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On the Rise of the Proton Structure Function F_2 Towards Low x

H1 Collaboration

Abstract:

A measurement of the derivative $(\partial \ln F_2 / \partial \ln x)_{Q^2} \equiv -\lambda(x, Q^2)$ of the proton structure function F_2 is presented in the low x domain of deeply inelastic positron–proton scattering. For $5 \cdot 10^{-5} \leq x \leq 0.01$ and $Q^2 \geq 1.5\,\mathrm{GeV}^2$, $\lambda(x, Q^2)$ is found to be independent of x and to increase linearly with $\ln Q^2$.

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The inclusive cross section for deeply inelastic lepton-proton scattering is governed by the proton structure function $F_2(x, Q^2)$. Because of the large centre-of-mass energy squared, $s \simeq 10^5$ GeV^2, the ep collider HERA has accessed the region of low Bjorken x, $x > Q^2/s > 10^{-5}$, for four-momentum transfers squared $Q^2 > 1$ GeV^2. One of the first observations at HERA was of a substantial rise of F_2 with decreasing x [1]. However, this rise may be limited at very low x by unitarity constraints.

Perturbative Quantum Chromodynamics (QCD) provides a rigorous and successful theoretical description of the Q^2 dependence of $F_2(x, Q^2)$ in deeply inelastic scattering. In the double asymptotic limit, the DGLAP evolution equations [2] can be solved [3] and F_2 is expected to rise approximately as a power of x towards low x. A power behaviour is also predicted in BFKL theory [4]. The rise is expected eventually to be limited by gluon self interactions in the nucleon [5].

Recently the H1 Collaboration has presented [6] a new measurement of $F_2(x, Q^2)$ in the kinematic range $3 \cdot 10^{-5} \leq x \leq 0.2$ and $1.5 \leq Q^2 \leq 150$ GeV^2 based on data taken in the years 1996/97 with a positron beam energy $E_e = 27.6$ GeV and a proton beam energy $E_p = 820$ GeV. The high accuracy of these data allows the derivative

$$\left(\frac{\partial \ln F_2(x, Q^2)}{\partial \ln x}\right)_{Q^2} \equiv -\lambda(x, Q^2) \tag{1}$$

to be measured as a function both of Q^2 and of x for the first time. Use of this quantity for investigating the behaviour of F_2 at low x was suggested in [7].

Here results are presented of a measurement of this derivative in the full kinematic range available. Data points at adjacent values of x and at fixed Q^2 are used [6] taking account of the full error correlations and the spacing between the x values. The results[1] obtained are presented in Table 1. The sensitivity of the derivative to the uncertainty of the structure function F_L [6] throughout the measured kinematic range is estimated to be much smaller than the total systematic error at the lowest values of x and is negligible elsewhere.

As can be seen in Figure 1, the derivative $\lambda(x, Q^2)$ is independent of x for $x \lesssim 0.01$ to within the experimental accuracy. This implies that the x dependence of F_2 at low x is consistent with a power law, $F_2 \propto x^{-\lambda}$, for fixed Q^2, and that the rise of F_2 , i.e. $(\partial F_2/\partial x)_{Q^2}$, is proportional to F_2/x. There is no experimental evidence that this behaviour changes in the measured kinematic range.

The derivative is well described by the NLO QCD fit to the H1 cross-section data [6], see Figure 1. In DGLAP QCD, for $Q^2 > 3$ GeV^2, the low x behaviour is driven solely by the gluon field, since quark contributions to the scaling violations of F_2 are negligible. At larger x the transition to the valence-quark region causes a strong dependence of λ on x as indicated by the QCD curves in Figure 1.

Figure 2 shows the measured derivative as a function of Q^2 for different x values. The derivative is observed to rise approximately logarithmically with Q^2. It can be represented by a function $\lambda(Q^2)$ which is independent of x within the experimental accuracy.

[1]Note that derivatives at adjacent x values are thus anti-correlated. The data points at $Q^2 = 150$ GeV^2 are obtained from the H1 measurement [8].

The function $\lambda(Q^2)$ is determined from fits of the form $F_2(x,Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ to the H1 structure function data, restricted to the region $x \leq 0.01$. The results for $c(Q^2)$ and $\lambda(Q^2)$ are presented in Table 2 and shown in Figure 3. The coefficients $c(Q^2)$ are approximately independent of Q^2 with a mean value of 0.18. As can be seen, $\lambda(Q^2)$ rises approximately linearly with $\ln Q^2$. This dependence can be represented as $\lambda(Q^2) = a \cdot \ln[Q^2/\Lambda^2]$, see Figure 3. The coefficients are $a = 0.0481 \pm 0.0013(\mathrm{stat}) \pm 0.0037(\mathrm{syst})$ and $\Lambda = 292 \pm 20(\mathrm{stat}) \pm 51(\mathrm{syst})$ MeV, obtained for $Q^2 \geq 3.5$ GeV^2. The values of $\lambda(Q^2)$ are more accurate than data hitherto published by the H1 [9] and ZEUS [10] Collaborations.

Below the deeply inelastic region, for fixed $Q^2 < 1$ GeV^2, the simplest Regge phenomenology predicts that $F_2(x,Q^2) \propto x^{-\lambda}$ where $\lambda = \alpha_{I\!P}(0) - 1 \simeq 0.08$ is given by the Pomeron intercept independently of x and Q^2 [11]. When extrapolating the function $\lambda(Q^2)$ into the lower Q^2 region it has the value of 0.08 at $Q^2 = 0.45$ GeV^2, see also [10].

To summarise, the derivative $(\partial \ln F_2/\partial \ln x)_{Q^2}$ is measured as a function both of x and of Q^2 and is observed to be independent of Bjorken x for $x \lesssim 0.01$ and Q^2 between 1.5 and 150 GeV^2. Thus the behaviour of F_2 at low x is consistent with a dependence $F_2(x,Q^2) = c(Q^2)\, x^{-\lambda(Q^2)}$ throughout that region. At low x, the exponent λ is observed to rise linearly with $\ln Q^2$ and the coefficient c is independent of Q^2 to within the experimental accuracy. There is no sign that this behaviour changes within the kinematic range of deeply inelastic scattering explored.

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References

[1] I. Abt *et al.* [H1 Collaboration], Nucl. Phys. B **407** (1993) 515.
M. Derrick *et al.* [ZEUS Collaboration], Phys. Lett. B **316** (1993) 412.

[2] Y. L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641 [Zh. Eksp. Teor. Fiz. **73** (1977) 1216];
V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15** (1972) 1218 [Sov. J. Nucl. Phys. **15** (1972) 675];
V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15** (1972) 781 [Sov. J. Nucl. Phys. **15** (1972) 438];
G. Altarelli and G. Parisi, Nucl. Phys. B **126** (1977) 298.

[3] A. De Rujula, S. L. Glashow, H. D. Politzer, S. B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D **10** (1974) 1649;
R. D. Ball and S. Forte, Phys. Lett. B **335** (1994) 77 [hep-ph/9405320].

[4] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **44** (1976) 443;
E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45** (1977) 199;
Y. Y. Balitsky and L. N. Lipatov, Sov. Journ. Nucl. Phys. **28** (1978) 822.

[5] L. V. Gribov, E. M. Levin and M. G. Ryskin, Nucl. Phys. B **188** (1981) 555;
L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100** (1983) 1.

[6] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **21** (2001) 33 [hep-ex/0012053].

[7] H. Navelet, R. Peschanski and S. Wallon, Mod. Phys. Lett. A **9** (1994) 3393 [hep-ph/9402352].

[8] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **13** (2000) 609 [hep-ex/9908059].

[9] S. Aid *et al.* [H1 Collaboration], Nucl. Phys. B **470** (1996) 3 [hep-ex/9603004].
C. Adloff *et al.* [H1 Collaboration], Nucl. Phys. B **497** (1997) 3 [hep-ex/9703012].

[10] J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. C **7** (1999) 609 [hep-ex/9809005].

[11] A. Donnachie and P. V. Landshoff, Z. Phys. C **61** (1994) 139 [hep-ph/9305319].

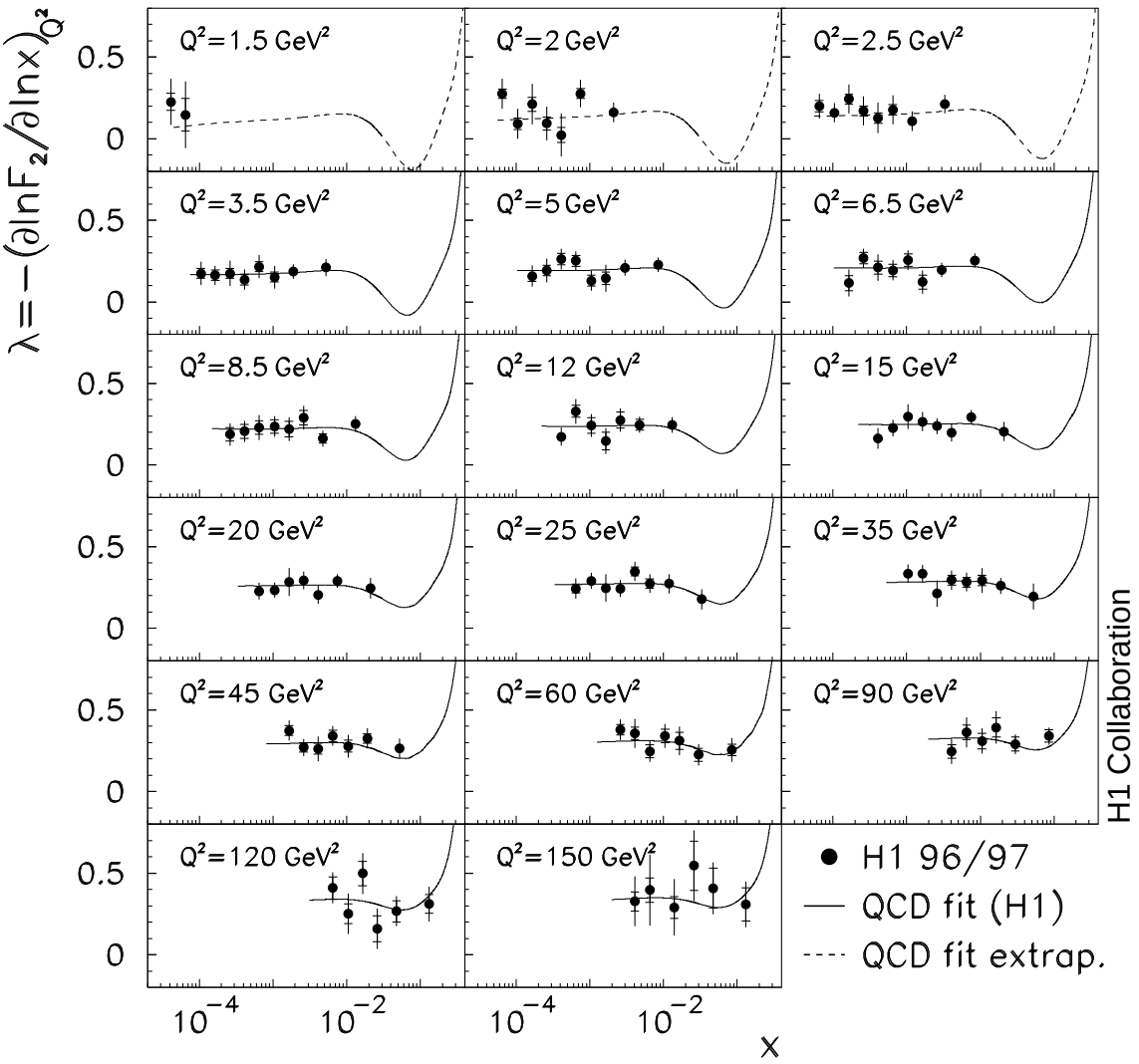

Figure 1: Measurement of the function $\lambda(x, Q^2)$: the inner error bars represent the statistical uncertainty; the full error bars include the systematic uncertainty added in quadrature; the solid curves represent the NLO QCD fit to the H1 cross section data described in [6]; the dashed curves represent the extrapolation of the QCD fit below $Q^2 = 3.5$ GeV^2.

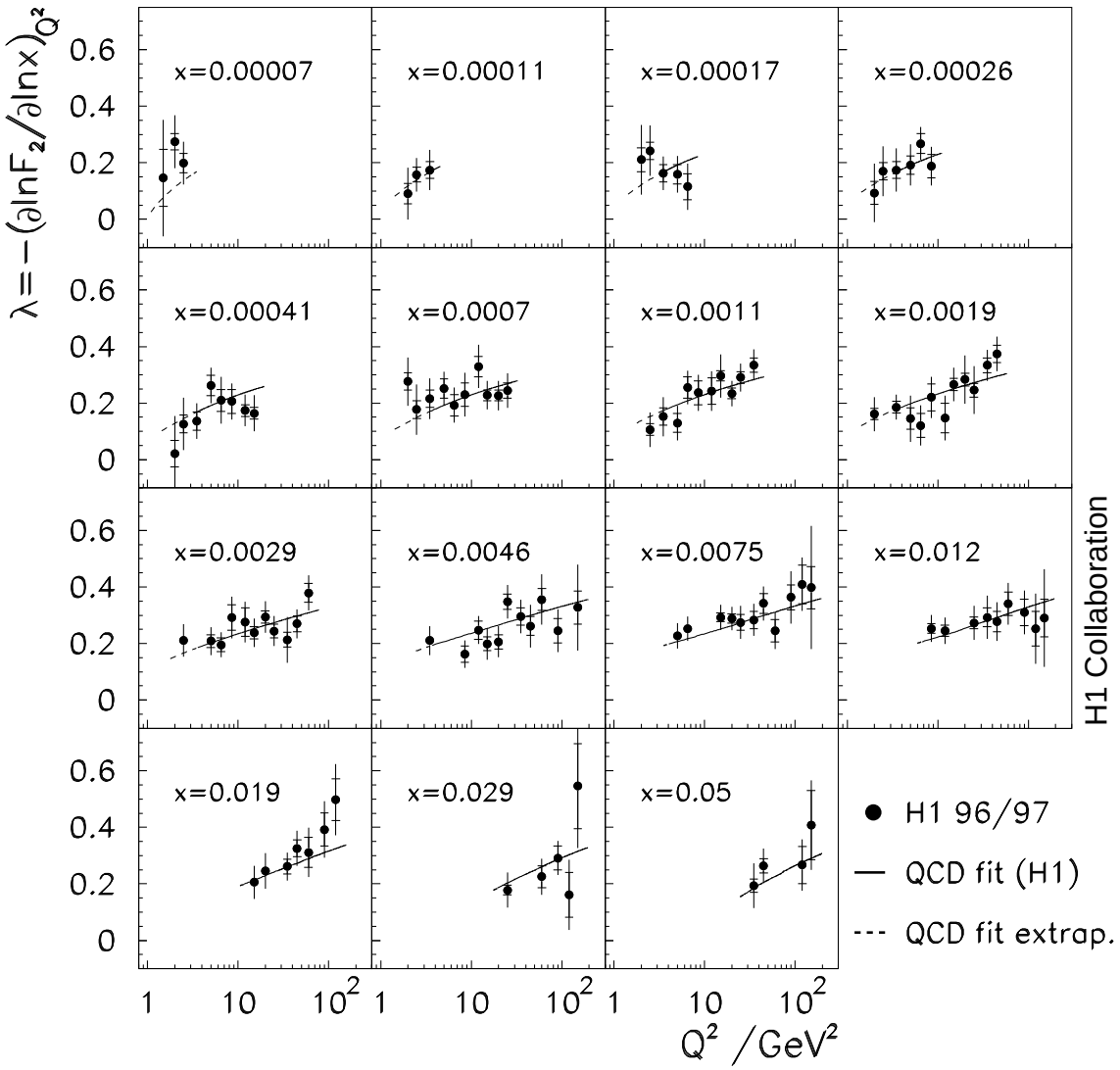

Figure 2: Measurement of the function $\lambda(x, Q^2)$: the inner error bars represent the statistical uncertainty; the full error bars include the systematic uncertainty added in quadrature; the solid curves represent the NLO QCD fit to the H1 cross section data described in [6]; the minimum Q^2 value of the data included in this fit is $Q^2 = 3.5$ GeV^2.

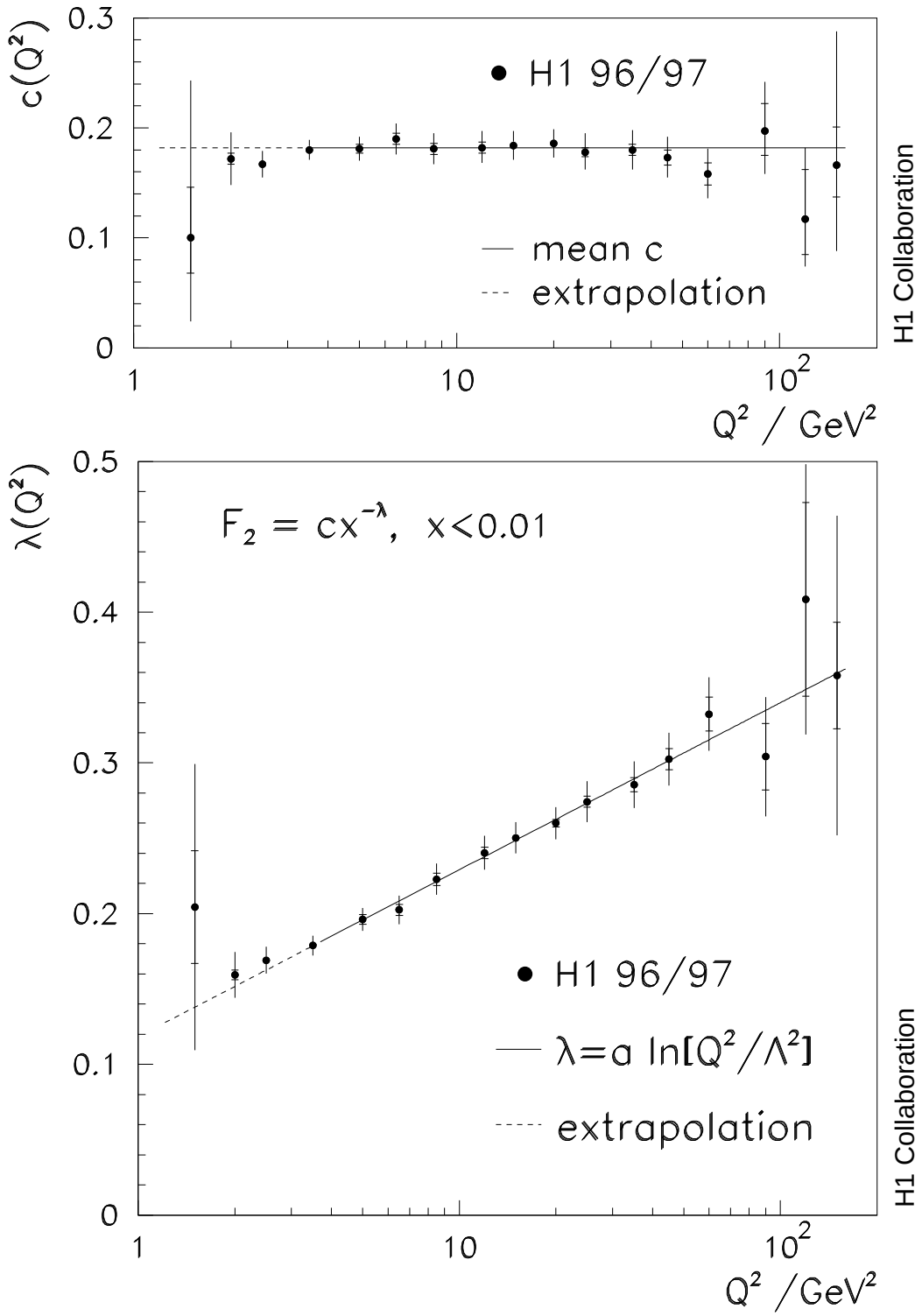

Figure 3: Determination of the coefficients $c(Q^2)$ (upper plot) and of the exponents $\lambda(Q^2)$ (lower plot) from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ to the H1 structure function data [6] for $x \leq 0.01$; the inner error bars illustrate the statistical uncertainties, the full error bars represent the statistical and systematic uncertainties added in quadrature. The straight lines represent the mean coefficient c (upper plot) and a fit of the form $a \ln[Q^2/\Lambda^2]$ (lower plot), respectively, using data for $Q^2 \geq 3.5$ GeV^2.

Q^2 [GeV^2]	x	λ	δ_{sta}	δ_{sys}	δ_{tot}	Q^2 [GeV^2]	x	λ	δ_{sta}	δ_{sys}	δ_{tot}
1.5	0.000041	0.225	0.052	0.131	0.141	15.0	0.00260	0.238	0.021	0.045	0.050
1.5	0.000065	0.146	0.101	0.180	0.206	15.0	0.00410	0.199	0.024	0.050	0.056
2.0	0.000065	0.274	0.029	0.089	0.093	15.0	0.00750	0.292	0.016	0.041	0.044
2.0	0.000105	0.090	0.036	0.085	0.092	15.0	0.0210	0.206	0.013	0.057	0.059
2.0	0.000165	0.211	0.043	0.116	0.124	20.0	0.000650	0.227	0.019	0.048	0.052
2.0	0.000260	0.093	0.040	0.096	0.104	20.0	0.00105	0.234	0.019	0.041	0.045
2.0	0.000410	0.022	0.047	0.124	0.132	20.0	0.00165	0.284	0.022	0.082	0.085
2.0	0.000750	0.276	0.030	0.080	0.086	20.0	0.00260	0.293	0.022	0.051	0.055
2.0	0.00210	0.161	0.020	0.057	0.060	20.0	0.00410	0.206	0.025	0.049	0.055
2.5	0.000065	0.199	0.034	0.068	0.076	20.0	0.00750	0.289	0.016	0.038	0.041
2.5	0.000105	0.158	0.026	0.053	0.059	20.0	0.0210	0.246	0.012	0.063	0.064
2.5	0.000165	0.242	0.031	0.084	0.089	25.0	0.000650	0.244	0.027	0.055	0.061
2.5	0.000260	0.170	0.030	0.082	0.088	25.0	0.00105	0.291	0.020	0.045	0.050
2.5	0.000410	0.126	0.032	0.088	0.093	25.0	0.00165	0.246	0.024	0.081	0.084
2.5	0.000650	0.177	0.032	0.084	0.090	25.0	0.00260	0.244	0.024	0.047	0.053
2.5	0.00119	0.106	0.021	0.058	0.061	25.0	0.00410	0.348	0.027	0.053	0.059
2.5	0.00329	0.211	0.013	0.057	0.058	25.0	0.00650	0.274	0.028	0.051	0.058
3.5	0.000105	0.174	0.030	0.065	0.071	25.0	0.0119	0.273	0.020	0.056	0.060
3.5	0.000165	0.163	0.031	0.050	0.059	25.0	0.0329	0.178	0.016	0.060	0.062
3.5	0.000260	0.174	0.030	0.071	0.077	35.0	0.00105	0.335	0.025	0.049	0.055
3.5	0.000410	0.136	0.032	0.054	0.063	35.0	0.00165	0.334	0.026	0.047	0.054
3.5	0.000650	0.216	0.031	0.065	0.072	35.0	0.00260	0.213	0.026	0.077	0.081
3.5	0.00105	0.152	0.031	0.063	0.070	35.0	0.00410	0.295	0.030	0.051	0.059
3.5	0.00190	0.186	0.021	0.039	0.044	35.0	0.00650	0.283	0.030	0.048	0.057
3.5	0.00525	0.210	0.012	0.050	0.051	35.0	0.0105	0.293	0.033	0.070	0.077
5.0	0.000165	0.159	0.035	0.055	0.065	35.0	0.0190	0.261	0.026	0.042	0.049
5.0	0.000260	0.192	0.033	0.065	0.073	35.0	0.0525	0.194	0.023	0.076	0.079
5.0	0.000410	0.262	0.036	0.052	0.063	45.0	0.00165	0.374	0.031	0.053	0.061
5.0	0.000650	0.251	0.034	0.049	0.060	45.0	0.00260	0.270	0.028	0.043	0.052
5.0	0.00105	0.129	0.033	0.051	0.061	45.0	0.00410	0.262	0.033	0.068	0.075
5.0	0.00165	0.145	0.038	0.072	0.081	45.0	0.00650	0.342	0.035	0.049	0.060
5.0	0.00299	0.209	0.022	0.044	0.049	45.0	0.01050	0.277	0.037	0.058	0.069
5.0	0.00849	0.227	0.013	0.042	0.044	45.0	0.0190	0.325	0.029	0.055	0.062
6.5	0.000165	0.115	0.046	0.068	0.082	45.0	0.0525	0.263	0.025	0.056	0.061
6.5	0.000260	0.268	0.036	0.047	0.059	60.0	0.00260	0.379	0.033	0.054	0.063
6.5	0.000410	0.210	0.039	0.073	0.082	60.0	0.00410	0.356	0.038	0.081	0.089
6.5	0.000650	0.193	0.038	0.049	0.062	60.0	0.00650	0.246	0.040	0.051	0.064
6.5	0.00105	0.255	0.037	0.047	0.060	60.0	0.0105	0.340	0.042	0.061	0.074
6.5	0.00165	0.121	0.042	0.058	0.072	60.0	0.0165	0.311	0.053	0.070	0.088
6.5	0.00299	0.195	0.024	0.037	0.044	60.0	0.0299	0.225	0.039	0.052	0.065
6.5	0.00849	0.253	0.014	0.040	0.043	60.0	0.0849	0.255	0.035	0.064	0.073
8.5	0.000260	0.188	0.042	0.054	0.068	90.0	0.00410	0.245	0.043	0.063	0.076
8.5	0.000410	0.206	0.042	0.048	0.064	90.0	0.00650	0.363	0.044	0.081	0.092
8.5	0.000650	0.230	0.042	0.064	0.077	90.0	0.0105	0.310	0.047	0.061	0.077
8.5	0.00105	0.237	0.042	0.049	0.064	90.0	0.0165	0.392	0.058	0.081	0.100
8.5	0.00165	0.220	0.047	0.054	0.072	90.0	0.0299	0.291	0.042	0.043	0.060
8.5	0.00260	0.292	0.045	0.058	0.073	90.0	0.0849	0.340	0.039	0.035	0.052
8.5	0.00475	0.162	0.028	0.040	0.049	120.0	0.00650	0.410	0.068	0.066	0.095
8.5	0.0132	0.253	0.017	0.045	0.048	120.0	0.0105	0.252	0.061	0.107	0.124
12.0	0.000410	0.174	0.020	0.052	0.056	120.0	0.0165	0.498	0.074	0.102	0.126
12.0	0.000650	0.330	0.036	0.067	0.076	120.0	0.0260	0.161	0.079	0.095	0.124
12.0	0.00105	0.243	0.047	0.052	0.070	120.0	0.0475	0.267	0.065	0.063	0.091
12.0	0.00165	0.147	0.053	0.059	0.079	120.0	0.132	0.312	0.056	0.091	0.106
12.0	0.00260	0.276	0.050	0.053	0.073	150.0	0.00410	0.327	0.059	0.141	0.153
12.0	0.00475	0.247	0.032	0.038	0.049	150.0	0.00650	0.397	0.075	0.204	0.218
12.0	0.0132	0.245	0.019	0.043	0.047	150.0	0.01400	0.291	0.067	0.159	0.173
15.0	0.000410	0.164	0.020	0.060	0.064	150.0	0.0260	0.545	0.150	0.158	0.218
15.0	0.000650	0.227	0.018	0.047	0.050	150.0	0.0475	0.408	0.122	0.102	0.159
15.0	0.00105	0.296	0.018	0.074	0.076	150.0	0.132	0.308	0.101	0.096	0.139
15.0	0.00165	0.266	0.021	0.055	0.059						

Table 1: Measurement of the derivative $\lambda = -(\partial \ln F_2 / \partial \ln x)_{Q^2}$ at fixed Q^2. For the systematic uncertainties the correlations between adjacent x values are taken into account. The total error is the squared sum of the statistical and systematic uncertainties, given as absolute values.

$Q^2[GeV^2]$	c	δ^c_{sta}	δ^c_{tot}	λ	δ^λ_{sta}	δ^λ_{tot}
1.5	0.10	$+$ 0.05 $-$ 0.03	$+$ 0.14 $-$ 0.08	0.20	0.04	0.10
2.0	0.172	0.005	0.024	0.159	0.003	0.015
2.5	0.167	0.003	0.012	0.169	0.002	0.009
3.5	0.180	0.003	0.009	0.179	0.002	0.007
5.0	0.181	0.004	0.011	0.196	0.003	0.008
6.5	0.190	0.005	0.014	0.202	0.004	0.009
8.5	0.181	0.005	0.014	0.223	0.004	0.010
12.0	0.182	0.005	0.015	0.240	0.004	0.011
15.0	0.184	0.003	0.013	0.250	0.002	0.010
20.0	0.186	0.003	0.013	0.260	0.003	0.011
25.0	0.178	0.004	0.017	0.274	0.004	0.014
35.0	0.180	0.005	0.018	0.286	0.005	0.016
45.0	0.173	0.007	0.019	0.302	0.007	0.017
60.0	0.158	0.010	0.023	0.332	0.011	0.024
90.0	0.197	$+$ 0.025 $-$ 0.022	$+$ 0.045 $-$ 0.039	0.304	0.022	0.040
120.0	0.117	$+$ 0.045 $-$ 0.032	$+$ 0.065 $-$ 0.043	0.408	0.064	0.089
150.0	0.17	$+$ 0.04 $-$ 0.03	$+$ 0.12 $-$ 0.08	0.36	0.04	0.11

Table 2: The coefficients c and exponents λ from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ using H1 F_2 data [6], for $x \leq 0.01$, taking into account the systematic error correlations. Here δ_{sta} denotes the statistical uncertainty and δ_{tot} comprises all uncertainties added in quadrature. The uncertainties are given as absolute values. They are symmetric to very good approximation, apart from the uncertainties of the coefficient $c(Q^2)$ at the edges of the Q^2 region.