Preprint DESY/00--140

Heavy Quarkonium Physics beyond the Next-to-Next-to-Leading Order of NRQCD *

Alexander A. Penin [a†]

[a]II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, D-22761 Hamburg, Germany

arXiv:hep-ph/0009324 28 Sep 2000

In this talk I briefly review the recent progress in calculation of the high order corrections to the parameters of the nonrelativistic heavy quark-antiquark system in the effective theory approach.

1. INTRODUCTION

The theoretical study of the nonrelativistic heavy quark-antiquark system [1] and its application to bottomonium [2] and toponium [3] physics is of special interest because it relies entirely on the first principles of QCD. The system in principle allows for a perturbative treatment with the nonperturbative effects being well under control and with no crucial model dependence. This makes the heavy quarkonium to be an ideal place to determine the fundamental QCD parameters such as the heavy quark mass m_q and the strong coupling constant α_s. Recently an essential progress has been made in the theoretical investigation of the nonrelativistic heavy quark dynamics based on the effective theory approach [4]. The analytical results for the main parameters of the nonrelativistic heavy quark-antiquark system are now available up to next-to-next-to-leading order (NNLO) in the strong coupling constant and the heavy quark velocity β [5–21]. The NNLO corrections have turned out to be so sizeable that it appears to be indispensable also to gain control over the next-to-next-to-next-to-leading order (N^3LO) both in regard of phenomenological applications and in order to understand the structure and the peculiarities of the nonrelativistic expansion. In this talk we review the first steps in this direction and consider two particular classes of N^3LO corrections: (i) the retardation effects arising from the emission and absorption of dynamical ultrasoft gluons by the heavy quarks and (ii) the corrections enhanced by a power of $\ln(1/\alpha_s)$ which are not generated by the renormalization group (RG) running of α_s.

*Talk presented at the High Energy Physics International Conference on Quantum Chromodynamics (QCD-00), Montpellier, France, 6-12 July 2000.

†Permanent address: Institute for Nuclear Research, 60th October Anniversary Pr., 7a, Moscow 117312, Russia

2. EFFECTIVE THEORY OF NONRELATIVISTIC HEAVY QUARKS

In this section we briefly outline the effective theory approach to the nonrelativistic heavy quark dynamics. Let us consider the near threshold behavior of the heavy quark vacuum polarization function

$$\left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi(q^2)$$

$$= i \int d^4x e^{iq\cdot x} \langle 0|T j_\mu(x) j_\nu(0)|0\rangle \,, \qquad (1)$$

where $j_\mu = \bar{q}\gamma_\mu q$ is the heavy quark vector current. Its imaginary part is related to the normalized cross section of $q\bar{q}$ production in the photon mediated e^+e^- annihilation at energy $s = q^2$

$$R(s) = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \,, \qquad (2)$$

by

$$R(s) = 12\pi Q_q^2 \mathrm{Im}\Pi(s + i\epsilon) \,, \qquad (3)$$

where Q_q is the fractional charge of quark q. The behavior of the vacuum polarization function near-below the threshold $s = 4m_q^2$ determines also the masses and leptonic widths of the perturbative heavy quarkonium bound states.

Near the threshold the heavy quarks are nonrelativistic so that one may consider the quark velocity $\beta = \sqrt{1-4m_q^2/s}$ as a small parameter. An expansion in β may be performed directly in the Lagrangian of QCD by using the framework of effective field theory. In the nonrelativistic problem there are four different scales [22]: (i) the hard scale (energy and momentum scale like m_q); (ii) the soft scale (energy and momentum scale like βm_q); (iii) the potential scale (energy scales like $\beta^2 m_q$ while momentum scales like βm_q); and (iv) the ultrasoft scale (energy and momentum scale like $\beta^2 m_q$). The ultrasoft scale is only relevant for gluons. By integrating out the hard scale of QCD one arrives at the effective theory of nonrelativistic QCD (NRQCD) [4]. If one also integrates out the soft scale and the potential gluons one obtains the effective theory of potential NRQCD (pNRQCD) which contains potential quarks and ultrasoft gluons as active particles [23]. The dynamics of the quarks is governed by the effective nonrelativistic Schrödinger equation and by their interaction with the ultrasoft gluons. To get a regular perturbative expansion within pNRQCD this interaction should be expanded in multipoles. The corrections from harder scales are contained in the Wilson coefficients leading to an expansion in α_s as well as in the higher-dimensional operators of the nonrelativistic Hamiltonian corresponding to an expansion in β. The nonrelativistic expansion in α_s and β provides us with the following representation of the heavy quark vacuum polarization function near threshold

$$\Pi(E) = \frac{N_c}{2m_q^2} C(\alpha_s) G(0,0,E) + \dots , \qquad (4)$$

where $E = \sqrt{s} - 2m_q$ is the $q\bar{q}$ energy counted from the threshold, $C(\alpha_s)$ is the square of the hard renormalization coefficient of the nonrelativistic vector current and the ellipsis stands for the higher-order terms in β. $G(\mathbf{x},\mathbf{y},E)$ is the nonrelativistic Green function which sums up the $(\alpha_s/\beta)^n$ terms singular near the threshold. It is determined by the Schrödinger equation which describes the the propagation of the nonrelativistic quark-antiquark pair in pNRQCD

$$(\mathcal{H} - E)\, G(\mathbf{x},\mathbf{y},E) = \delta^{(3)}(\mathbf{x}-\mathbf{y}), \qquad (5)$$

where $\mathcal{H}$ is the nonrelativistic Hamiltonian defined by

$$\mathcal{H} = -\frac{\partial_{\mathbf{x}}^2}{m_q} + V(x) + \dots , \quad V(x) = V_C(x) + \dots . \qquad (6)$$

Here $V_C(x) = -C_F\alpha_s/x$ is the Coulomb potential, $C_F = 4/3$ is the eigenvalue of the quadratic Casimir operator of the fundamental representation of the color group, $x = |\mathbf{x}|$ and the ellipses stands for the higher-order terms in α_s and β. The Green function has the spectral representation

$$G(\mathbf{x},\mathbf{y},E) = \sum_{n=1}^{\infty} \frac{\psi_n^*(\mathbf{x})\psi_n(\mathbf{y})}{E_n - E} + \int_0^\infty \frac{d^3k}{(2\pi)^3} \frac{\psi_{\mathbf{k}}^*(\mathbf{x})\psi_{\mathbf{k}}(\mathbf{y})}{k^2/m_q - E} , \qquad (7)$$

where ψ_m and $\psi_{\mathbf{k}}$ are the wave functions of the $q\bar{q}$ bound and continuum states respectively.

Below the threshold the vacuum polarization function of a stable heavy quark is determined by the bound state parameters. For the leading order Coulomb (C) Green function the energy levels and wave functions at the origin read

$$E_n^C = -\frac{\lambda_s^2}{m_q n^2}, \qquad \left|\psi_n^C(0)\right|^2 = \frac{\lambda_s^3}{\pi n^3} , \qquad (8)$$

where $\lambda_s = \alpha_s C_F m_q/2$. Note that for the study of the bound-state parameters we have $\beta \approx \alpha_s$ so that we are only dealing with one expansion parameter.

The current status of the theoretical research can be summarized as follows. The effective nonrelativistic Hamiltonian is known in the NNLO approximation including the two-loop corrections to the static potential [5] and the corresponding corrections to the Green function have been obtained in analytical form in [7–13]. The $\mathcal{O}(\alpha_s^2)$ contribution to the hard renormalization coefficient has been computed in [6]. Thus the complete NNLO analytical expression for the nonrelativistic heavy quark vacuum polarization function is now available. In the next section we present the recent results of the calculation of some N^{3}LO order corrections to the heavy quarkonium parameters.

Figure 1. (a) Feynman diagram giving rise to the ultrasoft contribution at N^{3}LO. The single and double lines stand for the singlet and octet Green functions, respectively, the wavy line represents the ultrasoft-gluon propagator in the Coulomb gauge and the vertices correspond to the chromoelectric dipole interaction. (b) Feynman diagram giving rise to the leading nonperturbative corrections. The crossed wavy lines represent the vacuum fluctuations of the gluonic field.

3. HEAVY QUARKONIUM PARAMETERS BEYOND THE NNLO OF NRQCD

3.1. Retardation effects

Let us start with a few general remarks concerning the structure of higher-order corrections in pNRQCD. At next-to-leading order (NLO) the only source of corrections is the perturbative corrections to the hard renormalization coefficient and the static potential. In NNLO the higher-dimensional operators start to contribute. In N^{3}LO the retardation effects which cannot be described by operators of instantaneous interaction enter the game. The retardation effects are induced by the emission and absorption of virtual ultrasoft gluons which "feel" the binding effects of quark-antiquark interaction [24,25]. This constitutes a genuinely new feature which is absent in NLO and NNLO and is supposed to be the last qualitatively new source of the corrections.

The leading retardation effects which are under consideration here arise from the chromoelectric dipole interaction $g_s(\mathbf{r_q}-\mathbf{r_{\bar q}})\cdot\mathbf{E}$ of heavy quarkonium with virtual ultrasoft gluons. The corresponding diagram is shown in Fig. 1a. An interesting distinction from the similar QED process is that after emitting the ultrasoft gluon the quark-antiquark pair converts into the color octet sate with repulsive Coulomb potential $V_C^o(x)=(C_A/2-C_F)\alpha_s/x$ where $C_A=3$ is the eigenvalue of the quadratic Casimir operator of the adjoint representation of the color group. As a consequence the intermediate color octet states belong to the continuum part of the spectrum only.

The result for this ultrasoft contribution diverges in the ultraviolet (UV) region. This divergence is spurious. It arises in the process of scale separation due to the use of pNRQCD perturbation theory at short distances where it is inapplicable. We use the dimensional regularization (DR) with $d=4-2\epsilon$ space-time dimensions to handle the UV and infrared (IR) divergences of the effective theory which are of the form $1/\epsilon^n$ $(n=1,2,\ldots)$ as $\epsilon\to 0$ [22,26,27]. Compared to the "traditional" NRQCD approach endowed with an explicit momentum cutoff and a fictitious photon mass to regulate the ultraviolet and infrared behavior [4] this scheme has the advantage that the contributions from the different scales are matched automatically. In the total N^{3}LO result the poles in $1/\epsilon$ in the ultrasoft contribution are canceled by the infrared poles coming from the hard and soft scale corrections. However since these corrections are still unknown we subtract the divergent part according to the $\overline{\rm MS}$ scheme. This means that the same scheme must be used for the calculation of the hard and soft scale corrections. As a consequence the partial result for the ultrasoft contribution depends on the auxiliary ultrasoft scale μ_{us} which drops out in the total result.

The corresponding corrections to the Coulomb energy levels and the wave functions at the origin

$$E_n = E_n^C + \Delta E_n\,,$$

$$|\psi_n(0)|^2 = \left|\psi_n^C(0)\right|^2\left(1+\Delta\psi_n^2\right) \qquad (9)$$

read [24]

$$\Delta E_n = -\frac{2\alpha_s^3}{3\pi}E_n^C\left\{\left[\frac{1}{4}C_A^3+\frac{2}{n}C_A^2C_F\right.\right.$$

$$\left.+\left(\frac{6}{n}-\frac{1}{n^2}\right)C_AC_F^2+\frac{4}{n}C_F^3\right] \qquad (10)$$

$$\left.\times\left(\ln\frac{\mu_{us}}{E_1^C}+\frac{5}{6}-\ln 2\right)+C_F^3L_n^E\right\}\,,$$

$$\Delta\psi_n^2 = -\frac{2\alpha_s^3}{\pi}\left[\left(\frac{1}{4}C_A^2C_F+\frac{1}{n^2}C_AC_F^2+\frac{1}{n^2}C_F^3\right)\right.$$

$$\times \left(\ln \frac{\mu_{us}}{E_1^C} + \frac{1}{2} - \ln 2 \right) + C_F^3 L_n^\psi \Bigg] \,. \qquad (11)$$

Here L_n^E and L_n^ψ are the QCD analogs of the famous QED "Bethe logarithms". They represent a pure retardation effect which cannot be reduced to the instantaneous interaction contribution. For $n = 1,\ 2,\ 3$ their numerical values are [24]

$$\begin{aligned} L_1^E &= -81.5379\,, & L_1^\psi &= -5.7675\,, \\ L_2^E &= -37.6710\,, & L_2^\psi &= \ \ 0.7340\,, \\ L_3^E &= -22.4818\,, & L_3^\psi &= \ \ 2.2326\,. \end{aligned} \qquad (12)$$

3.2. Non-RG logarithmic corrections

The origin of the logarithmic corrections which are not generated by the RG running of α_s is the presence of several scales in the nonrelativistic dynamics. The logarithmic integration over a loop momentum between different scales yields a power of $\ln(1/\beta)$ and in the approximately Coulomb system we have $\beta \propto \alpha_s$. These corrections can always be associated with the divergences of the effective theory [28–30]. By contrast the RG logarithms are well known and may be resummed by an appropriate scale choice.

In the heavy quarkonium the non-RG logarithms first arise in NNLO corrections to the wave functions at the origin. They are generated by the following $\mathcal{O}(\beta^2)$ operators in the effective nonrelativistic Hamiltonian:

$$-\frac{C_F C_A \alpha_s^2}{2 m_q x^2} + \frac{C_F \alpha_s}{2 m_q^2} \left\{ \partial_{\mathbf{x}}^2, \frac{1}{x} \right\}$$

$$+ \left(1 + \frac{4}{3} \mathbf{S}^2 \right) \frac{\pi C_F \alpha_s}{m_q^2} \delta(\mathbf{x}) - \frac{\partial_{\mathbf{x}}^4}{4 m_q^3} \,, \qquad (13)$$

where $\mathbf{S}$ represents the spin of the quark-antiquark system and $\{.,.\}$ denotes the anticommutator. The corresponding corrections to the wave functions are proportional to the Coulomb Green function at the origin which is UV divergent and in DR is of the following form

$$G_C(0,0,E) = \frac{m_q \lambda_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{-\mu^2}{m_q E} + \ldots \right) \,, \qquad (14)$$

where the ellipsis stands for the nonlogarithmic contribution. The pole term in Eq. (14) is canceled by the $\mathcal{O}(\alpha_s^2)$ IR pole of the hard matching coefficient $C(\alpha_s)$. Thus the scale μ in the logarithm is to be identified with the hard scale m_q and the sought corrections for the spin one states which are of the main interest read [11,13]

$$\Delta\psi_n^2(0) = C_F \alpha_s^2 \left(\frac{2}{3} C_F + C_A \right) \ln \frac{1}{\alpha_s} \,. \qquad (15)$$

The residual operators in the NNLO effective nonrelativistic Hamiltonian which are not contained in Eq. (13) correspond to the purely perturbative corrections to the static Coulomb potential. The corresponding corrections to the wave functions at the origin [11,13,14] contain RG logarithms of the form $\alpha_s^2 \ln^m(\mu/\alpha_s m_q)$ $(m = 1, 2)$ which vanish for $\mu = \alpha_s m_q$. This also holds in N^{3}LO for the RG logarithms of the form $\alpha_s^3 \ln^m(\mu/\alpha_s m_q)$ $(m = 1, 2, 3)$ because the ultrasoft effects enter the stage only in N^{3}LO so that the corresponding running of the strong coupling constant at the ultrasoft scale only becomes relevant in N^{4}LO. An important point here is that starting from NNLO the hard matching coefficient $C(\alpha_s)$ receives a nonvanishing anomalous dimension. Therefore starting from N^{3}LO the running of α_s in $C(\alpha_s)$ should be taken into account to match the scale dependence of the wave functions at $\mu = \alpha_s m_q$.

In N^{3}LO the non-RG leading logarithmic corrections are produced by the one-loop renormalization of the $\mathcal{O}(\beta^2)$ operators. In dimensional regularization the pole part of the correction is

$$\frac{1}{2\epsilon} \frac{C_F \alpha_s}{\pi} \left\{ -\frac{C_A^3 \alpha_s^3}{12 x} - \left(\frac{4}{3} C_F + \frac{2}{3} C_A \right) \frac{C_A \alpha_s^2}{m_q x^2} \right.$$

$$+ \frac{2}{3} \frac{C_A \alpha_s}{m_q^2} \left\{ \partial_{\mathbf{x}}^2, \frac{1}{x} \right\} - \left(\frac{16}{3} C_F - \frac{8}{3} C_A \right) \frac{\pi \alpha_s}{m_q^2} \delta(\mathbf{x})$$

$$\left. + \left[\frac{2}{3} C_F + \left(\frac{17}{3} - \frac{7}{3} \mathbf{S}^2 \right) C_A \right] \frac{\pi \alpha_s}{m_q^2} \delta(\mathbf{x}) \right\} \,, \qquad (16)$$

where the first three terms contained within the parentheses represent the IR divergence while the fourth one embodies the UV divergence of the potential. The IR poles are canceled by the ultrasoft contribution with the characteristic scale $\alpha_s^2 m_q$ and may be read off from [24,28] while the UV poles are canceled by the IR poles of the hard

coefficients and may be extracted from [31,32]. Thus the divergences of the effective theory endow the operators in the nonrelativistic Hamiltonian with anomalous dimensions. This results in the logarithmic corrections to the spin one $l = 0$ energy levels [28,29]

$$\Delta E_n = -E_n \frac{\alpha_s^3}{\pi} \left\{ \frac{3}{n} C_F^3 + \left[\frac{9}{2n} - \frac{2}{3n^2} \right] C_F^2 C_A + \frac{4}{3n} C_F C_A^2 + \frac{1}{6} C_A^3 \right\} \ln \frac{1}{\alpha_s} . \quad (17)$$

Note that the IR poles of Eq. (16) introduce a factor $\ln(E_1^C/\lambda_s) \approx \ln \alpha_s$ while the UV poles contribute a factor $\ln(m_q/\lambda_s) \approx \ln(1/\alpha_s)$.

Some of the operators with singular coefficients in Eq. (16) also result in the contributions to the wave functions which is proportional to the singular Coulomb Green function at the origin. The overlapping logarithmic divergences lead to the double pole in $1/\epsilon$ and therefore to the double logarithmic contributions which for the spin one states read [29]

$$\Delta\psi_n^2(0) = -\frac{C_F \alpha_s^3}{\pi} \left[\frac{3}{2} C_F^2 + \frac{9}{4} C_F C_A + \frac{2}{3} C_A^2 \right] \ln^2 \frac{1}{\alpha_s} . \quad (18)$$

The calculation of the subleading single logarithmic contributions to the wave functions is much more involved and the complete result for QCD bound states is not available. However two important parts of these corrections are known. The Abelian part of the correction to the ground sate wave function can be read of the positronium lifetime analysis [30]. Not that one has to exclude the one photon annihilation contribution from the positronium result because this is absent in QCD due to the color conservation. The trivial non-Abelian contribution originates from the interference between the non-Abelian part of Eq. (15) and the $\mathcal{O}(\alpha_s)$ term in the hard renormalization coefficient. The total known single logarithmic contribution to the spin one ground state wave function reads

$$\Delta\psi_n^2(0) = \frac{C_F^2 \alpha_s^3}{\pi} \left[\left(\frac{7}{90} - 8 \ln 2 \right) C_F - 4 C_A \right] \ln \frac{1}{\alpha_s} . \quad (19)$$

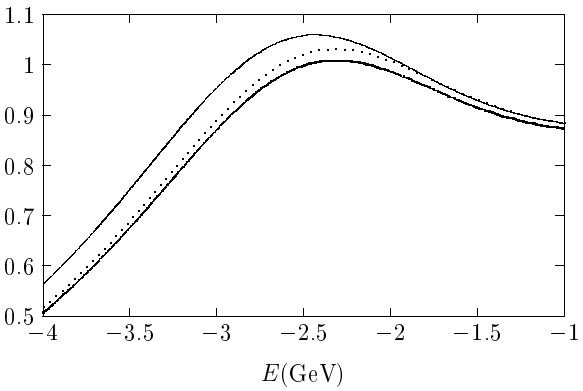

Figure 2. The normalized cross section $R(E)$ in NNLO (solid line) and with the complete leading logarithmic and known part of the subleading logarithmic N^{3}LO corrections included (dotted line and bold solid line respectively).

Several attempts have been made to sum up the high order RG and non-RG logarithmic corrections [16,33]. The problem however is not completely solved yet.

4. PHENOMENOLOGICAL APPLICATIONS

4.1. Top quark phenomenology

The relatively large electroweak top quark width Γ_t and characteristic scale of the nonrelativistic Coulomb dynamics $\alpha_s^2 m_t$ are considerably larger than $\Lambda_{\rm QCD}$ and serve as an effective infrared cutoff for long distance nonperturbative strong interaction effects. This makes perturbative QCD applicable for the theoretical description of the threshold top quark production. At the same time numerically $\Gamma_t \sim \alpha_s^2 m_t$ and the Coulomb effects are not completely dumped by the non-zero top quark width that should be properly taken into account.

In order to analyze the significance of the N^{3}LO logarithmic corrections to the cross section $R(E)$ we start from the NNLO result of [15] and add the contributions from Eqs. (17, 18) and Eq. (19). The results are plotted on Fig. 2. The input parameters are taken to be $\alpha_s(M_Z) = 0.118$,

$m_t = 175$ GeV and $\Gamma_t = 1.43$ GeV. The soft normalization scale of α_s in the nonrelativistic Coulomb problem is determined from the condition $\mu_s = 2\alpha_s(\mu_s)m_t$. The relatively large soft normalization point is taken to improve the convergence on the perturbative series for the spectral density in NNLO. The instability of the top quark is implemented by the complex energy shift $E \to E + i\Gamma_t$ in Eq. (6). This accounts for the leading imaginary electroweak contribution to the pNRQCD Hamiltonian. Numerically the non-RG logarithmic corrections are comparable to the contribution from the N^{3}LO RG logarithms which may be estimated from the renormalization scale dependence of the NNLO result. The effect of the N^{3}LO logarithms is twofold. The normalization of the cross section is reduced by about 10% around the $1S$ peak and the energy gap between the $1S$ peak and the nominal threshold is decreased by roughly 10%. This partially compensates the effect of huge NNLO corrections.

Since the ultrasoft corrections are scheme dependent we do not include them into the numerical analysis. However it is quite interesting to estimate their numerical importance. If we take $\mu_{us} = \alpha_s^2 m_t$ to cancel the logarithms of α_s in Eqs. (10, 11) the ultrasoft correction to the $1S$ peak energy is about -200 MeV and to the $1S$ peak normalization is only about $+2\%$ due to some cancellations. Note that one power of α_s in these equations refers to the ultrasoft gluon interaction and should be evaluated at the ultrasoft scale $\alpha_s^2 m_t$ while the two residual powers of α_s originate from the Coulomb Green function and should be evaluated at the soft scale $\alpha_s m_t$.

The stability of the perturbation theory for the $1S$ peak energy (but not for its normalization!) up to NNLO can be manifestly improved by using an infrared safe mass parameter instead of the pole mass in the analysis [13,16–20]. We should note that the corrections under consideration are nor related to the RG running of α_s and cannot be taken into account by the renormalon based mass redefinition. On the other hand in some cases the behavior of the perturbation perturbation theory can be improved by using the direct relations between the physical observables [11].

4.2. Bottom quark phenomenology

In the case of bottom quark-antiquark production the nonperturbative effects are much more significant and one is led to use the sum rule approach [2] to get them under control. Specifically appealing quark-hadron duality one matches the theoretical results for the moments of the spectral density

$$\mathcal{M}_n = (4m_q^2)^n \int_0^\infty ds \frac{R(s)}{s^{n+1}}\,. \qquad (20)$$

For sufficiently large n the moments are saturated by the near-threshold region. Then the main contribution to the experimental moments comes from the Υ resonances which are measured with high precision. On the other hand for n of order $\mathcal{O}\left(1/\alpha_s^2\right)$ the Coulomb effects should be properly taken into account on the theoretical side. In order to analyze the N^{3}LO logarithmic corrections to the Υ sum rules we upgrade the NNLO result of [11] by including Eqs. (17, 18) and Eq. (19). We fix the strong coupling constant and focus on the determination of the bottom quark mass. At present this seems to be the most interesting application of the sum rules. In the bottom-quark case α_s at the nominal ultrasoft scale seems to be too large for a reliable perturbative calculation. Thus to be on the safe side we redefine the ultrasoft scale to be $3\alpha_s^2 m_b$ so that in both cases $\alpha_s = 0.34$. We find that the inclusion of the N^{3}LO logarithms in the sum rules leads to a reduction of the extracted $\overline{\rm MS}$ mass value by approximately 150 MeV for moderate values of n, $5 < n < 15$. The result essentially depends on μ_s and μ_{us} because the scale dependence of the correction is compensated only by higher-order terms. For example, for $\mu_s \sim \mu_{us} \sim m_b$ the perturbative series for the moments converges much better and the correction to the $\overline{\rm MS}$ mass is only about -30 MeV. This result implies that the unsertainty of the value $m_{\overline{\rm MS}}(m_{\overline{\rm MS}}) \approx 4.2$ GeV obtained from NNLO analysis of Υ sum rules [11,13,19,20] is at least about 100 MeV.

In the case of the $\Upsilon(1S)$ meson the local duality is expected to work though the nonperturbative effects are much more important than in the sum rules approach. If we assume the $\Upsilon(1S)$ meson to be a perturbative system the results of Sect. 3 can

be applied to compute the corrections to its mass which is determined by the binding energy and leptonic width which is determined by the wave function at the origin. In this way we find that the N^{3}LO logarithmic correction to the $\Upsilon(1S)$ mass is about -170 MeV and to its leptonic width is about -70% (the leading and subleading logarithms are approximately equal). The ultrasoft correction to the $\Upsilon(1S)$ mass is about -110 MeV and to its leptonic width is about -10%.

It is interesting to compare the perturbative ultrasoft contribution to the leading nonperturbative contribution of the gluonic condensate due to vacuum fluctuations of the gluonic field at the scale $\Lambda_{\rm QCD}$ which is generated by the similar diagram with the broken gluon propagator shown in Fig. 1b in order to conclude how "perturbative" the heavy quarkonium is. In the case of the energy level the leading nonperturbative contribution is given by [34]

$$\Delta E_1 = \frac{117 m_q}{1275 \lambda_s^4} \left\langle \alpha_s G^a_{\mu\nu} G^{a\mu\nu} \right\rangle . \qquad (21)$$

Using the standard literature value $\langle \alpha_s G^2 \rangle \approx 0.06$ GeV^4 we have $\Delta E_1 \approx 60$ MeV which is of the same scale as the ultrasoft contribution.

5. CONCLUSION

In this talk we reviewed the first steps towards the N^{3}LO analysis of the nonrelativistic heavy quark-antiquark system. Two special classes of N^{3}LO contributions to the key parameters of heavy quark-antiquark bound states were considered, namely the non-RG corrections enhanced by a power of $\ln(1/\alpha_s)$ and the retardation effects. They are not related to the RG running of α_s and can be considered as typical representatives of the N^{3}LO corrections.

We gave the numerical estimates of the above corrections for the top and bottom quark systems. The corrections turn out to be comparable to the NNLO ones and reach 10% in magnitude even in the case of top where $\alpha_s \approx 1/10$. For $\Upsilon(1S)$ meson they are out of control. This tells us that the NRQCD expansion is not a fast convergent series for the physical value of the strong coupling constant. It is highly desirable to complete the calculation of N^{3}LO corrections to get more insight of the nonrelativistic heavy quark dynamics and the structure of the nonrelativistic perturbation theory. Although there is no conceptual problem on the theoretical side this analysis is extremely difficult from the technical point of view and includes, for example, the calculation of the three-loop hard matching coefficient and the three-loop static potential.

Finally we would like to mention that the subleading nonperturbative contributions [35] and charm quark mass effects [36] which are relevant for the bottom quark physics but numerically are not so important as the perturbative corrections.

Acknowledgments

The author thanks the organizers of QCD-00 conference for making this fruitful meeting. This work is supported in part by the Bundesministerium für Bildung und Forschung under Contract No. 05 HT9GUA 3 and by the Volkswagen Foundation under Contract No. I/73611.

REFERENCES

1. T. Appelquist and H. D. Politzer Phys. Rev. Lett. **34** (1975) 43.
2. V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, V. I. Zakharov, Phys. Rev. Lett. **38** (1977) 626; Erratum *ibid.*, 38 (1977) 791; Phys. Rep. **C41** (1978) 1.
3. V. S. Fadin, V. A. Khoze, JETP Lett. 46 (1987) 525 [Pis'ma Zh. Eksp. Teor. Fiz. 46 (1987) 417]; Sov. J. Nucl. Phys. 48 (1988) 309 [Yad. Fiz. 48 (1988) 487].
4. W. E. Caswell, G. P. Lepage, Phys. Lett. 167 B (1986) 437; G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, K. Hornbostel, Phys. Rev. D 46 (1992) 4052; G. T. Bodwin, E. Braaten, G. P. Lepage, Phys. Rev. D 51 (1995) 1125; Erratum *ibid.*, 55 (1997) 5855.
5. Y. Schröder, Phys. Lett. **B447** (1999) 321; M. Peter, Phys. Rev. Lett. **78** (1997) 602; Nucl. Phys. **B501** (1997) 471.
6. A. Czarnecky and K. Melnikov, Phys. Rev. Lett. **80** (1998) 2531; M. Beneke, A. Signer and V. A. Smirnov, Phys. Rev. Lett. **80**

(1998) 2535.
7. A. Pineda and F.J. Yndurain, Phys. Rev. **D58** (1998) 094022.
8. J. H. Kühn, A. A. Penin and A. A. Pivovarov, Nucl. Phys. **B534** (1998) 356.
9. A. H. Hoang and T. Teubner, Phys. Rev. **D58** (1998) 114023.
10. K. Melnikov, A. Yelkhovsky, Nucl. Phys. **B528** (1998) 59.
11. A. A. Penin and A. A. Pivovarov, Phys. Lett. **B435** (1998) 413; Phys. Lett. **B443** (1998) 264; Nucl. Phys. **B549** (1999) 217.
12. A. H. Hoang, Phys. Rev. **D59** (1999) 014039.
13. K. Melnikov and A. Yelkhovsky, Phys. Rev. **D59** (1999) 114009.
14. A. A. Penin and A. A. Pivovarov, Nucl.Phys. **B550** (1999) 375.
15. A. A. Penin, A. A. Pivovarov, Preprint No. MZ-TH/98-61 and hep-ph/9904278, to appear in Sov. J. Nucl. Phys. [Yad. Fiz.].
16. M. Beneke, A. Singer and V. A. Smirnov, Phys. Lett. **B454** (1999) 137.
17. T. Nagano, A. Ota and Y. Sumino, Phys. Rev. **D60** (1999) 114014.
18. A. H. Hoang and T. Teubner, Phys. Rev. **D60** (1999) 114027.
19. M. Beneke, A. Signer, Phys. Lett. **B471** (1999) 233.
20. A. H. Hoang, Phys. Rev. **D61** (2000) 034005.
21. A. H. Hoang, M. Beneke, K. Melnikov, T. Nagano, A. Ota, A. A. Penin, A. A. Pivovarov, A. Signer, V. A. Smirnov, Y. Sumino, T. Teubner, O. Yakovlev, A. Yelkhovsky, Eur. Phys. J. direct **C3** (2000) 1.
22. M. Beneke, V.A. Smirnov, Nucl. Phys. **B522** (1998) 321.
23. A. Pineda, J. Soto, Nucl. Phys. Proc. Suppl. **64** (1998) 428.
24. B. A. Kniehl and A. A. Penin, Nucl. Phys. **B563** (1999) 200.
25. N. Brambilla, A. Pineda, J. Soto, A. Vairo, Phys. Rev. **D60** (1999) 091502; Nucl. Phys. **B566** (2000) 275.
26. A. Pineda and J. Soto, Phys. Lett. **B420** (1998) 391; Phys. Rev. **D59** (1999) 016005.
27. A. Czarnecki, K. Melnikov, A. Yelkhovsky, Phys. Rev. **A59** (1999) 4316.
28. N. Brambilla, A. Pineda, J. Soto, A. Vairo, Phys. Lett. **B470** (1999) 215;
29. B. A. Kniehl and A. A. Penin, Nucl. Phys. **B577** (2000) 197.
30. B. A. Kniehl and A. A. Penin, Phys. Rev. Lett. **85** (2000) 1210; Erratum *ibid.*, to be published, hep-ph/0004267; R. Hill and G. P. Lepage, Report No. hep-ph/0003277; K. Melnikov and A. Yelkhovsky, Report No. SLAC-PUB-8557 and hep-ph/0008099.
31. A. V. Manohar, Phys. Rev. **D56** (1997) 230.
32. A. Pineda, J. Soto, Phys. Rev. **D58** (1998) 114011.
33. M. E. Luke, A. V. Manohar and I. Z. Rothstein, Phys. Rev. **D61** (2000) 074025. A. V. Manohar and I. W. Stewart, Phys. Rev. **D62** (2000) 014033. A. Pineda, J. Soto, Report No. CERN-TH-2000-205 and hep-ph/0007197.
34. M. B. Voloshin, Nucl. Phys. B 154 (1979) 365; Sov. J. Nucl. Phys. **36** (1982) 143 [Yad. Fiz. **36** (1982) 247]; H. Leutwyler, Phys. Lett. **B98** (1981) 447.
35. A. Pineda. Nucl. Phys. **B494** (1997) 213.
36. D. Eiras and J. Soto, Report No. UB-ECM-PF-00-07 and hep-ph/0005066; M. Melles, Report No. PSI-PR-00-03 and hep-ph/0001295; A. H. Hoang, Report No. CERN-TH-2000-227 and hep-ph/0008102.