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CP Violation, Neutrino Mixing and the Baryon Asymmetry

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Abstract

We study neutrino masses and mixings based on the simplest SO(10) mass relations and the seesaw mechanism. We find that the requirement of large neutrino mixings determines the relative magnitude of the heavy Majorana neutrino masses in terms of the known quark mass hierarchy. This leads to specific predictions for the structure of the neutrino mixing matrix, the light neutrino masses, CP violation in neutrino oscillations, neutrinoless double β-decay and the baryon asymmetry.

Recent results from the Sudbury Neutrino Observatory [1] and from the Super-Kamiokande experiment [2] provide further evidence for the neutrino oscillation hypothesis as the solution of the solar neutrino problem. Neutrino oscillations can also account for the atmospheric neutrino anomaly [3, 4], and a consistent picture is obtained with just three neutrinos, ν_e, ν_μ and ν_τ, undergoing 'nearest neighbour' oscillations, $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$.

The experimental results on the ν_e deficit in the solar neutrino flux favour the LMA or LOW solutions [5] of of the MSW conversion with large mixing angle. A large mixing also fits the atmospheric neutrino oscillations. As a result, the leptonic mixing matrix $U_{\alpha i}$ seems to be very different from the familiar CKM quark mixing matrix $V_{\alpha i}$. One finds $|U_{\alpha i}| = \mathcal{O}(1)$ for all elements, except for $|U_{e3}| < 0.16$ [6]. Furthermore, a possible hierarchy among the neutrino masses m_i has to be much weaker than the known hierarchy of quarks and charged leptons.

The seesaw mechanism naturally explains the smallness of light Majorana neutrino masses m by the largeness of right-handed neutrino masses M [7],

$$m_\nu \simeq -m_D \frac{1}{M} m_D^T \ , \tag{1}$$

where m_D is the Dirac neutrino mass matrix. In unified theories one expects m_D to be related to the quark and charged lepton mass matrices. Since they have a large hierarchy, the almost non-hierarchical structure of the leptonic mixing matrix is quite surprising.

The simplest grand unified theory (GUT) which unifies one generation of quarks and leptons including the right-handed neutrino in a single irreducible representation is based on the gauge group SO(10) [8]. In the following we shall demonstrate that, given the known properties of the up-quark mass matrix, the puzzle of large neutrino mixings can be resolved in SO(10) theories provided the heavy neutrino masses also obey a specific hierarchy. We then explore the consequences for several observables in neutrino physics including the cosmological baryon asymmetry. The role of the heavy neutrino mass hierarchy for the light neutrino mixings has already been discussed in different contexts [9, 10, 11, 12].

In SO(10) theories quark and lepton masses are obtained by coupling the fermion multiplet $\mathbf{16} = (q_L, u_R^c, e_R^c, d_R^c, l_L, \nu_R)$ to the Higgs multiplets $H_1(\mathbf{10})$, $H_2(\mathbf{10})$ and $\Phi(\mathbf{126})$,

$$\mathcal{L} = h_{uij}\mathbf{16}_i\mathbf{16}_j H_1(\mathbf{10}) + h_{dij}\mathbf{16}_i\mathbf{16}_j H_2(\mathbf{10}) + h_{Nij}\mathbf{16}_i\mathbf{16}_j\Phi(\mathbf{126}) \ . \tag{2}$$

Here we have assumed that the two Higgs doublets of the standard model are contained

in H_1 and H_2, respectively[1]. The corresponding Yukawa couplings are

$$\begin{aligned}\mathcal{L}_m &= h_{uij}\overline{q}_{Li}u_{Rj}H_1 + h_{dij}\overline{q}_{Li}d_{Rj}H_2 \\ &\quad + h_{\nu ij}\overline{l}_{Li}\nu_{Rj}H_1 + h_{eij}\overline{l}_{Li}e_{Rj}H_2 + \frac{1}{2}h_{Nij}\overline{\nu}^c_{Ri}\nu_{Rj}\Phi + h.c.\,. \end{aligned} \tag{3}$$

The quark and lepton mass matrices $m_u = h_u v_1$, $m_d = h_d v_2$, $m_D = h_\nu v_1$ and $m_e = h_e v_2$, with $v_1 = \langle H \rangle_1$ and $v_2 = \langle H \rangle_2$, satisfy the relations

$$m_u = m_D \ , \quad m_d = m_e \ . \tag{4}$$

Note, that all mass matrices are symmetric. The incorrect relation $m_s = m_\mu$ can be corrected by contributions from higher dimensional Higgs representations [14]. The Majorana mass matrix $M = h_N \langle \Phi \rangle$ is a priori independent of m_u and m_d.

From the phenomenology of weak decays we know that the quark matrices have approximately the form (see, e.g. [15, 16])

$$m_{u,d} \propto \begin{pmatrix} 0 & \epsilon^3 e^{i\phi} & 0 \\ \epsilon^3 e^{i\phi} & \rho\epsilon^2 & \eta\epsilon^2 \\ 0 & \eta\epsilon^2 & e^{i\psi} \end{pmatrix} \ . \tag{5}$$

Here $\epsilon \ll 1$ is the parameter which determines the flavour mixing, and

$$\rho = |\rho| e^{i\alpha} \ , \quad \eta = |\eta| e^{i\beta} \ , \tag{6}$$

are complex parameters $\mathcal{O}(1)$. We have chosen a 'hierarchical' basis, where off-diagonal matrix elements are small compared to the product of the corresponding eigenvalues, $|m_{ij}|^2 \leq \mathcal{O}(|m_i m_j|)$. In contrast to the usual assumption of hermitian mass matrices [15, 16], SO(10) invariance dictates the matrices to be symmetric. All parameters may take different values for up- and down- quarks. Typical choices for ϵ are $\epsilon_u = 0.07$, $\epsilon_d = 0.21$ [16]. The agreement with data can be improved by adding in the 1-3 element a term $\mathcal{O}(\epsilon^4)$ [17, 18] which, however, is not important for our analysis.

Three of the four phases in the quark mass matrix (5) can be absorbed into a phase matrix P,

$$m_{u,d} = P \ \widetilde{m}_{u,d} \ P \ , \tag{7}$$

where $\widetilde{m}_{u,d} = m_{u,d}(\phi = \alpha = \psi = 0)$ and

$$P = \begin{pmatrix} e^{i(\phi - \alpha/2)} & 0 & 0 \\ 0 & e^{i\alpha/2} & 0 \\ 0 & 0 & e^{i\psi/2} \end{pmatrix} \ . \tag{8}$$

[1]Note, that this is unavoidable in models with SO(10) breaking by orbifold compactification [13].

It is then straightforward, but more tedious than in the hermitean case (cf. [16]), to relate the phases of the mass matrix to those in the CKM matrix. We obtain $\beta_{CKM} = -\chi - \omega$ and $\gamma_{CKM} = \pi + \chi - \Delta$, where $\Delta = \phi_u - \alpha_u - \phi_d + \alpha_d$ and χ is defined in ref. [16]. The angle ω is a function of the phases and the real parameters. For $(\epsilon_u)^2 \ll (\epsilon_d)^2$ we have $\omega = \omega_d = \beta_d - (\alpha_d + \psi_d)/2$. Data implies $\Delta \simeq \pi/2$ with correspondingly smaller values for χ and ω.

Further information on the phases, in particular on relations between phases in the up- and down-quark mass matrices can come from theoretical consistency conditions. In this connection it might be interesting that the QCD Θ-parameter, which controls strong CP violation, is not renormalized if the quark mass matrices satisfy the condition $\mathrm{Im}\{\det(m_u m_d)\} = 0$ [19]. This suggests the phase relation

$$2\phi_u + \psi_u + 2\phi_d + \psi_d = n\ \pi \tag{9}$$

with integer n. It would go beyond the purpose of this paper to discuss all the possible solutions of this equation. We note, however, that if also ω_u is small and relation (18) below holds, $\phi_u \simeq \psi_u$ and there are two solutions, $\phi_u \simeq \pm\pi/4$, with $\alpha_u = 0$, depending on the value of n in eq. (9).

We do not know the structure of the Majorana mass matrix $M = h_N \langle\Phi\rangle$. It may be independent of the Higgs field, as in models with family symmetries. In this case, one expects the same texture zeroes as in the quark mass matrices,

$$M = \begin{pmatrix} 0 & M_{12} & 0 \\ M_{12} & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{pmatrix}, \tag{10}$$

with $M_{12} \ll M_{22} \sim M_{23} \ll M_{33}$. M is diagonalized by a unitary matrix $U^{(N)}$,

$$U^{(N)\dagger} M U^{(N)*} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}. \tag{11}$$

Using the seesaw formula we can now evaluate the light neutrino mass matrix. Since the choice of the Majorana matrix m_N fixes a basis for the right-handed neutrinos the allowed phase redefinitions of the Dirac mass matrix m_D are restricted. In eq. (5) we have therefore kept the phases of all matrix elements.

The ν_μ-ν_τ mixing angle is known to be large. This leads us to require $m_{\nu_{i,j}} = \mathcal{O}(1)$ for $i, j = 2, 3$. It is remarkable that this determines the hierarchy of the heavy Majorana

mass matrix to be[2]

$$M_{12} : M_{22} : M_{33} = \epsilon^5 : \epsilon^4 : 1 \ . \tag{12}$$

With $M_{33} \simeq M_3$, $M_{22} = \sigma\epsilon^4 M_3$, $M_{23} = \zeta\epsilon^4 M_3 \sim M_{22}$ and $M_{12} = \epsilon^5 M_3$, one obtains for masses and mixings to order $\mathcal{O}(\epsilon^4)$

$$M_1 \simeq -\frac{\epsilon^6}{\sigma} M_3 \ , \quad M_2 \simeq \sigma\epsilon^4 M_3 \ , \tag{13}$$

$$U_{12}^{(N)} = -U_{21}^{(N)} = \frac{\epsilon}{\sigma} \ , \quad U_{23}^{(N)} = \mathcal{O}(\epsilon^4) \ , \quad U_{13}^{(N)} = 0 \ . \tag{14}$$

Note, that σ can always be chosen real whereas ζ is in general complex. The inverse matrix reads, to leading order in ϵ,

$$M^{-1} = \begin{pmatrix} -\sigma & \epsilon & -\zeta\epsilon^3 \\ \epsilon & 0 & 0 \\ -\zeta\epsilon^3 & 0 & \epsilon^6 \end{pmatrix} \frac{1}{\epsilon^6 M_3} \ . \tag{15}$$

This yields for the light neutrino mass matrix

$$m_\nu = -\begin{pmatrix} 0 & \epsilon e^{2i\phi} & 0 \\ \epsilon e^{2i\phi} & -\sigma e^{2i\phi} + 2\rho e^{i\phi} & \eta e^{i\phi} \\ 0 & \eta e^{i\phi} & e^{2i\psi} \end{pmatrix} \frac{v_1^2}{M_3} \ . \tag{16}$$

The complex parameter ζ does not enter because of the hierarchy. Since, as required, all elements of the 2-3 submatrix are $\mathcal{O}(1)$, the mixing angle Θ_{23} is naturally large. A large mixing angle Θ_{12} may occur in case of a small determinant of the 2-3 submatrix [20],

$$(-\sigma + 2\rho e^{-i\phi})e^{2i\psi} - \eta^2 \equiv \delta e^{2i\gamma} = \mathcal{O}(\epsilon) \ . \tag{17}$$

Such a condition can be fullfilled without fine tuning if $\sigma, \rho, \eta = \mathcal{O}(1)$. It implies relations between the moduli as well as the phases of ρ and η. In the special case of a somewhat smaller mass of the second heavy neutrino, i.e., $|\sigma| < |\rho|$, the condition (17) becomes

$$\psi - \beta \simeq \frac{1}{2}(\phi - \alpha) \ , \quad |\eta|^2 \simeq 2|\rho| \ . \tag{18}$$

The mass matrix m_ν can again be diagonalized by a unitary matrix $U^{(\nu)}$,

$$U^{(\nu)\dagger} m_\nu U^{(\nu)*} = -\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \ . \tag{19}$$

[2] We also note that this result is independent of the zeroes in the mass matrix (5) if its 1-3 element is smaller than ϵ^3, as required by data.

A straightforward calculation yields ($s_{ij} = \sin\Theta_{ij}$, $c_{ij} = \cos\Theta_{ij}$, $\xi = \epsilon/(1+|\eta|^2)$),

$$U^{(\nu)} = \begin{pmatrix} c_{12}e^{i(\phi-\beta+\psi-\gamma)} & s_{12}e^{i(\phi-\beta+\psi-\gamma)} & \xi s_{23}e^{i(\phi-\beta+\psi)} \\ -c_{23}s_{12}e^{i(\phi+\beta-\psi+\gamma)} & c_{23}c_{12}e^{i(\phi+\beta-\psi+\gamma)} & s_{23}e^{i(\phi+\beta-\psi)} \\ s_{23}s_{12}e^{i(\gamma+\psi)} & -s_{23}c_{12}e^{i(\gamma+\psi)} & c_{23}e^{i\psi} \end{pmatrix} , \tag{20}$$

with the mixing angles,

$$\tan 2\Theta_{23} \simeq \frac{2|\eta|}{1-|\eta|^2} , \quad \tan 2\Theta_{12} \simeq 2\sqrt{1+|\eta|^2}\frac{\epsilon}{\delta} . \tag{21}$$

Note, that the 1-3 element of the mixing matrix is small, $U_{13}^{(\nu)} = \mathcal{O}(\epsilon)$. The masses of the light neutrinos are

$$m_1 \simeq -\frac{\epsilon}{(1+|\eta|^2)^{3/2}}\frac{(1-\cos 2\Theta_{12})}{\sin 2\Theta_{12}}\, m_3 , \tag{22}$$

$$m_2 \simeq \frac{\epsilon}{(1+|\eta|^2)^{3/2}}\frac{(1+\cos 2\Theta_{12})}{\sin 2\Theta_{12}}\, m_3 , \tag{23}$$

$$m_3 \simeq (1+|\eta|^2)\,\frac{v_1^2}{M_3} . \tag{24}$$

This corresponds to the weak hierarchy,

$$m_1 : m_2 : m_3 = \epsilon : \epsilon : 1 , \tag{25}$$

with $m_2^2 \sim m_1^2 \sim \Delta m_{21}^2 = m_2^2 - m_1^2 \sim \epsilon^2$. Since $\epsilon \sim 0.1$, this pattern is consistent with the LMA solution of the solar neutrino problem, but not with the LOW solution.

We have obtained the large ν_μ-ν_τ mixing as consequence of the required very large mass hierarchy (13) of the heavy Majorana neutrinos. The large ν_e-ν_μ mixing follows from the particular values of parameters $\mathcal{O}(1)$ for which we have not found a particular reason. Hence, one expects two large mixing angles, but single maximal or bi-maximal mixing would require strong fine tuning within our framework. On the other hand, a definite prediction is exactly one small matrix element, $U_{13}^{(\nu)} = \mathcal{O}(\epsilon)$.

This pattern of neutrino mixings is a direct consequence of the hierarchy of the heavy Majorana masses and is independent of the off-diagonal elements of the mass matrix M. For instance, replacing the texture (10) by a diagonal matrix, $M = \mathrm{diag}(M_1, M_2, M_3)$, yields the light neutrino mass matrix

$$m_\nu = \begin{pmatrix} \epsilon^2 e^{2i\phi} & \rho\epsilon e^{i\phi} & \eta\epsilon e^{i\phi} \\ \rho\epsilon e^{i\phi} & a & b \\ \eta\epsilon e^{i\phi} & b & c \end{pmatrix} \frac{v_1^2}{M_3} . \tag{26}$$

For the hierarchy,

$$M_1 : M_2 : M_3 = \epsilon^6 : \epsilon^4 : 1 \ , \tag{27}$$

the parameters a, b and c are again all $\mathcal{O}(1)$. The mass matrix (26) was previously obtained from a U(1) flavour symmetry [21] where its structure is a consequence of the U(1) charges of the lepton doublets and is unrelated to the mass hierarchy of the heavy neutrinos. This is in stark contrast to the SO(10) framework used here, where the structure of m_ν is intimately related to the hierarchy (27). Correspondingly, the assignment of U(1) charges is incompatible with the SO(10) multiplet structure which may appear as an unsatisfactory feature of models with U(1) family symmetry.

In order to calculate various observables in neutrino physics we need the leptonic mixing matrix

$$U = U^{(e)\dagger} U^{(\nu)} \ , \tag{28}$$

where $U^{(e)}$ is the charged lepton mixing matrix. In our framework we expect $U^{(e)} \simeq V^{(d)}$, and also $V = V^{(u)\dagger} V^{(d)} \simeq V^{(d)}$ for the CKM matrix since $\epsilon_u < \epsilon_d$. This yields for the leptonic mixing matrix

$$U \simeq V^\dagger U^{(\nu)} \ . \tag{29}$$

To leading order in the Cabibbo angle $\lambda \simeq 0.2$ we only need the off-diagonal elements $V_{12}^{(d)} = \overline{\lambda} = -V_{21}^{(d)*}$. Since the matrix m_d is complex, the Cabibbo angle is modified by phases, $\overline{\lambda} = \lambda \exp\{i(\phi_d - \alpha_d)\}$. The leptonic mixing matrix then reads explicitly,

$$U = \left(\begin{array}{ccc} U_1 & U_2 & U_3 \end{array} \right) \ , \tag{30}$$

with the column vectors

$$U_1 = \left(\begin{array}{c} c_{12} e^{i(\phi-\beta+\psi-\gamma)} + \overline{\lambda} c_{23} s_{12} e^{i(\phi+\beta-\psi+\gamma)} \\ -c_{23} s_{12} e^{i(\phi+\beta-\psi+\gamma)} + \overline{\lambda}^* c_{12} e^{i(\phi-\beta+\psi-\gamma)} \\ s_{23} s_{12} e^{i(\gamma+\psi)} \end{array} \right) \ , \tag{31}$$

$$U_2 = \left(\begin{array}{c} s_{12} e^{i(\phi-\beta+\psi-\gamma)} - \overline{\lambda} c_{23} c_{12} e^{i(\phi+\beta-\psi+\gamma)} \\ c_{23} c_{12} e^{i(\phi+\beta-\psi+\gamma)} + \overline{\lambda}^* s_{12} e^{i(\phi-\beta+\psi-\gamma)} \\ -s_{23} c_{12} e^{i(\gamma+\psi)} \end{array} \right) \ , \tag{32}$$

$$U_3 = \left(\begin{array}{c} \xi s_{23} e^{i(\phi-\beta+\psi)} - \overline{\lambda} s_{23} e^{i(\phi+\beta-\psi)} \\ s_{23} e^{i(\phi+\beta-\psi)} + \overline{\lambda}^* \xi s_{23} e^{i(\phi-\beta+\psi)} \\ c_{23} e^{i\psi} \end{array} \right) \ . \tag{33}$$

Note, that all matrix elements are $\mathcal{O}(1)$ except U_{13}, where we have counted the Cabibbo angle $\lambda = \mathcal{O}(\epsilon)$. This matrix element is predicted to be close to the experimental limit,

$$|U_{13}| = \mathcal{O}(\lambda, \epsilon) \sim 0.1 \ . \tag{34}$$

Next, we consider CP violation in neutrino oscillations. Obervable effects are controlled by the Jarlskog parameter J_l [22] ($\widetilde{\epsilon}_{ij} = \sum_{k=1}^{3} \epsilon_{ijk}$)

$$\mathrm{Im}\{U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}\} = \widetilde{\epsilon}_{\alpha\beta} \widetilde{\epsilon}_{ij} J_l \; , \tag{35}$$

for which we find

$$J_l \simeq \lambda s_{12} c_{12} c_{23} s_{23}^2 \sin\left(2(\beta - \psi + \gamma) + \phi_d - \alpha_d\right) . \tag{36}$$

In the case of a small mass difference Δm^2_{12} the CP asymmetry $P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e)$ is proportinal to δ (cf. (17)). Hence, the dependence of J_l on the angle γ is not surprising.

For large mixing, $c_{ij} \simeq s_{ij} \simeq 1/\sqrt{2}$, and in the special case (18) we find from the SO(10) phase relation $\phi - \alpha = \phi_u - \alpha_u$ and $\phi_u - \alpha_u - \phi_d + \alpha_d = \Delta \simeq \pi/2$,

$$J_l \simeq \frac{\lambda}{4\sqrt{2}} \sin\left(-\frac{\pi}{2} + 2\gamma\right) . \tag{37}$$

For small γ this corresponds to maximal CP violation, but without a deeper understanding of the fermion mass matrices this case is not singled out. Due to the large neutrino mixing angles, J_l is much bigger than the Jarlskog parameter in the quark sector, $J_q = \mathcal{O}(\lambda^6) \sim 10^{-5}$, which may lead to observable effects at future neutrino factories [23].

According to the seesaw mechanism neutrinos are Majorana fermions. This can be directly tested in neutrinoless double β-decay. The decay amplitude is proportional to the complex mass

$$\begin{aligned} \langle m \rangle &= \sum_i U_{ei}^2 m_i = -(U U^{(\nu)\dagger} m_\nu U^{(\nu)*} U^T)_{ee} \simeq -(V^{(d)\dagger} m_\nu V^{(d)*})_{ee} \\ &= -\frac{1}{1+|\eta|^2} \left(\lambda^2 |\eta|^2 e^{2i(\phi_d - \alpha_d + \beta + \phi - \psi)} - 2\lambda\epsilon e^{i(\phi_d - \alpha_d + 2\phi)}\right) m_3 \; . \end{aligned} \tag{38}$$

With $m_3 \simeq \sqrt{\Delta m^2_{atm}} \simeq 5 \times 10^{-2}$ eV this yields $\langle m \rangle \sim 10^{-3}$ eV, more than two orders of magnitude below the present experimental upper bound [24].

We now turn to the cosmological matter-antimatter asymmetry. An attractive mechanism to generate it is leptogenesis [25] which involves both, CP violation and the Majorana nature of the neutrinos. This connection has already been discussed in different contexts [31, 32]. The baryon asymmetry is given by

$$Y_B = \frac{n_B - n_{\overline{B}}}{s} = \kappa c_S \frac{\varepsilon_1}{g_*} \; . \tag{39}$$

Here n_B and s are baryon number and entropy densities, respectively. $g_* \sim 100$ is the number of degrees of freedom in the plasma of the early universe, ε_1 is the CP asymmetry

in the decay of the lightest of the heavy Majorana neutrinos and c_S is the conversion factor from lepton asymmetry to baryon asymmetry due to sphaleron processes. For three quark-lepton generations and two Higgs doublets one has $c_S = -8/15$ [26]. The effects of washout processes are accounted for by $\kappa < 1$.

It is convenient to express the CP asymmetry directly in terms of the light neutrino mass matrix. In a flavour diagonal basis for the heavy neutrinos one has [27]

$$\varepsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{(h_\nu^\dagger h_\nu)_{11}}\mathrm{Im}\left(h_\nu^\dagger h_\nu \frac{1}{M} h_\nu^T h_\nu^*\right)_{11} . \tag{40}$$

In an arbitrary basis for light and heavy neutrinos this can be rewritten as

$$\varepsilon_1 \simeq \frac{3}{16\pi}\mathrm{sign}(M_1)\frac{\mathrm{Im}\left(U^{(N)T} m_D^\dagger m_\nu m_D^* U^{(N)}\right)_{11}}{v_1^2 \widetilde{m}_1} , \tag{41}$$

where $U^{(N)}$ is the heavy neutrino mixing matrix defined in eq. (11). The effective neutrino mass $\widetilde{m}_1 = \left(U^{(N)T} m_D^\dagger m_D U^{(N)*}\right)_{11}/|M_1|$ is a sensitive parameter for successful leptogenesis [28]. From eqs. (5), (14) and (16) one then obtains

$$\varepsilon_1 \simeq \frac{3}{16\pi}\epsilon^6 \frac{|\eta|^2}{\sigma}\frac{(|\rho|^2 \sin\left(2(\phi-\alpha+\beta-\psi)\right) + 2|\rho|\sin\left(\phi-\alpha\right) + \sin\left(2(\psi-\beta)\right))}{\sigma^2 + |\eta|^2 + |\rho|^2 - 2|\rho|\sigma\cos(\phi-\alpha)} . \tag{42}$$

In the special case (18) this expression simplifies to

$$\varepsilon_1 \simeq \frac{3}{16\pi}\epsilon^6 \frac{|\eta|^2}{\sigma}\frac{(1+|\rho|)^2}{|\eta|^2+|\rho|^2}\sin(\phi-\alpha) . \tag{43}$$

With $\epsilon \sim 0.1$ one has $\varepsilon_1 \sim 10^{-7}$, $|M_1| \simeq (\epsilon^6/|\sigma|)(1+|\eta|^2)v_1^2/m_3 \sim 10^9$ GeV and $\widetilde{m}_1 \sim (|\eta|^2+|\rho|^2)/(\sigma(1+|\eta|^2))m_3 \sim 10^{-2}$ eV. The baryon asymmetry is then given by

$$Y_B \sim -\kappa\ \mathrm{sign}(\sigma)\ \sin\left(\phi-\alpha\right) \times 10^{-9} . \tag{44}$$

From the SO(10) symmetry one obtains $\phi - \alpha = \phi_u - \alpha_u$. According to the qualitative discussion below eq. (9), there are two solutions with $\alpha_u \simeq 0$, $\phi_u \simeq \pi/4$ and $\phi_u \simeq -\pi/4$. Thus, depending on the sign of σ, there is always a positive baryon asymmetry, in agreement with observation. Without further assumptions, the values of $\phi_u - \alpha_u$ and σ cannot be fixed.

The parameters ε_1, M_1 and $\widetilde{m}_1$ are rather similar to those considered previously in a leptogenesis scenario [29, 30] with hierarchical heavy Majorana neutrinos and with $B-L$ broken at the GUT scale. We therefore expect that a solution of the full Boltzmann equations will yield a baryon asymmetry which is consistent with the observed asymmetry $Y_B \simeq (0.6-1)\times 10^{-10}$.

In summary, we have considered the consequences of large neutrino mixing, as indicated by data, in connection with SO(10) symmetry and the seesaw mechanism. This determines uniquely the hierarchy of the heavy Majorana neutrino masses. The resulting light neutrino mass hierarchy is consistent with the LMA solution of the solar neutrino problem but incompatible with the LOW solution. Furthermore, the U_{13} element of the leptonic mixing matrix is predicted to be $U_{13} = \mathcal{O}(\lambda, \epsilon) \sim 0.1$. CP violation in neutrino oscillations may be maximal, and the correct order of magnitude for the baryon asymmetry is obtained. In the case of a very large heavy neutrino mass hierarchy, i.e. $(M_2/M_3) < (m_{charm}/m_{top})^2$, the baryon asymmetry is more closely related to the CP violating phases in the quark sector and the correct sign can be obtained. However, a complete determination of the magnitude and the relative sign of both CP violating observables requires a deeper understanding of the quark and lepton mass matrices.

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