Revised Version

DESY 01-054
TUM-HEP-412/01

arXiv:hep-ph/0104238 v2 1 Aug 2001

DESY 01–054
TUM–HEP–412/01
hep–ph/0104238
August 2001

Bounds on the Unitarity Triangle, $\sin 2\beta$ and $K \to \pi\nu\overline{\nu}$ Decays in Models with Minimal Flavour Violation

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Abstract

We present a general discussion of the unitarity triangle from ε_K, $\Delta M_{d,s}$ and $K \to \pi\nu\overline{\nu}$ in models with minimal flavour violation (MFV), allowing for arbitrary signs of the generalized Inami–Lim functions F_{tt} and X relevant for $(\varepsilon_K, \Delta M_{d,s})$ and $K \to \pi\nu\overline{\nu}$, respectively. In the models in which F_{tt} has a sign opposite to the one in the Standard Model, i.e. $F_{tt} < 0$, the data for $(\varepsilon_K, \Delta M_{d,s})$ imply an absolute lower bound on the $B_d \to \psi K_{\rm S}$ CP asymmetry $a_{\psi K_{\rm S}}$ of 0.69, which is substantially stronger than 0.42 arising in the case of $F_{tt} > 0$. An important finding of this paper is the observation that for given $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $a_{\psi K_{\rm S}}$ only *two* values for $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$, corresponding to the two signs of X, are possible in the full class of MFV models, independently of any new parameters arising in these models. This provides a powerful test for this class of models. Moreover, we derive *absolute* lower and upper bounds on $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ as functions of $Br(K^+ \to \pi^+\nu\overline{\nu})$. Using the present experimental upper bounds on $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $|V_{ub}/V_{cb}|$, we obtain the absolute upper bound $Br(K_{\rm L} \to \pi^0\nu\overline{\nu}) < 7.1 \cdot 10^{-10}$ (90% C.L.).

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1 Introduction

The exploration of CP violation in $B_d \to \psi K_{\rm S}$ decays and the related determination of the angle β in the usual unitarity triangle of the Cabibbo–Kobayashi–Maskawa (CKM) matrix are hot topics in present particle physics [1]-[17]. The corresponding time-dependent CP asymmetry takes the following general form:

$$a_{\psi K_{\rm S}}(t) \equiv \frac{\Gamma(B^0_d(t) \to \psi K_{\rm S}) - \Gamma(\overline{B^0_d}(t) \to \psi K_{\rm S})}{\Gamma(B^0_d(t) \to \psi K_{\rm S}) + \Gamma(\overline{B^0_d}(t) \to \psi K_{\rm S})} = \mathcal{A}_{\rm CP}^{\rm dir}\, \cos(\Delta M_d t) + \mathcal{A}_{\rm CP}^{\rm mix}\, \sin(\Delta M_d t), \tag{1}$$

where the rates correspond to decays of initially, i.e. at time $t = 0$, present B^0_d- or $\overline{B^0_d}$-mesons, and $\Delta M_d > 0$ denotes the mass difference between the mass eigenstates of the B^0_d–$\overline{B^0_d}$ system. The quantities $\mathcal{A}_{\rm CP}^{\rm dir}$ and $\mathcal{A}_{\rm CP}^{\rm mix}$ are usually referred to as "direct" and "mixing-induced" CP-violating observables, respectively. In the Standard Model (SM), (1) simplifies as follows [18]:

$$a_{\psi K_{\rm S}}(t) = -\sin 2\beta \sin(\Delta M_d t) \equiv -\, a_{\psi K_{\rm S}} \sin(\Delta M_d t), \tag{2}$$

thereby allowing the extraction of $\sin 2\beta$. It should be noted that a measurement of a non-vanishing value of $\mathcal{A}_{\rm CP}^{\rm dir}$ at the level of 10% would be a striking indication for new physics, as emphasized in a recent analysis of the $B \to \psi K$ system [13]. However, for the particular kind of physics beyond the SM considered in the present paper, direct CP violation in $B_d \to \psi K_{\rm S}$ decays is negligible.

In the future, $\sin 2\beta$ can also be determined through the measurement of the branching ratios for the rare decays $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_{\rm L} \to \pi^0 \nu\overline{\nu}$ [19]. In the SM, we have to an excellent approximation

$$\sin 2\beta = \frac{2 r_s}{1 + r_s^2}, \tag{3}$$

with

$$r_s = \sqrt{\sigma}\, \frac{\sqrt{\sigma(B_1 - B_2)} - P_c(\nu\overline{\nu})}{\sqrt{B_2}}\,. \tag{4}$$

Here B_1 and B_2 are the following "reduced" branching ratios:

$$B_1 = \frac{Br(K^+ \to \pi^+ \nu\overline{\nu})}{4.42 \cdot 10^{-11}}, \qquad B_2 = \frac{Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})}{1.93 \cdot 10^{-10}}, \tag{5}$$

the quantity $P_c(\nu\overline{\nu}) = 0.40 \pm 0.06$ [20] describes the internal charm-quark contribution to $K^+ \to \pi^+ \nu\overline{\nu}$, and

$$\sigma \equiv \frac{1}{(1 - \lambda^2/2)^2}, \tag{6}$$

with λ being one of the Wolfenstein parameters [21]. In writing (3), we have assumed that $\sin 2\beta > 0$, as expected in the SM. The numerical values in (5) and the value for

$P_c(\nu\overline{\nu})$ differ slightly from those given in [19, 20] due to $\lambda = 0.222$ used here instead of $\lambda = 0.22$ used in these papers. We will return to this point below.

The strength of formulae (2) and (3) is their theoretical cleanness, allowing a precise determination of $\sin 2\beta$ free of hadronic uncertainties that is independent of other parameters like $|V_{cb}|$, $|V_{ub}/V_{cb}|$ and m_t. Therefore the comparison of these two determinations of $\sin 2\beta$ with each other is particularly well suited for tests of CP violation in the SM, and offers a powerful tool to probe the physics beyond it [19, 22].

The simplest class of extensions of the SM are those models with "minimal flavour violation" (MFV) in which the contributions of any new operators beyond those present in the SM are negligible. In these models, all flavour-changing transitions are still governed by the CKM matrix, with no new complex phases beyond the CKM phase [23, 24]. If one assumes, in addition, that all new-physics contributions which are not proportional to $V_{td(s)}$ are negligible [24], then all the SM expressions for the decay amplitudes and particle–antiparticle mixing can be generalized to the MFV models by simply replacing the m_t-dependent Inami–Lim functions [25] by the corresponding functions F_i in the extensions of the SM. The latter functions acquire now additional dependences on the parameters present in these extensions. Examples are the Two-Higgs-Doublet Model II (THDM) and the constrained MSSM if $\tan\bar{\beta} = v_2/v_1$ is not too large. For MFV models, direct CP violation in $B_d \to \psi K_S$ is negligible and the $\cos(\Delta M_d t)$ term in (1) vanishes.

Let us consider the off-diagonal element of the B_q^0–$\overline{B_q^0}$ mixing matrix as an example ($q \in \{d, s\}$). In the SM, we have (for a detailed discussion, see [26])

$$M_{12}^{(q)} = \frac{G_F^2 M_W^2}{12\pi^2}\eta_B m_{B_q}\hat{B}_{B_q}F_{B_q}^2(V_{tq}^* V_{tb})^2 S_0(x_t)\, e^{i(\pi-\phi_{CP}(B_q))}, \tag{7}$$

where $\hat{B}_{B_q}$ is a non-perturbative parameter, F_{B_q} the B_q-meson decay constant, and $\eta_B = 0.55$ a perturbative QCD factor [27, 28], which is common to $M_{12}^{(d)}$ and $M_{12}^{(s)}$. Finally, the convention-dependent phase $\phi_{CP}(B_q)$ is defined through

$$(\mathcal{CP})|B_q^0\rangle = e^{i\phi_{CP}(B_q)}|\overline{B_q^0}\rangle. \tag{8}$$

In the MFV models, we have just to replace the Inami–Lim function $S_0(x_t)$ resulting from box diagrams with $(t, W^\pm)$ exchanges through an appropriate new function, which we denote by F_{tt} [5, 24]:

$$S_0(x_t) \to F_{tt}. \tag{9}$$

Expression (7) plays a key role for (2), as $\Delta M_d = 2|M_{12}^{(d)}|$, and 2β results from the difference of $\arg(M_{12}^{(d)})$ and the weak phase of the $B_d \to \psi K_S$ decay amplitude, where the convention-dependent quantity $\phi_{CP}(B_q)$ cancels.

Two interesting properties of the MFV models have recently been pointed out [24, 12]:

- There exists a universal unitarity triangle (UUT) [24] common to all these models and the SM that can be constructed by using measurable quantities that depend on the CKM parameters but are not polluted by the new parameters present in the extensions of the SM. These quantities simply do not depend on the functions F_i.

- There exists an absolute lower bound on $\sin 2\beta$ [12] that follows from the interplay of ΔM_d and ε_K, measuring "indirect" CP violation in the neutral kaon system. It depends only on $|V_{cb}|$ and $|V_{ub}/V_{cb}|$, as well as on the non-perturbative parameters $\hat{B}_K$, $F_{B_d}\sqrt{\hat{B}_d}$ and ξ entering the standard analysis of the unitarity triangle.

The UUT can be constructed, for instance, by using $\sin 2\beta$ from (2) or (3), and the ratio $\Delta M_s/\Delta M_d$. The relevant formulae can be found in [24], where also other quantities suitable for the determination of the UUT are discussed. Concerning the lower bound on $\sin 2\beta$, a conservative scanning of all relevant input parameters gives [12, 15]

$$(\sin 2\beta)_{\rm min} = 0.42, \tag{10}$$

corresponding to $\beta \geq 12^\circ$. This bound could be considerably improved when the values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $\hat{B}_K$, $F_{B_d}\sqrt{\hat{B}_d}$, ξ and – in particular of ΔM_s – will be known better [12, 15]. A handy approximate formula for $\sin 2\beta$ as a function of these parameters has recently been given in [17]. Using less conservative ranges of parameters, these authors find $(\sin 2\beta)_{\rm min} = 0.52$.

There is also an upper bound on $\sin 2\beta$, which is valid for the Standard Model and the full class of MFV models. It is simply given by [29]

$$(\sin 2\beta)_{\rm max} = 2R_b^{\rm max}\sqrt{1-(R_b^{\rm max})^2} \approx 0.82, \tag{11}$$

where

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar{\varrho}^2+\bar{\eta}^2} = \left(1-\frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \tag{12}$$

is one side of the unitarity triangle. Here [29],

$$\bar{\varrho} \equiv \varrho(1-\lambda^2/2), \qquad \bar{\eta} \equiv \eta(1-\lambda^2/2), \tag{13}$$

where λ, ϱ and η are Wolfenstein parameters [21]. In obtaining the numerical value in (11), which corresponds to $\beta \lesssim 28^\circ$, we have used $R_b^{\rm max} = 0.46$.

In this paper, we would like to point out that the analyses of the MFV models performed in [24, 12, 15, 17] have implicitly assumed that the new functions F_i, summarizing the SM and new-physics contributions to ε_K, $\Delta M_{d,s}$ and $K \to \pi\nu\overline{\nu}$, have the same sign as the standard Inami–Lim functions. This assumption is certainly correct in the THDM

and the MSSM. On the other hand, it cannot be excluded at present that there exist MFV models in which the functions F_i relevant for ε_K, ΔM_s and $K \to \pi\nu\overline{\nu}$ have a sign *opposite* to the corresponding SM Inami–Lim functions. In fact, in the case of the $B \to X_s\gamma$ decay, such a situation is even possible in the MSSM if particular values of the supersymmetric parameters are chosen. Beyond MFV, scenarios in which the new-physics contributions to neutral meson mixing and rare K decays were larger than the SM contributions and had opposite sign have been considered in [30]. Due to the presence of new complex phases in these general scenarios and new sources of flavour violation, the predictive power of the corresponding models is much smaller than of the MFV models considered here.

In the following, we would like to generalize the existing formulae for the MFV models to arbitrary signs of the generalized Inami–Lim functions F_i and investigate the implications of the sign reversal in question for the determination of $\sin 2\beta$ and the unitarity triangle (UT) through $a_{\psi K_S}$, ε_K, $\Delta M_{d,s}$ and $K \to \pi\nu\overline{\nu}$. In this context, we will also discuss strategies, allowing a direct determination of the sign of F_{tt}. However, the major findings of this paper deal with the rare kaon decays $K^+ \to \pi^+\nu\overline{\nu}$ and $K_{\rm L} \to \pi^0\nu\overline{\nu}$. In particular, we point out that – for given $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $a_{\psi K_S}$ – only *two* values for $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$, corresponding to the two possible signs of the generalized Inami–Lim function X, are possible in the full class of MFV models, independently of any new parameters present in these models. This feature provides an elegant strategy to check whether a MFV model is actually realized in nature and – if so – to determine the sign of X. Moreover, we derive *absolute* lower and upper bounds on the branching ratio $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ as a function of $Br(K^+ \to \pi^+\nu\overline{\nu})$, and emphasize the utility of $B \to X_s\nu\overline{\nu}$ decays to obtain further constraints. The branching ratio $Br(K^+ \to \pi^+\nu\overline{\nu})$ and the CP asymmetry $a_{\psi K_S}$ should be known rather accurately prior to the measurement of $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$.

Our paper is organized as follows: in Section 2, we analyse the unitarity triangle and $\sin 2\beta$ using $\Delta M_{d,s}$, ε_K and $a_{\psi K_S}$. Section 3 is devoted to the $K \to \pi\nu\overline{\nu}$ decays, and our conclusions are summarized in Section 4.

2 $\sin 2\beta$ and the UT from $\Delta M_{d,s}$, ε_K and $a_{\psi K_S}$

2.1 $\sin 2\beta$ from $\Delta M_{d,s}$ and ε_K

In MFV models, the new-physics contributions to $\Delta M_{d,s}$ can be parametrized by a single function F_{tt}, as we have noted in (9). The same "universal" function enters also the observable ε_K [5, 12, 24]. In the SM, it reduces to the Inami–Lim function $S_0(x_t) \approx 2.38$.

An important quantity for our discussion is the length of one side of the unitarity

triangle, R_t, defined by

$$R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1-\bar{\varrho})^2+\bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{14}$$

From ΔM_d and $\Delta M_d/\Delta M_s$, one finds [24, 12, 15]

$$R_t = 1.10\,\frac{R_0}{A}\frac{1}{\sqrt{|F_{tt}|}} \quad \text{with} \quad R_0 \equiv \sqrt{\frac{\Delta M_d}{0.50/\text{ps}}}\left[\frac{230\,\text{MeV}}{\sqrt{\hat{B}_d}F_{B_d}}\right]\sqrt{\frac{0.55}{\eta_B}} \tag{15}$$

and

$$R_t = 0.83\,\xi\sqrt{\frac{\Delta M_d}{0.50/\text{ps}}}\sqrt{\frac{15.0/\text{ps}}{\Delta M_s}} \quad \text{with} \quad \xi \equiv \frac{F_{B_s}\sqrt{\hat{B}_{B_s}}}{F_{B_d}\sqrt{\hat{B}_{B_d}}}, \tag{16}$$

respectively. The corresponding hadronic parameters were introduced after (7). The Wolfenstein parameter A is defined by $|V_{cb}| = A\lambda^2$. These formulae show very clearly that the sign of F_{tt} is immaterial for the analysis of $\Delta M_{d,s}$.

On the other hand, the constraint from ε_K reads [15]

$$\bar{\eta}\left[(1-\bar{\varrho})A^2\eta_2 F_{tt} + P_c(\varepsilon)\right]A^2\hat{B}_K = 0.204\,, \tag{17}$$

where $\eta_2 = 0.57$ is a perturbative QCD factor [27], and $P_c(\varepsilon) = 0.30\pm0.05$ [31] summarizes the contributions not proportional to $V_{ts}^*V_{td}$.

Following [12], but not assuming F_{tt} to be positive, we find from (15) and (17)

$$\sin 2\beta = \text{sgn}(F_{tt})\frac{1.65}{R_0^2\eta_2}\left[\frac{0.204}{A^2B_K} - \bar{\eta}P_c(\varepsilon)\right], \tag{18}$$

where the first term in the parenthesis is typically by a factor 2–3 larger than the second term. We observe that the sign of F_{tt} determines the sign of $\sin 2\beta$. Moreover, as (17) implies $\bar{\eta} < 0$ for $F_{tt} < 0$, also the sign of the second term in the parenthesis is changed. This means that, for a given set of input parameters, not only the sign of $\sin 2\beta$, but also its magnitude is affected by a reversal of the sign of F_{tt}.

At this point the following remark is in order. When using analytic formulae like (15), (16) and (17) one should remember that the numerical constants given there are sensitive functions of λ. Consequently, varying λ but keeping these values fixed would result in errors. On the other hand, for fixed $|V_{cb}|$ any change of λ modifies the parameter A and consequently the impact of the variation of λ within its uncertainties on $\sin 2\beta$ and the unitarity triangle is very small. The numerical values in (15), (16) and (17) and the value for $P_c(\varepsilon)$ differ slightly from those given in [12, 15] due to $\lambda = 0.222$ used here instead of $\lambda = 0.22$ used in these papers. Moreover, we have redefined R_0. This increase of λ in question is made in order to be closer to the experimental value of $|V_{ud}|$ [6].

Quantity	Central	Error
λ	0.222	
$\lvert V_{cb}\rvert$	0.041	± 0.002
$\lvert V_{ub}/V_{cb}\rvert$	0.085	± 0.018
$\lvert V_{ub}\rvert$	0.00349	± 0.00076
$\hat{B}_K$	0.85	± 0.15
$\sqrt{\hat{B}_d}F_{B_d}$	230 MeV	± 40 MeV
m_t	166 GeV	± 5 GeV
$(\Delta M)_d$	0.487/ps	± 0.014/ps
$(\Delta M)_s$	> 15.0/ps	
ξ	1.15	± 0.06

Table 1: The ranges of the input parameters.

The lower bound in (10) has been obtained by varying over all positive values of F_{tt} consistent with the experimental values of $\Delta M_{d,s}$, $|V_{ub}/V_{cb}|$ and $|V_{cb}|$, and scanning all the relevant input parameters in the ranges given in Table 1. Repeating this analysis for $F_{tt} < 0$, we find

$$(-\sin 2\beta)_{\rm min} = 0.69\,. \tag{19}$$

This result is rather sensitive to the minimal value of $\sqrt{\hat{B}_{B_d}}F_{B_d}$. Taking $(\sqrt{\hat{B}_{B_d}}F_{B_d})_{\rm min} = 170\,{\rm MeV}$ instead of 190 MeV used in (19), we obtain the bound of 0.51. For the same choice, the bound in (10) is decreased to 0.35. For $(\sqrt{\hat{B}_{B_d}}F_{B_d})_{\rm min} \geq 195\,{\rm MeV}$ there are no solutions for $\sin 2\beta$ for the ranges of parameters given in Table 1. Finally, only for $\hat{B}_K \geq 0.96$, $|V_{cb}| \geq 0.0414$ and $|V_{ub}/V_{cb}| \geq 0.094$ solutions for $\sin 2\beta$ exist.

We conclude that in the case of $F_{tt} < 0$ the lower bound on $|\sin 2\beta|$ is substantially stronger than for a positve F_{tt}. This is not surprising because in this case the contributions to ε_K proportional to $V_{ts}^* V_{td}$ interfere destructively with the charm contribution. Consequently, $|\sin 2\beta|$ has to be larger to fit ε_K. Our discussion also shows that the decrease in the uncertainties of the parameters in Table 1 could well soon exclude all MFV models with $F_{tt} < 0$.

2.2 $a_{\psi K_{\rm S}}$

Concerning $a_{\psi K_{\rm S}}$, the situation is a bit more involved. As we have noted after (9), the angle 2β in (2) originates from

$$2\beta = \arg(M_{12}^{(d)}) - \phi_{\rm D}(B_d \to \psi K_{\rm S}), \tag{20}$$

where $\phi_{\rm D}(B_d \to \psi K_{\rm S})$ denotes a characteristic weak phase of the $B_d \to \psi K_{\rm S}$ amplitude. In the SM expression (2), it has been taken into account that $S_0(x_t) > 0$, and it has been assumed implicitly that the bag parameter $\hat{B}_{B_d}$ is positive. As emphasized in [32], for $\hat{B}_{B_d} < 0$, the sign in (2) would flip. However, this case appears very unlikely to us. Indeed, all existing non-perturbative methods give $\hat{B}_{B_d} > 0$, which we shall also assume in our analysis. A similar comment applies to $\hat{B}_K$. However, since $S_0(x_t)$ is replaced by the new parameter F_{tt} in the case of the MFV models, which needs not be positive, the following phase ϕ_d is actually probed by the CP asymmetry of $B_d \to \psi K_{\rm S}$:

$$\phi_d = 2\beta + \arg(F_{tt}). \tag{21}$$

Consequently, formula (2) is generalized as follows:

$$a_{\psi K_{\rm S}} = \sin\phi_d = {\rm sgn}(F_{tt}) \sin 2\beta. \tag{22}$$

On the other hand, if we use (18) to predict $a_{\psi K_{\rm S}}$, the sign of the resulting CP asymmetry is unaffected:

$$a_{\psi K_{\rm S}} = \frac{1.65}{R_0^2 \eta_2} \left[\frac{0.204}{A^2 B_K} - \bar{\eta} P_c(\varepsilon)\right]. \tag{23}$$

However, its absolute value will generally be larger for $F_{tt} < 0$.

This analysis demonstrates that in the MFV models $\sin 2\beta$ can either be positive, as in the SM, or negative. This implies that, in addition to the universal unitarity triangle proposed in [24], there exists another universal unitarity triangle with $\sin 2\beta < 0$, which is valid for MFV models with $F_{tt} < 0$. This also means that the "true" CKM angle β in the MFV models can only be determined from $a_{\psi K_{\rm S}}$ and $\Delta M_s/\Delta M_d$ up to a sign that depends on the sign of F_{tt}. In the spirit of [24], one can distinguish these two cases by studying simultaneously ε_K and ΔM_d. If the data on $a_{\psi K_{\rm S}}$ should violate the bound in (19) but satisfy (10), the full class of MFV models with $F_{tt} < 0$ would be excluded by the measurement of $a_{\psi K_{\rm S}}(t)$ alone. If also the bound (10) should be violated, all MFV models would be excluded. The present experimental situation is given as follows:

$$a_{\psi K_{\rm S}} = \begin{cases} 0.59 \pm 0.14 \pm 0.05 & \text{(BaBar [1])} \\ 0.99 \pm 0.14 \pm 0.06 & \text{(Belle [2])} \\ 0.79^{+0.41}_{-0.44} & \text{(CDF [3]).} \end{cases} \tag{24}$$

Combining these results with the earlier measurement by ALEPH $(0.84^{+0.82}_{-1.04} \pm 0.16)$ [4] gives the grand average

$$a_{\psi K_{\rm S}} = 0.79 \pm 0.10, \tag{25}$$

which does not yet allow us to draw any definite conclusions. In particular, the most recent B-factory results in (24) are no longer in favour of a small value of $a_{\psi K_{\rm S}}$, so that

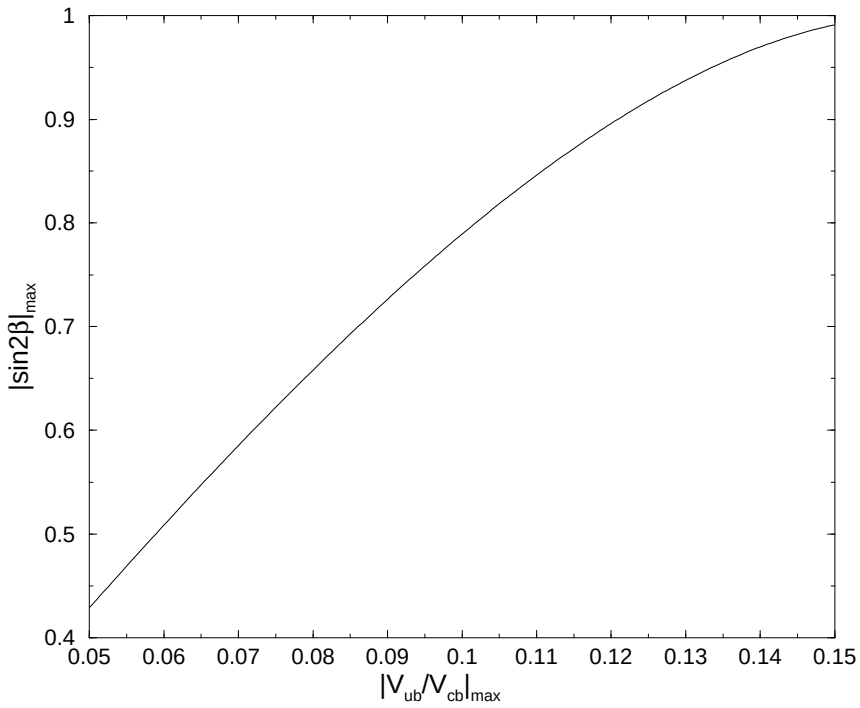

Figure 1: $|\sin 2\beta|_{\rm max}$ as a function of $|V_{ub}/V_{cb}|_{\rm max}$.

not even the case corresponding to negative F_{tt} can be excluded. On the other hand, in view of the Belle result [2], the upper bound given in (11) may play an important role to search for new physics in the future. We observe that whereas the BaBar result [1] is fully consistent with $|\sin 2\beta|_{\rm max} = 0.82$, corresponding to $|V_{ub}/V_{cb}|_{\rm max} = 0.105$, the Belle result violates this bound. This can also be seen in Fig. 1, where we show $|\sin 2\beta|_{\rm max}$ as a function of $|V_{ub}/V_{cb}|_{\rm max}$. Only for values of $|V_{ub}/V_{cb}|$ that are substantially higher than the ones given in Table 1 could the Belle result be valid within the MFV models. Finally, as seen from (19) and Fig. 1, a decrease of $|V_{ub}/V_{cb}|_{\rm max}$ down to 0.085 would put the MFV models with $F_{tt} < 0$ into difficulties, independently of other input parameters in Table 1.

2.3 Direct Determination of sgn(F_{tt})

It would of course be important to measure the sign of the parameter F_{tt} directly and to check the consistency with the bounds discussed above. Several strategies were proposed to extract the phase ϕ_d introduced in (21) unambiguously [33]. This information would be very useful to distinguish between $F_{tt} > 0$ and $F_{tt} < 0$. Let us illustrate this by considering an example, where we assume that $a_{\psi K_{\rm S}} = 0.75$ has been measured, corresponding to $\phi_d = 48.6^\circ$ or 131.4°. The strategies for the distinction between these two possibilities are

discussed in the next paragraph. Let us then assume that the unambiguous determination of ϕ_d gives 48.6°. For $\arg(F_{tt}) = 0$, we would then obtain $\beta = 24.3^\circ$ or $\beta = 204.3^\circ$, where the latter solution would be excluded by the data on $|V_{ub}/V_{cb}|$, requiring $\sqrt{\bar{\varrho}^2 + \bar{\eta}^2} \lesssim 0.5$ (see our discussion below (13)). For $\arg(F_{tt}) = 180^\circ$, we would get $\beta = 114.3^\circ$ or $\beta = 294.3^\circ$, which would both be excluded by $|V_{ub}/V_{cb}|$. Consequently, we would conclude $\beta = 24.3^\circ$ and $\arg(F_{tt}) = 0$ in this case, which could also accommodate the Standard Model. On the other hand, if ϕ_d is found to be 131.4°, the situation is as follows: for $\arg(F_{tt}) = 0$, we would get $\beta = 65.7^\circ$ or $\beta = 245.7^\circ$, which would both be excluded by $|V_{ub}/V_{cb}|$. In the case of $\arg(F_{tt}) = 180^\circ$, we would obtain $\beta = -24.3^\circ$ or $\beta = 155.7^\circ$, where the latter solution would again be excluded by $|V_{ub}/V_{cb}|$. In this case, we would then conclude that $\beta = -24.3^\circ$ and $\arg(F_{tt}) = 180^\circ$. Since the Standard Model cannot be included in this category, we would have an unambiguous signal for new physics.

The key element for the resolution of the twofold ambiguity in the extraction of ϕ_d from $a_{\psi K_S} = \sin\phi_d$ is the determination of $\cos\phi_d$. For the example given in the previous paragraph, $\cos\phi_d = +0.66$ would imply MFV models with $F_{tt} > 0$, containing also the Standard Model, whereas $\cos\phi_d = -0.66$ would imply unambiguously the presence of new physics, corresponding to $F_{tt} < 0$ in MFV scenarios. The quantity $\cos\phi_d$ can be probed through the angular distribution of $B_d \to \psi K^*[\to \pi^0 K_S]$ decays [34], allowing us to extract

$$\cos\delta_f \cos\phi_d. \tag{26}$$

Here δ_f is a strong phase corresponding to a given final-state configuration f of the ψK^* system. Theoretical tools, such as "factorization", may be sufficiently accurate to determine $\mathrm{sgn}(\cos\delta_f)$, thereby allowing the direct extraction of $\cos\phi_d$. In the case of B_s decays, even information on the sign of F_{tt} can be obtained in a direct way, as the SM "background" is negligibly small in

$$\phi_s = -2\lambda^2\eta + \arg(F_{tt}) \approx \arg(F_{tt}). \tag{27}$$

In analogy to the $B_d \to \psi K^*[\to \pi^0 K_S]$ case, the quantity

$$\cos\tilde{\delta}_f \cos\phi_s = \cos\tilde{\delta}_f \,\mathrm{sgn}(F_{tt}) \tag{28}$$

can be probed through the observables of the $B_s \to \psi\phi$ angular distribution [35]. These modes are very accessible at hadron machines. Using again a theoretical input, such as "factorization", to determine $\mathrm{sgn}(\cos\tilde{\delta}_f)$, the sign of F_{tt} can be extracted. If ϕ_d is known unambiguously, $SU(3)$ flavour-symmetry arguments can be used to fix $\mathrm{sgn}(\cos\tilde{\delta}_f)$ from $B_d \to \psi K^*$ decays [35]; alternative ways to determine $\cos\phi_s = \mathrm{sgn}(F_{tt})$ from B_s decays were also noted in that paper.

2.4 UUT from $a_{\psi K_S}$ and $\Delta M_s/\Delta M_d$

In [36, 24], a construction of the UUT by means of $a_{\psi K_S}$ and R_t following from $\Delta M_s/\Delta M_d$ has been presented. Generally, for given values of $(a_{\psi K_S}, R_t)$, there are eight solutions for $(\bar{\varrho}, \bar{\eta})$. However, only two solutions are consistent with the bound in (11), corresponding to the two possible signs of F_{tt}.

For the derivation of explicit expressions for $\bar{\varrho}$ and $\bar{\eta}$, it is useful to consider

$$\mathrm{sgn}(F_{tt})\,\mathrm{ctg}\beta = \frac{1-\bar{\varrho}}{|\bar{\eta}|} \equiv f(\beta), \tag{29}$$

as (14) implies

$$R_t^2 = (1-\bar{\varrho})^2 + \bar{\eta}^2 = \left[f(\beta)^2 + 1\right]\bar{\eta}^2. \tag{30}$$

Consequently, admitting also negative F_{tt}, we obtain

$$\bar{\eta} = \mathrm{sgn}(F_{tt})\left[\frac{R_t}{\sqrt{f(\beta)^2+1}}\right], \quad \bar{\varrho} = 1 - f(\beta)|\bar{\eta}|. \tag{31}$$

If we take into account the constraint from $|V_{ub}/V_{cb}|$, yielding $\bar{\varrho} < 1$, we conclude that $f(\beta)$ is always positive. Moreover, as $a_{\psi K_S} = \mathrm{sgn}(F_{tt})\sin 2\beta$, we may write

$$f(\beta) = \frac{1 \pm \sqrt{1-a_{\psi K_S}^2}}{a_{\psi K_S}} = \mathrm{sgn}(F_{tt})\left[\frac{1 \pm |\cos 2\beta|}{\sin 2\beta}\right]. \tag{32}$$

Now the upper bound $|\beta| \lesssim 28^\circ$ (see (11)) implies $|\mathrm{ctg}\beta| = f(\beta) \gtrsim 1.9$. As $0 < a_{\psi K_S} < 1$, the "$-$" solution in (32) is hence ruled out, and the measurement of $a_{\psi K_S}$ determines $f(\beta)$ *unambiguously* through

$$f(\beta) = \frac{1+\sqrt{1-a_{\psi K_S}^2}}{a_{\psi K_S}}. \tag{33}$$

Finally, with the help of (31), we arrive at

$$\bar{\eta} = \mathrm{sgn}(F_{tt})R_t\sqrt{\frac{1-\sqrt{1-a_{\psi K_S}^2}}{2}}, \quad \bar{\varrho} = 1 - \left[\frac{1+\sqrt{1-a_{\psi K_S}^2}}{a_{\psi K_S}}\right]|\bar{\eta}|. \tag{34}$$

The function $f(\beta)$ plays also a key role for the analysis of the $K \to \pi\nu\overline{\nu}$ system, which is the topic of Section 3.

2.5 Lower and Upper Bounds on J_{CP} and $\mathrm{Im}\lambda_t$

The areas A_Δ of all unitarity triangles are equal and related to the measure of CP violation J_{CP} [37]:

$$|J_{\mathrm{CP}}| = 2A_\Delta = \lambda\left(1 - \frac{\lambda^2}{2}\right)|\mathrm{Im}\lambda_t|, \tag{35}$$

where $\lambda_t = V_{ts}^* V_{td}$. The cleanest measurement of $\mathrm{Im}\lambda_t$ is offered by $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$ [19], which is discussed in the following section. The importance of the measurement of $J_{\rm CP}$ has been stressed in particular in [38].

From ε_K and $\Delta M_{d,s}$, we find the following absolute upper and lower bounds on $|\mathrm{Im}\lambda_t|$ in the MFV models:

$$|\mathrm{Im}\lambda_t|_{\rm max} = \begin{cases} 1.74 \cdot 10^{-4} & F_{tt} > 0 \\ 1.70 \cdot 10^{-4} & F_{tt} < 0 \end{cases} \tag{36}$$

and

$$|\mathrm{Im}\lambda_t|_{\rm min} = \begin{cases} 0.55 \cdot 10^{-4} & F_{tt} > 0 \\ 1.13 \cdot 10^{-4} & F_{tt} < 0, \end{cases} \tag{37}$$

with $\mathrm{sgn}(\mathrm{Im}\lambda_t) = \mathrm{sgn}(F_{tt})$. In the SM, $0.94 \cdot 10^{-4} \le \mathrm{Im}\lambda_t \le 1.60 \cdot 10^{-4}$, and the unitarity of the CKM matrix implies $|\mathrm{Im}\lambda_t|_{\rm max} = 1.83 \cdot 10^{-4}$.

3 $\sin 2\beta$ and UT from $K \to \pi\nu\overline{\nu}$ in MFV Models

3.1 Preface

In MFV models, the short-distance contributions to $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_{\rm L} \to \pi^0 \nu\overline{\nu}$ proportional to $V_{ts}^* V_{td}$ are described by a function X, resulting from Z^0 penguin and box diagrams. In evaluating $\sin 2\beta$ in terms of the branching ratios for $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_{\rm L} \to \pi^0 \nu\overline{\nu}$, the function X drops out [19]. Being determined from two branching ratios, there is a four-fold ambiguity in $\sin 2\beta$ that is reduced to a two-fold ambiguity if $\bar{\varrho} < 1$, as required by the size of $|V_{ub}/V_{cb}|$. The left over solutions correspond to two signs of $\sin 2\beta$ that can be adjusted to agree with the analysis of ε_K. In the SM, the THDM and the MSSM, the functions F_{tt} and X are both positive, resulting in $\sin 2\beta$ given by (3)–(5). We would now like to generalize this discussion and the SM formulae for $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_{\rm L} \to \pi^0 \nu\overline{\nu}$ to MFV models with arbitrary signs of F_{tt} and X. As one of our major findings, we point out the interesting feature that – for given $Br(K^+ \to \pi^+ \nu\overline{\nu})$ and $a_{\psi K_{\rm S}}$ – only *two* values for $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$, corresponding to the two signs of X, are possible in the full class of MFV models, independently of any new parameters arising in these models.

3.2 $K^+ \to \pi^+ \nu\overline{\nu}$

The reduced branching ratio B_1 defined in (5) is given by

$$B_1 = \left[\frac{\mathrm{Im}\lambda_t}{\lambda^5}|X|\right]^2 + \left[\frac{\mathrm{Re}\lambda_c}{\lambda}\mathrm{sgn}(X) P_c(\nu\overline{\nu}) + \frac{\mathrm{Re}\lambda_t}{\lambda^5}|X|\right]^2, \tag{38}$$

where $\lambda_t = V_{ts}^* V_{td}$ with

$$\mathrm{Im}\lambda_t = \eta A^2\lambda^5, \quad \mathrm{Re}\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right) A^2\lambda^5(1-\bar{\varrho}), \tag{39}$$

and $\lambda_c = -\lambda(1-\lambda^2/2)$. Therefore, the standard analysis of the unitarity triangle by means of $K^+ \to \pi^+\nu\overline{\nu}$ [19, 29] can be generalized to arbitrary signs of X and F_{tt} through the replacements

$$X \to |X|, \quad P_c(\nu\overline{\nu}) \to \mathrm{sgn}(X) P_c(\nu\overline{\nu}), \quad \bar{\eta} \to \mathrm{sgn}(F_{tt})|\bar{\eta}|. \tag{40}$$

We find then that the measured value of $Br(K^+ \to \pi^+\nu\overline{\nu})$ determines an ellipse in the $(\bar{\varrho}, \bar{\eta})$ plane,

$$\left(\frac{\bar{\varrho} - \varrho_0}{\bar{\varrho}_1}\right)^2 + \left(\frac{\bar{\eta}}{\bar{\eta}_1}\right)^2 = 1, \tag{41}$$

centered at $(\varrho_0, 0)$ with

$$\varrho_0 = 1 + \mathrm{sgn}(X)\frac{P_c(\nu\overline{\nu})}{A^2|X|}, \tag{42}$$

and having the squared axes

$$\bar{\varrho}_1^2 = r_0^2, \quad \bar{\eta}_1^2 = \left(\frac{r_0}{\sigma}\right)^2 \quad \text{with} \quad r_0^2 = \frac{\sigma B_1}{A^4|X|^2}\,. \tag{43}$$

The ellipse (41) intersects with the circle (12). This allows us to determine $\bar{\varrho}$ and $\bar{\eta}$:

$$\bar{\varrho} = \frac{1}{1-\sigma^2}\left[\varrho_0 \mp \sqrt{\sigma^2\varrho_0^2 + (1-\sigma^2)(r_0^2 - \sigma^2 R_b^2)}\right], \quad \bar{\eta} = \mathrm{sgn}(F_{tt})\sqrt{R_b^2 - \bar{\varrho}^2}, \tag{44}$$

and consequently

$$R_t^2 = 1 + R_b^2 - 2\bar{\varrho}\,. \tag{45}$$

Given $\bar{\varrho}$ and $\bar{\eta}$, one can determine V_{td}:

$$V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}), \quad |V_{td}| = A\lambda^3 R_t. \tag{46}$$

The deviation of ϱ_0 from unity measures the relative importance of the internal charm contribution. For $X > 0$, we have, as usual, $\varrho_0 > 1$ so that the "+" solution in (44) is excluded because of $\varrho < 1$. On the other hand, for $X < 0$, the center of the ellipse is shifted to $\varrho_0 < 1$, and for $|X| \le P_c(\nu\overline{\nu})/A^2$ can even be at $\varrho_0 \le 0$.

3.3 $K_{\rm L} \to \pi^0\nu\overline{\nu}$, $K^+ \to \pi^+\nu\overline{\nu}$ and the Unitarity Triangle

The reduced branching ratio B_2 defined in (5) is given by

$$B_2 = \left[\frac{{\rm Im}\lambda_t}{\lambda^5}|X|\right]^2. \tag{47}$$

Following [19], but admitting both signs of X and F_{tt}, we find

$$\bar{\varrho} = 1 + \left[\frac{\pm\sqrt{\sigma(B_1 - B_2)} + {\rm sgn}(X)P_c(\nu\overline{\nu})}{A^2|X|}\right], \quad \bar{\eta} = {\rm sgn}(F_{tt})\frac{\sqrt{B_2}}{\sqrt{\sigma}A^2|X|}, \tag{48}$$

where σ was defined in (6). Introducing

$$r_s \equiv \frac{1-\bar{\varrho}}{\bar{\eta}} = {\rm ctg}\beta, \tag{49}$$

we then find

$$r_s = {\rm sgn}(F_{tt})\sqrt{\sigma}\left[\frac{\mp\sqrt{\sigma(B_1 - B_2)} - {\rm sgn}(X)P_c(\nu\overline{\nu})}{\sqrt{B_2}}\right], \tag{50}$$

with (3) and (5) unchanged. We observe that r_s is independent of $|X|$ but the sign of the interference between the $V_{ts}^*V_{td}$ contribution and the charm contribution $P_c(\nu\overline{\nu})$ to $K^+ \to \pi^+\nu\overline{\nu}$ matters.

In order to deal with the ambiguities present in (50), we consider

$${\rm sgn}(F_{tt})\, r_s = \sqrt{\sigma}\left[\frac{\mp\sqrt{\sigma(B_1 - B_2)} - {\rm sgn}(X)P_c(\nu\overline{\nu})}{\sqrt{B_2}}\right] = f(\beta), \tag{51}$$

where $f(\beta)$ was introduced in (29). As we have noted after (31), $f(\beta)$ has to be positive. Consequently, for $X > 0$, only the "+" solution is allowed. On the other hand, in the case of $X < 0$, the "$-$" solution gives also a positive value of $f(\beta)$ if

$$B_1 - B_2 < \frac{P_c(\nu\overline{\nu})^2}{\sigma} \approx 0.15. \tag{52}$$

Numerical studies show that both $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ have to be below $1\cdot 10^{-11}$ to satisfy (52). As such low values are extremely difficult to measure , we will not consider this possibility further, which leaves us with the "+" solution in (50).

In Table 2, we show the resulting values of ${\rm sgn}(F_{tt})\sin 2\beta = a_{\psi K_{\rm S}}$ for several choices of $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$, setting $P_c(\nu\overline{\nu}) = 0.40$. We observe that the sign of X is important; we also note that certain values violate the bounds in (10) and (11). This implies that certain combinations of the two branching ratios are excluded within the MFV models. Let us then find out which combinations are still allowed.

Table 2: $\mathrm{sgn}(F_{tt})\sin 2\beta = a_{\psi K_S}$ in MFV models for specific values of $Br(K_L \to \pi^0\nu\overline{\nu}) \equiv Br(K_L)$ and $Br(K^+ \to \pi^+\nu\overline{\nu}) \equiv Br(K^+)$ for $\mathrm{sgn}(X) = +1$ (-1) and $P_c(\nu\overline{\nu}) = 0.40$.

$Br(K_L)$ $[10^{-11}]$	$Br(K^+) = 8.0$ $[10^{-11}]$	$Br(K^+) = 16$ $[10^{-11}]$	$Br(K^+) = 24$ $[10^{-11}]$
2.0	0.60 (0.35)	0.40 (0.27)	0.31 (0.22)
3.0	0.71 (0.43)	0.48 (0.32)	0.38 (0.27)
4.0	0.79 (0.49)	0.55 (0.37)	0.43 (0.32)
5.0	0.86 (0.54)	0.60 (0.42)	0.48 (0.35)
6.0	0.91 (0.59)	0.65 (0.45)	0.52 (0.38)
7.0	0.94 (0.64)	0.70 (0.49)	0.56 (0.41)
8.0	0.97 (0.68)	0.73 (0.52)	0.60 (0.44)

3.4 $Br(K_L \to \pi^0\nu\overline{\nu})$ from $a_{\psi K_S}$ and $Br(K^+ \to \pi^+\nu\overline{\nu})$

As $a_{\psi K_S}$ and $Br(K^+ \to \pi^+\nu\overline{\nu})$ will be known rather accurately prior to the measurement of $Br(K_L \to \pi^0\nu\overline{\nu})$, it is of interest to calculate $Br(K_L \to \pi^0\nu\overline{\nu})$ as a function of $a_{\psi K_S}$ and $Br(K^+ \to \pi^+\nu\overline{\nu})$. From (51), we obtain

$$B_1 = B_2 + \left[\frac{f(\beta)\sqrt{B_2} + \mathrm{sgn}(X)\sqrt{\sigma}P_c(\nu\overline{\nu})}{\sigma}\right]^2 . \tag{53}$$

The important virtue of (53) when compared with (50) is the absence of the ambiguity due to the $\mp$ in front of $\sqrt{\sigma(B_1 - B_2)}$.

As we have seen in (33), the measurement of $a_{\psi K_S}$ determines $f(\beta)$ unambiguously. This finding, in combination with (53), implies the following interesting property of the MFV models:

- For given $a_{\psi K_S}$ and $Br(K^+ \to \pi^+\nu\overline{\nu})$ only two values of $Br(K_L \to \pi^0\nu\overline{\nu})$, corresponding to the two possible signs of X, are possible in the full class of MFV models, independently of any new parameters present in these models.

Consequently, measuring $Br(K_L \to \pi^0\nu\overline{\nu})$ will either select one of these two possible values or rule out all MFV models. We would like to emphasize that the latter possibility could take place even if the lower bound on $|\sin 2\beta|$ [12] is satisfied by the data on $a_{\psi K_S}$, which is favoured by the most recent B-factory results given in (24).

In Table 3, we show values of $Br(K_L \to \pi^0\nu\overline{\nu})$ in the MFV models for specific values of $a_{\psi K_S}$ and $Br(K^+ \to \pi^+\nu\overline{\nu})$ and the two signs of X. Note that the second column gives the *absolute* lower bound on $Br(K_L \to \pi^0\nu\overline{\nu})$ in the MFV models as a function of $Br(K^+ \to \pi^+\nu\overline{\nu})$. This bound follows simply from the lower bound in (10). On the

Table 3: Values of $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ in the MFV models in units of 10^{-11} for specific values of $a_{\psi K_{\rm S}}$ and $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $\mathrm{sgn}(X) = +1$ (-1). We set $P_c(\nu\overline{\nu}) = 0.40$.

$Br(K^+ \to \pi^+\nu\overline{\nu})$ $[10^{-11}]$	$a_{\psi K_{\rm S}} = 0.42$	$a_{\psi K_{\rm S}} = 0.69$	$a_{\psi K_{\rm S}} = 0.82$
5.0	0.45 (2.0)	1.4 (5.8)	2.2 (8.6)
10.0	1.2 (3.5)	3.8 (10.0)	5.9 (15.0)
15.0	2.1 (4.8)	6.3 (14.0)	9.9 (21.1)
20.0	3.0 (6.2)	9.0 (17.9)	14.1 (27.0)
25.0	3.9 (7.5)	11.8 (21.7)	18.4 (32.8)
30.0	4.9 (8.7)	14.6 (25.4)	22.7 (38.6)

other hand, the last column gives the corresponding *absolute* upper bound. This bound is the consequence of the upper bound in (11). The third column gives the lower bound on $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ corresponding to the bound in (19) that applies for a negative F_{tt}.

A more detailed presentation is given in Figs. 2 and 3. In Fig. 2, we show $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ as a function of $Br(K^+ \to \pi^+\nu\overline{\nu})$ for chosen values of $a_{\psi K_{\rm S}}$ and $\mathrm{sgn}(X) = +1$. The corresponding plot for $\mathrm{sgn}(X) = -1$ is shown in Fig. 3. It should be emphasized that the plots shown in Figs. 2 and 3 are universal for all MFV models. Table 3 and Figs. 2 and 3 make it clear that the measurements of $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$, $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $a_{\psi K_{\rm S}}$ will easily allow the distinction between the two signs of X. The uncertainty due to $P_c(\nu\overline{\nu})$ is non-negligible but it should be decreased with the improved knowledge of the charm-quark mass.

We would like to emphasize that the upper bound on $Br(K_{\rm L} \to \pi^0\nu\overline{\nu})$ in the last column of Table 3 is substantially stronger than the model-independent bound following from isospin symmetry [39]

$$Br(K_{\rm L} \to \pi^0\nu\overline{\nu}) < 4.4 \cdot Br(K^+ \to \pi^+\nu\overline{\nu}). \tag{54}$$

Indeed, taking the experimental bound $Br(K^+ \to \pi^+\nu\overline{\nu}) \le 5.9 \cdot 10^{-10}$ (90% C.L.) from AGS E787 [40], we find

$$Br(K_{\rm L} \to \pi^0\nu\overline{\nu})_{\rm MFV} \le \begin{cases} 4.9 \cdot 10^{-10} & \mathrm{sgn}(X) = +1 \\ 7.1 \cdot 10^{-10} & \mathrm{sgn}(X) = -1. \end{cases} \tag{55}$$

This should be compared with $Br(K_{\rm L} \to \pi^0\nu\overline{\nu}) < 26 \cdot 10^{-10}$ (90% C.L.) following from (54), and with the present upper bound from the KTeV experiment at Fermilab [41], yielding $Br(K_{\rm L} \to \pi^0\nu\overline{\nu}) < 5.9 \cdot 10^{-7}$. The corresponding predictions within the SM read [15]

$$Br(K^+ \to \pi^+\nu\overline{\nu}) = (7.5 \pm 2.9) \cdot 10^{-11}\ , \quad Br(K_{\rm L} \to \pi^0\nu\overline{\nu}) = (2.6 \pm 1.2) \cdot 10^{-11}\ . \tag{56}$$

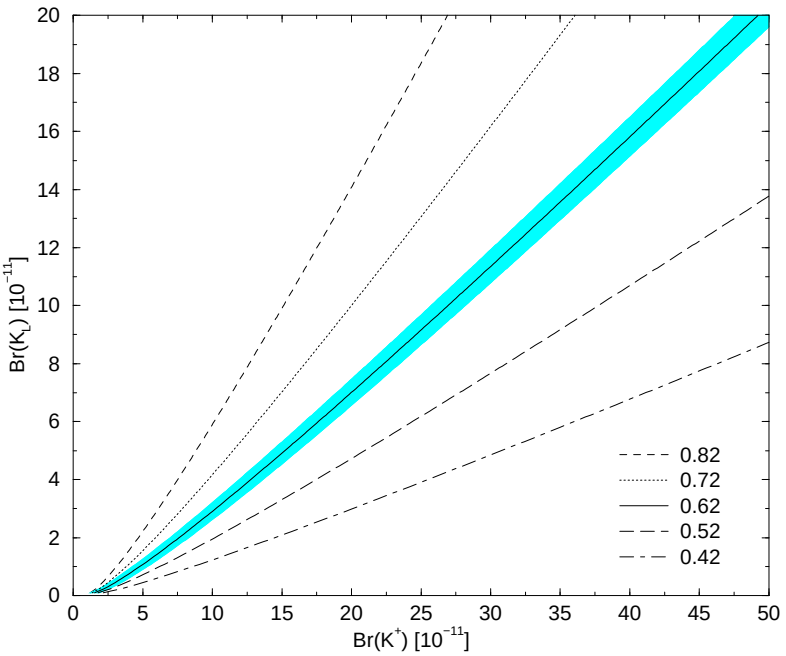

Figure 2: $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$ as a function of $Br(K^+ \to \pi^+ \nu\overline{\nu})$ for several values of $a_{\psi K_{\rm S}}$ in the case of $\mathrm{sgn}(X) = +1$. For $a_{\psi K_{\rm S}} = 0.62$, also the uncertainty due to $P_c(\nu\overline{\nu}) = 0.40 \pm 0.06$ has been shown.

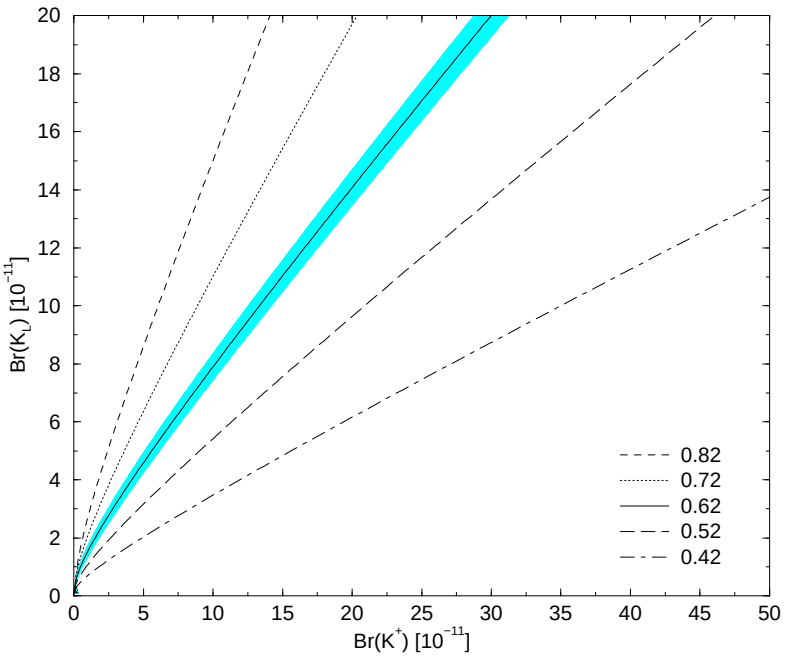

Figure 3: $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$ as a function of $Br(K^+ \to \pi^+ \nu\overline{\nu})$ for several values of $a_{\psi K_{\rm S}}$ in the case of $\mathrm{sgn}(X) = -1$. For $a_{\psi K_{\rm S}} = 0.62$, also the uncertainty due to $P_c(\nu\overline{\nu}) = 0.40 \pm 0.06$ has been shown.

As can be seen in Table 3 and in Figs. 2 and 3, the bounds in (55) will be considerably improved when $Br(K^+ \to \pi^+\nu\overline{\nu})$ and $a_{\psi \mathrm{K_S}}$ will be known better. The experimental outlook for both decays has recently been reviewed by Littenberg [42]. The existing measurement [40]

$$Br(K^+ \to \pi^+\nu\overline{\nu}) = \left(1.5^{+3.4}_{-1.2}\right) \cdot 10^{-10} \tag{57}$$

should be considerably improved already this year.

3.5 An Upper Bound on $Br(K_{\mathrm{L}} \to \pi^0\nu\overline{\nu})$ from $Br(B \to X_s\nu\overline{\nu})$

The branching ratio for the inclusive rare decay $B \to X_s\nu\overline{\nu}$ can be written in the MFV models as follows [15]:

$$Br(B \to X_s\nu\overline{\nu}) = 1.57 \cdot 10^{-5} \left[\frac{Br(B \to X_c e\overline{\nu})}{0.104}\right] \left|\frac{V_{ts}}{V_{cb}}\right|^2 \left[\frac{0.54}{f(z)}\right] \; X^2, \tag{58}$$

where $f(z) = 0.54 \pm 0.04$ is the phase-space factor for $B \to X_c e\overline{\nu}$ with $z = m_c^2/m_b^2$, and $Br(B \to X_c e\overline{\nu}) = 0.104 \pm 0.004$.

Formulae (47) and (58) imply an interesting relation between the decays $K_{\mathrm{L}} \to \pi^0\nu\overline{\nu}$ and $B \to X_s\nu\overline{\nu}$:

$$Br(K_{\mathrm{L}} \to \pi^0\nu\overline{\nu}) = 42.3 \cdot (\mathrm{Im}\lambda_t)^2 \left[\frac{0.104}{Br(B \to X_c e\overline{\nu})}\right] \left|\frac{V_{cb}}{V_{ts}}\right|^2 \left[\frac{f(z)}{0.54}\right] Br(B \to X_s\nu\overline{\nu}), \tag{59}$$

which is valid in all MFV models. Equation (59) constitutes still another connection between K- and B-meson decays, in addition to those discussed already in this paper and in [19, 20, 22, 17, 43].

Now, the experimental upper bound on $Br(B \to X_s\nu\overline{\nu})$ reads [44]

$$Br(B \to X_s\nu\overline{\nu}) < 6.4 \cdot 10^{-4} \quad (90\% \text{ C.L.}). \tag{60}$$

Using this bound and setting $\mathrm{Im}\lambda_t = 1.74 \cdot 10^{-4}$ (see (36)), $|V_{ts}| = |V_{cb}|$, $f(z) = 0.58$ and $Br(B \to X_c e\overline{\nu}) = 0.10$, we find from (59) the upper bound

$$Br(K_{\mathrm{L}} \to \pi^0\nu\overline{\nu}) < 9.2 \cdot 10^{-10} \quad (90\% \text{ C.L.}), \tag{61}$$

which is not much weaker than the bound in (55). As the bound in (60) should be improved in the B-factory era, also the latter bound should be improved in the next years.

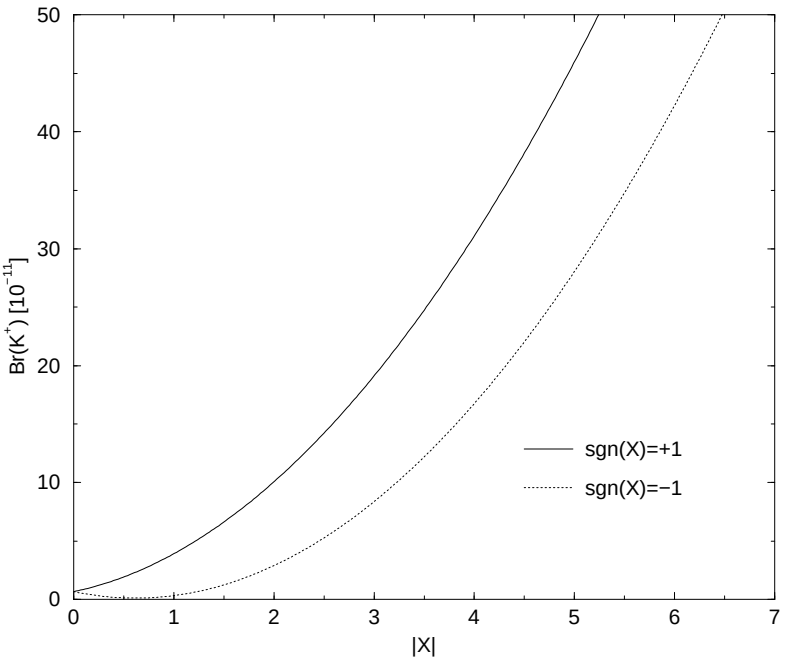

Figure 4: $Br(K^+ \to \pi^+ \nu\overline{\nu})$ as a function of $|X|$ for $\mathrm{sgn}(X) = \pm 1$ in the case of $A = 0.83$, $(\bar{\varrho}, \bar{\eta}) = (0.23, 0.35)$ and $P_c(\nu\overline{\nu}) = 0.40$.

3.6 Determination of X

The knowledge of the function X would be a very important information, providing constraints on the MFV models. In the SM, we have $X \approx 1.5$. Present bounds on the function X from $K^+ \to \pi^+ \nu\overline{\nu}$ and $B \to X_s \nu\overline{\nu}$ within MFV models were recently discussed in [17]. In particular, from (58) and (60) we find

$$|X| < 6.8, \tag{62}$$

which agrees well with [17].

In the future, a theoretically clean determination of X will be made possible by determining $\bar{\eta}$ and $\bar{\varrho}$ by means of $\Delta M_s/\Delta M_d$ and $a_{\psi K_S}$ (see (16) and (34)), and inserting them into (39) and (38). In this manner, we may calculate $Br(K^+ \to \pi^+ \nu\overline{\nu})$ as a function of X. The measurement of this branching ratio yields then two values of $|X|$, corresponding to $\mathrm{sgn}(X) = \pm 1$. We illustrate this in Fig. 4, where we plot $Br(K^+ \to \pi^+ \nu\overline{\nu})$ as a function of $|X|$ for $\mathrm{sgn}(X) = \pm 1$. Here we have assumed, as an example, $A = 0.83$, $(\bar{\varrho}, \bar{\eta}) = (0.23, 0.35)$, which corresponds to $a_{\psi K_S} = 0.75$, and $P_c(\nu\overline{\nu}) = 0.40$. As expected, $Br(K^+ \to \pi^+ \nu\overline{\nu})$ is substantially smaller in the case of a negative X.

Direct access to $|X|$ will also be provided by $Br(K_L \to \pi^0 \nu\overline{\nu})$, as can be seen from (47). If a MFV model is realized in nature, both determinations have to give the same

value of $|X|$. This requirement allows us to distinguish between the two branches in Fig. 4, thereby offering another way to fix the sign of X.

However, the strategy presented in Subsection 3.4, which is based on Figs. 2 and 3 and involves just $a_{\psi K_S}$, $Br(K^+ \to \pi^+ \nu\overline{\nu})$ and $Br(K_L \to \pi^0 \nu\overline{\nu})$, is much more elegant to check whether a MFV model is realized in the $K \to \pi\nu\overline{\nu}$ system and – if so – to determine $\mathrm{sgn}(X)$. In order to determine also $|X|$, $\Delta M_s/\Delta M_d$ is needed as an additional input, as we have seen above.

4 Conclusions

In this paper, we have explored the determination of $\sin 2\beta$ through the standard analysis of the unitarity triangle, the CP asymmetry $a_{\psi K_S}$, and the decays $K \to \pi\nu\overline{\nu}$ in MFV models, admitting new-physics contributions that reverse the sign of the corresponding generalized Inami–Lim functions F_{tt} and X. Our findings can be summarized as follows:

- There are bounds on $\sin 2\beta$, which can be translated into lower bounds on $a_{\psi K_S}$. For $F_{tt} > 0$, $(a_{\psi K_S})_{\rm min} = 0.42$ [12], whereas we obtain a stronger bound of $(a_{\psi K_S})_{\rm min} = 0.69$ in the case of $F_{tt} < 0$. Consequently, for $0.42 < a_{\psi K_S} < 0.69$, the full class of MFV models with $F_{tt} < 0$ would be excluded; for $a_{\psi K_S} < 0.42$, even all MFV models would be ruled out. The reduction of the uncertainties of the relevant input parameters could improve these bounds in the future. We have also discussed strategies to determine the sign of F_{tt} directly, allowing interesting consistency checks of the MFV models.

- The most recent B-factory data are no longer in favour of small values of $a_{\psi K_S}$, and the present world average of 0.79 ± 0.10 does not even allow us to exclude the case corresponding to $F_{tt} < 0$. Consequently, an important role may be played in the future by the upper bound on $a_{\psi K_S}$ that is implied by $|V_{ub}/V_{cb}|$. Since the BaBar and Belle results are not fully consistent with each other, the measurement of $a_{\psi K_S}$ will remain a very exciting issue. Let us hope that the situation will be clarified soon.

- We have generalized the SM analysis of the unitarity triangle through $K \to \pi\nu\overline{\nu}$ to MFV models, allowing negative values of X. In particular, we have explored the behaviour of $Br(K_L \to \pi^0 \nu\overline{\nu})$ as a function of $a_{\psi K_S}$ and $Br(K^+ \to \pi^+ \nu\overline{\nu})$ for the general MFV model. This is an important excercise, since the latter two quantities will be known rather precisely before $Br(K_L \to \pi^0 \nu\overline{\nu})$ will be accessible. In this context, we have pointed out that for given $Br(K^+ \to \pi^+ \nu\overline{\nu})$ and $a_{\psi K_S}$, only two

values for $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$ are possible in the full class of MFV models, which correspond just to the two signs of X and are independent of any new parameters present in these models. Consequently, the measurement of this branching ratio will either select one particular class of MFV models, or will exclude all of them.

- At present, the existing lower and upper bounds on $a_{\psi {\rm K_S}}$ in the MFV models allow us to find *absolute* lower and upper bounds on the branching ratio $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$ as a function of $Br(K^+ \to \pi^+ \nu\overline{\nu})$. We find that the present upper bounds on $Br(K^+ \to \pi^+ \nu\overline{\nu})$ and $|V_{ub}/V_{cb}|$ imply an absolute upper bound $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu}) < 7.1 \cdot 10^{-10}$ (90% C.L.), which is substantially stronger than the bound following from isospin symmetry. On the other hand, the experimental upper bound on $Br(B \to X_s \nu\overline{\nu})$ implies $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu}) < 9.2 \cdot 10^{-10}$ (90% C.L.).

The present paper, in conjunction with earlier analyses [24, 12, 15, 17], demonstrates the simplicity of the MFV models, allowing transparent and general tests of these models without the necessity to assume particular values for their new parameters.

It will be exciting to follow the development in the experimental values of $a_{\psi {\rm K_S}}$, $Br(K^+ \to \pi^+ \nu\overline{\nu})$, $Br(K_{\rm L} \to \pi^0 \nu\overline{\nu})$, $Br(B \to X_s \nu\overline{\nu})$ and $\Delta M_s/\Delta M_d$. Possibly already before the LHC era we will know whether any of the MFV models survives all tests discussed here and in [19, 22, 24, 12, 15, 17], or whether new operators and/or new complex phases are required to describe the data.

Acknowledgements

A.J.B. would like to thank Piotr Chankowski and Lucja Sławianowska for discussions. This work has been supported in part by the German Bundesministerium für Bildung and Forschung under the contract 05HT9WOA0.

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