arXiv:hep-ph/0112346 v1 28 Dec 2001

DGLAP and BFKL evolution equations in the $N = 4$ supersymmetric gauge theory.

A.V. Kotikov

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
141980 Dubna, Russia

and

L.N. Lipatov

Theoretical Physics Department
Petersburg Nuclear Physics Institute
Orlova Roscha, Gatchina
188300, St. Petersburg

Abstract

We discuss DGLAP and BFKL evolution equations in the $N = 4$ supersymmetric gauge theory in the leading and next-to-leading approximations. Eigenvalues of the BFKL kernel in this model turn out to be analytic functions of the conformal spin. It allows us to find the residues of the anomalous dimensions of the twist-2 operators in the points $j = 1, 0, -1, ...$ from the BFKL equation in an agreement with their direct calculation from the DGLAP equation. The holomorphic separability of the BFKL kernel and the integrability of the DGLAP dynamics in this model are also discussed.
PACS: 12.38.Bx

1 Introduction

The Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [1, 2] is used now together with the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [3] for a theoretical description of structure functions of the deep-inelastic ep scattering at small values of the Bjorken variable x. In this kinematical region the structure functions are measured by the H1 and ZEUS collaborations [4]. The radiative corrections to the splitting kernels for the DGLAP equation are well known [5]. Although the BFKL equation in the leading logarithmic approximation (LLA) was constructed many years ago, the calculation of the next-to-leading corrections to its kernel was started only in 1989 [6] and completed comparatively recently [7, 8, 9].

In supersymmetric gauge theories the structure of BFKL and DGLAP equations is simplified significantly. In the case of the extended $N = 4$ SUSY model the next-to-leading order (NLO) corrections to BFKL equation were calculated in ref. [9] for arbitrary values of the conformal spin n in a framework of the dimensional regularization (DREG) scheme. In the section 3 below the results are presented in the scheme of the dimensional reduction (DRED) which does not violate supersymmetry. The analyticity of the eigenvalue of the BFKL kernel over the conformal spin n gives a possibility to relate DGLAP and BFKL equations in this model, as we show below. Further, the eigenvalue in both schemes (DREG and DRED) can be written as a sum of the eigenvalues of holomorphic and anti-holomorphic operators (see Section 3).

Let us to introduce the unintegrated parton distributions (UnPD) $\varphi_a(x, k_\perp^2)$ (hereafter $a = q, g, \varphi$ for the quark and gluon densities respectively) and the (integrated) parton distributions (PD) $f_a(x, Q^2)$, where

$$f_a(x, Q^2) \;=\; \int_{k_\perp^2 < Q^2} dk_\perp^2 \varphi_a(x, k_\perp^2). \tag{1}$$

The DGLAP equation relates the (integrated) parton distributions having different values of Q^2. It has the form:

$$\begin{aligned} \frac{d}{d\ln Q^2} f_a(x, Q^2) &= -\tilde{W}_a f_a(x, Q^2) + \int_x^1 \frac{dy}{y} \sum_b \tilde{W}_{b\to a}(x/y) f_b(y, Q^2), \\ \tilde{W}_a &= \sum_b \int_0^1 dx\; x\; \tilde{W}_{a\to b}(x), \end{aligned} \tag{2}$$

where the last term in the r.h.s. is the Mellin convolution of the inclusive transition probabilities $\tilde{W}_{b\to a}(x)$ and PD $f_b(x, Q^2)$. Usually the first and second terms in the right-hand side of the equation are unified by modifying $\tilde{W}_{b\to a}$ in the form of the splitting kernel $W_{b\to a}$:

$$\frac{d}{d\ln Q^2} f_a(x, Q^2) \;=\; \int_x^1 \frac{dy}{y} \sum_b W_{b\to a}(x/y) f_b(y, Q^2) \tag{3}$$

It is known, that DGLAP equation (3) is simplified essentially after the Mellin transformation to the t-channel angular momentum j representation:

$$\frac{d}{d\ln Q^2} f_a(j, Q^2) \;=\; \sum_b \gamma_{ab}(j) f_b(j, Q^2), \tag{4}$$

where

$$f_a(j,Q^2) = \int_0^1 dx\ x^{j-1} f_a(x,Q^2) \tag{5}$$

are the Mellin moments of parton distributions. The Mellin moment of the splitting kernel

$$\gamma_{ab}(j) = \int_0^1 dx\ x^{j-1} W_{b\to a}(x) \tag{6}$$

coincides with the anomalous dimension matrix for twist-2 operators [1]. These operators are constructed as bilinear combinations of derivations of the fields describing the partons a (see Eq. (8) below).

The BFKL equation relates the unintegrated parton distributions having various values of the Bjorken variable x and has the form:

$$\frac{d}{d\ln(1/x)}\varphi_g(x,k_\perp^2) \ = \ 2\omega(-k_\perp^2)\,\varphi_g(x,k_\perp^2) + \int d^2k'_\perp K(k_\perp,k'_\perp)\varphi_g(x,k_\perp^2), \tag{7}$$

where $\omega(-k_\perp^2)$ is the gluon Regge trajectory.

Let us introduce the local twist-two operators:

$$\begin{aligned}
O^g_{\mu_1,\ldots,\mu_j} &= \hat{S} G_{\rho\mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} G_{\rho\mu_j}, \\
\tilde{O}^g_{\mu_1,\ldots,\mu_j} &= \hat{S} G_{\rho\mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} \tilde{G}_{\rho\mu_j}, \\
O^q_{\mu_1,\ldots,\mu_j} &= \hat{S}\overline{\Psi}\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_j}\Psi, \\
\tilde{O}^q_{\mu_1,\ldots,\mu_j} &= \hat{S}\overline{\Psi}\gamma_5\,\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_j}\Psi, \\
O^\varphi_{\mu_1,\ldots,\mu_j} &= \hat{S}\Phi D_{\mu_1} D_{\mu_2} \ldots D_{\mu_j}\Phi,
\end{aligned} \tag{8}$$

where the last operator is constructed from derivations of the scalar field Φ appearing in supersymmetric models. The symbol $\hat{S}$ implies a symmetrization of the tensor in the Lorenz indices $\mu_1,\ldots,\mu_j$ and a subsequent subtraction of its traces.

The matrix elements of $O^a_{\mu_1,\ldots,\mu_j}$ and $\tilde{O}^a_{\mu_1,\ldots,\mu_j}$ are related to the moments of parton distributions in a hadron h in the following way

$$\begin{aligned}
\int_0^1 dx x^{j-1} f_a(x,Q^2) &= < h|\tilde{n}^{\mu_1}\ldots\tilde{n}^{\mu_j} O^a_{\mu_1,\ldots,\mu_j}|h>, \qquad a=(q,g,\varphi), \\
\int_0^1 dx x^{j-1} \Delta f_a(x,Q^2) &= < h|\tilde{n}^{\mu_1}\ldots\tilde{n}^{\mu_j} \tilde{O}^a_{\mu_1,\ldots,\mu_j}|h>, \qquad a=(q,g),
\end{aligned} \tag{9}$$

where the vector $\tilde{n}^\mu$ is light-like: $\tilde{n}^2 = 0$. Note, that in the deep-inelastic ep scattering $\tilde{n}^\mu = q + xp$.

The conformal spin n and the quantity $1+\omega$, expressed in terms of the eigenvalue ω of the BFKL kernel, coincide respectively with the total numbers of transverse and

[1] As in Ref. [7, 9], the anomalous dimensions differ from ones used usually in DIS by a factor (-2), i.e. $\gamma_{ab}(j) = (-1/2)\gamma^{DIS}_{ab}(j)$

longitudinal indices of the Lorentz tensor with the rank $j = 1 + \omega + n$. Namely, we can introduce the following projectors of this tensor

$$n^{\mu_1}...n^{\mu_{j-|n|}}\, O^a_{\mu_1,...,\mu_{j-|n|}\,\sigma_1,...,\sigma_{|n|}}\, l_\perp^{\sigma_1}...l_\perp^{\sigma_{|n|}}, \tag{10}$$

where the complex transverse vector $l_\perp$ is given below

$$l_\perp^\sigma \;=\; \frac{1}{\sqrt{2}}\,(\delta_1^\sigma + i\delta_2^\sigma)\,, \quad l_\perp^2 = 0\,.$$

Anomalous dimension matrices $\gamma_{ab}(j)$ and $\tilde{\gamma}_{ab}(j)$ for the twist-2 operators $O^a_{\mu_1,...,\mu_j}$ and $\tilde{O}^a_{\mu_1,...,\mu_j}$ do not depend on the different projections of the tensors due to the Lorentz invariance.

In the gluon case the usual light-cone projections entering in the DGLAP equation (see Eqs.(10)) are

$$\tilde{n}^{\mu_1}...\tilde{n}^{\mu_j} \;<P|O^g_{\mu_1,...,\mu_j}|P>\;\; = \;\; \int_0^1 dx x^{j-1} f_g(x,Q^2). \tag{11}$$

The mixed projections

$$\tilde{n}^{\mu_1}...\tilde{n}^{\mu_{1+\omega}}\, l_\perp^{\mu_{1+\omega}}...l_\perp^{\mu_j} \;<P|O_g^{\mu_1,...,\mu_j}|P>\;\; \sim \;\; \int_0^1 dx x^\omega \int d^2k_\perp \left(\frac{k_\perp}{|k_\perp|}\right)^n \varphi_g(x,k_\perp^2), \tag{12}$$

can be expressed in terms of the solution of the BFKL equation. Generally the corresponding operators have higher twists.

Thus, it looks possible to obtain some additional information about the parton x-distributions satisfying the DGLAP equation from the analogous $k_\perp$-distributions satisfied the BFKL equation. Moreover, in the extended $N = 4$ SUSY the β-function equals zero and therefore the 4-dimensional conformal invariance could allow to relate the Regge and Bjorken asymptotics of scattering amplitudes.

Our presentation is organized as follows. In Section 2 we discuss the relation between DGLAP and BFKL equations in the leading logarithmic approximation. In Section 3 we review shortly the results of Ref. [9] and rewrite them in the framework of the DRED scheme. Section 4 contains the information about the anomalous dimensions calculated independently with the use of the renormalization group. A summary is given in Conclusion.

2 Anomalous dimensions of twist-2 operators and their singularities

In the leading logarithmic approximation (LLA) for the BFKL equation the contribution of fermions is not essential and therefore in this approximation the integral kernel is

the same for all supersymmetric gauge theories. In the impact parameter representation due to the conformal invariance the solution of the homogeneous BFKL equation has the form (see [10])

$$E_{\nu,n}(\overrightarrow{\rho_{10}},\overrightarrow{\rho_{20}}) \equiv < \phi(\overrightarrow{\rho_1})\, O_{m,\tilde{m}}(\overrightarrow{\rho_0})\, \phi(\overrightarrow{\rho_2}) > \;\; = \;\; \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^{m} \left(\frac{\rho^*_{12}}{\rho^*_{10}\rho^*_{20}}\right)^{\widetilde{m}}, \qquad (13)$$

where

$$m = 1/2 + i\nu + n/2,\ \widetilde{m} = 1/2 + i\nu - n/2.$$

are the conformal weights. Here we introduced the complex variables $\rho_k = x_k + iy_k$ and denoted $\rho_{kl} = \rho_k - \rho_l$

For the principal series of the unitary representations the quantities ν and n are correspondingly real and integer numbers. The projection n of the conformal spin $|n|$ can be positive or negative, but the eigenvalue of the BFKL equation in LLA

$$\begin{aligned} \omega = \omega^0(n,\nu) \;\; &= \;\; \frac{g^2 N_c}{8\pi^2}\Big(4\Psi\big(1\big) - \Psi\big(\frac{1}{2} + i\nu + \frac{n}{2}\big) - \Psi\big(\frac{1}{2} - i\nu - \frac{n}{2}\big) \\ &- \;\; \Psi\big(\frac{1}{2} + i\nu - \frac{n}{2}\big) - \Psi\big(\frac{1}{2} - i\nu + \frac{n}{2}\big)\Big) \end{aligned} \qquad (14)$$

depends only on $|n|$. We shall imply below that n is positive, i.e.

$$n \;\; = \;\; |n|\,. \qquad (15)$$

Note, that eq. (14) has the property of the holomorphic separability and corresponds to the pair hamiltonian of the integrable Heisenberg spin model (see [11]-[16]).

The solution of the inhomogeneous BFKL equation in the LLA approximation can be written as the four-point function of a two-dimensional field theory

$$< \phi(\overrightarrow{\rho_1})\, \phi(\overrightarrow{\rho_2})\, \phi(\overrightarrow{\rho_{1'}})\, \phi(\overrightarrow{\rho_{2'}}) > = \;\; \sum_n \int_{-\infty}^{\infty} d\nu C(\nu,n) \int d^2\rho_0 \, \frac{E_{\nu,n}(\overrightarrow{\rho_{10}},\overrightarrow{\rho_{20}}) E^*_{\nu,n}(\overrightarrow{\rho_{1'0}},\overrightarrow{\rho_{2'0}})}{\omega - \omega^0(n,\nu)}, \qquad (16)$$

where $C(\nu,n)$ is expressed through the inhomogeneous term of the equation with the use of the completeness condition for $E_{\nu,n}$ (see [10]).

For $\overrightarrow{\rho_{1'}} \to \overrightarrow{\rho_{2'}}$ we obtain from the integration region $\overrightarrow{\rho_0} \to \overrightarrow{\rho_{1'}}$:

$$\begin{aligned} < \phi(\overrightarrow{\rho_1})\, \phi(\overrightarrow{\rho_2})\, \phi(\overrightarrow{\rho_{1'}})\, \phi(\overrightarrow{\rho_{2'}}) > \;\; &\sim \;\; \sum_n \int_{-\infty}^{\infty} d\nu C(\nu,n)\, \frac{E_{\nu,n}(\overrightarrow{\rho_{11'}},\overrightarrow{\rho_{21'}})}{\omega - \omega^0(n,\nu)} \rho^m_{1'2'} \rho^{*\tilde{m}}_{1'2'} \\ &\sim \;\; \sum_n C(\nu_\omega,n)\, \frac{E_{\nu_\omega,n}(\overrightarrow{\rho_{11'}},\overrightarrow{\rho_{21'}})}{\omega'(n,\nu_\omega)} |\rho_{1'2'}|^{1+2i\nu_\omega} \left(\frac{\rho_{1'2'}}{\rho^*_{1'2'}}\right)^{n/2} \end{aligned} \qquad (17)$$

where ν_ω is a solution of the equation

$$\omega = \omega^0(n,\nu) \qquad (18)$$

with $Im\nu_\omega < 0$.

The above asymptotics has a simple interpretation in terms of the Wilson operator-product expansion

$$\lim_{\rho_{1'}\to\rho_{2'}} \phi(\overrightarrow{\rho_{1'}})\,\phi(\overrightarrow{\rho_{2'}}) \;=\; \sum_n \frac{C(\nu_\omega,n)}{\omega'(n,\nu_\omega)}|\rho_{1'2'}|^{2\Gamma_\omega}\left(\frac{\rho_{1'2'}}{\rho^*_{1'2'}}\right)^{n/2} O_{\nu_\omega,n}(\overrightarrow{\rho_{1'}})\,, \tag{19}$$

where

$$\Gamma_\omega = \frac{1}{2} + i\nu_\omega \tag{20}$$

is the transverse dimension of the operator $O_{\nu_\omega,n}(\overrightarrow{\rho_{1'}})$, calculated in units of squared mass. This operator is the following projection

$$O_{\nu_\omega,n}(\overrightarrow{\rho_{1'}}) \;=\; \tilde{n}^{\mu_1}...\tilde{n}^{\mu_{1+\omega}}\, l_\perp^{\sigma_1}...l_\perp^{\sigma_n}\, O_{\mu_1,...,\mu_{1+\omega}\sigma_1,...,\sigma_n} \tag{21}$$

of the gauge-invariant tensor with $1+\omega+n$ indices.

The anomalous dimension $\gamma(\omega)$ obtained from the BFKL equation in LLA has the poles

$$\Gamma_\omega = 1 + \frac{|n|}{2} - i\gamma(j), \quad \gamma(j)|_{\omega\to 0} = \frac{g^2 N_c}{4\pi^2\omega}. \tag{22}$$

The canonical contribution $1+|n|/2$ to the transverse dimension Γ_ω corresponds to the local operator

$$G_{\rho\mu_1} D^{||}_{\mu_2}...D^{||}_{\mu_\omega} D^{\perp}_{\sigma_1}...D^{\perp}_{\sigma_n} G_{\rho\mu_{1+\omega}}\,, \tag{23}$$

because in the light-cone gauge $A_\mu\tilde{n}_\mu = 0$ the tensor

$$G_{\rho\mu_1} G_{\rho\mu_{1+\omega}}\,\tilde{n}^{\mu_1}\,\tilde{n}^{\mu_2} \;\sim\; \partial_{\mu_1} A^\perp_\rho \partial_{\mu_2} A^\perp_\rho\,\tilde{n}^{\mu_1}\,\tilde{n}^{\mu_2} \tag{24}$$

has the transverse dimension equal to 1.

The above local operator $O_{\nu_\omega,n}$ for $|n|>0$ should have the twist higher than 2 because its anomalous dimension is singular at $\omega\to 0$. Indeed, such singularities are impossible for the twist-2 operator, because for $n>0$ and small ω the total number of its indices is integer and physical. The only way to obtain some information about the twist-2 operators is to continue analytically the anomalous dimension $\gamma(\omega)$ to negative integer points

$$|n| \;\to\; -r-1, \tag{25}$$

where r is a positive integer. Then in the limit $\omega\to 0$ the total number of indices $j = 1+\omega+|n|$ tends to $-r$ and therefore the pole of $\gamma(\omega)$ in ω can be interpreted as the pole $1/(j+r)$ for a non-physical value of j for the twist-2 operator.

In LLA one can obtain after the analytic continuation of $\omega^0(|n|,\nu)$

$$\gamma(j)|_{j\to -r} \;=\; \frac{g^2 N_c}{4\pi^2}\frac{1}{j+r} \tag{26}$$

We remind, that for the BFKL equation in LLA the fermions are not important, but generally they give non-vanishing contributions to the residues of the poles of $\gamma(j)$ in the DGLAP equation even in LLA. Therefore the above result for $\gamma(j)$ can be valid only for a definite generalization of QCD. It is known [9], that only for the extended N=4 supersymmetric Yang-Mills theory the anomalous dimension, calculated in the next-to-leading approximation for the BFKL equation, can be analytically continued to $|n| = -r - 1$. Therefore it is natural to expect, that the above result for $\gamma(j)$ in LLA is valid for the N=4 case.

Indeed, using the conservation of the stress tensor in this theory to fix the substruction constant in the expansion over the poles at $j = -r$, we obtain :

$$\gamma(j)^{LLA} \;=\; \frac{g^2 N_c}{4\pi^2}\Big(\Psi(1) - \Psi(j-1)\Big) \tag{27}$$

in an agreement with the direct calculation of $\gamma(j)$ in this theory (see [18, 19] and Section 4).

In the next-to-leading order (NLO) approximation it is needed to modify the above procedure of the derivation of $\gamma(j)$ from the BFKL equation, taking into account the possibility of the appearance of double-logarithmic terms leading to triple poles at $j = -k$. We shall return to this problem in our future publication.

3 NLO corrections to BFKL kernel in $N = 4$ SUSY

To begin with, we review shortly the results of Ref. [9], where the NLO corrections to the BFKL integral kernel at $t = 0$ were calculated in the case of QCD and supersymmetric gauge theories. We discuss only the case of the $N = 4$ supersymmetric gauge theory and write the formulae important for our analysis.

3.1 The set of eigenvalues

The set of eigenvalues for eigenfunctions of the homogeneous BFKL equation for the $N = 4$ supersymmetric theory

$$\omega = 4\,\overline{a}\left[\chi(n,\gamma) + \left(\frac{1}{3}\chi(n,\gamma) + \delta(n,\gamma)\right)\,\overline{a}\right] \tag{28}$$

has been found in [9] in the following form

$$\chi(n,\gamma) \;=\; 2\Psi(1) - \Psi\Big(\gamma + \frac{n}{2}\Big) - \Psi\Big(1 - \gamma + \frac{n}{2}\Big) \tag{29}$$

$$\begin{aligned}\delta(n,\gamma) \;&=\; -\Big[2\Phi(n,\gamma) + 2\Phi(n,1-\gamma) + 2\zeta(2)\chi(n,\gamma) \\ &-\; 6\zeta(3) - \Psi''\Big(\gamma + \frac{n}{2}\Big) - \Psi''\Big(1 - \gamma + \frac{n}{2}\Big)\Big],\end{aligned} \tag{30}$$

where $\Psi(z)$, $\Psi'(z)$ and $\Psi''(z)$ are respectively the Euler Ψ -function and its derivatives; $\overline{a} = g^2 N_c/(16\pi^2)$ is coupling constant in DREG scheme. The function $\Phi(n,\gamma)$ is given below[2]

$$\begin{aligned}
\Phi(n,\gamma) &= -\int_0^1 dx\,\frac{x^{\gamma-1+n/2}}{1+x}\Biggl[\frac{1}{2}\Bigl(\Psi'\Bigl(\frac{n+1}{2}\Bigr)-\zeta(2)\Bigr)+\mathrm{Li}_2(-x)+\mathrm{Li}_2(x)\\
&+ \ln(x)\Bigl(\Psi(n+1)-\Psi(1)+\ln(1+x)+\sum_{k=1}^{\infty}\frac{(-x)^k}{k+n}\Bigr)\\
&+ \sum_{k=1}^{\infty}\frac{x^k}{(k+n)^2}(1-(-1)^k)\Biggr]\\
&= \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{k+\gamma+n/2}\Biggl[\Psi'(k+1)-\Psi'(k+n+1)+(-1)^{k+1}\Bigl(\beta'(k+n+1)+\beta'(k+1)\Bigr)\Bigr)\\
&+ \frac{1}{k+\gamma+n/2}\Bigl(\Psi(k+n+1)-\Psi(k+1)\Bigr)\Biggr], \qquad (31)
\end{aligned}$$

and

$$\beta'(z)=\frac{1}{4}\Bigl[\Psi'\Bigl(\frac{z+1}{2}\Bigr)-\Psi'\Bigl(\frac{z}{2}\Bigr)\Bigr],\quad \beta''(z)=\frac{1}{8}\Bigl[\Psi''\Bigl(\frac{z+1}{2}\Bigr)-\Psi''\Bigl(\frac{z}{2}\Bigr)\Bigr],$$

Note, that the term

$$\frac{1}{3}\chi(n,\gamma) \qquad (32)$$

appears as a result of the use of the DREG scheme (see (29) in [9]). It is well known, that DREG violates SUSY. The proper procedure, which is very close to DREG and satisfies the supersymmetry requirements, is the DRED scheme. The results in the framework of the DRED scheme [3] can be found from (28) by the redefinition of the coupling constant

$$\overline{a} \to \hat{a} = \overline{a} + \frac{1}{3}\overline{a}^2, \qquad (33)$$

which eliminates the above term (32) in (28). For the new coupling constant a the above expression for ω can be written in the following form

$$\omega = 4\,\hat{a}\,\Bigl[\chi(n,\gamma)+\delta(n,\gamma)\,\hat{a}\Bigr] \qquad (34)$$

3.2 Generalized holomorphical separability

Following the method of Refs. [18, 19] and using the results of the previous section we can present the above NLO corrections to the BFKL equation in the form having a (generalized) holomorphical separability. We can split the function $\Phi(n,\gamma)$ in two parts

[2] Notice that the representation of $\Phi(n,\gamma)$ contains a misprint in [9]: the factor $(-1)^{k+1}$ in the last sum of (31) was substituted by $(-1)^k$

[3] In the calculations given in [9] the result obtained in the DRED scheme with 6 scalars and pseudoscalars is equal to that for DREG with $6+2\hat{\varepsilon}$ scalars and pseudoscalars, where $\hat{\varepsilon}=(4-D)/2$ and D is the space-time dimension.

$$\Phi(n,\gamma) = \Phi_1(n,\gamma) + \Phi_2(n,\gamma)\,,$$

where

$$\Phi_1(n,\gamma) = \sum_{k=0}^{\infty} \frac{\Big(\beta'(k+n+1) - (-1)^k \Psi'(k+n+1)\Big)}{k+\gamma+n/2} + \sum_{k=0}^{\infty} \frac{(-1)^k \left(\Psi(n+k+1) - \Psi(1)\right)}{(k+\gamma+n/2)^2}\,,$$

and

$$\Phi_2(n,\gamma) = \sum_{k=0}^{\infty} \frac{\Big(\beta'(k+1) + (-1)^k \Psi'(k+1)\Big)}{k+\gamma+n/2} - \sum_{k=0}^{\infty} \frac{(-1)^k \left(\Psi(k+1) - \Psi(1)\right)}{(k+\gamma+n/2)^2} \equiv \Phi_2(\gamma+n/2)\ .$$

Here $\Phi_2(n,\gamma)$ depends only on $m = \gamma + n/2$ and therefore the corresponding contributions to ω have the property of the holomorphic separability. Further, for $\Phi_1(n,\gamma)$ we obtain the simple relation

$$\Phi_1(n,\gamma) + \Phi_1(n,1-\gamma) = \beta'(\gamma+n/2)\left[\Psi(1) - \Psi(1-\gamma+n/2)\right] + \beta'(1-\gamma+n/2)\left[\Psi(1) - \Psi(\gamma+n/2)\right]\,,$$

which can be verified by calculating the residues at $\gamma = -r - n/2$ and $\gamma = 1 + r + n/2$ and the asymptotic behavior at $\gamma \to \infty$. Therefore one has

$$\Phi(n,\gamma) + \Phi(n,1-\gamma) = \chi(n,\gamma)\ \Big(\beta'(\gamma+\frac{n}{2}) + \beta'(1-\gamma+\frac{n}{2})\Big) + \Phi_2(\gamma+\frac{n}{2}) -$$

$$\beta'(\gamma+\frac{n}{2})\left[\Psi(1) - \Psi(\gamma+\frac{n}{2})\right] + \Phi_2(1-\gamma+\frac{n}{2}) - \beta'(1-\gamma+\frac{n}{2})\left[\Psi(1) - \Psi(1-\gamma+\frac{n}{2})\right],$$

where $\chi(n,\gamma)$ is given by Eq.(29).

Thus, we can rewrite the NLO corrections $\delta(n,\gamma)$ in the generalized holomorphic separable form (providing that ω_0 is substituted by ω, which is valid with our accuracy):

$$\delta(n,\gamma) = \phi\Big(\gamma+\frac{n}{2}\Big) + \phi\Big(1-\gamma+\frac{n}{2}\Big) - \frac{\omega_0}{2\hat{a}}\Big(\rho\Big(\gamma+\frac{n}{2}\Big) + \rho\Big(1-\gamma+\frac{n}{2}\Big)\Big) \qquad (35)$$

$$\omega_0 = 4\hat{a}\Big(2\Psi(1) - \Psi\Big(\gamma+\frac{n}{2}\Big) - \Psi\Big(1-\gamma+\frac{n}{2}\Big)\Big) \qquad (36)$$

and

$$\rho(\gamma) = \beta^{(1)}(\gamma) + \frac{1}{2}\zeta(2) \qquad (37)$$

$$\phi(\gamma) = 3\zeta(3) + \Psi^{(\prime\prime)}(\gamma) - 2\Phi_2(\gamma) + 2\beta^{(1)}(\gamma)\Big(2\Psi(1) - \Psi(\gamma)\Big)\,. \qquad (38)$$

3.3 The asymptotics of "cross-sections" at $s \to \infty$

As an example we consider the cross-sections for the inclusive production of two pairs of particles with mass m in the polarised $\gamma\gamma$ collision (see [2, 9]):

$$\sigma(s) \;=\; \alpha_{em}^2 \hat{a}^2 \frac{1}{m^2}\frac{32}{81}\Big(\sigma_0(s) + \Big(\cos^2\vartheta - \frac{1}{2}\Big)\sigma_2(s)\Big),$$

where α_{em} is the electromagnetic coupling constant, the coefficient $\sigma_0(s)$ is proportional to the cross-section for the scattering of unpolarized photons and $\sigma_2(s)$ describes the spin correlation depending on the relative azimutal angle ϑ between the polarization vectors of colliding photons.

The asymptotic behavior of the cross-sections $\sigma_k(s)$ $(k = 0, 2)$ at $s \to \infty$ leads in the t-channel to unmoving singularities $f_\omega(t) \sim (\omega - \omega_k)^{1/2}$, where

$$\omega_k \;=\; 4\hat{a}\Big[\chi(k,\frac{1}{2}) + \hat{a}\delta(k,\frac{1}{2})\Big] \equiv 4\hat{a}\chi(k,\frac{1}{2})\Big[1 - \hat{a}c(k,\frac{1}{2})\Big] \quad \Big(c(n,\gamma) = -\frac{\delta(n,\gamma)}{\chi(n,\gamma)}\Big)\,,$$

$$\sigma_0(s) \;=\; \frac{9\pi^{5/2}}{32\sqrt{7\zeta(3)}}\frac{s^{\omega_0}}{\Big(\ln(s/s_0)\Big)^{1/2}}\cdot\Big(1 + O(a)\Big), \tag{39}$$

$$\sigma_2(s) \;=\; \frac{\pi^{5/2}}{9\cdot 32\sqrt{7\zeta(3)-8}}\frac{s^{\omega_2}}{\Big(\ln(s/s_0)\Big)^{1/2}}\cdot\Big(1 + O(a)\Big). \tag{40}$$

Here the symbol $O(\hat{a})$ denotes unknown next-to-leading corrections to the impact factors.

Using our results (26), (27), we obtain in the $N = 4$ case the following values for $\chi(k, \frac{1}{2})$ and $c(k, \frac{1}{2})$ $(k = 0, 2)$:

$$\chi(0,\frac{1}{2}) \;=\; 4\ln 2, \quad \chi(2,\frac{1}{2}) \;=\; 4(\ln 2 - 1)\,, \tag{41}$$

$$c(0,\frac{1}{2}) \;=\; 2\zeta(2) + \frac{1}{2\ln 2}\Big[11\zeta(3) - 32\mathrm{Ls}_3\Big(\frac{\pi}{2}\Big) - \frac{165}{16}\pi\zeta(2)\Big] \;=\; 9.5812\,, \tag{42}$$

$$\begin{aligned} c(2,\frac{1}{2}) \;&=\; 2\zeta(2) + \frac{1}{2(\ln 2 - 1)}\Big[11\zeta(3) + 32\mathrm{Ls}_3\Big(\frac{\pi}{2}\Big) + 14\pi\zeta(2) - 32\ln 2\Big] \\ &=\; 6.0348\,, \end{aligned} \tag{43}$$

where (see [20, 21])

$$\mathrm{Ls}_3(x) = -\int_0^x \ln^2\Big|2\sin\Big(\frac{y}{2}\Big)\Big|\,dy\,.$$

Note, that the function $\mathrm{Ls}_3(x)$ appears also in calculations of some massive diagrams (see, for example, the recent papers [22] and references therein).

The LO results (41) coincide with ones obtained in ref. [2]. As it was shown in [7, 9], in the framework of QCD the NLO correction $c^{QCD}(0, 1/2)$ is large and leads to a quite strong reduction of the value of the Pomeron intercept (see recent analyses [23]-[25] of various effective resummations of the large NLO terms). Contrary to $c^{QCD}(0, 1/2)$, the correction

$c(0,1/2)$ is not large ($c^{QCD}(0,1/2)/c(0,1/2) \approx 2.7$), which seems to support the results of Ref [23], where a quite large value of the non-conformal contribution to $c^{QCD}(0,1/2)$ was found in the physical renormalization schemes. The values of $c^{QCD}(2,1/2)$ (see [7, 9]) and $c(2,1/2)$ are small and do not change significantly the small LO value [2] of the angle-dependent contribution.

3.4 Non-symmetric choice of the energy normalization

Analogously to refs. [7, 9] one can calculate the eigenvalues of the kernel in the case of a non-symmetric choice of the energy normalization parameter s_0 in eq.(15). For the scale $s_0 = q^2$, which is natural for the deep-inelastic scattering process, we obtain

in DREG-scheme

$$\omega = 4\,\overline{a}\left[\chi(n,\gamma) + \left(\frac{1}{3}\chi(n,\gamma) + \delta(n,\gamma) - 2\chi(n,\gamma)\chi'(n,\gamma)\right)\overline{a}\right] \tag{44}$$

and in DRED-scheme

$$\omega = 4\,\hat{a}\left[\chi(n,\gamma) + \left(\delta(n,\gamma) - 2\chi(n,\gamma)\chi'(n,\gamma)\right)\hat{a}\right], \tag{45}$$

where

$$\chi'(n,\gamma) \equiv \frac{d}{d\gamma}\chi(n,\gamma) = -\Psi'\Big(\gamma + \frac{n}{2}\Big) + \Psi'\Big(1 - \gamma + \frac{n}{2}\Big)$$

3.5 The limit $\gamma \to 0$ for $n = 0$

By considering the limit $\gamma \to 0$ in the Eq.(45) we have for $n = 0$ (see also the analysis in [9])

$$\begin{aligned}
\chi(0,\gamma) &= \frac{1}{\gamma} + O(\gamma^2), \\
\frac{1}{3}\chi(n,\gamma) &+ \delta(0,\gamma) - 2\chi(0,\gamma)\chi'(0,\gamma) = \frac{B^{DREG}}{\gamma} + C + O(\gamma^2), \\
\delta(0,\gamma) &- 2\chi(0,\gamma)\chi'(0,\gamma) = \frac{B^{DRED}}{\gamma} + C + O(\gamma^2),
\end{aligned} \tag{46}$$

where

$$B^{DREG} = \frac{1}{3}, \quad B^{DRED} = 0, \quad \text{and} \quad C = 2\zeta(3). \tag{47}$$

Analogous to ref.[7, 9] with the use of eqs.(46) and (47) one can obtain the expression for anomalous dimensions of twist-2 operators γ at $\omega \to 0$ (i.e. near $j = 1$)

in DREG-scheme

$$\gamma = 4\,\overline{a}\left[\left(\frac{1}{\omega} + O(\omega)\right) + a\left(\frac{B^{DREG}}{\omega} + O(1)\right) + \overline{a}^2\left(\frac{C}{\omega^2} + O\left(\omega^{-1}\right)\right)\right] \tag{48}$$

and in DRED-scheme

$$\gamma = 4\,\hat{a}\left[\left(\frac{1}{\omega}+O(\omega)\right)+\hat{a}\left(\frac{B^{DRED}}{\omega}+O(1)\right)+\hat{a}^2\left(\frac{C}{\omega^2}+O\left(\omega^{-1}\right)\right)\right]. \tag{49}$$

Thus, in the framework of DRED scheme the vanishing contribution at $j \to 1$ is obtained for the NLO contribution to the anomalous dimension.

3.6 Symmetry between γ and $j-\gamma$

Using the scale $s_0 = q^2$, the expression for ω as a function of the correctly defined anomalous dimension γ at general n can be written in the following form

$$\omega = 4\hat{a}\left(\chi\left(n,\gamma-\frac{n}{2}\right)+\hat{a}\left[\delta\left(n,\gamma-\frac{n}{2}\right)-\chi\left(n,\gamma-\frac{n}{2}\right)\cdot\chi'\left(n,\gamma-\frac{n}{2}\right)\right]\right), \tag{50}$$

where

$$\begin{aligned}
\chi\left(n,\gamma-\frac{n}{2}\right) &= 2\,\Psi(1)-\Psi(\gamma)-\Psi(n+1-\gamma) \qquad (51)\\
\delta\left(n,\gamma-\frac{n}{2}\right) &= \Psi''(\gamma)+\Psi''(n+1-\gamma)-2\,\Phi\left(n,\gamma-\frac{n}{2}\right)-2\,\Phi\left(n,1-\gamma+\frac{n}{2}\right)\\
&+ 6\zeta(3)\,.
\end{aligned}$$

To calculate the anomalous dimension γ we write ω in the "Lorentz invariant" form

$$\omega = 4\hat{a}\left(2\,\Psi(1)-\Psi(\gamma)-\Psi(j-\gamma)+\Delta(n,\gamma)\,\hat{a}\right),\ j = n+1+\omega\,, \tag{52}$$

where

$$\Delta(n,\gamma) = \delta\left(n,\gamma-\frac{n}{2}\right)+2\left[\Psi'(\gamma)+\Psi'(n+1-\gamma)\right]\chi\left(n,\gamma-\frac{n}{2}\right). \tag{53}$$

Using the analysis of the subsection 3.2, Eq.(52) can be presented as follows

$$\omega = 4\hat{a}\ \left(2\,\Psi(1)-\Psi(\gamma)-\Psi(n+1+\omega-\gamma)+\varepsilon\right), \tag{54}$$

where ε can be written in the "holomorphically separable" form

$$\begin{aligned}
\varepsilon &= \frac{\omega}{2}\Big(p(\gamma)+p(1+n-\gamma)\Big)+\hat{a}\Big(\phi(\gamma)+\phi(1+n-\gamma)\Big),\\
\omega &= 4\hat{a}\Big(2\,\Psi(1)-\Psi(\gamma)-\Psi(n+1-\gamma)\Big)+O(\hat{a}^2)\,. \qquad (55)
\end{aligned}$$

Here

$$p(\gamma) = -\beta'(\gamma)+\Psi'(\gamma)-\frac{1}{2}\zeta(2) \ = \ 2\sum_{k=0}^{\infty}\frac{1}{(\gamma+2k)^2}-\frac{1}{2}\zeta(2)$$

and $\phi(\gamma)$ is given by Eq.(38).

Let us calculate $\Delta(n,\,\gamma)$ near its singularities. To begin with, we consider $\gamma \to 0$ for physical $n \geq 0$:

$$\Delta(n,\,\gamma) \to \frac{4}{\gamma^2}\,(\,\Psi(1) - \Psi(n+1)) \,+ \frac{2}{\gamma}\,c(n)\,,$$

$$c(n) = 2\,\Psi'(n+1) - 2\Psi'(1) - \beta'(n+1) - \beta'(1).$$

Therefore

$$\gamma = \frac{4\hat{a}}{\omega}\Big(1 + \omega\,(\,\Psi(1) - \Psi(n+1))\Big) + \frac{(4\hat{a})^2}{\omega^2}\left(\Psi(1) - \Psi(n+1) + \frac{\omega}{2}c(n)\right)\,.$$

At $n = 0$ the correction $\sim \hat{a}$ is absent, but for other n, especially for $n \to -r-1$, we have the large correction to γ

$$\Delta\gamma = 4\hat{a}\;(\Psi(1) - \Psi(n+1))\;,$$

which has the pole at $j \to -r$ and leads to a contribution changing even singularities of the Born term. It is related with the fact, that for positive n we calculate the anomalous dimensions of the higher twist operators (with the anti-symmetrization between n transverse and $1+\omega$ longitudinal indices).

Because in the case $N = 4$ SUSY the result is analytic in $|n|$, one can continue the anomalous dimensions to the negative values of $|n|$. It gives a possibility to find the singular contributions of the anomalous dimensions of the twist-2 operators not only at $j = 1$ but also at other integer points $j = 0,\,-1,\,-2....$ In particular, as it was discussed above, in the Born approximation for the anomalous dimension of the supermultiplet of the twist-2 operators we obtain $\gamma = 4\,\hat{a}\,(\Psi(1) - \Psi(j-1))$ which coincides with the result of the direct calculations (see [18, 19] and the discussion below). Thus, in the case $N = 4$ the BFKL equation presumably contains the information sufficient for restoring the kernel of the DGLAP equation.

4 The anomalous dimensions matrix in the $N = 4$ SUSY

The DGLAP evolution equation for the moments of the parton distributions for $N = 4$ SUSY has the form

$$\frac{d}{d\ln Q^2} f_a(j,Q^2) \;=\; \sum_k \gamma_{ab}(j) f_b(j,Q^2) \qquad (a,b = q,g,\varphi), \tag{56}$$

$$\frac{d}{d\ln Q^2} \Delta f_a(j,Q^2) \;=\; \sum_k \tilde{\gamma}_{ab}(j) \Delta f_b(j,Q^2) \qquad (a,b = q,g), \tag{57}$$

where the anomalous dimension matrices $\gamma_{ab}(j)$ an $\tilde{\gamma}_{ab}(j)$ can be written as expansions over the coupling constant $\hat{a}$ in the form:

$$\gamma_{ab}(j) \;=\; \hat{a}\cdot\gamma^{(0)}_{ab}(j) + \hat{a}^2\cdot\gamma^{(1)}_{ab}(j), \quad \tilde{\gamma}_{ab}(j) \;=\; \hat{a}\cdot\tilde{\gamma}^{(0)}_{ab}(j) + \hat{a}^2\cdot\tilde{\gamma}^{(1)}_{ab}(j). \tag{58}$$

In the following subsections we will present the results of exact calculations for the leading order (LO) coefficients $\gamma_{ab}^{(0)}(j)$ and $\tilde{\gamma}_{ab}^{(0)}(j)$ and construct the anomalous dimensions of the multiplicatively renormalizable operators of the twist-2. In the NLO approximation the corresponding coefficients $\gamma_{ab}^{(1)}(j)$ and $\tilde{\gamma}_{ab}^{(1)}(j)$ are yet unknown and their calculation is in progress [26]. However, the form of the LO anomalous dimensions of the multiplicatively renormalizable operators in $N = 4$ SUSY is rather simple because they are expressed in terms of one function. Taking into account the universality of this result, the knowledge of the anomalous dimensions in the QCD case, the NLO corrections to the BFKL kernel [9] and an experience in integrating some types of the Feynman diagrams, one can write an ansatz for the NLO anomalous dimensions of the multiplicatively renormalizable operators in the $N = 4$ SUSY. This result will be checked by us later with direct calculations of the matrix elements $\gamma_{ab}^{(1)}(j)$ and $\tilde{\gamma}_{ab}^{(1)}(j)$.

4.1 The results of exact calculations of the anomalous dimensions matrix in the $N = 4$ SUSY

The elements of the LO anomalous dimension matrix in the $N = 4$ SUSY have the following form (see [19]):

for usual tensor twist-2 operators

$$\begin{aligned}
\gamma_{gg}^{(0)}(j) &= 4\left(\Psi(1)-\Psi(j-1)-\frac{2}{j}+\frac{1}{j+1}-\frac{1}{j+2}\right), \\
\gamma_{qg}^{(0)}(j) &= 8\left(\frac{1}{j}-\frac{2}{j+1}+\frac{2}{j+2}\right), \qquad \gamma_{\varphi g}^{(0)}(j) = 12\left(\frac{1}{j+1}-\frac{1}{j+2}\right), \\
\gamma_{gq}^{(0)}(j) &= 2\left(\frac{2}{j-1}-\frac{2}{j}+\frac{1}{j+1}\right), \qquad \gamma_{q\varphi}^{(0)}(j) = \frac{8}{j}, \\
\gamma_{qq}^{(0)}(j) &= 4\left(\Psi(1)-\Psi(j)+\frac{1}{j}-\frac{2}{j+1}\right), \quad \gamma_{\varphi q}^{(0)}(j) = \frac{6}{j+1}, \\
\gamma_{\varphi\varphi}^{(0)}(j) &= 4\left(\Psi(1)-\Psi(j+1)\right), \qquad \gamma_{g\varphi}^{(0)}(j) = 4\left(\frac{1}{j-1}-\frac{1}{j}\right)
\end{aligned} \tag{59}$$

and for the pseudo-tensor operators:

$$\begin{aligned}
\tilde{\gamma}_{gg}^{(0)}(j) &= 4\left(\Psi(1)-\Psi(j+1)-\frac{2}{j+1}-\frac{2}{j+2}\right), \\
\tilde{\gamma}_{qg}^{a,(0)}(j) &= 8\left(-\frac{1}{j}+\frac{2}{j+1}\right), \qquad \tilde{\gamma}_{gq}^{(0)}(j) = 2\left(\frac{2}{j}-\frac{1}{j+1}\right), \\
\tilde{\gamma}_{qq}^{(0)}(j) &= 4\left(\Psi(1)-\Psi(j+1)+\frac{1}{j-1}-\frac{1}{j}\right),
\end{aligned} \tag{60}$$

Note, that in $N = 4$ SUSY there are also the twist-2 operators with the fermion quantum numbers but the anomalous dimension of the corresponding multiplicatively renormalizable operators is the same as that for the bosonic components of the supermultiplet.

4.2 Anomalous dimensions and twist-2 operators with a multiplicative renormalization

Let us denote the distributions of partons with the spin S by $f_S(x)$ and their corresponding momenta by $f_S(j)$. We introduce also the inclusive probabilities $\Delta f_S(x)$ and their momenta $\Delta f_S(j)$ which are the differences of the distributions of the partons with the helicities $\pm S$.

It is possible to construct 5 independent twist-two operators with the multiplicative renormalization. The corresponding parton distributions and their LO anomalous dimensions have the form (see [19]):

$$
\begin{aligned}
f_I(j) &= f_g(j) + f_q(j) + f_\varphi(j) \sim f^+_{q,g,\varphi}(j),\\
\gamma^{(0)}_I(j) &= 4\left(\Psi(1) - \Psi(j-1)\right) \equiv -4S_1(j-2) \equiv \gamma^{(0)}_+(j),\\
\\
f_{II}(j) &= -2(j-1)f_g(j) + f_q(j) + \frac{2}{3}(j+1)f_\varphi(j) \sim f^0_{q,g,\varphi}(j),\\
\gamma^{(0)}_{II}(j) &= 4\left(\Psi(1) - \Psi(j+1)\right) \equiv -4S_1(j) \equiv \gamma^{(0)}_0(j),\\
\\
f_{III}(j) &= -\frac{j-1}{j+2}f_g(j) + f_q(j) - \frac{j+1}{j}f_\varphi(j) \sim f^-_{q,g,\varphi}(j),\\
\gamma^{(0)}_{III}(j) &= 4\left(\Psi(1) - \Psi(j+3)\right) \equiv -4S_1(j+2) \equiv \gamma^{(0)}_-(j),\\
\\
f_{IV}(j) &= 2\Delta f_g(j) + \Delta f_q(j) \sim \Delta f^+_{q,g}(j),\\
\gamma^{(0)}_{IV}(j) &= 4\left(\Psi(1) - \Psi(j)\right) \equiv -4S_1(j-1) \equiv \tilde\gamma^{(0)}_+(j),\\
\\
f_V(j) &= -(j-1)\Delta f_1(j) + \frac{j+2}{2}\Delta f_{1/2}(j) \sim \Delta f^-_{q,g}(j),\\
\gamma^{(0)}_{IV}(j) &= 4\left(\Psi(1) - \Psi(j+2)\right) \equiv -4S_1(j+1) \equiv \tilde\gamma^{(0)}_-(j),
\end{aligned}
\qquad (61)
$$

Thus, we have one supermultiplet of operators with the same anomalous dimension $\gamma^{LO}(j)$, proportional to $\Psi(1) - \Psi(j-1)$. The momenta of the corresponding linear combinations of the parton distributions can be obtained from the above momenta by an appropriate shift of their argument j in accordance to this universal anomalous dimension $\gamma^{LO}(j)$. Moreover, the coefficients in these linear combinations for $N = 4$ SUSY can be obtained from the conformal supersymmetry (cf. Ref [17]) and should be the same for all orders of the perturbation theory.

4.3 The NLO anomalous dimensions and twist-two operators with the multiplicative renormalization

We have the following initial information to be able to predict the NLO anomalous dimensions of twist-two operators with the multiplicative renormalization in N=4 SUSY:

1. As it was shown in the previous subsections LO anomalous dimensions are meromorphic functions. Moreover, there is really only one basic anomalous dimension $\gamma^{LO}(j)$ and all other anomalous dimensions can be obtained as $\gamma^{LO}(j \pm m)$, where m is integer. It is useful to choose:

$$\gamma^{LO}(j) = 4\left(\Psi(1) - \Psi(j-1)\right) \equiv -4S_1(j-2), \tag{62}$$

Then, $\gamma^{LO}(j)$ has a pole at $j \to 1$ and vanishes at $j = 2$. It is natural to keep the above universality also for the NLO anomalous dimensions $\gamma_{ab}^{(1)}(j)$ and $\tilde{\gamma}_{ab}^{(1)}(j)$. Adding some other properties we will construct ansatz for the basic NLO anomalous dimension $\gamma^{NLO}(j)$.

2. There are well known results for NLO corrections to the QCD anomalous dimensions.

3. In $\overline{MS}$-scheme (with the coupling constant $\overline{a}$) and also in the $\overline{MS}$-like scheme with the coupling constant $\hat{a}$ (i.e. in the scheme based on DRED procedure), the terms $\sim \zeta(2)$ should be cancelled in the final result for the forward Compton scattering (see [27]-[29]). Therefore the terms $\sim \zeta(2)$ should be cancelled at even j in anomalous dimensions for the structure functions F_2 and F_L (and for the unpolarized parton distributions) and at odd j in anomalous dimensions for structure functions g_1 and F_3 (and for the polarized parton distributions).

4. From the BFKL equation in the framework of DRED scheme (see (34), (29), (30) and (31)) we know, that there is no mixing among the functions of different 'transcendentality level' [4] i, i.e. all special functions at the NLO correction contain the sums of the terms $\sim 1/n^i$ $(i = 3)$. Indeed, if we denote the following functions by the terms in the corresponding sums

$$\Psi \sim 1/n, \quad \Psi' \sim \beta' \sim \zeta(2) \sim 1/n^2, \quad \Psi'' \sim \beta'' \sim \zeta(3) \sim 1/n^3,$$

then for the BFKL equation the LO term and NLO one have the 'levels' $i = 1$ and $i = 3$, respectively.

Because in N=4 SUSY there is a relation between BFKL and DGLAP equations, the similar properties should be valid for anomalous dimensions themselves, i.e. the basic functions $\gamma^{LO}(j)$ and $\gamma^{NLO}(j)$ should be of the types $\sim 1/j^i$ with the levels $i = 1$ and $i = 3$, respectively. The LO basic anomalous dimension is given by Eq. (62). Then, the NLO basic anomalous dimension $\gamma^{NLO}(j)$ can be expressed through the functions:

$$S_i(j-2),\ K_i(j-2), S_{k,l}(j-2), K_{k,l}(j-2), \zeta(k)S_l(j-2),\ \zeta(k)K_i(j-2),\ \zeta(i)$$

[4] Note that similar arguments have been used also in [30] for finding results for some types of complicated massive Feynman diagrams. Their evaluation was based on the direct calculation of several terms in the series over the inversed mass and on the knowledge of the basic structure of the series obtained earlier in [31, 30] by an exact calculation of few special diagrams with the use of the differential equation method [32].

(here $i = 3$ and $j + l = i$), where

$$\begin{aligned}
S_i(j) &= \sum_{m=1}^{j} \frac{1}{m^i} \sim \Psi^{i-1}(j+1), \\
S_{2,1}(j) &= \sum_{m=1}^{j} \frac{1}{m^2} S_1(m), \\
K_2(j) &= \Bigl(1-(-1)^j\Bigr)\frac{1}{2}\,\zeta(2) + (-1)^j \sum_{m=1}^{j} \frac{(-1)^{m+1}}{m^2} \sim \beta'(j+1), \\
K_3(j) &= \Bigl(1-(-1)^j\Bigr)\frac{3}{4}\,\zeta(3) + (-1)^j \sum_{m=1}^{j} \frac{(-1)^{m+1}}{m^3} \sim \beta''(j+1), \\
K_{2,1}(j) &= \Bigl(1-(-1)^j\Bigr)\frac{5}{8}\,\zeta(3) + (-1)^j \sum_{m=1}^{j} \frac{(-1)^{m+1}}{m^2} S_1(m)
\end{aligned} \tag{63}$$

Note that the terms $\sim \zeta(2)$ should be absent in accordance with item **4**.

Moreover, the terms

$$S_l(j-2)/(j \pm m)^k, \qquad K_i(j-2)(j \pm m)^k \qquad (j + l = i) \tag{64}$$

should be absent, too. There are two reasons for this absence.

First one, these terms have additional poles at different values of j: $j = \mp m$. But such additional poles should be absent, if we start with the BFKL equation and obtain $\gamma^{NLO}(j)$ by replacement $n \to -1 - r$ because the BFKL equation does not dublicate the poles at different values of j.

The second reason comes from the consideration of linear combinations (61). If, for example, in the polarized case terms (64) contribute, then we will have the terms $\sim (j \pm m)^{-1}$ in one combination and the terms $\sim (j \pm m \pm 2)^{-1}$ in another combination. However, from the direct calculation of QCD NLO anomalous dimensions in the polarized case (see [33]) we know that only the terms $\sim j^{-1}$ and $\sim (j+1)^{-1}$ can contribute to these combinations.

So, terms (64) should be absent in the results for the universal NLO anomalous dimension in the N=4 SUSY case.

5. Further, the NLO anomalous dimension $\gamma^{NLO}(j)$ is equal to a combination of more complicated contributions (i.e. the contributions containing the functions with the maximal value of i: $i = 3$) for the QCD anomalous dimensions (with the SUSY relation for the QCD color factors $C_F = C_A = N_c$).

Note, that these most complicated contributions (with $i = 3$) are the same for all QCD anomalous dimensions with above SUSY relation (with a possible exception for the NLO scalar-scalar anomalous dimension which is not known yet).

Thus, for $N = 4$ SUSY the NLO universal anomalous dimension $\gamma^{NLO}(j)$ has the form

$$\gamma^{NLO}(j) \;=\; 16\,Q(j-2), \tag{65}$$

where

$$Q(j) \;=\; K_{2,1}(j) + \frac{1}{2}\Big(S_3(j) - K_3(j)\Big) + S_1(j)\Big(S_2(j) - K_2(j)\Big) \tag{66}$$

We would like to note that the sums $K_2(j)$, $K_3(j)$ and $K_{2.1}(j)$ (see Eq.(63)) have been calculated directly only at even values of j (or at odd ones, when $(-1)^j \to (-1)^{j+1}$). The analytical continuation to complex values of j can be done rather easily (see [28] and [34], respectively).

6. We can add the term $\sim \zeta(3)$ to the r.h.s. of (66), but due to the condition $\gamma^{NLO}(j=2) = 0$ it cancels.

So, for N=4 SUSY the universal anomalous dimension $\gamma(j)$ has the form

$$\gamma(j) \;=\; \hat{a}\gamma^{LO}(j) + \hat{a}^2\gamma^{NLO}(j), \tag{67}$$

where $\gamma^{LO}(j)$ and $\gamma^{NLO}(j)$ are given by Eqs. (62) and (65), respectively. All other anomalous dimensions can be obtained as $\gamma^{LO}(j \pm m)$ and $\gamma^{NLO}(j \pm m)$, where m is an integer number.

Thus, the above arguments allow us to construct the NLO corrections to anomalous dimensions in the N=4 SUSY, which were unknown earlier. We plan, however, to check these results by direct calculations [26].

4.4 DGLAP evolution

Using our knowledge of the anomalous dimensions we can construct the solution of the DGLAP equation in the Mellin moment space in the framework of N=4 SUSY.

A. Polarized case

The polarized parton distributions are splitted in the two contributions:

$$\Delta f_{q,g}(j,Q^2) \;=\; \Delta f^{+}_{q,g}(j,Q^2) + \Delta f^{-}_{q,g}(j,Q^2), \tag{68}$$

where

at LO

$$\Delta f^{\pm}_{q,g}(j,Q^2) \;=\; \Delta f^{\pm,LO}_{q,g}(j,Q^2_0)\left(\frac{Q^2}{Q^2_0}\right)^{\tilde{\gamma}^{(0)}_{\pm}a} \qquad \left(\tilde{\gamma}^{(0)}_{\pm} = 4S_1(j\mp 1)\right) \tag{69}$$

at NLO

$$\Delta f^{\pm}_{q,g}(j,Q^2) \;=\; \Delta f^{\pm,NLO}_{q,g}(j,Q^2_0)\left(\frac{Q^2}{Q^2_0}\right)^{\left(\tilde{\gamma}^{(0)}_{\pm}a + \tilde{\gamma}^{(1)}_{\pm\pm}a^2\right)} \qquad \left(\tilde{\gamma}^{(1)}_{\pm\pm} = 16Q(j\mp 1)\right), \tag{70}$$

where

$$\Delta f^{\pm,NLO}_{q,g}(j,Q_0^2) = \Delta f^{\pm,NLO}_{q,g}(j,Q_0^2)\left(1-\frac{\tilde\gamma^{(1)}_{\pm\mp}\ \hat a}{\tilde\gamma^{(0)}_{\pm}-\tilde\gamma^{(0)}_{\mp}}\right) + \frac{\tilde\gamma^{(1)}_{\mp\pm}\ \hat a}{\tilde\gamma^{(0)}_{\mp}-\tilde\gamma^{(0)}_{\pm}}\ \Delta f^{\mp,NLO}_{q,g}(j,Q_0^2) \tag{71}$$

Notice that only the anomalous dimensions $\tilde\gamma^{(1)}_{\pm\pm}$ are important for $N=4$ SUSY at the order $O(\hat a^2)$: they contribute to the Q^2-evolution of parton distributions.

The anomalous dimensions $\tilde\gamma^{(1)}_{\pm\mp}$ give contributions at $O(\hat a^2)$ only to the normalization factors $\Delta f^{\pm,NLO}_{q,g}(j,Q_0^2)$. Their contribution to the Q^2-dependent part of $\Delta f^{\pm}_{q,g}(j,Q^2)$ starts at $O(\hat a^3)$ level in the following form:

$$\hat a^3\ \frac{\tilde\gamma^{(1)}_{\pm\mp}\ \tilde\gamma^{(1)}_{\mp\pm}}{\tilde\gamma^{(0)}_{\pm}-\tilde\gamma^{(0)}_{\mp}} \tag{72}$$

B. Nonpolarized case

The polarized parton distributions are splitted in the three parts:

$$f_{q,g,\varphi}(j,Q^2) = \sum_{i=+,-,0} f^i_{q,g,\varphi}(j,Q^2), \tag{73}$$

where[5]

at LO

$$f^i_{q,g,\varphi}(j,Q^2) = f^{i,LO}_{q,g,\varphi}(j,Q_0^2)\left(\frac{Q^2}{Q_0^2}\right)^{\gamma_i^{(0)}a} \qquad \left(\gamma_i^{(0)}=4S_1(j-i2)\right) \tag{74}$$

at NLO

$$f^i_{q,g,\varphi}(j,Q^2) = f^{i,NLO}_{q,g,\varphi}(j,Q_0^2)\left(\frac{Q^2}{Q_0^2}\right)^{\left(\gamma_i^{(0)}a+\gamma_{ii}^{(1)}a^2\right)} \qquad \left(\gamma_{ii}^{(1)}=16Q(j-i2)\right) \tag{75}$$

As in the previous case **A**, only anomalous dimensions $\gamma^{(1)}_{ii}$ are important at $O(\hat a^2)$ in N=4 SUSY. The anomalous dimensions $\gamma^{(1)}_{ik}$ $(i\neq k)$ contribute at $O(\hat a^2)$ level only to normalization factors $f^{i,NLO}_{q,g,\varphi}(j,Q_0^2)$.

5 Conclusion

Above we reviewed the results for the next-to-leading corrections to the kernel of the BFKL equation and to the anomalous dimensions of twist-2 operators in the extended

[5] The arguments are $j-i2=\{j-2,j,j+2\}$ for respectively $i=\{+,0,-\}$.

$N = 4$ SUSY. The absence of the coupling constant renormalization in this model leads presumable to the Möbius invariance of the BFKL equation in higher orders of the perturbation theory. The cancellation of non-analytic contributions proportional to δ_n^0 and δ_n^2 in $N = 4$ SUSY is remarkable (such terms contribute to ω in the framework of QCD as it has been demonstrated in [9, 35]). This property could be a possible manifestation of the integrability of the reggeon dynamics in the Maldacena model [36] corresponding to the $N = 4$ SUSY in the limit $N_c \to \infty$. Note, that in this model the eigenvalues of the LLA pair kernels in the evolution equations for the matrix elements of the quasi-partonic operators are proportional to $\psi(j-1) - \psi(1)$ [18, 19], which means, that the corresponding Hamiltonian coincides with the local Hamiltonian for an integrable Heisenberg spin model. The residues of these eigenvalues at the points $j = -k$ are obtained from the BFKL equation by an analytic continuation of the anomalous dimensions to negative integer values of the conformal spin $|n|$. Therefore the DGLAP equation is not independent from the BFKL equation in $N = 4$ SUSY and their integrability properties at $N_c \to \infty$ are presumably related.

Acknowledgments

The authors are supported in part by the INTAS 00-366 grant. A. Kotikov thanks the Alexander von Humboldt Foundation for its support at the beginning of this work. L. Lipatov is thankful to the Hamburg University for its hospitality during the period of time when this work was completed. He was supported also by the NATO and CRDF grants.

We are indebted to J. Bartels, V. Fadin, V. Kim, R. Kirschner, R. Peschansky, V. Velizhanin and the participants of the PNPI Winter School for helpful discussions.

References

[1] L.N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 338;
E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Phys. Lett. **B60** (1975) 50; Sov. Phys. JETP **44** (1976) 443; **45** (1977) 199.

[2] Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822; Sov. Phys. JETP Lett. **30** (1979) 355.

[3] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 438, **15** (1972) 675;
L.N. Lipatov, Sov. J. Nucl. Phys. **20** (1975) 94;
G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298;
Yu.L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641.

[4] H1 Collaboration, S. Aid *et al.*, Nucl. Phys. **B470** (1996) 3;
ZEUS Collaboration, M. Derrick *et al.*, Zeit. Phys. **C69** (1996) 607.

[5] G. Gurci, W. Furmanski and R. Petronzio, Nucl. Phys. **B175** (1980) 27;
W. Furmanski and R. Petronzio, Phys. Lett. **B97** (1980) 437.

[6] L.N. Lipatov and V.S. Fadin, Sov. J. Nucl. Phys. **50** (1989) 712.

[7] V.S. Fadin and L.N. Lipatov, Phys. Lett. **B429** (1998) 127.

[8] G. Camici and M. Ciafaloni, Phys. Lett. **B430** (1998) 349.

[9] A.V. Kotikov and L.N. Lipatov, Nucl. Phys. **B582** (2000) 19.

[10] L.N. Lipatov, Sov. Phys. JETP **63** (1986) 904.

[11] L. N. Lipatov, Phys. Lett. **B309** (1993) 394, preprint DFPD/93/TH/70, hep-th/9311037.

[12] J. Bartels, Nucl. Phys. **B175** (1980) 365;
J. Kwiecinski and M. Prascalowich, Phys. Lett. **B94** (1980) 413.

[13] L. N. Lipatov, Sov. Phys. JETP Lett. **59** (1994) 596;
L. D. Faddeev and G. P. Korchemsky, Phys. Lett. **B342** (1995) 311.

[14] L. N. Lipatov, Nucl. Phys. **B548** (1999) 328.

[15] L. N. Lipatov, in: *Proceedings of the International Workshop DIS'99*, Zeuthen, 1999, pp. 207-209;
J. Bartels, G. P. Vacca and L. N. Lipatov, Phys. Lett. **B477** (2000) 178.

[16] L.N. Lipatov, Nucl. Phys. **B452** (1995) 369; Physics Reports **320** (1999) 249.

[17] A. P. Bukhvostov, G. V. Frolov, E. A. Kuraev and L. N. Lipatov, Nucl. Phys. **B258** (1985) 601.

[18] L. N. Lipatov, Perspectives in Hadronic Physics, in: *Proc. of the ICTP conf.* (World Scientific, Singapore, 1997).

[19] L. N. Lipatov, in: *Proc. of the Int. Workshop on very high multiplicity physics*, Dubna, 2000, pp.159-176; Nucl. Phys. Proc. Suppl. **99A** (2001) 175.

[20] L.Lewin, Polylogarithms and Associated Functions (North Holland, Amsterdam, 1981).

[21] A. Devoto and D.W. Duke, Riv. Nuovo Cim. **7** (1984) 1;
N. Nilsen, Nova Acta **90** (1909) 125;
K.S. Kolbig, J.A. Mignaco and E. Remiddi, BIT **10** (1970) 38; Nuovo Cim. **A11** (1972) 824.

[22] A.I. Davydychev and J.B. Tausk, Phys. Rev. **D53** (1996) 7381;
J. Fleischer, M.Yu. Kalmykov and A.V. Kotikov, Phys. Lett. **B462** (1999) 169; in: *6th Int. Workshop on Software Engineering, Artificial Intelligence, Neural Nets, Genetic Algorithms, Symbolic Algebra, Automatic Calculation (AIHENP 99)*, Heraklion, Crete, Greece, 12-16 April, 1999 (hep-ph/9905379);
A.I. Davydychev, Phys. Rev. **D61** (2000) 087701;
A.I. Davydychev and M.Yu. Kalmykov, Nucl. Phys. **B605** (2001) 266.

[23] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, JETP. Lett. **70** (1999) 155; in: *Proceedings of the PHOTON2001*, Ascona, Switzerland, 2001 (CERN-TH/2001-341, SLAC-PUB-9069, hep-ph/0111390);
V.T. Kim, L.N. Lipatov and G.B. Pivovarov, in: *Proceedings of the VIIIth Blois Workshop at IHEP*, Protvino,Russia, 1999 (IITAP-99-013, hep-ph/9911228); in: *Proceedings of the Symposium on Multiparticle Dynamics (ISMD99)*, Providence, Rhode Island, 1999 (IITAP-99-014, hep-ph/9911242).

[24] L3 Collaboration, M. Acciarri et al., Phys. Lett. **B453** (1999) 94.

[25] B. Andersson, G. Gustavson and J. Samuelson, Nucl. Phys. **B467** (1996) 443;
B. Andersson, G. Gustavson and H. Kharraziha, Phys. Rev. **D57** (1998) 5543;
G. Salam, JHEP **9807** (1998) 019;
M. Ciafaloni, D. Colferai and G.P. Salam, JHEP **9910** (1999) 017; Phys. Rev. **D60** (1999) 114036;
M. Ciafaloni and D. Colferai, Phys. Lett. **B452** (1999) 372;
R.S. Thorne, Phys. Rev. **D60** (1999) 054031;
G. Altarelli, R.D. Ball and S. Forte, Nucl. Phys. **B575** (2000) 313; **B599** (2001) 383.

[26] A.V. Kotikov, L.N. Lipatov and V.N. Velizhanin, work in progress.

[27] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, Nucl. Phys. **B174** (1980) 345.

[28] D.I. Kazakov and A.V. Kotikov, Theor. Math. Phys. **73** (1987) 1264; Nucl. Phys. **B307** (1988) 721; **B345** (1990) 299(E).

[29] A.V. Kotikov, Theor. Math. Phys. **78** (1989) 134; Phys. Lett. **B375** (1996) 240; in: *Proceedings of the XVth Int. Workshop "High Energy Physics and Quantum Field Theory"*, Tver, Russia, 2000 (hep-ph/0102177).

[30] J. Fleischer, A.V. Kotikov and O.L. Veretin, Nucl. Phys. **B547** (1999) 343; Acta Phys. Polon. **B29** (1998) 2611, hep-ph/9808243.

[31] J. Fleischer, A.V. Kotikov and O.L. Veretin, Phys. Lett. **B417** (1998) 163.

[32] A.V. Kotikov, Phys. Lett. **B254** (1991) 158; **B259** (1991) 314; **B267** (1991) 123; in: *Proceedings of the XVth Int. Workshop "High Energy Physics and Quantum Field Theory"*, Tver, Russia, 2000 (hep-ph/0102178).

[33] R. Merting and W.L. van Neerven, Z. Phys. **C70** (1996) 625.

[34] A.V. Kotikov, Phys. At. Nucl. **57** (1994) 133.

[35] V. N. Gribov, L. N. Lipatov and G. V. Frolov, Phys. Lett. **B31** (1970) 34; Sov. J. Nucl. Phys. **12** (1971) 543;
H. Cheng and T. T. Wu, Phys. Rev. **D1** (1970) 2775; *Expanding Protons: Scattering at High Energies* (MIT press, Cambridge, Massachusetts, 1987).

[36] J. Maldacena, Adv. Theor. Phys. **2** (1998) 231, Int. J. Theor. Phys. **38** (1998) 1113.