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Family replicated gauge groups and large mixing angle solar neutrino solution

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Abstract

We present a modification of our previous family replicated gauge group model, which now generates the Large Mixing Angle MSW solution rather than the experimentally disfavoured Small Mixing Angle MSW solution to the solar neutrino oscillation problem. The model is based on each family of quarks and leptons having its own set of gauge fields, each containing a replica of the Standard Model gauge fields plus a $(B-L)$-coupled gauge field. By a careful choice of the Higgs field gauge quantum numbers, we avoid our previous prediction that the solar neutrino mixing angle is equal order of magnitudewise to the Cabibbo angle, replacing it and the well-known Fritzsch relation with the relation $\theta_c \sim (\theta_\odot)^{-1/3}(m_d/m_s)^{2/3}$. At the same time we retain a phenomenologically successful structure for the charged quark and lepton mass matrices. A fit of all the seventeen quark-lepton mass and mixing angle observables, using just six new Higgs field vacuum expectation values, agrees with the experimental data within the theoretically expected uncertainty of about 64%, *i.e.* it fits perfectly order of magnitudewise.

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1 Introduction

The first results on the charge current interactions from the Sudbury Neutrino Observatory (SNO) collaboration [1] have provided an important signal confirming the existence of the solar neutrino anomaly puzzle [2, 3, 4, 5, 6]: SNO detected a flux of non-electron neutrinos, ν_μ and ν_τ, among solar neutrinos after travelling from the core of the Sun to the Earth. Combination of the SNO results with previous measurements from other experiments reveals a confirmation of the standard solar model [7], whose predictions of the total flux of active ^{8}B neutrinos in the Sun agree with the SNO and Super-Kamiokande [6] data. Furthermore, the measurement of the ^{8}B and *hep* solar neutrino fluxes shows no significant energy dependence of the electron neutrino survival probability in the Super-Kamiokande and SNO energy ranges. These results support the Large Mixing Angle MSW [8] solution (LMA-MSW) rather than the Small Mixing Angle MSW solution (SMA-MSW) to the solar neutrino problem.

Another important result on the solar neutrino problem, reported by the Super-Kamiokande collaboration [9], is that the day-night asymmetry data disfavour the SMA-MSW solution at the 95% C.L.. In fact, global analyses [10, 11, 12, 13] of solar neutrino data, including the first SNO results and the day-night effect, have confirmed that the LMA-MSW solution gives the best fit to the data and that the SMA-MSW solution is very strongly disfavoured and only accepted at the 3σ level. The best fit values of the mass squared difference and mixing angle parameters in the two flavour LMA-MSW solution[1] are $\Delta m^2_\odot \approx 4.5 \times 10^{-5}$ eV^2 and $\tan^2\theta_\odot \approx 0.35$.

We have previously attempted to fit all the fermion – quark and lepton – masses and mixing angles including baryogenesis [14, 15, 16] in a rather specific model without supersymmetry or grand unification. The model has the maximum number of gauge fields consistent with maintaining the irreduciblity of the usual Standard Model fermion representations, including three right-handed neutrinos. The predictions of this previous model are in order of magnitude agreement with all existing experimental data; however, only provided we use the SMA-MSW solution. But, for the reasons given above, the SMA-MSW solution is now disfavoured phenomenologically. So, in this article, we present a modified version of the previous model, which manages to accommodate the LMA-MSW solution for solar neutrino oscillations: all the fermion mass and mixing angle parameters are fitted within a factor of two, using 6 adjustable parameters.

This article is organised as follows: in the next section, we define our notation for the charged fermion Yukawa coupling matrices, mass matrices and mixing angles. Then, in section 3 we review the family replicated gauge model. In section 4 we discuss the reasons for the modification of our model and the introduction of new Higgs fields. The calculation is described in section 5 and the results are presented in section 6. Finally, section 7 contains our conclusion.

[1]The best fit parameter values for the LMA solution depend somewhat on the analysis method. However they do not change drastically from one two flavour analysis to another. We discuss the three flavour analyses in section 6.1.

2 Charged fermion masses and their mixing angles

In the Standard Model all fermions (apart from the neutrinos) get a mass via the electroweak spontaneous symmetry breaking – the Higgs mechanism. In extensions of the Standard Model containing right-handed neutrinos, the physical light neutrinos get a mass via the see-saw mechanism (see discussion of the see-saw mechanism in section 3.4). The Higgs mechanism generates charged fermion mass terms from their Yukawa couplings in the Standard Model Lagrangian:

$$-\mathcal{L}_{\text{charged-fermion-mass}} = \overline{Q_L} Y_U \tilde{\Phi}_{WS} U_R + \overline{Q_L} Y_D \Phi_{WS} D_R + \overline{L_L} Y_E \Phi_{WS} E_R + h.c. \quad (1)$$

Here Φ_{WS} is the Weinberg-Salam Higgs field, Q_L denotes the three $SU(2)$ doublets of left-handed quarks, U_R denotes the three singlets of right-handed up-type quarks and Y_U is the three-by-three Yukawa coupling matrix for the up-type quarks. Similarly Y_D and Y_E are the Yukawa coupling matrices for the down-type quarks and charged leptons respectively. The $SU(2)$ doublets Φ_{WS} and Q_L can be represented as 2 component column vectors and we then define:

$$\tilde{\Phi}_{WS} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Phi^{\dagger}_{WS} \quad (2)$$

and

$$\overline{Q_L} = \overline{\begin{pmatrix} U_L \\ D_L \end{pmatrix}} = (\overline{U_L}\ \overline{D_L}) \quad (3)$$

where $\overline{U_L}$ are the CP conjugates of the three left-handed up-type quarks. After electroweak symmetry breaking the Weinberg-Salam Higgs field gets a vacuum expectation value (VEV) and we obtain the following mass terms in the Lagrangian:

$$-\mathcal{L}_{\text{charged-fermion-mass}} = \overline{U_L}\, M_U\, U_R + \overline{D_L}\, M_D\, D_R + \overline{E_L}\, M_E\, E_R + h.c. \quad (4)$$

where the mass matrices are related to the Yukawa coupling matrices and Weinberg-Salam Higgs VEV by:

$$M = Y\, \frac{\langle \phi_{WS} \rangle}{\sqrt{2}} \quad (5)$$

We have chosen the normalisation from the Fermi coupling constant so that:

$$\langle \phi_{WS} \rangle = 246 \text{ GeV} \ . \quad (6)$$

In order to obtain the masses from the mass matrices, M_U, M_D and M_E, we must diagonalise them to find their eigenvalues. In particular we can find unitary matrices, V_U for the up-type quarks, V_D for the down-type quarks and V_E for the charged leptons:

$$V_U^{\dagger}\, M_U\, M_U^{\dagger}\, V_U = \text{diag}\left(m_u^2, m_c^2, m_t^2\right) \quad (7)$$

$$V_D^{\dagger}\, M_D\, M_D^{\dagger}\, V_D = \text{diag}\left(m_d^2, m_s^2, m_b^2\right) \quad (8)$$

$$V_E^{\dagger}\, M_E\, M_E^{\dagger}\, V_E = \text{diag}\left(m_e^2, m_\mu^2, m_\tau^2\right) \quad (9)$$

The quark mixing matrix is then defined with these unitary matrices as [17]:

$$V_{\text{CKM}} = V_U^{\dagger}\, V_D \ . \quad (10)$$

3 Model with many quantum numbers

We have already investigated a model [14, 15] which can predict not only quark and charged lepton quantities – masses and mixing angles – but also neutrino oscillations. This model has, as its back-bone, the property that there are generations (or families) not only for fermions but also for the gauge bosons, *i.e.* we have a generation (family) replicated gauge group namely

$$\overset{\times}{_{i=1,2,3}} (SMG_i \times U(1)_{B-L,i}) \ , \tag{11}$$

where SMG denotes the Standard Model gauge group $\equiv SU(3) \times SU(2) \times U(1)$, $\times$ denotes the Cartesian product and i runs through the generations. For the prediction of the charged particle masses and mixings, the important part of the gauge group is the repetition of the Standard Model gauge group plus one extra $U(1)$ called $U(1)_f$, where f denotes flavour [18, 19, 20]. But for the extension to neutrino masses and mixings using the see-saw picture, it is necessary to introduce a right-handed neutrino and a gauged $B-L$ charge for each generation with the associated abelian gauge groups $U(1)_{B-L,i}$ $(i = 1, 2, 3)$. The just mentioned $U(1)_f$ abelian factor gets absorbed as a linear combination of the $B-L$ charge and the weak hypercharge abelian gauge groups for the different generations. Note that this family replicated gauge group, eq. (11), is the maximal gauge group under the following assumptions:

1) We only consider that part of the gauge group of Nature which acts non-trivially on the known 45 Weyl fermions of the Standard Model and the additional three heavy see-saw (right-handed) neutrinos. That is our gauge group is assumed to be a subgroup of $U(48)$.

2) We avoid any new gauge transformation that would transform a Weyl state from one irreducible representation of the Standard Model group into another irreducible representation: there is no gauge coupling unification.

3) The gauge group does not contain any anomalies in the gauge symmetry – neither gauge nor mixed anomalies. Note that otherwise the model becomes non-renormalisable.

3.1 Gauge quantum numbers for the "proto" fermions at the fundamental scale

In our model at the fundamental scale, which we take to be the Planck scale, there exist many bosons and fermions with practically all quantum numbers we can ask for. But most of the fermions have vector couplings, in the sense that they are described as Dirac particles from the Weyl point of view: they are combinations of left-handed and right-handed states with the same (gauge) quantum numbers. The left-over Weyl particles (in other word those without chiral partners) in our model are specified in more detail and

are actually assumed to form a system of three proto-generations, each consisting of the 16 Weyl particles of a usual Standard Model generation plus one see-saw particle. In this way we can label these particle as proto-left-handed or proto-right-handed u-quark, d-quark, electron *etc.* To get the quantum numbers under our model gauge group for a given proto-irreducible representation, we proceed in the following way: We note the generation number of the particle for which we want quantum numbers and we look up, in the Standard Model, what are the quantum numbers of the irreducible representation in question and what is the $B-L$ quantum number. For instance, if we want to find the quantum numbers of the proto-right-handed strange quark, we note that the quantum numbers of the right-handed strange quark in the Standard Model are weak hypercharge $y/2 = -1/3$, singlet under $SU(2)$ and triplet under $SU(3)$, while $B-L$ is equal to the baryon number $= 1/3$. Moreover, ignoring mixing angles, the generation is denoted as number $i = 2$. The latter fact means that all the quantum numbers for SMG_i $i = 1, 3$ are trivial. Also the baryon number minus lepton number for the proto-generation number one and three are zero: only the quantum numbers associated with proto-generation two are non-trivial. Thus, in our model, the quantum numbers of the proto-right-handed strange quark are $y_2/2 = -1/3$, singlet under $SU(2)_2$, triplet under $SU(3)_2$ and $(B-L)_2 = 1/3$. For each proto-generation the following charge quantisation rule applies

$$\frac{t_i}{3} + \frac{d_i}{2} + \frac{y_i}{2} = 0 \pmod 1 \ , \tag{12}$$

where t_i and d_i are the triality and duality for the i'th proto-generation gauge groups $SU(3)_i$ and $SU(2)_i$ respectively.

Combining eq. (12) with the principle of taking the smallest possible representation of the groups $SU(3)_i$ and $SU(2)_i$, it is sufficient to specify the six Abelian quantum numbers $y_i/2$ and $(B-L)_i$ in order to completely specify the gauge quantum numbers of the fields, *i.e.* of the Higgs fields and fermion fields. Using this rule we easily specify the fermion representations as in Table 1 (the representations of the Higgs fields will be given in subsection 5.1, where we present the fermion mass matrices).

Note that each proto-generation gauge group $SMG_i \times U(1)_{B-L,i}$ is a subgroup of $SO(10)$, *i.e.* our gauge group eq. (11) is really a subgroup of $SO(10)^3$. That means the i'th proto-generation has its own subgroup of $SO(10)_i$. However, we do not take the gauge fields of these $SO(10)_i$ to exist, except for those corresponding to the subgroups $SMG_i \times U(1)_{B-L,i}$.

3.2 Breaking of the family replicated gauge group to the Standard Model

The gauge group $\overset{\times}{}_{i=1,2,3}(SMG_i \times U(1)_{B-L,i})$ is at first spontaneously broken down at one or two orders of magnitude below the Planck scale, by 5 different Higgs fields, to the gauge group $SMG \times U(1)_{B-L}$ which is the diagonal subgroup of the original one:

$$\{\ (U,U,U) \mid U \subset SMG \times U(1)_{B-L}\} \subseteq \overset{\times}{}_{i=1,2,3}(SMG_i \times U(1)_{B-L,i})\ . \tag{13}$$

Table 1: All $U(1)$ quantum charges for the proto-fermions in the model.

	SMG_1	SMG_2	SMG_3	$U_{B-L,1}$	$U_{B-L,2}$	$U_{B-L,3}$
u_L, d_L	$\frac{1}{6}$	0	0	$\frac{1}{3}$	0	0
u_R	$\frac{2}{3}$	0	0	$\frac{1}{3}$	0	0
d_R	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	0	0
e_L, ν_{e_L}	$-\frac{1}{2}$	0	0	-1	0	0
e_R	-1	0	0	-1	0	0
ν_{e_R}	0	0	0	-1	0	0
c_L, s_L	0	$\frac{1}{6}$	0	0	$\frac{1}{3}$	0
c_R	0	$\frac{2}{3}$	0	0	$\frac{1}{3}$	0
s_R	0	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	0
μ_L, ν_{μ_L}	0	$-\frac{1}{2}$	0	0	-1	0
μ_R	0	-1	0	0	-1	0
ν_{μ_R}	0	0	0	0	-1	0
t_L, b_L	0	0	$\frac{1}{6}$	0	0	$\frac{1}{3}$
t_R	0	0	$\frac{2}{3}$	0	0	$\frac{1}{3}$
b_R	0	0	$-\frac{1}{3}$	0	0	$\frac{1}{3}$
τ_L, ν_{τ_L}	0	0	$-\frac{1}{2}$	0	0	-1
τ_R	0	0	-1	0	0	-1
ν_{τ_R}	0	0	0	0	0	-1

We have to emphasize here that the gauge groups SMG and $U(1)_{B-L}$ act similarly on all three families, *i.e.* they are not any more family replicated gauge groups but correspond to the usual gauge group of the Standard Model and the usual baryon number minus lepton number. This diagonal subgroup is further broken down by yet two more Higgs fields — the Weinberg-Salam Higgs field Φ_{WS} and another Higgs field ϕ_{B-L} — to $SU(3) \times U(1)_{em}$. The vacuum expectation value (VEV) of the ϕ_{B-L} Higgs field is taken to be about 10^{11} GeV and is designed to break the gauged $B-L$ quantum number. In other words the VEV $\langle \phi_{B-L} \rangle$ gives the see-saw scale.

Let us stress that we have *only one* Weinberg-Salam Higgs field Φ_{WS}, *i.e.* it only has one irreducible representation in our family replicated gauge group. We freely use both Φ_{WS} and its Hermitian conjugate $\tilde{\Phi}_{WS}$, which means that we have no supersymmetry in the model preventing one or the other from giving masses to the quarks and leptons. Some of our predictions would be spoiled by introducing supersymmetry, because we need both a Higgs field and its Hermitian conjugate. With supersymmetry the number of Higgs fields and associated VEVs would have to be doubled; in the special case of the Weinberg-Salam Higgs field, this means introducing the unknown parameter $\tan\beta$.

3.2.1 Characterization of quark and charged lepton mass spectra

An important prediction of our model depends on the strongly non-supersymmetric feature of there being only one Weinberg-Salam Higgs field, but it is independent of the details of the other Higgs fields which break our gauge group down to the Standard Model. This predicted feature is that corresponding diagonal matrix elements in each of the three charged mass matrices, M_U, M_D, M_E and even for the Dirac neutrino mass matrix, M_ν^D, are order of magnitudewise the same [18, 19]. The quantum number differences between the left- and right-handed Weyl fermions, between which a transition is needed to get these diagonal elements in our model, are $y_i/2 = \pm 1/2$ and $SU(2)_i$ representation equal to doublet, the rest being trivial, where i is the proto-generation number of the diagonal element in question. Thus the quantum number violation needed, and therefore the order of magnitude resulting when all couplings are of order unity, will be the same for the diagonal element corresponding to a proto-family i in each of the four left-to-right mass matrices, M_U, M_D, M_E and M_ν^D (the Dirac neutrino mass matrix).

The second and third family physical up-type quarks, t and c, get their masses from two off-diagonal elements in M_U which dominate the diagonal ones in our model [18, 19]. So the above family degeneracy prediction then ends up becoming a prediction for the down-type quarks and leptons, simulating the simple $SU(5)$ GUT prediction ($m_b = m_\tau$, $m_s = m_\mu$, $m_d = m_e$), but *we only get it with respect to order of magnitude.* Thus our model can get the rough $SU(5)$ mass predictions, without having to suffer from the problem of needing, say, an extra $\underline{45}$ Higgs at the Weinberg-Salam Higgs scale and thereby varying the Clebsch-Gordon coefficients so as to cope with, what is honestly speaking, sheer disagreement for the simplest $SU(5)$ GUT. For the first family, in addition to the simulated GUT prediction, there is the degeneracy prediction that, when extrapolated to the Planck scale, $m_u \approx m_e$ order of magnitude-wise. This is an example of a prediction of our model that is sensitive to it $\underline{\text{not}}$ being supersymmetric, because with supersymmety we would have two Weinberg-Salam Higgs fields and, with our philosophy that Higgs VEV's are likely to have their own order of magnitude, it would be difficult ever to get the prediction $m_u \approx m_e$.

Another regularity predicted from our model is the "factorisation" of the quark mixing angles [19, 20]

$$V_{ub} \approx V_{us}\, V_{cb} \ . \tag{14}$$

This result mainly comes about because both V_{ub} and V_{us} contain, as a factor, similar Higgs field VEVs to take care of converting second family quantum numbers into first family ones. Really five of the eleven predictions of our model are made up from these general rules: four from the family degeneracy predictions and one from the above factorisation.

3.3 Introduction of Right-handed Majorana neutrinos

In order to explain the neutrino oscillations, we have introduced three very heavy right-handed neutrinos into our model, which are mass-protected by the Higgs field, ϕ_{B-L}, at

an energy scale of about 10^{11} GeV. We use the gauged $B-L$ charge to mass-protect the right-handed neutrinos; in fact we use the total – diagonal – one because we break $U(1)_{B-L,1} \times U(1)_{B-L,2} \times U(1)_{B-L,3} \supset U(1)_{B-L}$ at a much higher energy scale, say about 10^{18} GeV. Another new Higgs field, χ, was also introduced in our previous see-saw model. This field plays the role of helping the VEV $\langle \phi_{B-L} \rangle$ to give non-zero effective mass terms for the see-saw neutrinos, by providing a transition between the right-handed tau-neutrino and the right-handed mu-neutrino. This transition coupling means that, with the new Higgs field χ, we can obtain a large atmospheric neutrino mixing angle.

However, unavoidably in the previous model, the solar mixing angle is in the region of the small mixing angle MSW (SMA-MSW) solution, *i.e.* the solar mixing angle and the Cabibbo angle are characterised by the same parameter, ξ, of order 1/10. Furthermore the ratio of the solar neutrino mass squared difference to that for the atmospheric neutrino oscillations is given by ξ^4 [14] without technical corrections [21]. On the other hand, with these technical corrections – "factorial factor corrections" – we could manage to make a mass squared difference ratio consistent with data for the SMA-MSW solutions [15]: due to the presence of a Higgs field S, whose VEV is of order one in Planck units, there are many choices of the quantum numbers of the other Higgs fields that only change the number of occurrences of this field S in the fermion mass matrices. Moreover, one also has some freedom in the choice of the quantum numbers of the ϕ_{B-L} field, which spontaneously breaks the gauged $U(1)_{B-L}$ group and thereby gives the see-saw scale (about 10^{11} GeV). In this way, we managed to get the mass squared difference ratio to be of zeroth order in ξ and rather to be given by $S^8/4$, where the S field VEV is close to unity. However, the solar mixing angle could not be essentially changed, it remained of order ξ, and thus *only* fits the SMA-MSW region.

3.4 Neutrino masses and mixing angles

The assumption of the existence of three right-handed Majorana neutrinos at a high scale[2] gives rise to the addition of Majorana mass terms to the Lagrangian:

$$\begin{aligned} -\mathcal{L}_{\text{neutrino-mass}} &= \bar{\nu}_L \, M_\nu^D \, \nu_R + \frac{1}{2}(\overline{\nu_L})^c \, M_L \, \nu_L + \frac{1}{2}(\overline{\nu_R})^c \, M_R \, \nu_R + h.c. \\ &= \frac{1}{2}(\overline{n_L})^c \, M \, n_L + h.c. \end{aligned} \tag{15}$$

where

$$n_L \equiv \begin{pmatrix} \nu_L \\ (\nu_L)^c \end{pmatrix} \quad , \quad M \equiv \begin{pmatrix} M_L & M_\nu^D \\ M_\nu^D & M_R \end{pmatrix} \quad . \tag{16}$$

Here M_ν^D is the left-right transition mass term – Dirac neutrino mass term – and M_L and M_R are the isosinglet Majorana mass terms of left-handed and right-handed neutrinos, respectively.

[2] In the present model the right-handed neutrinos become massive by the action of the ϕ_{B-L} and χ Higgs fields together with another new Higgs field ρ. These massive right-handed neutrinos would all have decayed and be washed out completely by the present epoch in the evolution of the Universe.

Due to mass-protection by the Standard Model gauge symmetry, the left-handed Majorana mass terms, M_L, are negligible in our model with a fundamental scale set by the Planck mass [20]. Then, naturally, the light neutrino mass matrix – effective left-left transition Majorana mass matrix – can be obtained via the see-saw mechanism [22]:

$$M_{\rm eff} \approx M_\nu^D \, M_R^{-1} \, (M_\nu^D)^T \ . \tag{17}$$

In the framework of the three active neutrino model, the flavour eigenstates ν_α ($\alpha = e, \nu, \tau$) are related to the mass eigenstates ν_i ($i = 1, 2, 3$) in the vacuum by a unitary matrix $V_{\rm MNS}$,

$$|\nu_\alpha\rangle = \sum_i (V_{\rm MNS})_{\alpha i} \, |\nu_i\rangle \ . \tag{18}$$

Here $V_{\rm MNS}$ is the three-by-three Maki-Nakagawa-Sakata (MNS) mixing matrix [23] which is parameterised by

$$\begin{aligned} V_{\rm MNS} \ = \ & \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{13}} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta_{13}} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta_{13}} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta_{13}} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta_{13}} & c_{13}c_{23} \end{pmatrix} \\ & \times \begin{pmatrix} e^{i\varphi} & 0 & 0 \\ 0 & e^{i\psi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \ , \end{aligned} \tag{19}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ and δ_{13} is a CP-violating phase. Note that, due to the existence of Majorana neutrinos, we have two additional CP-violating Majorana phases φ, ψ, which are also included in the MNS unitary mixing matrix.

In order to get predictions for the neutrino masses from the effective mass matrix, $M_{\rm eff}$, we have to diagonalise this matrix using a unitary matrix, $V_{\rm eff}$, to find the mass eigenvalues:

$$V_{\rm eff} M_{\rm eff} M_{\rm eff}^\dagger V_{\rm eff}^\dagger = {\rm diag}(m_1^2, m_2^2, m_3^2) \ . \tag{20}$$

With the charged lepton unitary matrix V_E, eq. (9), we can then find the neutrino mixing matrix:

$$V_{\rm MNS} = V_{\rm eff}^\dagger \, V_E \ . \tag{21}$$

Obviously, we should compare these theoretical predictions with experimentally measured quantities, therefore we define:

$$\Delta m^2_\odot \ \equiv \ m_2^2 - m_1^2 \tag{22}$$

$$\Delta m^2_{\rm atm} \ \equiv \ m_3^2 - m_2^2 \tag{23}$$

$$\tan^2\theta_\odot \ \equiv \ \tan^2\theta_{12} \tag{24}$$

$$\tan^2\theta_{\rm atm} \ \equiv \ \tan^2\theta_{23} \ . \tag{25}$$

Note that since we use the philosophy of order of magnitudewise predictions (see section 5) with complex order one coupling constants, our model is capable of making predictions for these three phases, the CP-violating phase δ_{13} and the two Majorana phases; put simply, we assume all these phases are of order $\pi/2$, *i.e.* essentially maximal CP violations.

3.5 No-Go theorem for large mixing angle in previous model

We have traced the reluctance of our previous see-saw models to fit the large mixing angle MSW solution to the following feature of the Dirac neutrino mass matrix, M_ν^D: for every column, *i.e.* for all right-handed neutrinos in our notation, the first row elements – left electron ones – are smaller than the other matrix elements in the same column, by at least a factor of $\xi \approx 1/10$. With this property, we can indeed prove that the solar mixing angle cannot be bigger than of order ξ, if we do not fine-tune the right-handed neutrino sector.

Note that there is the possibility of getting a large solar neutrino mixing angle from the charged lepton sector, if it has big mixing relative to the proto-flavours [24]. Our model, however, has an almost diagonal charged lepton mass matrix. Therefore we unavoidably obtain a small solar mixing angle, unless we re-arrange the Dirac neutrino mass matrix. That means that both the solar and atmospheric mixing angles must come from the Dirac neutrino sector in our present model.

Our no-go theorem states that, provided there is essentially no mixing in the charged lepton sector and that the Dirac neutrino mass matrix, M_ν^D, obeys

$$\left|(M_\nu^D)_{1i}\right| \lesssim \left|(M_\nu^D)_{2i}\right| \xi \quad \text{and} \quad \left|(M_\nu^D)_{1i}\right| \lesssim \left|(M_\nu^D)_{3i}\right| \xi \quad \text{for } i = 1,2,3 \ , \tag{26}$$

the solar mixing angle cannot be larger than of order ξ ($\xi \approx 1/10$ in our previous model).

This no-go theorem is even harder to circumvent if one has an $SO(10)$ gauge group, because the up-type mass matrix is then very strongly related with the Dirac neutrino mass matrix and also the down-type mass matrix is similarly related with the charged lepton mass matrix [25]. Really though it is necessary to exclude higher dimensional $SO(10)$ representations than say $\underline{10}$ for Higgs fields at the Weinberg-Salam Higgs scale, in order to obtain the identity of the mass matrices [26]:

$$M_U = M_\nu^D \ , \quad M_D = M_E \ . \tag{27}$$

However, using this relationship, it is totally impossible to get a large solar mixing angle in the $SO(10)$ model.

4 Discussion of modification

In order to get an LMA-MSW solution to the solar neutrino problem, we have to re-arrange the Dirac neutrino sector [27] as we have already discussed in the previous section. We do this by the introduction of two new Higgs fields ρ and ω, which replace S and ξ.

In our present model, we manage to make all the three elements of the first column (coupling to the first right-handed neutrino – the lightest one in our case) in the Dirac neutrino mass matrix roughly equal in order of magnitude, *i.e.* the $(1,1)$, $(2,1)$ and even $(3,1)$ matrix elements are made the same order of magnitude. The transition from the

second to the first column corresponds to a shift in the generation $B-L$ quantum numbers, since it is given by the difference in the charges of the respective right-handed neutrinos. The new Higgs field, ρ, plays this role; more precisely the third power of ρ carries the quantum numbers required to make a transition form the first to the second column in the Dirac neutrino sector[3].

In our new version of the model, the second and the third column in the Dirac neutrino mass matrix still obey the condition of our "no-go" eq. (26), which is actually very nice since it is needed to have a hope of getting a small CHOOZ angle θ_{13}. If now all the elements in the first column would simply be obtained from the second by multiplication with $(\rho^\dagger)^3$ as the quantum numbers at first suggest, this column would inherit the property of the first row element being small and we would not be able to get an LMA-MSW solution. However, we managed to get a need for the use of the ρ field in the matrix element $(1,2)$ so that it has a factor of ρ^3 in it, and then in the transition to the first column we get rid of the ρ^3 factor rather than getting an extra factor of $(\rho^\dagger)^3$. In this way we succeeded in making the ratio of matrix element $(1,1)$ to $(2,1)$ become bigger by a factor of ρ^6 than the ratio of $(1,2)$ to $(2,2)$. We really want the ratio of matrix element $(1,1)$ to $(2,1)$ to be of order unity, in order to obtain a large solar neutrino mixing angle. This is arranged by introducing the Higgs field ω having a vacuum expectation value of about the same order of magnitude as ρ.

The value of the Cabibbo angle corresponded to the VEV of ξ in the previous model. In the present model, it is given by the product $\omega\rho^\dagger$ whose VEV should thus be of order of $\xi \sim 1/10$. From these considerations we can crudely estimate the VEVs of the new Higgs fields to be: $\omega \sim \rho \sim 1/3$.

5 Method of numerical computation

A very important assumption in our model is that, at the Planck scale, we find a lot of different particles with many imaginable quantum numbers and having coupling constants which, when they are allowed, are complex numbers of order unity [28]. This means that we assume essentially maximal CP violation in all sectors, including the neutrino sector. Since we do not know the exact values of all these couplings we are, in general, only able to make predictions order of magnitudewise. According to this philosophy [28], we evaluate the product of mass-protecting Higgs VEVs required for each mass matrix element and provide it with a random complex number of order one as a factor. In this way, we simulate a long chain of fundamental Yukawa couplings and propagators making the transition corresponding to an effective Yukawa coupling in the Standard Model. In the numerical computation we then calculate the masses and mixing angles time after time, using different sets of random numbers and, in the end, we take the logarithmic

[3]The quantum numbers of this Higgs field can be found in Table 2.

average of the calculated quantities according to the following formula:

$$\langle m \rangle = \exp\left(\sum_{i=1}^{N} \frac{\ln m_i}{N}\right) \quad . \tag{28}$$

Here $\langle m \rangle$ is what we take to be the prediction for one of the masses or mixing angles, m_i is the result of the calculation done with one set of random number combinations and N is the total number of random number combinations used.

In order to find the best possible fit, we define a quantity which we call the goodness of fit (g.o.f.). Since our model can only make predictions order of magnitudewise, this quantity g.o.f. should only depend on the ratios of the fitted masses (mass squared differences in the neutrino case) and mixing angles to the experimentally determined masses and mixing angles:

$$\text{g.o.f.} \equiv \sum \left[\ln\left(\frac{\langle m \rangle}{m_{\text{exp}}}\right)\right]^2 \tag{29}$$

Here $\langle m \rangle$ are the fitted masses and mixing angles defined in eq. (28) and m_{exp} are the corresponding experimental values. The Yukawa coupling matrices are calculated at the fundamental scale, which we take to be the Planck scale. We use the first order renormalisation group equations for the Standard Model to calculate the matrices at lower scales. Running masses are calculated in terms of the Yukawa couplings at 1 GeV (see section 5.2).

5.1 Quantum numbers of the Higgs fields

The model we present in this article has exactly the same gauge group and gauge quantum numbers for the fermions as in earlier versions [14, 15] of our see-saw model. It is only the system of Higgs fields which have different gauge quantum numbers and they are presented in Table 2. The only essential change, even of the Higgs system, is that the fields ω and ρ in the table replace the previous Higgs fields S and ξ and take on different quantum numbers. As can be seen from Table 2, the fields ω and ρ have only non-trivial quantum numbers with respect to the first and second families. This choice of quantum numbers makes it possible to express a fermion mass matrix element involving the first family in terms of the corresponding element involving the second family, by the inclusion of an appropriate product of powers of ρ and ω.

In previous versions of the model, this role of the ρ and ω fields was played by the fields S and ξ with the quantum number combinations (ordered as in Table 2):

$$S: \qquad (\frac{1}{6}, -\frac{1}{6}, 0, -1, -\frac{2}{3}, \frac{2}{3}) \tag{30}$$

$$\xi: \qquad (\frac{1}{6}, -\frac{1}{6}, 0, 0, \frac{1}{3}, -\frac{1}{3}) \tag{31}$$

It is with these quantum numbers that one gets the "no-go" situation for the LMA-MSW solution, since the solar neutrino mixing angle then satisfies $\theta_\odot \sim \xi \sim V_{us}$. It turns out

Table 2: All $U(1)$ quantum charges of the Higgs fields.

	SMG_1	SMG_2	SMG_3	$U_{B-L,1}$	$U_{B-L,2}$	$U_{B-L,3}$
ω	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	0	0
ρ	0	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	0
W	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{3}$
T	0	$-\frac{1}{6}$	$\frac{1}{6}$	0	0	0
χ	0	0	0	0	-1	1
ϕ_{WS}	0	$\frac{2}{3}$	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{1}{3}$
ϕ_{B-L}	0	0	0	0	0	2

that, fitting with these "old" quantum numbers, the vacuum expectation value of the field S is close to being unity in fundamental (Planck) units. Once the Higgs field S had a VEV of order unity, a large number of inessential modifications of the Higgs field quantum numbers became possible: one could add or subtract the quantum numbers of S to/from any of the other proposed Higgs fields a large number of times, without making any changes except in small details. Therefore, in the previous work, it was necessary to consider and make fits using these other possibilities.

The new Higgs fields ω and ρ turn out to have VEVs of the order of 1/3. So, in the present model, there are no fields with a VEV of the order of unity and thus no such ambiguities in the choice of Higgs field quantum numbers. In this way the "new" model escapes the "discrete" parameters of shuffling around the Higgs quantum numbers by multiples of those of S. So one now has a smaller amount of hidden fitting and a good fit should thus be considered a bit more impressive than in the previous model! The new model is in this way simplified compared to the old one.

With the system of quantum numbers in Table 2 one can easily evaluate, for a given mass matrix element, the numbers of Higgs field VEVs of the different types needed to perform the transition between the corresponding left- and right-handed Weyl fields. The results of calculating the products of Higgs fields needed, and thereby the order of magnitudes of the mass matrix elements in our model, are presented in the following mass matrices:

the up-type quarks:

$$M_U \simeq \frac{\left\langle (\phi_{\rm WS})^\dagger \right\rangle}{\sqrt{2}} \begin{pmatrix} (\omega^\dagger)^3 W^\dagger T^2 & \omega\rho^\dagger W^\dagger T^2 & \omega\rho^\dagger (W^\dagger)^2 T \\ (\omega^\dagger)^4 \rho W^\dagger T^2 & W^\dagger T^2 & (W^\dagger)^2 T \\ (\omega^\dagger)^4 \rho & 1 & W^\dagger T^\dagger \end{pmatrix} \tag{32}$$

the down-type quarks:

$$M_D \simeq \frac{\langle \phi_{\rm WS} \rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W (T^\dagger)^2 & \omega\rho^\dagger W (T^\dagger)^2 & \omega\rho^\dagger T^3 \\ \omega^2 \rho W (T^\dagger)^2 & W (T^\dagger)^2 & T^3 \\ \omega^2 \rho W^2 (T^\dagger)^4 & W^2 (T^\dagger)^4 & WT \end{pmatrix} \tag{33}$$

the charged leptons:

$$M_E \simeq \frac{\langle \phi_{\rm WS} \rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W (T^\dagger)^2 & (\omega^\dagger)^3 \rho^3 W (T^\dagger)^2 & (\omega^\dagger)^3 \rho^3 W T^4 \chi \\ \omega^6 (\rho^\dagger)^3 W (T^\dagger)^2 & W (T^\dagger)^2 & W T^4 \chi \\ \omega^6 (\rho^\dagger)^3 (W^\dagger)^2 T^4 & (W^\dagger)^2 T^4 & W T \end{pmatrix} \tag{34}$$

the Dirac neutrinos:

$$M_\nu^D \simeq \frac{\left\langle (\phi_{\rm WS})^\dagger \right\rangle}{\sqrt{2}} \begin{pmatrix} (\omega^\dagger)^3 W^\dagger T^2 & (\omega^\dagger)^3 \rho^3 W^\dagger T^2 & (\omega^\dagger)^3 \rho^3 W^\dagger T^2 \chi \\ (\rho^\dagger)^3 W^\dagger T^2 & W^\dagger T^2 & W^\dagger T^2 \chi \\ (\rho^\dagger)^3 W^\dagger T^\dagger \chi^\dagger & W^\dagger T^\dagger \chi^\dagger & W^\dagger T^\dagger \end{pmatrix} \tag{35}$$

and the Majorana (right-handed) neutrinos:

$$M_R \simeq \langle \phi_{\rm B-L} \rangle \begin{pmatrix} (\rho^\dagger)^6 (\chi^\dagger)^2 & (\rho^\dagger)^3 (\chi^\dagger)^2 & (\rho^\dagger)^3 \chi^\dagger \\ (\rho^\dagger)^3 (\chi^\dagger)^2 & (\chi^\dagger)^2 & \chi^\dagger \\ (\rho^\dagger)^3 \chi^\dagger & \chi^\dagger & 1 \end{pmatrix} \tag{36}$$

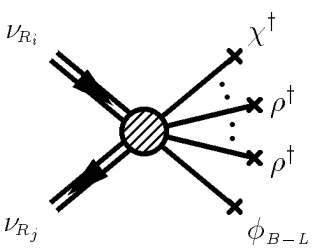
In order to get the true model matrix elements, one must imagine that each matrix element is provided with an order of unity factor, which is unknown within our system of assumptions and which, as described above, is taken in our calculation as a complex random number, later to be logarithmically averaged over as in eq. (28).

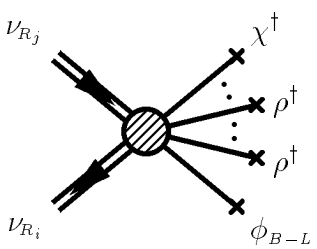
Note that the quantum numbers of our 6 Higgs fields are not totally independent. In fact there is a linear relation between the quantum numbers of the three Higgs fields W, T and χ:

$$\vec{Q}_\chi = 3\,\vec{Q}_W - 9\,\vec{Q}_T \tag{37}$$

where the 6 components of the charge vector $\vec{Q}$ correspond to the 6 columns of Table 2. Thus the Higgs field combinations needed for a given transition are not unique, and the largest contribution has to be selected for each matrix element in the above mass matrices.

Furthermore, there is another remark: the symmetric mass matrix – for the Majorana neutrinos – gives rise to the same off-diagonal term twice. The Feynman diagram for off-diagonal elements of the right-handed neutrino matrix is

$2\ (M_R)_{ij}\ =$ 

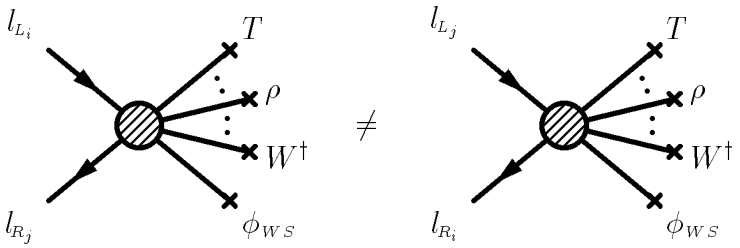
$=$ 

$= 2\ (M_R)_{ji}\ (i \neq j)\ .$ (38)

Thus to avoid overcounting we just have to multiply off-diagonal elements of the right-handed Majorana mass matrix by a factor of 1/2. However, in the Dirac mass matrix

columns and rows are related to completely different Weyl fields and, therefore, we do not need to worry about overcounting – the off-diagonal elements should not be multiplied by an extra factor of 1/2:

$$(M^D)_{ij} \;=\; \neq \;=\; (M^D)_{ji} \;\; (i \neq j) \;\; .$$



(39)

The previous versions of our model predicted the Fritzsch relation [29] $V_{us} = \theta_c \approx \sqrt{m_d/m_s}$ (however only order of magnitudewise), provided that the VEV of the field S was of order unity, $S \approx 1$. With the above mass matrices, this relation is now replaced by a relation involving the solar neutrino mixing angle:

$$V_{us} = \theta_c \sim (\theta_\odot)^{-\frac{1}{3}} \left(\frac{m_d}{m_s}\right)^{\frac{2}{3}} . \tag{40}$$

5.2 Renormalisation group running of coupling constants

It should be kept in mind that the effective Yukawa couplings for the Weinberg-Salam Higgs field, which are given by the Higgs field factors in the above mass matrices multiplied by some order unity factors (taken as random numbers), are the running (effective) Yukawa couplings at a scale *very close to the Planck scale*. Thus, in our calculations, we had to use the renormalisation group β-functions to run these couplings down to the experimentally observable scale, *i.e.* $\mu = 1$ GeV where μ is the renormalisation point. This is because we took the charged fermion masses to be compared to "measurements" at the conventional scale of 1 GeV. In other words, what we take as input quark masses are the current algebra masses, corresponding to running masses at 1 GeV, except for the top quark. We used the top quark pole mass instead:

$$M_t = m_t(M)\left(1 + \frac{4}{3}\frac{\alpha_s(M)}{\pi}\right) \;\; , \tag{41}$$

where we set $M = 180$ GeV as an input, for simplicity.

Using the notation in eq. (1), we can define the one-loop β functions for the gauge couplings and the charged fermion Yukawa matrices [30] as follows:

$$\begin{aligned} 16\pi^2 \frac{dg_1}{dt} &= \frac{41}{10}\, g_1^3 \\ 16\pi^2 \frac{dg_2}{dt} &= -\frac{19}{16}\, g_2^3 \end{aligned}$$

$$
\begin{aligned}
16\pi^2\frac{dg_3}{dt} &= -7\,g_3^3 \\
16\pi^2\frac{dY_U}{dt} &= \frac{3}{2}\left(Y_U(Y_U)^\dagger - Y_D(Y_D)^\dagger\right)\,Y_U + \left\{\,Y_S - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)\right\}\,Y_U \qquad (42)\\
16\pi^2\frac{dY_D}{dt} &= \frac{3}{2}\left(Y_D(Y_D)^\dagger - Y_U(Y_U)^\dagger\right)\,Y_D + \left\{\,Y_S - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)\right\}\,Y_D \\
16\pi^2\frac{dY_E}{dt} &= \frac{3}{2}\left(Y_E(Y_E)^\dagger\right)\,Y_E + \left\{\,Y_S - \left(\frac{9}{4}g_1^2 + \frac{9}{4}g_2^2\right)\right\}\,Y_E \\
Y_S &= \mathrm{Tr}(\,3\,Y_U^\dagger\,Y_U + 3\,Y_D^\dagger\,Y_D + Y_E^\dagger\,Y_E\,)\ ,
\end{aligned}
$$

where $t = \ln\mu$.

In order to run the the renormalisation group equations down to 1 GeV, we use the following initial values:

$$U(1): \quad g_1(M_Z) = 0.462\ , \quad g_1(M_{\rm Planck}) = 0.614 \qquad (43)$$

$$SU(2): \quad g_2(M_Z) = 0.651\ , \quad g_2(M_{\rm Planck}) = 0.504 \qquad (44)$$

$$SU(3): \quad g_3(M_Z) = 1.22\ , \quad g_3(M_{\rm Planck}) = 0.491 \qquad (45)$$

5.3 The renormalisation group equations for the effective neutrino mass matrix

The effective light neutrino masses are given by an irrelevant, non-renormalisable dimension 5 term [31, 32]:

$$\Delta L_{\rm eff} = \frac{1}{2}\,C^{ij}\,(\epsilon_{ab}H_a l_b^i)\,(\epsilon_{cd}H_c l_d^j)\ , \qquad (46)$$

where l_a^i are left-handed Weyl lepton fields with the flavour index i and $SU(2)$ weak isospin index a, and C^{ij} is a symmetric matrix of coefficients:

$$C^{ij}(\mu) = Y_\nu^{ki}(M_R^{-1})^{kl}Y_\nu^{lj}\ . \qquad (47)$$

Here M_R is the Majorana mass matrix of the right-handed Majorana neutrinos, and Y_ν is the Dirac neutrino Yukawa coupling matrix.

The renormalisation group equations for the symmetric matrix, C^{ij}, are given by

$$16\pi^2\frac{dC^{ij}}{dt} = (-3g_2^2 + 2\lambda + 2Y_S)\,C^{ij} - \frac{3}{2}\left(C^{ik}(Y_E^\dagger)^{kl}(Y_E)^{lj} + (Y_E^\dagger)^{lk}(Y_E)^{ki}\,C^{lj}\right)\ , \qquad (48)$$

where λ is the Weinberg-Salam Higgs self-coupling constant and

$$Y_S = \mathrm{Tr}(\,3\,Y_U^\dagger\,Y_U + 3\,Y_D^\dagger\,Y_D + Y_E^\dagger\,Y_E\,)\ . \qquad (49)$$

The mass of the Standard Model Higgs boson is given by $M_H^2 = \lambda\,\langle\phi_{WS}\rangle^2$ and, for definiteness, we take $M_H = 115$ GeV thereby fixing the value of the Higgs self-coupling

$\lambda = 0.2185$. These evolution equations can be rewritten using the elements of the light neutrino effective mass matrix $M_{\rm eff}$ as running quantities:

$$16\pi^2 \frac{dM_{\rm eff}}{dt} = \left(-3g_2^2 + 2\lambda + 2Y_{\scriptscriptstyle S}\right) M_{\rm eff} - \frac{3}{2}\left(M_{\rm eff}\left(Y_{\scriptscriptstyle E}Y_{\scriptscriptstyle E}^\dagger\right)^T + \left(Y_{\scriptscriptstyle E}Y_{\scriptscriptstyle E}^\dagger\right) M_{\rm eff}\right) \ . \qquad (50)$$

Note that the renormalisation group equations are used to evolve the effective neutrino mass matrix from the see-saw sale, set by $\langle\phi_{B-L}\rangle$ in our model, to 1 GeV. We should emphasize that we have used the approximation of ignoring the running of the Dirac neutrino Yukawa coupling constants between the Planck scale and the see-saw scale; however, this effect is small and so this approximation should be good enough for our order of magnitude calculations.

6 Numerical results

Using the three charged quark-lepton mass matrices and the effective neutrino mass matrix together with the renormalisation group equations, eqs. (42) and (50), we made a fit to all the fermion quantities in Table 3 varying just 6 Higgs fields VEVs. We averaged over $N = 10,000$ complex order unity random number combinations (see eq. 28). These complex numbers are chosen to be the exponential of a number picked from a Gaussian distribution, with mean value zero and standard deviation one, multiplied by a random phase factor. We varied the 6 free parameters and found the best fit, corresponding to the lowest value for the quantity g.o.f. defined in eq. (29), with the following values for the VEVs:

$$\begin{aligned} &\langle\phi_{W\,S}\rangle = 246 \text{ GeV} \ , \ \langle\phi_{B-L}\rangle = 1.64 \times 10^{11} \text{ GeV} \ , \ \langle\omega\rangle = 0.233 \ , \\ &\langle\rho\rangle = 0.246 \ , \ \langle W\rangle = 0.134 \ , \ \langle T\rangle = 0.0758 \ , \ \langle\chi\rangle = 0.0737 \ , \end{aligned} \qquad (51)$$

where, except for the Weinberg-Salam Higgs field and $\langle\phi_{B-L}\rangle$, the VEVs are expressed in Planck units. Hereby we have considered that the Weinberg-Salam Higgs field VEV is already fixed by the Fermi constant. The results of the best fit, with the VEVs in eq. (51), are shown in Table 3 and the fit has g.o.f. = 3.63.

We have $11 = 17 - 6$ degrees of freedom – predictions – leaving each of them with a logarithmic error of $\sqrt{3.63/11} \simeq 0.57$, which is very close to the theoretically expected value 0.64 [21]. This means, in other words, that we can fit all quantities within a factor $1.78 \simeq \exp\left(\sqrt{3.63/11}\right)$ of the experimental value. However, we do not count the 42 complex order unity random numbers entering the mass matrices eqs. (32)-(36) as parameters when discussing predictions, since we *do not adjust them*. We only use them as a calculational technique to avoid "unnatural" matrices which would be degenerate in our calculations, if we did not have random number coefficients.

Unlike in older versions of the model, the first and second family sub-matrix of $M_{\scriptscriptstyle D}$ is now dominantly diagonal. In previous versions of the model this submatrix satisfied the order of magnitude factorisation condition $(M_{\scriptscriptstyle D})_{12} \cdot (M_{\scriptscriptstyle D})_{21} \approx (M_{\scriptscriptstyle D})_{11} \cdot (M_{\scriptscriptstyle D})_{22}$; thus the

Table 3: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass. Note that we use the square roots of the neutrino data in this Table, as the fitted neutrino mass and mixing parameters $\langle m \rangle$, in our goodness of fit (g.o.f.) definition, eq. (29).

	Fitted	Experimental
m_u	4.4 MeV	4 MeV
m_d	4.3 MeV	9 MeV
m_e	1.0 MeV	0.5 MeV
m_c	0.63 GeV	1.4 GeV
m_s	340 MeV	200 MeV
m_μ	80 MeV	105 MeV
M_t	208 GeV	180 GeV
m_b	7.2 GeV	6.3 GeV
m_τ	1.1 GeV	1.78 GeV
V_{us}	0.093	0.22
V_{cb}	0.027	0.041
V_{ub}	0.0025	0.0035
$\Delta m^2_\odot$	9.5×10^{-5} eV^2	4.5×10^{-5} eV^2
$\Delta m^2_{\rm atm}$	2.6×10^{-3} eV^2	3.0×10^{-3} eV^2
$\tan^2\theta_\odot$	0.23	0.35
$\tan^2\theta_{\rm atm}$	0.65	1.0
$\tan^2\theta_{13}$	4.8×10^{-2}	$\lesssim 2.6 \times 10^{-2}$
g.o.f.	3.63	–

down quark mass m_d received two contributions (off-diagonal as well as diagonal) of the same order of magnitude as the up quark mass m_u. This extra off-diagonal contribution to m_d of course improved the goodness of the fit to the masses of the first family, since phenomenologically $m_d \approx 2\ m_u$. However, in the present version of the model with the ω and ρ Higgs fields, the off-diagonal element $(M_D)_{21}$ becomes smaller and we are left with a full order of magnitude degeneracy of the first family masses, even including the down quark. Furthermore, our expectation from section 4 that $\langle\omega\rangle \sim \langle\rho\rangle \sim 1/3$ tends to overestimate the first family masses. So the result of our fit, eq. (51), is to take $\langle W\rangle$ somewhat smaller than in our previous models and $\langle\omega\rangle \sim \langle\rho\rangle < 1/3$. Consequently our best fit values for the charm quark mass m_c and the Cabibbo angle V_{us} are smaller than in our previous fits to the charged fermion masses [19, 20, 21]. Nonetheless, as mentioned above, our present best fit agrees with the experimental data within the theoretically expected uncertainty of about 64% and is, therefore, as good as can be expected from an order of magnitude fit.

Experimental results on the values of neutrino mixing angles are often presented in terms of the function $\sin^2 2\theta$ rather than $\tan^2\theta$ (which, contrary to $\sin^2 2\theta$, does not have

a maximum at $\theta = \pi/4$ and thus still varies in this region). Transforming from $\tan^2\theta$ variables to $\sin^2 2\theta$ variables, our predictions for the neutrino mixing angles become:

$$\sin^2 2\theta_{\odot} = 0.61 \ , \tag{52}$$
$$\sin^2 2\theta_{\rm atm} = 0.96 \ , \tag{53}$$
$$\sin^2 2\theta_{13} = 0.17 \ . \tag{54}$$

We also give here our predicted hierarchical neutrino mass spectrum:

$$m_1 = 4.9 \times 10^{-4} \ \text{eV} \ , \tag{55}$$
$$m_2 = 9.7 \times 10^{-3} \ \text{eV} \ , \tag{56}$$
$$m_3 = 5.2 \times 10^{-2} \ \text{eV} \ . \tag{57}$$

Compared to the experimental data these predictions are excellent: all of our order of magnitude neutrino predictions lie inside the 99% C.L. border determined from phenomenological fits to the neutrino data, even including the CHOOZ upper bound. On the other hand, our prediction of the solar mass squared difference is about a factor of 2 larger than the global fit data even though the prediction is inside of the LMA-MSW region, giving a contribution to our goodness of fit of g.o.f. ≈ 0.14. Our CHOOZ angle also turns out to be about a factor of 2 larger than the experimental limit at 90% C.L., corresponding to another contribution of g.o.f. ≈ 0.14. In summary our predictions for the neutrino sector agree extremely well with the data, giving a contribution of only 0.34 to g.o.f. while the charged fermion sector contributes 3.29 to g.o.f..

6.1 CHOOZ angle and three flavour analysis

The combination of the results from atmospheric neutrino experiments [33] and the CHOOZ reactor experiment [34] constrains the first- and third-generation mixing angle to be small, *i.e.* the 3σ upper bound is given by $\tan^2\theta_{13} \lesssim 0.06$. This limit was obtained from a three flavour neutrino analysis (in the five dimensional parameter space – $\theta_{\odot}$, θ_{13}, $\theta_{\rm atm}$, $\Delta m^2_{\odot}$ and $\Delta m^2_{\rm atm}$), using all the solar and atmospheric neutrino data and based on the assumption that neutrino masses have a hierarchical structure, *i.e.* $\Delta m^2_{\odot} \ll \Delta m^2_{\rm atm}$ [35].

However, the solar neutrino data in Table 3 come from a global two flavour analysis, which means that the first- and third-generation mixing angle is essentially put equal to zero, *i.e.* the dependence of θ_{13} on the solar neutrino parameters have been ignored. In principle we should, of course, fit to neutrino parameters from a three flavour analysis. Recently, global three flavour analyses were performed [36, 37, 38] and they showed a significant influence of the non-zero CHOOZ angle on the solar neutrino mass squared difference and mixing angle[4] and vice versa: if the CHOOZ angle becomes far from zero

[4] In [38] the relatively large solar neutrino mass squared difference lying in the LMA-MSW region (with the condition $\Delta m^2_{\odot} \gtrsim 10^{-4}$ eV^2), the solar mixing angle and the CHOOZ reactor experiment data were analysed using the three flavour analysis method.

then the solar mixing angle becomes smaller. This effect is more significant for the larger $\Delta m^2_{\odot}$ values. Because of this correlation, our fit to the neutrino data is even somewhat better than that suggested by the g.o.f. value; even including the CHOOZ angle our neutrino fit is extremely good.

6.2 CP violation

We have fitted all the fermion masses and their mixing angles and therefore have predictions for the CKM and MNS mixing matrices, in the quark sector and in the lepton sector respectively, including CP violating phases of order unity. In this subsection, we will first consider the size of CP violation in the quark sector and then the electron "effective Majorana mass" responsible for neutrinoless double beta decay.

The Jarlskog invariant J_{CP} provides a measure of the amount of CP violation in the quark sector [39] and, in the approximation of setting cosines of mixing angles to unity, is just twice the area of the unitarity triangle:

$$J_{CP} = V_{us}\, V_{cb}\, V_{ub}\, \sin\delta \ , \tag{58}$$

where δ is the CP violation phase in the CKM matrix. In our model the quark mass matrix elements have random phases, so we expect δ (and also the three angles α, β and γ of the unitarity triangle) to be of order unity and, taking an average value of $|\sin\delta| \approx 1/2$, the area of the unitarity triangle becomes

$$J_{CP} \approx \frac{1}{2}\, V_{us}\, V_{cb}\, V_{ub} \ . \tag{59}$$

Using the best fit values for the CKM elements from Table 3, we predict $J_{CP} \approx 3.1 \times 10^{-6}$ to be compared with the experimental value $(2-3.5)\times 10^{-5}$. Since our result for the Jarlskog invariant is the product of four quantities, we do not expect the usual $\pm 64\%$ logarithmic uncertainty but rather $\pm\sqrt{4}\cdot 64\% = 128\%$ logarithmic uncertainty. This means our result deviates from the experimental value by $\log \frac{2.7\times 10^{-5}}{3.1\times 10^{-6}}/1.28 = 1.7$ "standard deviations".

Another prediction, which can also be made from this model, is the electron "effective Majorana mass" – the parameter in neutrinoless beta decay – defined by:

$$|\langle m\rangle| \equiv \left| \sum_{i=1}^{3} U_{ei}^2\, m_i \right| \ , \tag{60}$$

where m_i are the masses of the neutrinos ν_i and U_{ei} are the MNS mixing matrix elements for the electron flavour to the mass eigenstates i. We can substitute values for the neutrino masses m_i from eqs. (55-57) and for the fitted neutrino mixing angles from Table 3 into the left hand side of eq. (60). As already mentioned, the CP violating phases in the MNS mixing matrix are essentially random in our model. So we combine the three terms in eq. (60) by taking the square root of the sum of the modulus squared of each term, which gives our prediction:

$$|\langle m\rangle| \approx 3.1 \times 10^{-3} \ \text{ eV} \ . \tag{61}$$

Although the Jarlskog invariant and the effective Majorana electron neutrino mass have been calculated from the best fit parameters in Table 3, it is also possible to calculate them directly while making the fit. So we have calculated J_{CP} and $|\langle m\rangle|$ for $N = 10,000$ complex order unity random number combinations. Then we took the logarithmic average of these $10,000$ samples of J_{CP} and $|\langle m\rangle|$ and obtained the following results:

$$J_{CP} = 3.1 \times 10^{-6} \ , \tag{62}$$

$$|\langle m\rangle| = 4.4 \times 10^{-3} \ \text{eV} \ . \tag{63}$$

in good agreement with the values given above.

We should mention here that our effective Majorana mass parameter eq. (63), of course, respects the upper limit presented in ref. [40].

7 Conclusion

We have developed an older version of our model, with the purpose of making it fit the experimentally favored LMA-MSW solution rather than the SMA-MSW solution for solar neutrino oscillations. In the older version, the magnitudes of the solar mixing angle $\theta_\odot$ and the Cabibbo angle θ_c are both characterised by the VEV of the Higgs field $\xi \sim 1/10$ and thus the previous model could only be made compatible with the SMA-MSW solution. The required modification of the model was achieved by replacing the fields S and ξ in the previous model by another pair of Higgs fields: ω and ρ having non-trivial and opposite quantum numbers with respect to the family one and family two gauge groups, while having trivial family three gauge quantum numbers. In this way an excellent fit to the LMA-MSW solution is obtained. The price paid for the greatly improved neutrino mass matrix fit – the neutrino parameters now contribute only very little to the g.o.f. – is a slight deterioration in the fit to the charged fermion mass matrices. In particular the predicted values of the quark masses m_d and m_c and the Cabibbo angle V_{us} are reduced compared to our previous fits. However the overall fit agrees with the seventeen measured quark-lepton mass and mixing angle parameters in Table 3 within the theoretically expected uncertainty [21] of about 64%; it is a perfect fit order of magnitudewise.

It should be remarked that our model provides an order of magnitude fit/understanding of all the effective Yukawa couplings of the Standard Model and the neutrino oscillation parameters in terms of only 6 parameters – the Higgs field vacuum expectation values. So we can say that we fit all the parameters of the Standard Model and neutrino oscillations, except for the gauge coupling constants and the Higgs mass and its self-coupling. Actually we should note here that even the gauge coupling constants may be derived from order one quantities in the following sense: We postulate that the gauge couplings – and here, perhaps a bit arbitrarily at first, let us say that we take these to be g_1, g_2 and g_3 – are of order unity at the Planck scale. That means that the corresponding inverse α's (the inverse fine structure constants) are of order $4\pi = 12.5$. Now, according to our model, the Standard Model gauge groups which we see experimentally are the diagonal subgroups of

a cross product of three replicas of the same mathematical group; one cross product factor for each family. The formula for calculating the inverse α for the diagonal subgroup, α_{DS}, is:

$$\alpha_{DS}^{-1} = \sum_{i=1}^{3} {\alpha_i}^{-1} \ . \tag{64}$$

Thus the order unity assumption leads in our model to a Standard Model inverse α at the Planck scale being of the order of $3 \cdot 12.5 = 37.5$. This agrees very well with the experimentally measured Standard Model couplings, when extrapolated to the Planck scale using the renormalisation group equations of section 5.2, being respectively: $\alpha_1^{-1} = 55.5$, $\alpha_2^{-1} = 49$, $\alpha_3^{-1} = 54$. This observation can be further elaborated and converted into an exact prediction [41], by using the so-called Multiple Point Principle (MPP). The phase transition couplings used in this MPP can be taken as the definition of couplings being of order unity and, with this choice, it turns out that it is indeed the g_1, g_2 and g_3 that are really of order unity rather than, say, the α_1, α_2 and α_3.

In a similar way the application [42] of the MPP, requiring degenerate minima in the Weinberg-Salam Higgs effective potential, can lead to a Higgs field self-coupling which is of a similar order to the squares of the g_i's, *i.e.* $\lambda \approx g_i^2$.

Thus, including this sort of argument, our model gives an order of magnitude understanding of all the Standard Model gauge, Higgs and Yukawa couplings, and even also of the beyond the Standard Model neutrino oscillation parameters.

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