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The Drell-Yan-Levy Relation: ep vs e^+e^- Scattering to $O(\alpha_s^2)$*†

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We study the validity of a relation by Drell, Levy and Yan (DLY) connecting the deep inelastic structure (DIS) functions and the single-particle fragmentation functions in e^+e^- annihilation which are defined in the spacelike ($q^2 < 0$) and timelike ($q^2 > 0$) regions, with respect to physical evolution kernels for the two processes to $O(\alpha_s^2)$. We also comment on a relation proposed by Gribov and Lipatov.

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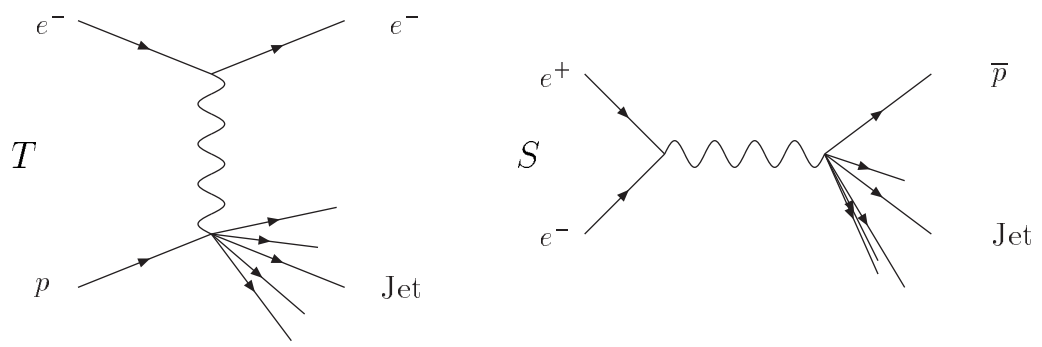
1 The DLY-Relation

Right after the discovery of the partonic structure of nucleons the question arose, whether at large virtualities of the exchanged photon the hard processes $e^- p \to e^- X$ and $e^+ e^- \to \overline{p} X$ are related by crossing from the $t-$ to the $s-$channel, [1,2].



DIS $\qquad\qquad$ e^+e^- annihilation

For the two–fold differential scattering cross sections for both processes,

$$\frac{d^2\sigma}{dx dQ^2} \sim L_{\mu\nu} W^{\mu\nu} \ , \tag{1}$$

one may express channel crossing by the hadronic tensors $W_{\mu\nu}$ as done by Drell, Levy, and Yan [1]

$$W^{(S)}_{\mu\nu}(q,p) = -W^{(T)}_{\mu\nu}(q,-p) \ . \tag{2}$$

At that time partons were assumed as fermionic particles, interacting via (pseudo) scalars, with $\delta(1-z)$-sources, where z denotes the longitudinal momentum fraction. Considering only ladder graphs at lowest order, such a crossing could be envisaged for the whole hadronic tensor.

Viewing these reactions in QCD, the picture changes. The sources of the partons are extended non-perturbative distributions with $z \, \epsilon \, [0,1]$, about which perturbative QCD cannot make a statement, even resumming whole classes of graphs. However, at large scales of Q^2 both processes factorize into the parton densities and perturbative evolution kernels, which rule the Q^2 behaviour. The question of crossing from the $t-$ to the $s-$ channel can thus be modified in studying it for the *factorized* evolution kernels at the one side within *perturbative QCD* and leaving the related question for the non-perturbative sources to *Lattice Gauge Theory*.

The scaling variables describing deep inelastic scattering at the one side and hadron fragmentation on the other side are x_B and x_E,

$$x_B = \frac{Q^2}{2p.q}, \quad 0 \le x_B \le 1 \quad \text{DIS} \tag{3}$$

$$x_E = \frac{2p.q}{Q^2}, \quad 0 \le x_E \le 1 \quad e^+e^- \text{ annihilation} . \tag{4}$$

The point $x = 1$ connects both domains and is usually a singular point. One may now calculate QCD evolution kernels for both domains. The central question of the present paper is, what are the conditions to *continue* the kernel obtained in one domain into that of the other. In general one cannot expect to find an analytic continuation in an arbitrarily chosen factorization scheme in which the process independent splitting functions are evaluated. However, one may form *physical evolution kernels*, in both domains, which are scheme–invariant and study their crossing behaviour. Thus the above question is directed to the connection of the physical evolution behaviour of *observables* as the structure and fragmentation functions. In the present paper we study this relation up to $O(\alpha_s^2)$. Early related investigations (partly before the advent of QCD) were performed in Refs. [3]–[11], and more recently in Refs. [12]–[15].

2 Scheme–invariant Evolution Equations

To investigate the crossing behaviour for the evolution kernels of structure and fragmentation functions we first derive physical evolution equations. The twist-2 contributions to these functions can be expressed in the form

$$F_i(x,Q^2) = \sum_{l=q,g} \left(C_{i,l}\Big(\alpha_s(\mu_r^2), \frac{Q^2}{\mu_f^2}, \frac{\mu_f^2}{\mu_r^2}\Big) \otimes f_l\Big(\alpha_s(\mu_r^2), \frac{\mu_f^2}{\mu^2}, \frac{\mu_f^2}{\mu_r^2}\Big) \right)(x), \tag{5}$$

where $C_{i,l}$ denote the Wilson coefficients, f_l the parton densities and $\otimes$ is the Mellin convolution. μ_f^2 and μ_r^2 are the factorization and renormalization scales, respectively. Beyond leading order the parton densities and Wilson coefficients obey factorization scheme–dependent evolution equations and are thus no observables. Their dependence on μ_f^2 can, however, be eliminated in expressing the non–singlet and singlet parton densities via physical observables, the scale dependence of which is finally studied. For the singlet case one obtains

$$\frac{\partial}{\partial t}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{pmatrix}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix} . \tag{6}$$

Here the evolution kernels K_{IJ}^N written in Mellin–moment space are no longer process–independent quantities for the evolution of the pair of observables $\{F_A^N, F_B^N\}$, but

scheme–independent quantities. Eq. (6) refers to the evolution variable $t = -(2/\beta_0) \times \ln(a_s(Q^2)/a_s(Q_0^2))$. β_0 is the lowest order β–function, and $a_s(Q^2) = \alpha_s(Q^2)/(4\pi)$. The physical evolution kernels read

$$K_{IJ}^N = \left[-4 \frac{\partial C_{I,m}^N(t)}{\partial t} \left(C^N\right)_{m,J}^{-1}(t) - \frac{\beta_0 a_s(Q^2)}{\beta(a_s(Q^2))} C_{I,m}^N(t) \gamma_{mn}^N(t) \left(C^N\right)_{n,J}^{-1}(t) \right] , \qquad (7)$$

with $\beta(a_s)$ the β–function and

$$K_{IJ}^N = \sum_{n=0}^{\infty} a_s^n(Q^2) \left(K^N\right)_{IJ}^{(n)} \ . \qquad (8)$$

One easily sees that the kernels Eq. (7) are very difficult to obtain in x–space, due to the inverse coefficient functions to be evaluated. Instead they take a simple form for the Mellin-transforms. The transformed coefficient functions are needed in *analytic form* in N, which are usually polynomials out of multiple alternating and non–alternating harmonic sums [16,17]. These expressions have to be analytically continued to complex values of N. It turns out that all Wilson coefficients to $O(\alpha_s^2)$ can be expressed by at most 26 basic functions of complex N, the analytic continuations of which can be found in Ref. [18].

Possible choices for the observables $\{F_A^N, F_B^N\}$ are $F_2, \partial F_2/\partial t$, $g_1, \partial g_1/\partial t$, and F_2, F_L. Here we denote by F_i and g_i the respective unpolarized and polarized structure *and* fragmentation functions. The physical evolution kernels, as obtained from the anomalous dimensions and coefficient functions, read :

System: $F_2, \partial F_2/\partial t$

Leading Order [19]:

$$\begin{aligned}
K_{22}^{N(0)} &= 0 \\
K_{2d}^{N(0)} &= -4 \\
K_{d2}^{N(0)} &= \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \\
K_{dd}^{N(0)} &= \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}
\end{aligned} \qquad (9)$$

Next-to-Leading Order [19]:

$$\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right]
\end{aligned}$$

$$
\begin{aligned}
&-\frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)}-\gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right)\\
&+\frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)}+\gamma_{gg}^{N(0)}-2\beta_0\right)\\
&-\frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[(\gamma_{qq}^{N(0)})^2-\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)}+2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}-2\beta_0\gamma_{qq}^{N(0)}\right]\\
&-\frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)}-\frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right)
\end{aligned}
\tag{10}
$$

The same structures apply to $g_1, \partial g_1/\partial t$.

System: $\boldsymbol{F_2}, \boldsymbol{\hat{F}_L}$

For convenience we define $\hat{F}_L^N \equiv F_L^N/(a_s(Q^2)C_{L,g}^{N(1)})$.
Leading Order [20]:

$$
\begin{aligned}
K_{22}^{N(0)} &= \gamma_{qq}^{N(0)}-\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\\
K_{2L}^{N(0)} &= \gamma_{qg}^{N(0)}\\
K_{L2}^{N(0)} &= \gamma_{gq}^{N(0)}-\left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2\gamma_{qg}^{N(0)}\\
K_{LL}^{N(0)} &= \gamma_{gg}^{N(0)}+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)
\end{aligned}
\tag{11}
$$

Next-to-Leading Order [15]:

$$
\begin{aligned}
K_{22}^{N(1)} &= \gamma_{qq}^{N(1)}-\frac{\beta_1}{\beta_0}\gamma_{qq}^{N(0)}-\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qg}^{N(1)}-\frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)}\right)\\
&\quad+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{qq}^{N(0)}-\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{gg}^{N(0)}\\
&\quad-\left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}+\left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2C_{2,g}^{N(1)}-\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right]\gamma_{qg}^{N(0)}
\end{aligned}
$$

$$
\begin{aligned}
& + C_{2,g}^{N(1)} \gamma_{gq}^{N(0)} + 2\beta_0 \left(C_{2,q}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} \right) \\
K_{2L}^{N(1)} &= \gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qg}^{N(0)} - C_{2,g}^{N(1)} (\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}) + 2\beta_0 C_{2,g}^{N(1)} \\
& + \left(C_{2,q}^{N(1)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} - \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right) \gamma_{qg}^{N(0)} \\
K_{L2}^{N(1)} &= \gamma_{gq}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qq}^{N(0)} \right) \\
& - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 \left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qg}^{N(0)} \right) - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{gg}^{N(0)} \right) \\
& + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,q}^{N(1)} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,g}^{N(1)} \right] \gamma_{qq}^{N(0)} \\
& - \left[\left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^3 C_{2,g}^{N(1)} + 2 \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right. \\
& \left. - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,q}^{N(1)} \right] \gamma_{qg}^{N(0)} + 2\beta_0 \left(\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right) \\
& + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} - C_{2,q}^{N(1)} + \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right) \gamma_{gq}^{N(0)} \\
& - \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,q}^{N(1)} \right] \gamma_{gg}^{N(0)} \\
K_{LL}^{N(1)} &= \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0} \gamma_{qg}^{N(0)} \right) \\
& - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} \gamma_{qq}^{N(0)} + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \right. \\
& \left. + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 C_{2,g}^{N(1)} \right] \gamma_{qg}^{N(0)}
\end{aligned}
$$

$$-C_{2,g}^{N(1)}\gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)}\gamma_{gg}^{N(0)} + 2\beta_0 \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \tag{12}$$

3 DLY–Relations for Evolution Kernels

The original crossing relation [1]

$$W_{\mu\nu}^{T}(q,p) = -W_{\mu\nu}^{S}(q,-p) \tag{13}$$

is modified to [11]

$$F_i^{(S)}(x_B) = -(-1)^{2(s_1+s_2)} x_E F_i^{(T)}\left(\frac{1}{x_E}\right) \quad , \quad i = 1,2,L \tag{14}$$

if particles of different spin s_i contribute, again considering ladder approaches to the problem with idealized sources.

In the following we give the analytic continuation relations, cf. Ref. [15], which yield the correct transformations for physical kernels up to $O(\alpha_s^2)$. They read :

$$\begin{aligned}
P(z) &\to z P(1/z) && (15)\\
P_{ii} &\to -P_{ii} && (16)\\
P_{qg}, P_{gq} &\to \text{cross color pre} - \text{factor} && (17)\\
\ln\left(Q^2/\mu_f^2\right)_{\text{space-like}} &\to \ln\left(Q^2/\mu_f^2\right)_{\text{time-like}} - i\pi \ . && (18)\\
\delta(1-z) &\to -\delta(1-z) && (19)\\
\ln(1-z) &\to \ln(1-z) - \ln(z) + i\pi && (20)\\
\ln(\varepsilon) &\to \ln(\varepsilon) + i\pi && (21)
\end{aligned}$$

Due to Eq. (19), Eq. (14) does not hold for $x_B = x_E = 1$, even in leading order, where the physical evolution kernels are the splitting functions.

In next–to–leading order the differences between the analytically continued space–like splitting functions and the time–like splitting functions are :

$$\begin{aligned}
\overline{P}_{qq}^{(1)S} - P_{qq}^{(1)T} &= -2\beta_0 Z_{qq}^{T(1)} + Z_{qg}^{T(1)} \otimes \overline{P}_{gq}^{(0)} - Z_{gq}^{T(1)} \otimes \overline{P}_{qg}^{(0)} \,,\\
\overline{P}_{qg}^{(1)S} - P_{gq}^{(1)T} &= -2\beta_0 Z_{qg}^{T(1)} + Z_{qg}^{T(1)} \otimes (\overline{P}_{gg}^{(0)} - \overline{P}_{qq}^{(0)})\\
&\quad + \overline{P}_{qg}^{(0)} \otimes (Z_{qq}^{T(1)} - Z_{gg}^{T(1)}) \,,\\
\overline{P}_{gq}^{(1)S} - P_{qg}^{(1)T} &= -2\beta_0 Z_{gq}^{T(1)} + Z_{gq}^{T(1)} \otimes (\overline{P}_{qq}^{(0)} - \overline{P}_{gg}^{(0)})\\
&\quad + \overline{P}_{gq}^{(0)} \otimes (Z_{gg}^{T(1)} - Z_{qq}^{T(1)}) \,,\\
\overline{P}_{gg}^{(1)S} - P_{gg}^{(1)T} &= -2\beta_0 Z_{gg}^{T(1)} + Z_{gq}^{T(1)} \otimes \overline{P}_{qg}^{(0)} - Z_{qg}^{T(1)} \otimes \overline{P}_{gq}^{(0)} \ , \qquad (22)
\end{aligned}$$

and do *not* vanish. Here we defined

$$Z_{ij}^{T(1)} = P_{ji}^{(0)} \cdot \Big(\ln(z) + a_{ji} \Big) \; ,$$

where for unpolarized scattering

$$a_{qq} = a_{gg} = 0 \quad , \quad a_{qg} = -\frac{1}{2} \quad , \quad a_{gq} = \frac{1}{2} \; , \tag{23}$$

and for polarized scattering

$$a_{ij} = 0 \; , \tag{24}$$

cf. also [13].

The transformation of the NLO coefficient functions $C_{1,q(g)}$ and $C_{L,q(g)}$ are :

$$\begin{aligned} C_{1,q}^{(T)(1)}(z) + \left\{ z\, C_{1,q}^{(S)(1)} \left(\frac{1}{z} \right) \right\} &= Z_{qq}^{(T)(1)} \\ \frac{1}{2} \left[C_{1,g}^{(T)(1)}(z) - \frac{C_F}{2 N_f T_f} \left\{ 2z C_{1,g}^{(S)(1)} \left(\frac{1}{z} \right) \right\} \right] &= Z_{qg}^{(T)(1)} \; , \end{aligned} \tag{25}$$

and

$$\begin{aligned} C_{L,q}^{(T)(1)}(z) - \frac{z}{2} C_{L,q}^{(S)(1)} \left(\frac{1}{z} \right) &= 0 \, , \\ \frac{1}{2} \left[C_{L,g}^{(T)(1)}(z) + \frac{C_F}{2 N_f T_f} \left\{ z\, C_{L,g}^{(S)(1)} \left(\frac{1}{z} \right) \right\} \right] &= 0 \; . \end{aligned} \tag{26}$$

Finally the NNLO unpolarized longitudinal coefficient functions [21] transform as [15] :

$$\begin{aligned} & C_{L,q}^{(T)(2)}(z) + \left\{ -\frac{z}{2}\, C_{L,q}^{(S)(2)} \left(\frac{1}{z} \right) \right\} = \\ & \qquad Z_{qq}^{(T)(1)} \otimes \frac{z}{2} C_{Lq}^{(1)S} \left(\frac{1}{z} \right) + Z_{gq}^{(T)(1)} \otimes \frac{C_F}{2 N_f T_f} \left\{ -\frac{z}{2}\, C_{L,g}^{(S)(1)} \left(\frac{1}{z} \right) \right\} \; , \\ & \frac{1}{2} \left[C_{L,g}^{(T)(2)}(z) + \frac{C_F}{2 N_f T_f} \left\{ z C_{L,g}^{(S)(2)} \left(\frac{1}{z} \right) \right\} \right] = \\ & \qquad Z_{qg}^{(T)(1)} \otimes \frac{z}{2} C_{L,q}^{(S)(1)} \left(\frac{1}{z} \right) + Z_{gg}^{(T)(1)} \otimes \frac{C_F}{2 N_f T_f} \left\{ -\frac{z}{2}\, C_{L,g}^{(1)S} \left(\frac{1}{z} \right) \right\} \; . \end{aligned}$$

The transformations for the other NNLO coefficient functions [21,22] are given in Ref. [15].

Let us define the difference between the time–like physical evolution kernel and the analytically continued space–like evolution kernel by

$$\delta K_{IJ} := K_{IJ}^{T} - \overline{K}_{IJ}^{S}. \tag{27}$$

Using identities for pair–convolutions of higher order Nielsen integrals, see Ref. [15], one finally obtains that

$$\begin{aligned} \delta K_{d2} &= 0 \\ \delta K_{dd} &= 0 \ , \end{aligned} \tag{28}$$

and

$$\begin{aligned} \delta K_{22}^{N(1)} &= 0 \\ \delta K_{L2}^{N(1)} &= 0 \\ \delta K_{2L}^{N(1)} &= 0 \\ \delta K_{LL}^{N(1)} &= 0 \ , \end{aligned} \tag{29}$$

showing explicitely the validity of the Drell–Levy–Yan relation for the physical evolution kernels in leading and next–to–leading order at the above choice of observables.

We finally would like to comment on a relation suggested by Gribov and Lipatov [23],

$$\overline{K}(x_E, Q^2) = K(x_B, Q^2)$$

This relation holds for the LO non-singlet contributions and some pieces in the NLO non-singlet contributions, but is generally violated beyond LO.

4 Conclusions

The scale evolution of structure and fragmentation functions can be represented in terms of physical evolution kernels and observable non-perturbative input distributions. The physical evolution kernels of either choice of observables are related for the evolution of structure and fragmentation functions by an analytic continuation (DLY relation) from $0 \leq x < 1$ to $1 < x < \infty$ up to $O(\alpha_s^2)$, for which transformation rules were derived. The Gribov–Lipatov relation is violated beyond LO. An extension of the present investigation to $O(\alpha_s^3)$ requires the knowledge of the hitherto unknown 3–loop singlet anomalous dimensions. The DLY relation for the evolution kernels is not necessarily expected to hold to arbitrary high orders due to the emergence of new production thresholds for the s-channel process. An interesting test of QCD can be carried out in comparing the scaling violations of structure and fragmentation functions using factorization scheme–independent evolution equations.

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