{-3}$ at recombination is 3.5 and 2.4 for comoving galactic size scale $R_G = 1$, and 2 Mpc, respectively.

For a flat universe the numerical factor in $\delta_{i,c}$ is given by (Martel 1999)

$$\frac{3}{5} \frac{(3-n)}{(2-n)^{(2-n)/(3-n)}} \quad , \quad n = 3(1 + w_Q). \tag{2}$$

Thus $\delta_{i,c} = 1.1339 x^*/(1 + z_i)$ for both $n = 0$ and $n = 3/2$. The collapsed mass parameter $\beta = (1.1339/2) \cdot [x^*/\sigma_i(R_G, \mathcal{R}_0)]^2$, depends explicitly on $\mathcal{R}_0$ for LCDM and QCDM. It also depends implicitly on $\mathcal{R}_0$ through σ_i. Nevertheless, in going from LCDM to $w_Q = -1/2$ QCDM, the argument of $f_{c,\infty}$ scales simply as $\beta_{QCDM} = \beta_{LCDM} \cdot \mathcal{R}_0^{1/3}$.

Both asymptotic mass fractions are practically unity for large Ω_{m0}, but fall off with increasing ratio $\mathcal{R}_0 > 1$. For any $\mathcal{R}_0 > 1$ or $\Omega_{m0} < 0.5$, QCDM always leads to a smaller

asymptotic mass fraction than LCDM. For ratio $\mathcal{R}_0 < 1$, $f_{c,\infty}$ changes slowly and the differences between QCDM and LCDM are not large.

3.2. Asymptotic Collapsed Mass Fraction Distribution Function

As function of the ratio $\mathcal{R}_0$, the asymptotic mass fraction defines a distribution function

$$f_{c,\infty} = d\mathcal{P}/d\mathcal{R}_0.$$

In Figure 2, instead of $f_{c,\infty}$ we plot the logarithmic distribution function in the ratio $\mathcal{R}_0$

$$F(\Omega_{m0}) = \mathcal{R}_0 \cdot f_{c,\infty} = d\mathcal{P}/d\log\mathcal{R}_0, \tag{3}$$

for LCDM and for QCDM and galactic size scale 1 Mpc. (Even for LCDM, this differs by a factor $\sigma_i^3(\Omega_{m0})$ from the logarithmic distribution in β, $d\mathcal{P}/d\log(\beta^{3/2})$ that is plotted in Martel (1998) and Garriga (1999).) $F(\Omega_{m0})$ may be thought of as the ratio $\mathcal{R}_0$ weighted by the number density of galaxies $f_{c,\infty}$.

Figure 2 shows broad peaks in the logarithmic distributions in Ω_{m0} at $(\Omega_{m0}, F, R_G) =$ $(0.23, 1.09, 1$ Mpc) and $(0.28, 0.78, 2$ Mpc) for LCDM, and at $(0.32, 0.80, 1$ Mpc) and $(0.37, 0.61, 2$ Mpc) for QCDM. At the observed $\Omega_{m0} = 1/3$, the LCDM asymptotic mass fraction logarithmic distributions in $\mathcal{R}_0$ fall 13%, 5% below the LCDM peaks for $R_G = 1$,2 Mpc respectively. The QCDM distributions are peaked nearer $\Omega_{m0} = 1/3$, and peak only 0.3%, 1.7% below the QCDM peaks for $R_G = 1$, 2 Mpc respectively. These asymptotic collapsed mass curves have not yet been normalized to unit area.

In order to interpret these distributions as differential probabilities $\mathcal{P}(\Omega_{m0})$, we now normalize the function $F(\Omega_{m0})$ in Eq.(3, by dividing the $R_G = 1$ Mpc curves by 0.378, 0.485 for QCDM, LCDM respectively and the $R_G = 1$ Mpc curves by 0.289, 0.364 respectively. In Figure 3, we plot this conditional probability at $\mathcal{R}_0 = 2$ as a function of w_Q for $R_G = 1$

and 2 Mpc. At every w_Q, $\mathcal{R}_0 = 2$ is more probable in QCDM than in LCDM, particularly for the smaller galactic mass smoothing scale. For $R_G = 1$ Mpc, $w_Q = -1/2$ QCDM is 10% more probable than LCDM.

It is not surprising that our universe, containing at least one habitable galaxy, has $\mathcal{R}_0 = \mathcal{O}(1)$. What is impressive is that our observed low-density universe, is almost exactly that which will maximize the number density of habitable galaxies. Our existence does not explain Ω_{m0}, but the observed value makes our existence (and that of other evolved galaxies) most likely.

4. $\Omega_{m0} \sim 1/3$ is Quite Likely for Our Universe

4.1. The Datum Implies that the Prior is Flat

What inference should we draw from the datum that our observed universe lies at or near the peak in the asymptotic mass fraction logarithmic distributions for a QCDM or LCDM universe?

Bayes' Theorem makes the posterior probability $\mathcal{P}_{post}(\Omega_{m0})$ of observing a particular value of Ω_{m0}

$$\mathcal{P}_{post}(\Omega_{m0}) = \mathcal{P}(\Omega_{m0}) \cdot \mathcal{P}_{prior}(\Omega_{m0}) \ , \tag{4}$$

where $\mathcal{P}(\Omega_{m0})$ is $F(\Omega_{m0})$ divided by the normalization factors given in Sec. 3.2. The posterior probability always depends directly on the assumed prior $\mathcal{P}_{prior}(\Omega_{m0})$, which measures our subjective hypotheses about Ω_{m0}, and should ultimately be determined by the initial conditions. From the fact that our universe falls at or near the peak of the logarithmic asymptotic mass distribution, we can infer that the prior for the logarithmic

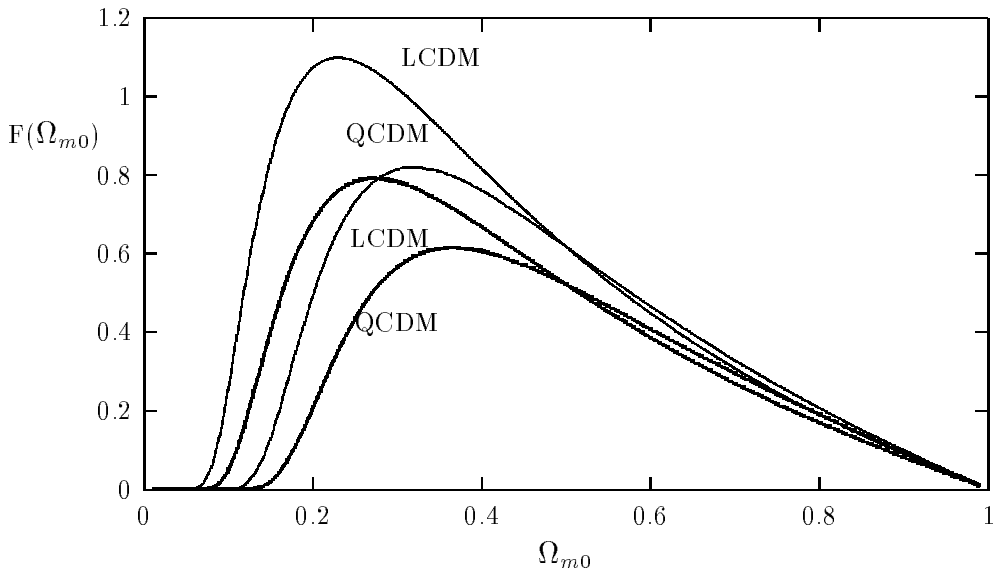

Fig. 2.— Logarithmic distribution function for the asymptotic collapsed mass fraction as function of hypothetical present matter density Ω_{m0}. The smoothing scale is taken to be $R_G = 1$ Mpc for the thin line curves, and 2 Mpc for the thick line curves. Our observed universe with $\Omega_{m0} \sim 1/3$, $\mathcal{R}_0 \sim 2$ falls within the broad peak of the LCDM distributions and close to the peak of the QCDM distributions.

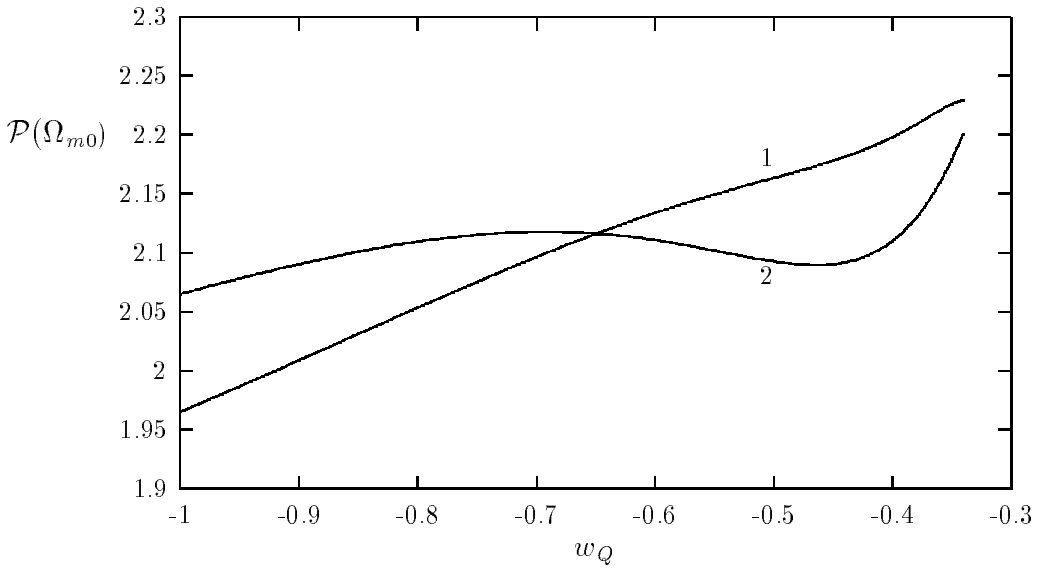

Fig. 3.— The effect of quintessence equation of state on the asymptotic mass fraction logarithmic distribution function for the observed ratio $\mathcal{R}_0 = 2$. The normalized logarithmic distribution function is shown as function of w_Q for each of the two smoothing scales $R_G =$ 1, 2 Mpc. Particularly for $R_G = 1$ Mpc, the observed ratio is more likely in QCDM than in LCDM.

distribution

$$\mathcal{P}_{prior}(\Omega_{m0}) = \mathcal{P}_{post}(\Omega_{m0})/\mathcal{P}(\Omega_{m0}) \ . \tag{5}$$

is flat, at least for $\mathcal{R}_0 = \mathcal{O}(1)$. (Alternatively, if we chose $\mathcal{P}(\Omega_{m0})$ to be simply the asymptotic mass fraction $f_{c,\infty}$, we would infer that the prior must be proportional to $\mathcal{R}_0$. Quasi-philosophical arguments depending on "simplicity" or Occam's Razor (Jefferys 1992) favor defining $\mathcal{P}(\Omega_{m0})$ by the logarithmic distribution, rather than by such a linear distribution, particularly when the data is sparse.)

Garriga and Vilenkin (Garriga 2000) argue that, for many theories, the prior is *not* flat. MSW, assuming nothing about initial conditions, assume a prior flat in Ω_{m0}. Indeed, (Weinberg 2000) finds that the prior will be flat for a large class of theories: those with slow-roll potential $V(\phi) = V_1 f(\lambda\phi)$ where $V_1 \sim \mathcal{O}(M_P^4)$ is a large energy density, $f(x)$ is a dimensionless function involving no very large or very small parameters, and λ is a very small dimensional parameter.

4.2. Anthropic Interpretation of the Flat Prior

The Anthropic Principle asserts that the probability of habitable galaxies, solar systems and intelligent observers is proportional to the posterior probability $\mathcal{P}_{post}(\Omega_{m0})$. The datum $\Omega_{m0} \sim 1/3$, at or near the peak of the logarithmic asymptotic mass distribution, then infers only that its prior is flat, i.e. is indifferent to hypotheses concerning Ω_{m0}, at least for $\mathcal{R}_0 = \mathcal{O}(1)$. It is essential to recognize that the prior is a functional of *hypotheses*, not of the data sample (Loredo 1995). What is the observation $\mathcal{R}_0 \sim 2$ telling us that we didn't believe before-hand?

A popular anthropic interpretation has already been given (Efstathiou 1995; Vilenkin 1995; Martel 1998; Garriga 1999) to "explain" the observed non-vanishing cosmological

constant. These authors take seriously a meta-universe containing an infinite ensemble of real subuniverses with all possible values for the vacuum energy $\Omega_\Lambda = 1 - \Omega_{m0}$. In each of these subuniverses, Ω_Λ determines $\mathcal{P}(\Omega_{m0})$, the normalized probability for habitable galaxies to have emerged before the present epoch. Our habitable subuniverse is rare, only one of many more subuniverses with inhospitable values for Ω_{m0}. Many theories of cosmology and of quantum mechanics do predict other sub-universes, with different values of the fundamental constants, or even of the physical laws. Nevertheless, some of these other sub-universes need to be ultimately observable by us, at least in principle, if this VAnthropic Principle is to be a falsifiable physical theory. This many-world interpretation of the flat prior is suggested by eternal inflation and by quantum cosmology, but close to a frequentist probability view and practically impossible to test. It is not required by the data.

Indeed, the present situation in cosmology is an ideal case for the proper use of Bayes' Theorem: in the face of (presently) incomplete information, to make statistical inference concerning *hypotheses*. A modest inverse probability application of Bayes' Theorem does not require a present distribution of potentially observable subuniverses. (Our own universe might, of course, ultimately evolve from or towards different universes with different values for the cosmological parameters.) Instead, our calculation of the flat prior merely asserts that our knowledge of Ω_{m0} is consistent with those slow-roll scalar field theories (Weinberg 2000) which are indifferent to Ω_{m0}.

In summary, our anthropic interpretation of the ratio of smooth energy to clustered mass in our own universe, $\mathcal{R}_0 \sim 2$, predicts a large class of generic quintessence models and slightly prefers $w_Q = -1/2$ QCDM over LCDM.

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