DESY 01–107
July 2001

hep-ph/0107317

QCD Analysis of Polarized Deep Inelastic Scattering Data

J. Blümlein* and H. Böttcher*

*DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany

arXiv:hep-ph/0107317 31 Jul 2001

Abstract. A QCD analysis of the world data on inclusive polarized deep inelastic scattering of leptons on nucleons is presented in leading and next–to–leading order. New parameterizations are derived for the quark and gluon distributions and the value of $\alpha_s(M_Z)$ is determined. Emphasis is put on the derivation of fully correlated error bands for these distributions which are directly applicable to determine experimental errors of other polarized observables. The impact of the variation of both the renormalization and factorization scales on the value of α_s is studied. Finally a factorization–scheme invariant QCD analysis based on the observables $g_1(x,Q^2)$ and $dg_1(x,Q^2)/d\log(Q^2)$ is performed in next–to–leading order, which is compared to the standard analysis.

INTRODUCTION

The remarkable growth of experimental data on inclusive polarized deep inelastic scattering of leptons on nucleons over the last years [1–9] allows to perform refined QCD analyses of polarized structure functions in order to reveal the spin–dependent partonic structure of the nucleon. A number of such analyses has already been worked out. The most recent ones are [10–13] [1]. In this talk results from a new QCD analysis in leading (LO) and next–to–leading (NLO) order [14] [2] are presented. New parameterizations of the polarized quark and gluon distributions are derived including the parameterizations of fully correlated 1σ error bands for these distributions, which are directly applicable to calculate errors of other polarized observables. Furthermore the value of $\alpha_s(M_Z)$ is determined. Finally and for the first time a factorization–scheme independent QCD evolution based on the observables $g_1(x,Q^2)$ and $dg_1(x,Q^2)/d\log(Q^2)$ in next–to–leading order is performed.

FORMALISM

In LO the polarized structure function $g_1(x,Q^2)$ is expressed as the sum of the polarized quark distributions $\Delta q_i(x,Q^2)$ weighted by the square of the quark charges. In NLO the

[1] For a more complete list of references see the references therein and Ref. [14]
[2] All details of the analysis are given in Ref. [14].

expression for $g_1(x,Q^2)$ involves the polarized singlet $\Delta\Sigma(x,Q^2)$, the gluon $\Delta G(x,Q^2)$, and the non–singlet $\Delta q^{NS}(x,Q^2)$ distributions and reads

$$g_1(x,Q^2) = \frac{1}{2}\left[\left(\frac{1}{n_f}\sum_{i=1}^{n_f} e_i^2\right)[\delta C_S \otimes \Delta\Sigma + \delta C_G \otimes \Delta G] + \delta C_{NS} \otimes \Delta q^{NS}\right], \qquad (1)$$

where n_f is the number of active quark flavors and e_i is the quark charge. The symbol $\otimes$ denotes the Mellin convolution w.r.t. x of the polarized parton densities $\Delta q_i(x,Q^2)$ with the corresponding polarized Wilson coefficient functions $\delta C_i(x,\alpha_s(Q^2))$. The polarized singlet and non–singlet distributions are certain combinations of the polarized quark distributions $\Delta q_i(x,Q^2)$.

The evolution equations used to evolve the parton densities to different Q^2 values contain the polarized splitting functions $\Delta P_{ij}(x,\alpha_s(Q^2))$. Both the polarized Wilson coefficient [15] and the polarized splitting functions [16] are known in the $\overline{MS}$ scheme up to order $O(\alpha_s^2)$.

METHOD

The shape chosen for the parameterization of the polarized parton distributions at the input scale of $Q^2 = 4.0\ GeV^2$ is :

$$x\Delta q_i(x,Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x+\rho_i x^{\frac{1}{2}}). \qquad (2)$$

The normalization constant A_i is chosen such that η_i is the first moment of $\Delta q_i(x,Q_0^2)$. The densities to be fitted are Δu_v[3], Δd_v, $\Delta\bar{q}$, and ΔG.

Assuming $SU(3)$ flavor symmetry the first moments of Δu_v and Δd_v are determined by the $SU(3)$ parameters F and D measured in neutron and hyperon β–decays and can be fixed to $\eta_{u_v} = 0.926$ and $\eta_{d_v} = -0.341$. In addition we assume a flavor symmetric sea, i.e. only one general sea distribution $\Delta\bar{q}(x,Q^2)$ is required. No assumptions are made concerning positivity and helicity retention. Given the present accuracy of the data we set a number of parameters to zero, namely $\rho_{u_v} = \rho_{d_v} = 0$, $\gamma_{\bar{q}} = \rho_{\bar{q}} = 0$, and $\gamma_G = \rho_G = 0$. This choice reduces the number of parameters to be fitted for each parton distribution to three. In addition the parameter Λ_{QCD} was determined. The relative normalizations of the different data sets were fitted and then fixed. Doing so part of the experimental systematics was taken into account.

RESULTS

The results reported here are based on 433 data points of asymmetry data, i.e. g_1/F_1 or A_1, above $Q^2 = 1.0\ GeV^2$, the world statistics published so far. The QCD fits are performed on g_1 which is evaluated from the asymmetry data using parameterizations for

[3] Note that: $\Delta q + \Delta\bar{q} = \Delta q_v + 2\Delta\bar{q}$.

the unpolarized structure functions F_2 [17] and R [18]. We realized that the 4 parameters γ_{u_v}, γ_{d_v}, $b_{\bar{q}}$, and b_G had to be fixed in addition at their values at $\chi^2_{\rm min}$ since the data do not constrain these parameters well enough. Only fits with a positive definite covariance matrix were accepted in order to be able to calculate the fully correlated 1σ error bands. The NLO polarized parton densities at the input scale are presented in Fig. 1.

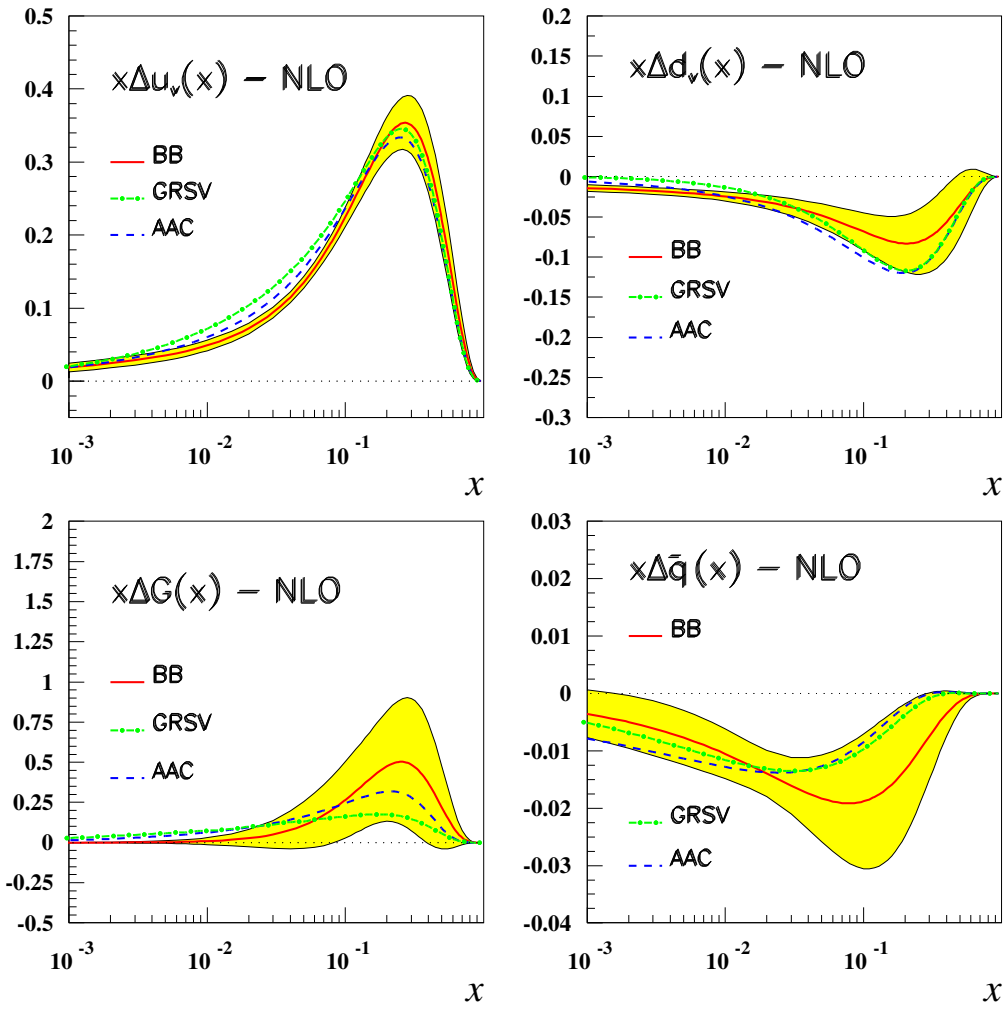

FIGURE 1. Polarized parton distribution at the input scale $Q_0^2 = 4.0\ GeV^2$ (solid line) compared to results obtained by GRSV (dashed–dotted line) [12] and AAC (dashed line) [10]. The shaded areas represent the fully correlated 1σ error bands calculated by Gaussian error propagation, Ref. [14].

While the quality of the data is sufficient to determine Δu_v and Δd_v with good accuracy, ΔG and $\Delta\bar{q}$ have much broader error bands. This is essentially due to the lack of data at low x. The agreement with the results of the analyses of Refs. [10] and [12] is satisfactory within the error bands. The measured structure function g_1^p is well described both as function of x and of Q^2. The derived parton distributions and its error bands have been evolved to Q^2 values up to $10,000\ GeV^2$. As an example the evolution of ΔG is shown in Fig. 2. One observes that even within the error band ΔG stays *positive* up to the highest Q^2 value. It should be mentioned that $\Delta\bar{q}$ develops a trend to change sign and becomes slightly positive towards higher Q^2 values and for $x \gtrsim 0.1$ within the errors.

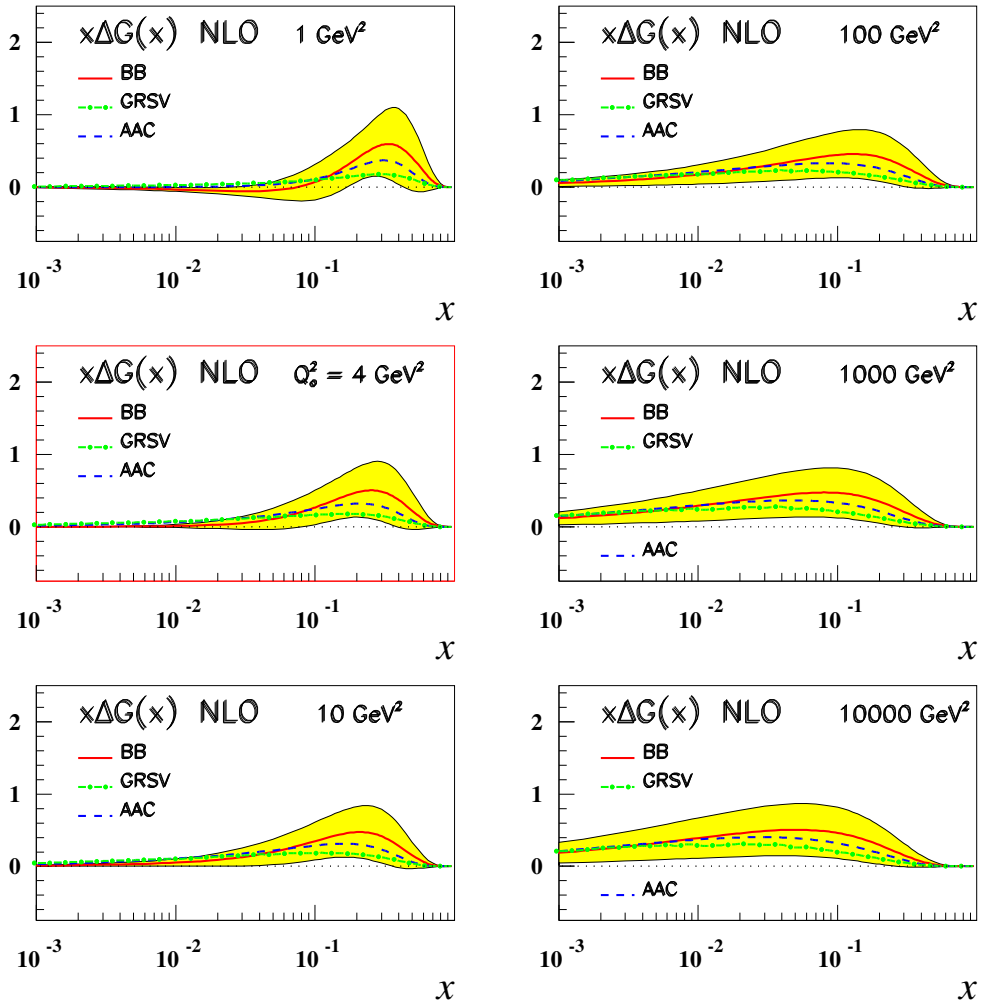

FIGURE 2. The polarized parton distribution ΔG evolved up to Q^2 values up to $Q^2 - 10,000\ Gev^2$ (solid line) compared to results obtained by GRSV (dashed–dotted line) [12] and AAC (dashed line) [10]. The shaded areas represent the fully correlated 1σ error bands calculated by Gaussian error propagation, Ref. [14].

In determining $\alpha_s(M_z^2)$ the parameter Λ_{QCD} was fitted. The impact of the variation of both the renormalization and factorization scales on the value of α_s was studied. The following value for $\Lambda_{\rm QCD}$ was obtained

$$\Lambda^{(4)}_{\rm QCD} = 241 \pm 58\ (\text{fit})\ {}^{+65}_{-44}\ (\text{fac})\ {}^{+117}_{-58}\ (\text{ren}) \quad MeV,$$

which results into a value of

$$\alpha_s(M_Z^2) = 0.114\ {}^{+0.004}_{-0.005}\ (\text{fit})\ {}^{+0.005}_{-0.004}\ (\text{fac})\ {}^{+0.008}_{-0.005}\ (\text{ren}).$$

This value of $\alpha_s(M_Z^2)$ is compatible within the errors with the world average of 0.118 ± 0.002 [19] and with values from other QCD analyses [20], although the central value tends to be lower, as also in Ref. [20b].

Finally a factorization–scheme invariant QCD analysis based on the observables $g_1(x,Q^2)$ and $dg_1(x,Q^2)/d\log(Q^2)$ in next-to-leading order was performed. The corresponding evolution equations have been worked out in Ref. [21] [4]. Such an analysis has the advantage of direct control over the input since it comes from measured quantities. The only parameter to be determined is Λ_{QCD}. Unfortunately, the present data do not yet allow to determine the slope $\partial g_1(x,Q^2)/\partial\log(Q^2)$ as an input density from measurements of g_1, but it is derived here from the fit result for g_1 described above. The evolution of the so determined slope is shown in Fig. 3.

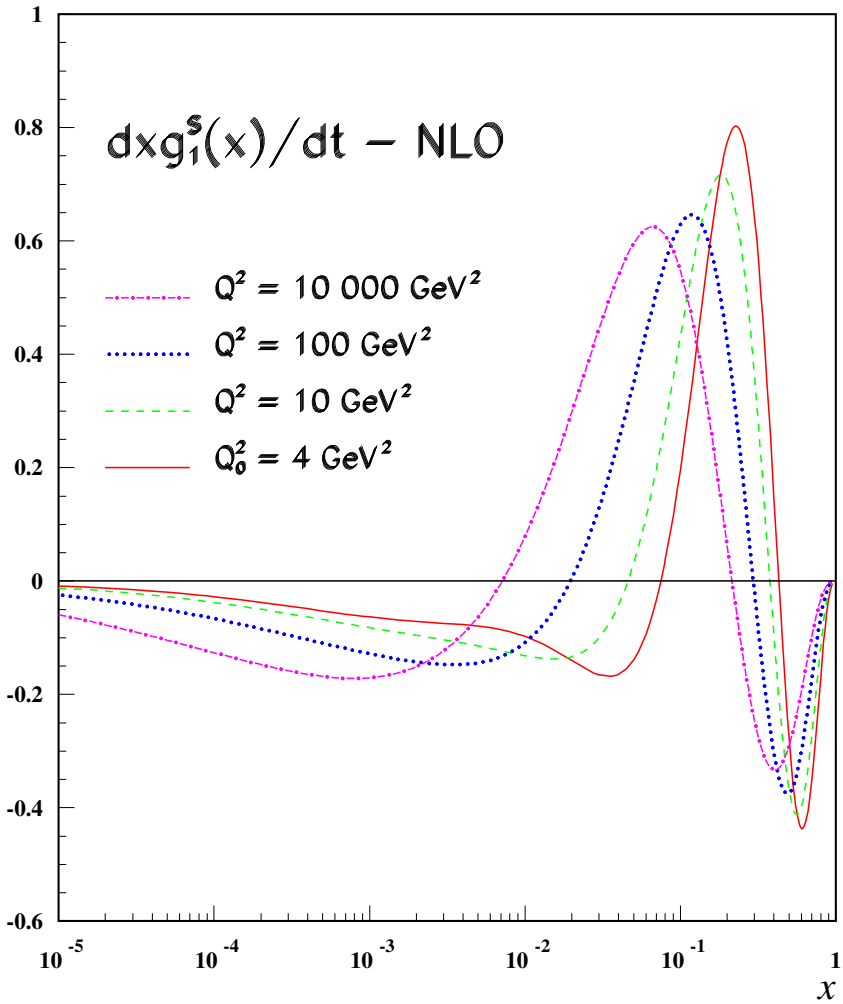

FIGURE 3. The evolution of $dg_1^s(x,Q^2)/dt$ (singlet contribution) with $t = -2/\beta_0 \ln(\alpha_s(Q^2)/\alpha_s(Q_0^2))$. The slope of g_1 was determined from a fitted g_1, see text.

[4] The same case has already been considered in Ref. [22].

A downward shift of 12 MeV in Λ_{QCD} was found yielding a similar result for $\alpha_s(M_Z^2)$ as obtained in the standard analysis.

CONCLUSIONS

An LO and NLO QCD Analysis of the current World–Data on Polarized Structure Functions was performed. New parameterizations of the polarized parton densities including their errors were derived. They are available via a fast `FORTRAN` code for the range: $1 < Q^2 < 10^6$ GeV^2 and $10^{-4} < x < 1$. The value determined for $\alpha_s(M_Z^2)$ is compatible with the world average, although the central value obtained is lower. First steps in a factorization–scheme invariant QCD evolution based on the structure function $g_1(x,Q^2)$ and $\partial g_1(x,Q^2)/\partial \log Q^2$ were performed yielding similar results for $\alpha_s(M_Z^2)$. This latter analysis is a very promising way to proceed in the future, since it allows to extract $\Lambda_{\rm QCD}$ fixing all the input distributions by direct measurements.

ACKNOWLEDGMENTS

This work was supported in part by EU contract FMRX-CT98-0194 (DG 12 - MIHT). For discussions in an early phase of this work we would like to thank A. Vogt.

REFERENCES

1. J. Ashman et al. (EMC), Phys. Lett. **B206** (1988) 364; Nucl. Phys. **B328** (1989) 1.
2. P.L. Anthony et al. (E142), Phys. Rev. **D54** (1996) 6620.
3. K. Ackerstaff et al. (HERMES), Phys. Lett. **B404** (1997) 383.
4. K. Abe et al. (E154), Phys. Rev. Lett. **79** (1997) 26.
5. B. Adeva et al. (SMC), Phys. Rev. **D58** (1998) 112001.
6. K. Abe et al. (E143), Phys. Rev. **D58** (1998) 120003.
7. A. Airapetian et al. (HERMES), Phys. Lett. **B442** (1998) 484,
   D. Hasch, PhD thesis, Humboldt-Univ. Berlin, (1999) and DESY-THESIS-2000-032.
8. P.L. Anthony et al. (E155), Phys. Lett. **B463** (1999) 339.
9. P.L. Anthony et al. (E155), Phys. Lett. **B493** (2000) 19.
10. Y. Goto et al., Phys. Rev. **D62** (2000) 034017.
11. Dilip Kumar Gosh et al., Phys. Rev. **D62** (2000) 094112.
12. M. Glück et al., Phys. Rev. **D63** (2001) 094005.
13. E. Leader et al., hep-ph/0106214, to be published in Proceed. of DIS2001.
14. J. Blümlein and H. Böttcher, DESY 01-087 (2001), in preparation.
15. W. L. van Neerven and E. B. Zijlstra, Nucl. Phys. **B417** (1994) 61, Nucl. Phys. **B426** (1994) 245.
16. R. Mertig and W. L. van Neerven, Z. Phys. **C70** (1996) 637,
    W. Vogelsang, Phys. Rev. **D54** (1996) 2023.
17. M. Arneodo et al., Phys. Lett. **B364** (1995) 107.
18. L. Withlow et al., Phys. Lett. **B250** (1990) 193.
19. Eur. Phys. J. **C15** (2000) .
20. a) G. Altarelli et al., Nucl. Phys. **B496** (1997) 337,
    b) K. Abe et al. (E154), Phys. Lett. **B405** (1997) 180,
    c) B. Adeva et al. (SMC), Phys. Rev. **D58** (1998) 112002.
21. J. Blümlein, V. Ravindran, and W.L. van Neerven, Nucl. Phys. **B586** (2000) 349.
22. W. Furmanski and R. Petronzio, Z. Phys. **C11** (1982) 293.