BI-TP 2001/24
DESY 01-153

arXiv:hep-lat/0110132 v1 16 Oct 2001

BI-TP 2001/24
DESY 01-153
October 2001

Meson Spectral Functions at finite Temperature

I. Wetzorke* with F. Karsch, E. Laermann, P. Petreczky and S. Stickan †

Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

The Maximum Entropy Method provides a Bayesian approach to reconstruct the spectral functions from discrete points in Euclidean time. The applicability of the approach at finite temperature is probed with the thermal meson correlation function. Furthermore the influence of fuzzing/smearing techniques on the spectral shape is investigated. We present first results for meson spectral functions at several temperatures below and above T_c. The correlation functions were obtained from quenched calculations with Clover fermions on large isotropic lattices of the size $(24-64)^3 \times 16$. We compare the resulting pole masses with the ones obtained from standard 2-exponential fits of spatial and temporal correlation functions at finite temperature and in the vacuum. The deviation of the meson spectral functions from free spectral functions is examined above the critical temperature.

1. INTRODUCTION

The maximum entropy method (MEM) [1] has been applied successfully to reconstruct hadronic spectral functions from correlation functions in Euclidean time calculated at zero temperature in quenched QCD [2,3]. It was demonstrated that the spectral shape reflects ground and excited state contributions, which yields reliable results for the respective masses and decay constants.

At finite temperature, where very little is known about the spectral shape, MEM would provide new insight in the thermal changes of hadronic properties. Moreover, this approach requires no a priori assumptions or specific ansaetze and thus allows a reconstruction of the spectral functions from first principles. A first application of MEM at finite temperature could show that a reconstruction of the continuum part of the free thermal meson spectral function is possible [4]. In the following a similar test is reported for free thermal meson correlators calculated on the lattice. On the basis of such knowledge the MEM analysis is extended to meson correlation functions at several temperatures below and above the deconfinement transition.

*Present address: NIC/DESY Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany
†The work has been supported by the TMR network ERBFMRX-CT-970122 and the DFG under grant FOR 339/1-2.

2. APPLICABILITY OF MEM AT FINITE TEMPERATURE

Instead of sharp ground state peaks at $T = 0$ broad bumps and continuum-like structures are expected to dominate the spectral shape at sufficiently high temperature. In order to test the applicability of MEM at finite temperature we have calculated free thermal meson correlation functions $G(\tau) = \int d^3x \ \langle J(\tau,\vec{x}) J^\dagger(0,\vec{0})\rangle$ in the (pseudo-)scalar channel for different lattice sizes projected to zero momentum. A Gaussian noise with the variance $\Delta(\tau) \sim \tau G(\tau)$ was added to the exact values to simulate the statistical error for correlators at finite temperature [2,4]. The correlator is then related to the respective spectral function $\sigma(\omega)$ through the integral equation

$$G(\tau,T) = \int_0^\infty d\omega \ \sigma(\omega) \ K(\tau,\omega) \ ,$$

where the continuum integral kernel is given by

$$K(\tau,\omega) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.$$

The thermal correlators and reconstructed spectral functions are illustrated in figure 1. Strong finite size effects are visible for the correlators at small time separations compared to the continuum curve (solid line). In the reconstructed spectral function this results in unphysical bumps (dotted curve).

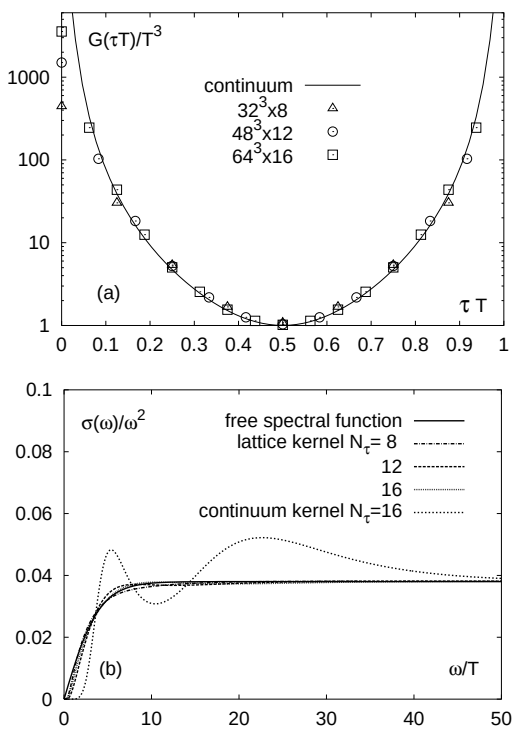

Figure 1. Thermal correlators (a) and reconstructed spectral functions (b) with integral kernel K and K_L

The application of a lattice adapted kernel

$$K_L(\tau,\omega) = \frac{2\omega}{T} \sum_{n=0}^{N_\tau - 1} \frac{\exp(-i2\pi n\tau T)}{(2N_\tau \sin(n\pi/N_\tau))^2 + (\omega/T)^2}$$

is absolutely mandatory to obtain the correct spectral shape. It can be observed in figure 1 that this choice of the kernel combined with a restriction of the ω/T-range up to about $4N_\tau$ absorbs the cut-off effects and yields an almost perfect reconstruction of the free spectral function $\sigma(\omega) = \frac{3}{8\pi^2}\omega^2 \tanh(\omega/4T)$ for $N_\tau = 8, 12$ and 16.

2.1. Smearing at finite Temperature

At zero temperature the application of fuzzing [5] and/or smearing techniques to optimize the projection onto the ground state leads to a reduction of the peak width in the spectral function and an almost complete elimination of excited states. Therefore one might be tempted to use this method also at finite temperature, where the

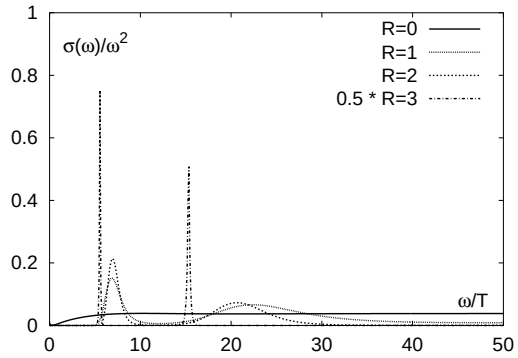

Figure 2. Influence of fuzzing [5] on the thermal spectral function for different fuzzing radii R

peaks are less pronounced compared to the continuum. However, the entire concept of smearing has to fail when a single state is not well separated from higher excited states. The situation will naturally be even worse when only a continuum exists. In the case $T > T_c$ one can no longer be sure to project on an actual ground state, since even the fuzzing of the free meson correlation function leads to sharp peaks instead of the broad continuum. This is illustrated in figure 2 for different fuzzing radii R. In order not to be biased in the MEM analysis we therefore use unmodified point-point correlators only, which preserve the full information about ground and excited states as well as the continuum contribution.

3. MESON SPECTRAL FUNCTIONS

Having tested the applicability of MEM at finite temperature, we use the approach to analyze meson spectral functions in the temperature range 0.4 to 3 T_c. Gauge field configurations on lattice sizes $(24-64)^3 \times 16$ were generated with the plaquette gauge action, while the Clover action with non-perturbatively improved coefficients [6] was used in the fermion sector. Temporal and spatial meson correlators were calculated for four quark mass values below T_c and at almost zero quark mass (in the vicinity of κ_c) above T_c.

In general a broadening of the peaks and a reduction in height can be observed at finite temperature due to the short extent in the temporal direction. Nevertheless, a good agreement of the

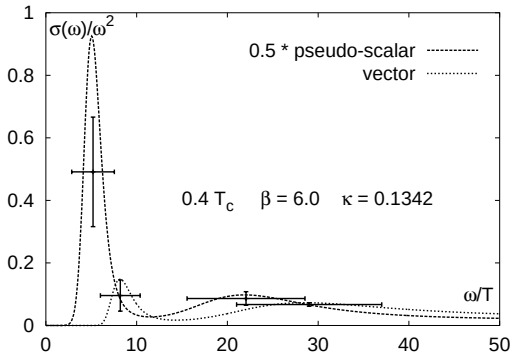

Figure 3. Meson spectral functions at 0.4 T_c

meson ground state masses is obtained comparing the MEM results at 0.4 T_c with zero temperature data ($24^3 \times 32$ lattice [7]) and conventional exponential fits results of the spatial correlator. The advantage of MEM is most obvious for the temporal vector meson correlator, where the exponential fits lead to overestimated mass values. MEM allows a distinction between the pole and continuum contributions (see fig. 3) and thus detects precisely the actual ground state mass. Error bars are indicated for the average spectral functions in the given ω/T-range. While the situation at 0.4 T_c and 0.6 T_c is similar and almost no thermal effects are visible, the situation is clearly different at 0.9 T_c. Here we observe a broadening of the spectral functions as well as a shift in the location of the peaks. To what extent this is a physical or statistical effect has to be clarified with larger statistics.

Increasing the temperature above T_c a gradual approach towards the free curve is found in the (pseudo-)scalar channel (see fig. 4a). The vector meson correlator is instead much closer to free quark behavior. In this channel we have summed up the contributions with all four γ-matrices, which yields twice the (pseudo-)scalar correlator in the free case. These properties are similarly evident from the reconstructed spectral shape in figure 4b. However, the vector meson spectral function shows an enhancement around $\omega/T \sim 6$, while a suppression compared to the free curve is found at smaller energies. This observation is in contrast to HTL-resummed perturbation theory [8], where the spectral function is IR-

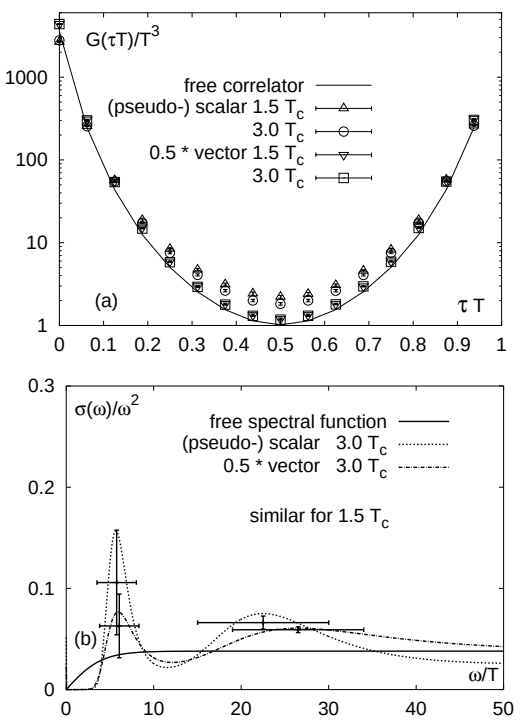

Figure 4. Meson correlators (a) and reconstructed spectral functions (b) above T_c

divergent. The deviations in the (pseudo-)scalar channel cannot be sufficiently explained by HTL medium effects like thermal quark masses and Landau damping. This leads to the conclusion that the behavior in the deconfined plasma phase is characterized by strongly correlated quarks and gluons in the temperature range up to 3 T_c.

REFERENCES

1. M.Jarrell and J.E.Gubernatis, Phys. Rep. 269 (1996) 133
2. M.Asakawa et al., Prog. Part. Nucl. Phys. 46 (2001) 459
3. T.Yamzaki et al., hep-lat/0105030
4. I.Wetzorke and F.Karsch, Proc. SEWM 2000, World Scientific (2001) 193, hep-lat/0008008
5. UKQCD, P.Lacock et al., Phys. Rev. D51 (1995) 6403
6. M.Lüscher et al., Nucl. Phys. B491 (1997) 323
7. M.Göckeler et al., Phys. Rev. D57 (1998) 5562
8. F.Karsch et al., Phys. Lett. B497 (2001) 249