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Probing generalized parton distributions in $\pi N \to \ell^+\ell^- N$

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Abstract

We study the exclusive reactions $\pi^- p \to \ell^+\ell^- n$ and $\pi^+ n \to \ell^+\ell^- p$ in view of possible future experiments with high-intensity pion beams. For large invariant mass of the lepton pair $\ell^+\ell^-$ and small squared momentum transfer to the nucleon these are hard-scattering processes providing access to generalized parton distributions. We estimate the cross section for these reactions, explore their connection with the pion form factor, and discuss the role they can play in improving our understanding of the relevant reaction mechanisms.

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1 Introduction

Hard exclusive scattering on nucleon targets has been studied in the last few years with the aim of extracting generalized parton distributions, quantities that contain a wealth of information on the quark and gluon structure of hadrons [1, 2, 3]. Among the channels most studied is the production of mesons ($\gamma^* N \to \pi N$, $\gamma^* N \to \rho N$, ...) or real photons ($\gamma^* N \to \gamma N$) induced by highly virtual spacelike photons from a lepton beam, cf. [4] for a recent review. High-intensity neutrino beam facilities under discussion [5] may permit the study of charged current induced processes such as $W^* N \to D_s N$ [6]. The secondary pion beams at such accelerators or at standalone proton beam facilities [7], with energies in the range of a few 10 GeV, could at the same time be used for experiments on hadron targets. Here we consider the exclusive production of a high-mass timelike photon decaying into a lepton pair, $\pi N \to \gamma^* N \to \ell^+ \ell^- N$ with $\ell = e, \mu$, and argue that it would be a valuable complement to the previously mentioned processes. It may be seen as the analog of the timelike Compton process, $\gamma N \to \gamma^* N \to \ell^+ \ell^- N$, recently investigated by us [8].

Two-photon processes like $\gamma^* N \to \gamma N$ or $\gamma N \to \gamma^* N$ are in many ways complementary to their meson counterparts such as $\gamma^* N \to \pi N$ or $\pi N \to \gamma^* N$. The theory description is simpler for Compton scattering, and one can expect the hard-scattering regime to be reached at lower values of the photon virtuality. The analysis of meson production, on the other hand, is simpler because there is no competition with an electromagnetic Bethe-Heitler process. Pions further simplify the structure of the cross section as they are spinless. Information beyond the Compton channels is indispensable to disentangle the flavor and spin degrees of freedom of the generalized parton distributions. Finally, the process we study here involves beams of hadrons instead of leptons or photons, and will thus allow studies under quite different experimental conditions. We consider both $\pi^- p \to \ell^+ \ell^- n$ and $\pi^+ n \to \ell^+ \ell^- p$, which make use of different beams and targets and present different requirements when the outgoing nucleon is to be detected.

2 The scaling limit

A factorization theorem [9] can been proven for pion production $\gamma^* N \to \pi N$. Its contents is represented in Fig. 1a, where we also define the relevant four-momenta. In the limit of large photon virtuality $Q^2 = -q^2$ at fixed scaling variable $x_B = Q^2/(2p \cdot q)$ and invariant momentum transfer $t = (p - p')^2$ the amplitude can be written in terms of a hard-scattering process at parton level, a distribution amplitude ϕ_π describing the formation of the pion from a $q\bar{q}$ pair, and generalized parton distributions $\tilde{H}$ and $\tilde{E}$ encoding nonperturbative physics in the nucleon. The arguments for factorization do not rely on the photon being spacelike and can be extended to the case $\pi N \to \gamma^* N$, shown in Fig. 1b, with the same nonperturbative input. The appropriate kinematical limit is now that of large timelike virtuality $Q'^2 = q'^2$ at fixed t and fixed scaling variable

$$\tau = \frac{Q'^2}{2p \cdot q} \approx \frac{Q'^2}{s - M^2}, \tag{1}$$

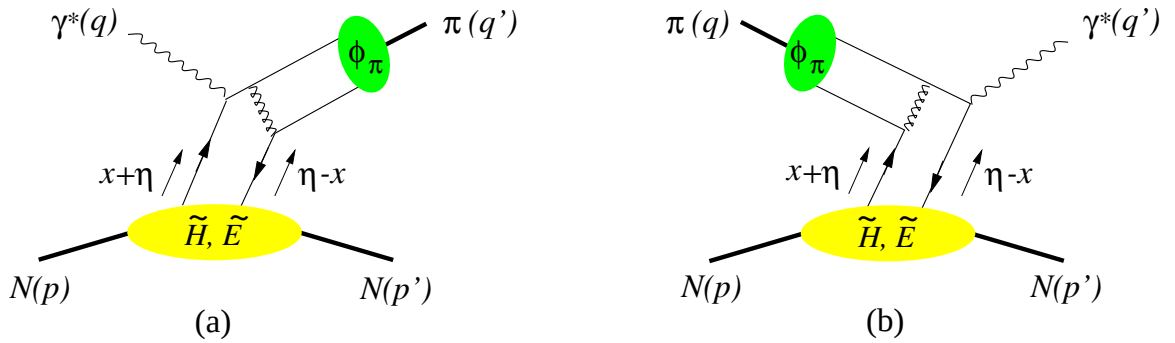

Figure 1: Sample Feynman diagrams at leading order in α_s for pion electroproduction (a) and its timelike counterpart (b) in the scaling limit. In both cases three other diagrams are obtained by attaching the photon to the quark lines in all possible ways. The plus-momentum fractions x and η refer to the average nucleon momentum $\frac{1}{2}(p+p')$.

where $s = (p+q)^2$ is the squared c.m. energy. Here and in the following we neglect the masses of the pion and the final-state leptons compared with the nucleon mass M.

Among the predictions of the factorization theorem is that in the limit of large virtuality the dominant polarization of the γ^* is longitudinal in the collision c.m. The corresponding amplitude for $\pi N \to \gamma^* N$ scales like $1/Q'$ at fixed t and τ, up to logarithmic modifications due to radiative corrections. Transverse photon helicity is suppressed by an extra factor of $1/Q'$ in the amplitude. In the limit where it can be neglected the cross section for the overall process $\pi N \to \ell^+\ell^- N$ is simply

$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{\rm em}}{256\,\pi^3}\,\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\,\sin^2\theta, \tag{2}$$

where the superscript 0 stands for a longitudinal photon and we have respectively taken the average and sum over the initial and final nucleon helicities λ and λ'. The decay angles θ and φ of the photon in its rest frame are defined in analogy to timelike Compton scattering (cf. Fig. 5 of [8]), and the $\sin^2\theta$ behavior in (2) is the sign of the purely longitudinal γ^* polarization. In general the distribution in these angles allows separation of the contributions to the cross section from longitudinal and transverse photons, as well as their different interference terms. Along the lines of [10] one can thus test whether Q'^2 is large enough to ensure the Q' behavior and suppression pattern of the different helicity transitions predicted by the factorization theorem. With polarized nucleon targets one has further access to different combinations of nucleon helicities, in analogy with the case of $\ell N \to \ell\pi N$ [11].

In the large Q' limit the helicity amplitudes $M^{0\lambda',\lambda}$ for $\pi^- p \to \gamma^* n$ read

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{4\pi}{3}\,\frac{f_\pi}{Q'}$$
$$\times\quad \frac{1}{(p+p')^+}\,\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(-\eta,\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(-\eta,\eta,t)\right]u(p,\lambda). \tag{3}$$

An analogous equation holds for $\pi^+ n \to \gamma^* p$ with $\tilde{\mathcal{H}}^{du}$ replaced by $\tilde{\mathcal{H}}^{ud}$ and $\tilde{\mathcal{E}}^{du}$ by $\tilde{\mathcal{E}}^{ud}$. Here e is the positron charge and $f_\pi \approx 132$ MeV the pion decay constant. We have introduced plus- and minus-components $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ for any four-vector v and work in a reference frame where the average nucleon momentum $\frac{1}{2}(p+p')$ points along the positive 3-axis. $\eta = (p-p')^+/(p+p')^+$ parameterizes the plus-momentum transfer to the nucleon and has a value $\eta = \tau/(2-\tau)$ in our kinematical limit. To leading order in α_S the convolution integrals $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$, normalized as in [12], are given by

$$\begin{aligned}
\tilde{\mathcal{H}}^{du}(\xi,\eta,t) &= \frac{8}{3}\,\alpha_S \int_{-1}^{1} dz\, \frac{\phi_\pi(z)}{1-z^2} \\
&\quad \times \int_{-1}^{1} dx \left[\frac{e_d}{\xi - x - i\epsilon} - \frac{e_u}{\xi + x - i\epsilon}\right] [\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)] \\
\tilde{\mathcal{H}}^{ud}(\xi,\eta,t) &= \frac{8}{3}\,\alpha_S \int_{-1}^{1} dz\, \frac{\phi_\pi(z)}{1-z^2} \\
&\quad \times \int_{-1}^{1} dx \left[\frac{e_u}{\xi - x - i\epsilon} - \frac{e_d}{\xi + x - i\epsilon}\right] [\tilde{H}^u(x,\eta,t) - \tilde{H}^d(x,\eta,t)]
\end{aligned} \tag{4}$$

with $e_u = \frac{2}{3}$ and $e_d = -\frac{1}{3}$, and the pion distribution amplitude normalized according to $\int_{-1}^{+1} dz\, \phi_\pi(z) = 1$. Analogous expressions give $\tilde{\mathcal{E}}^{du}$ and $\tilde{\mathcal{E}}^{ud}$. Following [13] we have used isospin invariance to express the generalized parton distributions for $p \to n$ and $n \to p$ transitions in terms of the flavor diagonal generalized u and d quark distributions $\tilde{H}^u$, $\tilde{E}^u$, $\tilde{H}^d$, $\tilde{E}^d$ in the proton, defined in [2]. For ease of writing we have not explicitly indicated the logarithmic Q'^2 dependence of $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$ due to the running of α_S and the factorization scale dependence of the parton distributions and the pion distribution amplitude. The factorization scale dependence is governed by evolution equations, which for generalized parton distributions are a hybrid of the usual DGLAP equations for parton densities and the ERBL equations [14] for meson distribution amplitudes, cf. [1, 2, 3, 15].

It is instructive to compare (3) with the leading amplitudes for the spacelike processes. In our approximation $M^{\lambda',0\lambda}(\gamma^* n \to \pi^- p)$ is obtained from $M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n)$ by replacing Q' with Q and changing the first argument in $\tilde{\mathcal{H}}^{du}$ and in $\tilde{\mathcal{E}}^{du}$ from $-\eta$ to η, now given by $\eta = x_B/(2-x_B)$. The amplitudes for $\gamma^* p \to \pi^+ n$ and $\pi^+ n \to \gamma^* p$ are connected in the same way. We thus obtain the simple relations

$$\begin{aligned}
M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) &= \Big[M^{\lambda',0\lambda}(\gamma^* p \to \pi^+ n)\Big]^*, \\
M^{0\lambda',\lambda}(\pi^+ n \to \gamma^* p) &= \Big[M^{\lambda',0\lambda}(\gamma^* n \to \pi^- p)\Big]^*
\end{aligned} \tag{5}$$

at leading power in $1/Q'$ and leading order in α_S, to be evaluated at the same t and equal values of τ, x_B and Q', Q. The corresponding unpolarized cross sections will thus be the same for timelike and spacelike processes, up to appropriate phase space factors. We anticipate that the relations (5) no longer hold at the level of corrections in α_S or in $1/Q$ and $1/Q'$.

3 Numerical estimates

In order to estimate cross sections we need to specify the nonperturbative input functions. Among all mesons the distribution amplitude of the pion is best constrained. In particular the integral needed in $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$ is approximately determined by data on $\gamma^*\gamma \to \pi$, and found close to the value it takes for the asymptotic form $\phi_\pi(z) = \frac{3}{4}(1-z^2)$ of the distribution amplitude under ERBL evolution. This is the form we will use here, the associated uncertainties (cf. e.g. [16]) being modest compared with others in the process at hand.

Our knowledge of generalized parton distributions is still rather limited. $\tilde{H}^u$ and $\tilde{H}^d$ are known for $\eta = 0$ and $t = 0$, where they become the usual spin dependent parton densities $\Delta u(x)$ and $\Delta d(x)$, and their integral over x is related to the axial form factor $g_A(t)$ of the nucleon. An ansatz incorporating these constraints is

$$\tilde{H}^u(x,\eta,t) - \tilde{H}^d(x,\eta,t) = \Big[\tilde{h}^u(x,\eta) - \tilde{h}^d(x,\eta)\Big]\, g_A(t)/g_A(0). \tag{6}$$

We take the parameterization $g_A(t)/g_A(0) = (1 - t/M_A^2)^{-2}$ with $M_A = 1.06$ GeV from [17]. The functions $\tilde{h}^u$ and $\tilde{h}^d$ are constructed from $\Delta u(x)$ and $\Delta d(x)$ using a model prescription based on double distributions [3] and can be found in [8]. For $\Delta u(x)$ and $\Delta d(x)$ we use set A of the LO parameterization by Gehrmann and Stirling [18], retaining only the valence part since the polarized quark sea is still poorly constrained by data. We note that the factorization of the t-dependence in (6) is a convenient ansatz because of its simplicity, but must be taken with a grain of salt.

For the distributions $\tilde{E}$ we take a form given in [19], motivated by results obtained in the chiral soliton model of the nucleon:

$$\tilde{E}^u(x,\eta,t) - \tilde{E}^d(x,\eta,t) = \Theta(\eta - |x|)\, \frac{1}{\eta}\, \phi_\pi\Big(\frac{x}{\eta}\Big)\, F(t) \tag{7}$$

with the step function Θ, the asymptotic pion distribution amplitude given above, and

$$F(t) = \frac{4.4\,\mathrm{GeV}^2}{m_\pi^2 - t}\left[1 - \frac{B\,(m_\pi^2 - t)}{(1 - Ct)^2}\right] \tag{8}$$

with $B = 1.7\,\mathrm{GeV}^{-2}$ and $C = 0.5\,\mathrm{GeV}^{-2}$. Note that (7) has only support for $|x| < \eta$, termed ERBL region because of the form taken by the evolution equations there. In this region the generalized parton distributions describe the emission of a $q\bar{q}$ pair from the initial nucleon as shown in Fig. 1. The physics expressed in (7) is that this $q\bar{q}$ pair comes from an off-shell pion. As t approaches m_π^2 the form factor (8) is well approximated by a pion pole form

$$F_{\mathrm{pole}}(t) = \frac{4M^2 g_A(0)}{m_\pi^2 - t}, \tag{9}$$

where $g_A(0) \approx 1.25$. For larger values the difference between $F(t)$ and $F_{\mathrm{pole}}(t)$ may be seen as a correction to the simple pole approximation or, in a different language, as a

correction for the off-shellness of the exchanged pion. We note that the model calculation in [19] also found a contribution to $\tilde{E}^u - \tilde{E}^d$ in the DGLAP region $|x| > \eta$, but this was small compared to (7).

In our estimates we evaluate α_S at the scale Q'^2, using the one-loop expression for the running coupling with 4 active flavors and $\Lambda^{(4)} = 200$ MeV [20]. This gives $\alpha_S \approx 0.31$ for the value of $Q'^2 = 5\,\mathrm{GeV}^2$ we consider in the following. In Fig. 2a we show the unpolarized cross section

$$\frac{d\sigma}{dQ'^2\,dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{\rm em}^2}{27}\,\frac{\tau^2}{Q'^8}\,f_\pi^2 \times \left[(1-\eta^2)|\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2\,\mathrm{Re}\Big(\tilde{\mathcal{H}}^{du*}\,\tilde{\mathcal{E}}^{du}\Big) - \eta^2\frac{t}{4M^2}\,|\tilde{\mathcal{E}}^{du}|^2\right], \qquad (10)$$

obtained with our models (6) and (7). We also plot the separate contributions from the terms with $|\tilde{\mathcal{H}}|^2$, $\mathrm{Re}(\tilde{\mathcal{H}}^*\,\tilde{\mathcal{E}})$, and $|\tilde{\mathcal{E}}|^2$, and see that their relative importance is strongly t dependent. Notice that since our ansatz for $\tilde{E}^u - \tilde{E}^d$ vanishes in the DGLAP region, $\tilde{\mathcal{E}}$ is purely real in the Born approximation (4), in contrast to $\tilde{\mathcal{H}}$. Fig. 2b shows the results obtained when instead of the full form factor $F(t)$ we only take its pole term $F_{\rm pole}(t)$. Clearly, contributions to $\tilde{E}$ beyond the simple pion pole are important in a wide range of t.

Fig. 3a shows the cross section and its components at fixed Q'^2 and t as a function of τ and thus of the collision energy $\sqrt{s}$. We see that the relative importance of $\tilde{\mathcal{E}}$ and $\tilde{\mathcal{H}}$ depends on both t and τ, in agreement with the findings of [21] for the spacelike process $\gamma^* p \to \pi^+ n$. Where exactly one or the other dominates depends on the details of the generalized parton distributions. In this respect our results should be taken with due care, especially since we have used an oversimplified ansatz for the t-dependence of $\tilde{H}$ in (6). In Fig. 3b we show the corresponding curves for $\pi^+ n \to \gamma^* p$. With our ansatz (7) the contribution from $\tilde{E}$ is the same for $\pi^- p$ and $\pi^+ n$, whereas the one from $\tilde{H}$ comes out somewhat smaller for $\pi^+ n$.

4 The pion pole

We have seen that part of the amplitude in our process is due to the pion pole. This is represented in Fig. 4, where the blob representing $\tilde{E}$ in Fig. 1 has been replaced by a nucleon-pion vertex and the pion distribution amplitude. We recognize in the upper part of the diagrams the space- and timelike pion form factor F_π in the hard-scattering formalism [14]. The connection between the pion form factors and the processes $\gamma^* N \to \pi N$ and $\pi N \to \gamma^* N$ goes of course beyond our approximations valid at large Q'^2. For the amplitude of the timelike process at small t one may in general write

$$M^{0\lambda',\lambda}(\pi N \to \gamma^* N) = -ie\,Q' F_\pi(Q'^2)\,\frac{F_{\rm pole}(t)}{2Mf_\pi}\,\bar{u}(p',\lambda')\gamma_5\,u(p,\lambda) + \text{ non-pole terms.} \qquad (11)$$

The "non-pole terms" include both off-shell corrections for pion exchange (an example of which is the difference between $F(t)$ and $F_{\rm pole}(t)$ discussed above) and contributions from

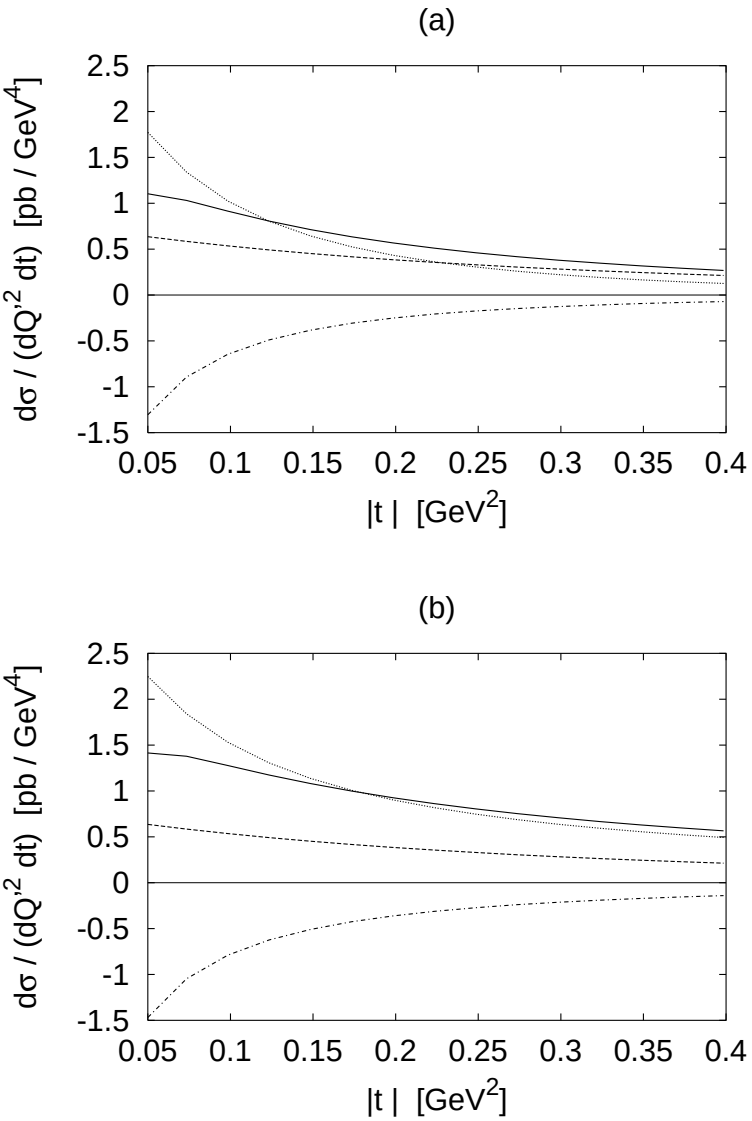

Figure 2: (a) Cross section estimates (full lines) for $\pi^- p \to \gamma^* n$ for $Q'^2 = 5\,\mathrm{GeV}^2$ and $\tau = 0.2$, calculated from (10) with the models (6) and (7). Separate contributions are shown for the terms with $|\tilde{\mathcal{H}}|^2$ (dashed), $\mathrm{Re}(\tilde{\mathcal{H}}^*\,\tilde{\mathcal{E}})$ (dash-dotted), and $|\tilde{\mathcal{E}}|^2$ (dotted). (b) The same calculated with the pole-term form factor $F_{\mathrm{pole}}(t)$ instead of $F(t)$.

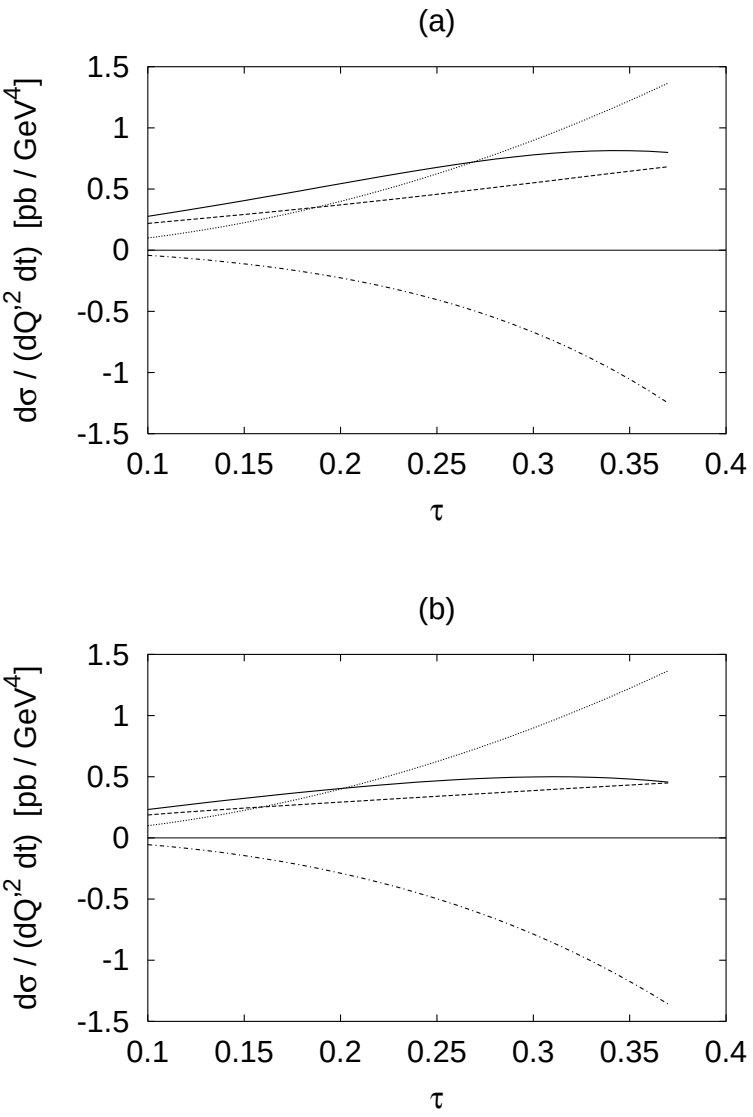

Figure 3: (a) As Fig. 2a but as a function of τ at fixed $Q'^2 = 5\,\mathrm{GeV}^2$ and $|t| = 0.2\,\mathrm{GeV}^2$. (b) The same for $\pi^+ n \to \gamma^* p$.

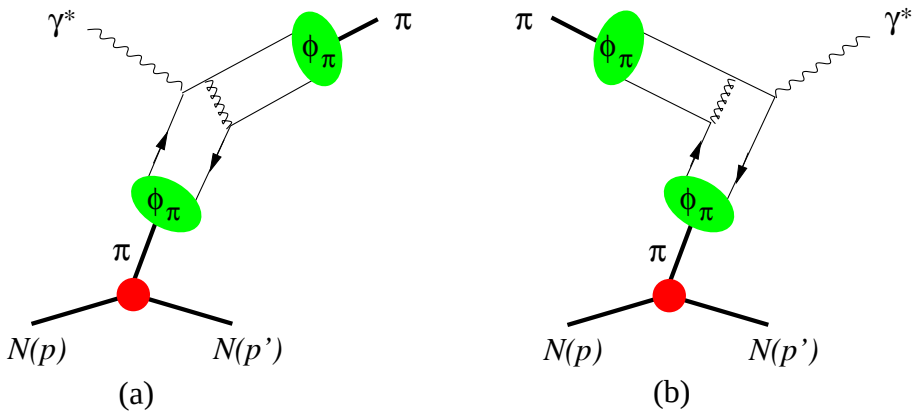

Figure 4: The part of the diagrams in Fig. 1 due to the pion pole contribution to the distribution $\tilde{E}$.

other sources (such as the DGLAP region of the distribution $\tilde{E}$ or the contribution of $\tilde{H}$).

An equation analogous to (11) relates the electroproduction process $\gamma^* N \to \pi N$ with $F_\pi(-Q^2)$, and is in fact being used to measure the spacelike pion form factor at large momentum transfer. The extraction of $F_\pi(-Q^2)$ is however not trivial because the "non-pole terms" in the amplitude need not be small in the accessible kinematics and typically have to be modeled and subtracted. Models developed and tested for small photon virtualities may not be adequate to describe physics at large Q^2 where the photon scatters not on a full off-shell pion but only on the small-size $q\bar{q}$ component of its wave function shown in Fig. 1 [22]. As we have seen, even the departure of the form factor $F(t)$ from a pure pole form is numerically important in a wide range of t, as has been pointed out earlier [23]. The timelike process presents a unique opportunity here, since one may directly compare data for the timelike form factor from $e^+e^- \to \pi^+\pi^-$ with data from $\pi N \to \ell^+\ell^- N$ and thus *test* the quality of the pion pole approximation or of models aiming to describe corrections to it.

5 Radiative corrections

The $\mathcal{O}(\alpha_S)$ corrections to the pion form factor have been fully calculated, cf. [24] for a recent discussion. By a straightforward rescaling of the longitudinal momentum variables they also give the NLO corrections to $\gamma^* N \to \pi N$ [12]. The corresponding expressions can be analytically continued to the timelike region; for this one needs to replace the logarithms $\log(Q^2/\mu^2)$ by $\log(Q'^2/\mu^2) - i\pi$ in the hard-scattering kernel, where μ is either the renormalization or the factorization scale.

Numerical studies of the spacelike form factor [24] and of pion electroproduction [12] indicate that the size of NLO corrections can be substantial and strongly depends on the choice of renormalization scale μ_R in α_S. If the asymptotic pion distribution amplitude is taken, Ref. [24] found NLO corrections to be quite small for $\mu_R^2 \approx Q^2/20$. Further

arguments for such a choice have been given in [25]. Notice that for a wide range of Q^2 this requires one to modify the running of $\alpha_S(\mu_R)$ in the infrared and thus to take into account effects beyond perturbation theory. This is highly nontrivial, and in our numerical studies here we preferred to stay with the naive choice $\mu_R^2 = Q'^2$. It is however understood that the absolute values of our cross sections can only be taken as rough estimates.

6 Beyond the limit of large Q'^2

In all processes we have discussed there are corrections to the hard-scattering description of the factorization theorems as the hard scale Q'^2 or Q^2 is not infinitely large. An obvious issue in reactions with timelike photons are resonance effects. Based mainly on the data for $e^+e^- \to hadrons$ we have estimated in [8] that a description in terms of quarks and gluons may be working for photon masses Q' above, say, 1.5 or 2 GeV, excluding the regions of the $c\bar{c}$ and the $b\bar{b}$ resonances. We insist however that this is only a guideline since parton-hadron duality can be realized to different degrees in different processes. In $\pi N \to \gamma^* N$ one will further impose that Q' is not too close to the total available energy $\sqrt{s}$ in order to avoid reinteractions between the hadronic part of the photon and the nucleon becoming important.

We cannot give a comprehensive discussion of power corrections to the scaling limit here, but wish to point out two distinct mechanisms that have been studied in the literature. One concerns contributions from the one-gluon exchange mechanism of Figs. 1 and 4 that go beyond the approximations giving the leading power behavior in $1/Q$ or $1/Q'$. They can be estimated in the modified hard-scattering picture of [26] which takes into account the relative transverse momentum k_T of the partons in the hard-scattering subprocess, or using parton-hadron duality, for instance in the framework of light-cone sum rules. In the spacelike processes $\gamma^*\pi \to \pi$ and $\gamma^* N \to \pi N$ these effects tend to decrease the scattering amplitude [27, 28, 29], whereas for the timelike form factor an enhancement over the leading term in $1/Q'$ was found [30].

A different contribution to the scattering amplitude comes from the soft overlap mechanism, represented in Fig. 5, where a low-momentum parton directly connects two quark-hadron vertices. Note that in the case of Fig. 5a this can be evaluated using the formalism of light-cone wave functions [27, 28], but not in cases b and d where one parton in the quark-pion vertex is incoming and the other outgoing. Sudakov-type corrections to the quark-photon vertex in this mechanism were investigated within parton-hadron duality [25] and found to suppress the spacelike form factor, whereas the timelike form factor was enhanced and also acquired an imaginary part. Little is known about soft overlap in the DGLAP region of meson electroproduction, Fig. 5b.

Many estimates find that both types of corrections just discussed can be quite large for photon virtualities of several GeV^2. In the spacelike region they partly cancel each other, but by how much depends on the particular model used. The process $\pi N \to \gamma^* N$ may help to further the understanding of these corrections. It allows for an unambiguous

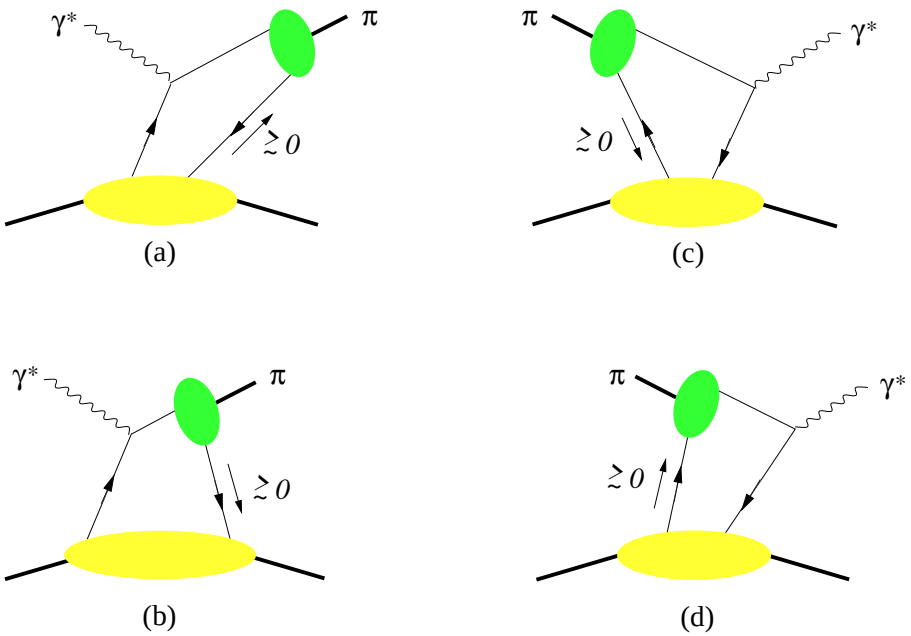

Figure 5: The soft overlap in $\gamma^* N \to \pi N$ (a and b) and $\pi N \to \gamma^* N$ (c and d). Plus-momenta $\gtrsim 0$ of the soft partons refer to the average nucleon momentum $\frac{1}{2}(p + p')$. Diagrams a, c, and d have analogs in the space- or timelike pion form factor, with the lower blob replaced by a quark-pion vertex.

comparison of time- and spacelike data as the latter are unaffected by the problems of extracting $F_\pi(-Q^2)$ discussed above, and the different mechanisms act differently in the space- and timelike cases as we have seen. One can expect that they also show differences between the ERBL and DGLAP regions and between pion pole and non-pole contributions, whose relative weight can be varied by external kinematics as our above estimates suggest.

7 Conclusions

We argue that data on high-mass lepton pair production in the timelike process $\pi N \to \gamma^* N$ can make important contributions to the study of the bound-state structure of hadrons, in particular through generalized parton distributions. In the limit of large photon virtuality and to leading order in α_S, the processes $\pi N \to \gamma^* N$ and $\gamma^* N \to \pi N$ have essentially the same scattering amplitude, reflecting the analogous relation between the time- and spacelike pion form factor at the same accuracy. We have estimated the cross section for $\pi^- p \to \ell^+ \ell^- n$ and $\pi^+ n \to \ell^+ \ell^- p$ in view of possible experiments with high-luminosity pion beams at neutrino factories or elsewhere. We find that the relative contributions from pion exchange and other mechanisms strongly depend on the external kinematics, although the details are model dependent.

Various theoretical studies as well as comparison with existing data suggest that power corrections to the hard-scattering description of the above processes are important, even for photon virtualities of several GeV^2. Their quantitative understanding will thus be necessary in order to extract generalized parton distributions. We argue that the comparison of data on $\gamma^* N \to \pi N$ and $\pi N \to \gamma^* N$ could offer help towards this goal, since various correction mechanisms behave differently under the transition from space- to timelike kinematics. An additional handle is provided by exploring these corrections as a function of the variables controlling the relative importance of the DGLAP and ERBL regimes in the generalized parton distributions.

Finally, the comparison of data on $\pi N \to \gamma^* N$ and on $e^+e^- \to \pi\pi$ would present a unique possibility to explore pion pole dominance in processes with highly virtual photons in a quantitative and model independent way. This would be of great benefit for the endeavors to extract the spacelike pion form factor from $\gamma^* N \to \pi N$.

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