Revised Version

DESY-02-010
BN-TH-2002-01

BN–TH–2002–01
DESY–02–010
hep-ph/0201247

arXiv:hep-ph/0201247 v2 7 Feb 2002

Softly broken supersymmetric Yang-Mills theories: Renormalization and non-renormalization theorems

E. Kraus[a] and D. Stöckinger[b] *

[a] *Physikalisches Institut, Universität Bonn, Nußallee 12, D–53115 Bonn, Germany*

[b] *Deutsches Elektronen-Synchrotron DESY, D–22603 Hamburg, Germany*

Abstract

We present a minimal version for the renormalization of softly broken Super-Yang-Mills theories using the extended model with a local gauge coupling. It is shown that the non-renormalization theorems of the case with unbroken supersymmetry are valid without modifications and that the renormalization of the soft-breaking parameters is completely governed by the renormalization of the supersymmetric parameters. The symmetry identities in the present context are peculiar, since the extended model contains two anomalies: the Adler-Bardeen anomaly of the axial current and an anomaly of supersymmetry in the presence of the local gauge coupling. From the anomalous symmetries we derive the exact all-order expressions for the β functions of the gauge coupling and of the soft-breaking parameters. They generalize earlier results to arbitrary normalization conditions and imply the NSVZ expressions for a specific normalization condition on the coupling.

*E-mail addresses:
kraus@th.physik.uni-bonn.de,
dominik@mail.desy.de.

1 Introduction

Renormalization of softly broken supersymmetric gauge theories is an important issue for a better understanding of supersymmetry and its possible realization in nature. Proofs of the all-order renormalizability of these theories in the Wess-Zumino gauge have been proposed in refs. [1] and [2]. However, renormalization of softly broken supersymmetric theories remains unsatisfactory in both approaches. There are far-reaching relations and non-renormalization theorems that govern the renormalization of the supersymmetric and the soft-breaking parameters, but these relations are not prescribed by the symmetries used in [1, 2] and thus escape the algebraic renormalization procedure.

The existence of relations between the renormalization of softly broken and supersymmetric theories has been suggested by [4] and has been further elaborated in refs. [5, 6, 7] using superfield techniques in superspace. Most of the corresponding calculations have been performed assuming a supersymmetric and gauge-invariant regularization scheme. But that these symmetries by themselves do not lead to the special renormalization properties is seen from the algebraic renormalization of soft supersymmetry-breakings in the spurion formalism [2], where the symmetries are accounted for but restrictions on parameters are not present. Therefore, in these old approaches it was difficult to identify those aspects of the symmetries that are the underlying reasons for the special renormalization properties.

The renormalization properties of the soft-breaking parameters are close to the non-renormalization theorems in models with unbroken supersymmetry [8, 9]. These non-renormalization theorems have been first derived in superspace, but not as a direct consequence of supersymmetry. Recently it has been shown in a series of papers [10, 11, 12, 13] that the non-renormalization theorems are in fact consequences of the special structure of supersymmetric Lagrangians. Every supersymmetric part of the Lagrangian is the highest component of a supermultiplet and thus can be expressed as the supersymmetry variation of a lower-dimensional field polynomial. This aspect is the origin of the improved renormalization properties in supersymmetric theories.

The way to make use of the multiplet structure of supersymmetric Lagrangians is to couple the multiplets to external fields. These external fields are chiral or vector supermultiplets according to the structure of the respective Lagrangian multiplets, and they have an immediate interpretation (see also [14]): The external field coupled to the Super-Yang-Mills part corresponds to an extension of the coupling to an external superfield and the external multiplet coupled to the matter part is the axial vector multiplet, whose vector component couples to the axial $U(1)$ current. Hence, the derivation of non-renormalization theorems is traced back to renormalization of supersymmetric field theories in presence of local couplings and by taking into account axial symmetry.

It is obvious that this construction intrinsically includes soft breakings of the Girardello-Grisaru class [3], since these breakings are just the lowest components of the Lagrangian multiplets and are automatically present in the extended construction of supersymmetric theories with local coupling. The purpose of the present paper is to work out the consequences of the extended construction for softly broken supersymmetry. It will indeed turn out that the divergency structure is unchanged by passing from the unbroken to the softly broken case, and we will find explicit results for the specific renormalization properties of softly broken theories.

Renormalization of supersymmetric Yang-Mills theories in presence of the local coupling has to be performed with care: We have shown that not only the axial current is anomalous [15], but that also supersymmetry has an anomaly [13, 16]. Both anomalies can be consistently absorbed into redefined symmetry identities, the Adler-Bardeen anomaly by an anomalous axial transformation of the space-time dependent Θ angle of QCD, the supersymmetry anomaly by an anomalous supersymmetry transformation of the local gauge coupling. The resulting symmetry identities govern the renormalization of Super-Yang-Mills theories with soft breakings to all orders and imply the non-renormalization theorems as well as the relations between soft-breaking and supersymmetric parameters.

The most important implications of the anomalous symmetries are their effects on the renormalization group functions. The supersymmetry anomaly induces the two-loop term in the β function of pure Super-Yang-Mills theories, whereas the Adler-Bardeen anomaly induces the matter part contributions to the gauge-β function. Finally, the complete all-order construction implies a closed expression for the gauge-β function [11, 13], which takes with particular normalization conditions the NSVZ form [17]. Similarly, we prove here that the anomaly coefficients show up in the β functions of the gaugino mass [18, 19] and in the β function of the scalar mass [20, 21, 22]. These results also clarify the reason of the well-known failure of superfield techniques for the derivation of the scalar-mass β function.

The analysis of the present paper is similar to the one we have carried out in SQED [12], but it differs in details due to the appearance of the supersymmetry anomaly [13]. Hence, we give all results in a very condensed form and refer for technical details to [12]. In section 2 we present the classical supersymmetric and gauge invariant action with local coupling and soft supersymmetry-breaking. As a direct consequence, the non-renormalization theorems and the relations for the soft-breaking parameters are formulated on the level of the invariant counterterms in section 3. In section 4 we renormalize the model by constructing the anomalous Slavnov-Taylor identity that governs the renormalization of softly broken Super-Yang-Mills theories in the Wess-Zumino gauge. In section 5 we derive the closed expressions of soft β functions from the symmetry identities. Generalizations to more complicated models and discussions of the results can be found in the

conclusions. In the appendix we summarize the BRS transformations of the fields in presence of local couplings. Notations and conventions are the ones given in appendix A of [11].

2 The extended model

The basic observation behind the non-renormalization theorems is that every supersymmetric part in a supersymmetric Lagrangian is the highest component of a supermultiplet. This fact can be made visible in an extended model of Super-Yang-Mills (SYM) theories [13], where three kinds of external supermultiplets are introduced that couple to the different parts of the Lagrangian. The most important of these is the local gauge coupling $g(x)$ with its supermultiplet $G(x,\theta,\overline{\theta})$:

$$G(x,\theta,\overline{\theta}) = (\boldsymbol{\eta}(x,\theta,\overline{\theta}) + \overline{\boldsymbol{\eta}}(x,\theta,\overline{\theta}))^{-\frac{1}{2}} \equiv g(x) + \mathcal{O}(\theta,\overline{\theta}) \ . \tag{1}$$

Due to the chiral structure of the SYM action the multiplet of the local coupling is decomposed into a dimensionless chiral and antichiral field multiplet, $\boldsymbol{\eta}$ and $\overline{\boldsymbol{\eta}}$, with the following component expansion in chiral and antichiral representation, respectively:

$$\boldsymbol{\eta}(x,\theta) = \eta + \theta^\alpha \chi_\alpha + \theta^2 f \ , \qquad \overline{\boldsymbol{\eta}}(x,\overline{\theta}) = \overline{\eta} + \theta_{\dot\alpha}\overline{\chi}^{\dot\alpha} + \overline{\theta}^2 \overline{f} \ . \tag{2}$$

As shown in refs. [11, 13] the extension to local coupling requires at the same time to introduce the multiplet of the axial vector field,

$$\phi^V = \theta\sigma^\mu\overline{\theta}V^\mu - i\overline{\theta}^2\theta^\alpha\tilde{\lambda}_\alpha + i\theta^2\overline{\theta}_{\dot\alpha}\overline{\tilde{\lambda}}^{\dot\alpha} + \frac{1}{4}\theta^2\overline{\theta}^2\tilde{D} \ , \tag{3}$$

whose vector component V^μ couples to the anomalous axial $U(1)$ current, and a chiral and an antichiral multiplet, $\mathbf{q}$ and $\overline{\mathbf{q}}$, which couple to the matter mass term and govern the soft breaking of axial symmetry:

$$\mathbf{q} = q + \theta q^\alpha + \theta^2 q_F \quad \text{and} \quad \overline{\mathbf{q}} = \overline{q} + \overline{\theta}_{\dot\alpha}\overline{q}^{\dot\alpha} + \overline{\theta}^2\overline{q}_F. \tag{4}$$

The classical action of SYM with local coupling is decomposed into the supersymmetric Yang-Mills part, the matter part and the matter mass term and q-field interaction,

$$\Gamma_{\text{susy}} = \Gamma_{\text{YM}} + \Gamma_{\text{matter}} + (\Gamma_{\text{mass}} + \Gamma_q) \ . \tag{5}$$

The individual contributions are invariant under non-abelian gauge transformations, supersymmetry and softly broken $U(1)$ axial symmetry. For simplification

we restrict the construction to non-abelian gauge theories with a Susy-QCD-like structure. We assume a simple gauge group with generators τ_a and consider left- and right-handed matter fields, each in an irreducible representation generated by T_a and $(-T_a{}^*)$, respectively.

The contributions to $\Gamma_{\rm susy}$ can be efficiently written by using the following superfield expressions for the gauge multiplet ϕ^A and for the left- and right-handed matter fields A_L, A_R and $\bar{A}_L, \bar{A}_R$:

$$\phi^A = \theta\sigma^\mu\overline{\theta}A^\mu - i\overline{\theta}^2\theta^\alpha\lambda_\alpha + i\theta^2\overline{\theta}_{\dot\alpha}\overline{\lambda}^{\dot\alpha} + \frac{1}{4}\theta^2\overline{\theta}^2 D \ , \qquad \phi^A = \phi^A_a\tau_a \ , \tag{6}$$

$$A_X = \varphi_X + \psi_X\theta + F_X\theta^2 \ , \quad \bar{A}_X = \overline{\varphi}_X + \overline{\psi}_X\overline{\theta} + \bar{F}_X\overline{\theta}^2 \ , \quad X = L, R, \tag{7}$$

with the normalization $\mathrm{Tr}(\tau_a\tau_b) = \delta_{ab}$ for the generators. Expressed in terms of superspace integrals, the invariant classical action in the Wess-Zumino gauge reads

$$\Gamma_{\rm YM} = -\frac{1}{4}\int dS\ \boldsymbol{\eta}\mathcal{L}_{\rm YM} - \frac{1}{4}\int d\bar{S}\ \overline{\boldsymbol{\eta}}\bar{\mathcal{L}}_{\rm YM} \ , \tag{8}$$

$$\Gamma_{\rm matter} = \frac{1}{16}\int dV\ \mathcal{L}_{\rm matter} \ , \tag{9}$$

$$\Gamma_{\rm mass} + \Gamma_q = -\frac{1}{4}\int dS\ (\mathbf{q}+m)\mathcal{L}_{\rm mass} - \frac{1}{4}\int d\bar{S}\ (\overline{\mathbf{q}}+m)\bar{\mathcal{L}}_{\rm mass} \ . \tag{10}$$

with

$$\mathcal{L}_{\rm YM} = \mathrm{Tr}W^\alpha W_\alpha = W^\alpha_a W_{\alpha a} \ , \quad W_\alpha \equiv \frac{1}{8\sqrt{2}}\bar{D}\bar{D}(e^{-2g(x)\phi^A}D_\alpha e^{2g(x)\phi^A}) \ ; \tag{11}$$

$$\mathcal{L}_{\rm matter} = \bar{A}_{Lk}\left(e^{2g\phi^A_a T_a + 2\phi^V}\right)^k{}_j A^j_L + A_{Rk}\left(e^{-2g\phi^A_a T_a + 2\phi^V}\right)^k{}_j \bar{A}^j_R \ , \tag{12}$$

$$\mathcal{L}_{\rm mass} = A^k_R A_{Lk} \ . \tag{13}$$

Gauge and supersymmetry transformations of the individual fields can be read off from the corresponding BRS transformations of the appendix (see (22)). In the limit to constant supercoupling $G(x,\theta,\overline{\theta}) \to g = \text{const.}$ we obtain the usual classical action of SYM theories with gauged axial symmetry.

The SYM action with local coupling can immediately be extended to a consistent description of softly broken supersymmetry. The external fields have exactly the properties of the spurion fields used in [3] to describe soft breaking. Indeed,

the only change we have to apply to the model of [13] is to shift the highest components of the external multiplets by constant mass parameters,

$$\begin{aligned} & f \to f + \frac{M_\lambda}{g^2} \ , \qquad \bar{f} \to \bar{f} + \frac{M_\lambda}{g^2} \ , \\ & q_F \to q_F - b \ , \qquad \overline{q}_F \to \overline{q}_F - b, \\ & \tilde{D} \to \tilde{D} - 2M^2. \end{aligned} \tag{14}$$

Through these shifts all parity conserving soft-breaking terms of the Girardello-Grisaru class [3] are generated in the classical action, i.e.

$$\Gamma_{\rm susy} \to \Gamma_{\rm susy} + \Gamma_{\rm soft} \tag{15}$$

with

$$\begin{aligned} \Gamma_{\rm soft} = \int d^4x \ \Big(& -\frac{1}{2} M_\lambda (\lambda\lambda + \overline{\lambda\lambda}) \\ & - M^2(\varphi_L \overline{\varphi}_L + \varphi_R \overline{\varphi}_R) - b(\varphi_L \varphi_R + \overline{\varphi}_L \overline{\varphi}_R) \Big) \ . \end{aligned} \tag{16}$$

If the shifts (14) are also included in the supersymmetry transformations and in the axial transformations of the external fields, the softly broken model is characterized by the same symmetries as the original unbroken model.

The crucial point of the present construction of soft supersymmetry-breaking are the highly restrictive symmetry properties of the extended model. They give access not only to the non-renormalization theorems [13] but also to the renormalization properties of the soft-breaking parameters.

3 Improved renormalization properties

The explicit form of the divergences in any quantum field theory generally depends on the regularization scheme. However, after the possible symmetry-breaking effects of the regularization are cancelled by suitable counterterms, the structure of the remaining divergences is scheme-independent. These divergences are the superficial divergences and correspond order by order to local field monomials that are invariant with respect to the classical symmetries. Hence, they can be cancelled by invariant counterterms and an investigation of invariant counterterms is equivalent to an investigation of the structure of the divergences.

The properties of the extended model put severe constraints on the invariant counterterms of physical parameters. These terms are characterized by being

invariant under the same symmetry transformations as $\Gamma_{\rm susy}$ and can be derived already at the present stage. The results will exhibit all the special renormalization properties of softly broken SYM theories. We will continue the discussion of invariant counterterms in sec. 4.3. There we will find further invariant counterterms, which correspond to field renormalizations and include a redefinition of the classical symmetry transformations.

In order to obtain full control over the renormalization, it is necessary to use two further constraints. On the one hand, $(\eta + \overline{\eta})^{-1/2}$ is identified with the local gauge coupling (1). Since for constant gauge coupling the loop expansion is a power series in the coupling and an l-loop counterterm is of the order g^{2l}, this identification fixes the powers of $\boldsymbol{\eta}$ in the counterterms (see (27) later for an explicit formula). On the other hand, $(\eta - \overline{\eta})$ takes the role of a space-time dependent Θ angle of the model. Thus, the classical action depends on it only via a total derivative, and this property can be expressed by the following identity:

$$\int d^4x \, \Big(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\Big)\Gamma_{\rm susy} = 0 \ . \tag{17}$$

The same identity has to hold for the invariant counterterms.

Using furthermore an additional R-invariance of the classical action (for the explicit form see (28), (29)) the results for the gauge invariant and supersymmetric field monomials of l-loop order are given by

$$\begin{aligned}
\Gamma^{(l)}_{\rm ct,phys} = & -\hat{z}^{(l)}_{\rm YM}\frac{1}{4}\int dS \, \Big(\boldsymbol{\eta} + \theta^2\frac{M_\lambda}{g^2}\Big)^{-l+1}\mathcal{L}_{\rm YM} + \text{c.c.} \\
& -\hat{z}^{(l)}_{\rm mass}\frac{1}{4}\int dS \, \Big(\boldsymbol{\eta} + \theta^2\frac{M_\lambda}{g^2}\Big)^{-l}(\mathbf{q} + m - \theta^2 b)\mathcal{L}_{\rm mass} + \text{c.c.} \\
& +\frac{1}{16}\int dV \, \mathcal{K}^{(l)}\Big(\boldsymbol{\eta} + \theta^2\frac{M_\lambda}{g^2}, \overline{\boldsymbol{\eta}} + \overline{\theta}^2\frac{M_\lambda}{g^2}\Big)\big(1 - \theta^2\overline{\theta}^2 M^2\big)\mathcal{L}_{\rm matter},
\end{aligned} \tag{18}$$

with the restriction $\mathcal{K}^{(l)}(\boldsymbol{\eta}, \overline{\boldsymbol{\eta}}) \to \hat{z}^{(l)}_{\rm matter} g^{2l}$.

Applying the identity (17) on $\Gamma_{\rm ct,phys}$ we find the following constraints that have to be satisfied by the invariant counterterms:

$$\begin{aligned}
& \hat{z}^{(l)}_{\rm YM} = 0 \quad \text{for} \quad l \geq 2 \ , \qquad \hat{z}^{(l)}_{\rm mass} = 0 \quad \text{for} \quad l \geq 1 \ , \\
& \mathcal{K}^{(l)}(\boldsymbol{\eta}, \overline{\boldsymbol{\eta}}) = \hat{z}^{(l)}_{\rm matter}(\boldsymbol{\eta} + \overline{\boldsymbol{\eta}})^{-l} \ .
\end{aligned} \tag{19}$$

Hence, the counterterm to the Yang-Mills part is restricted to one-loop order and invariant counterterms to the supersymmetric mass term are absent. These

restrictions on physical counterterms state the non-renormalization theorems of chiral vertices [8, 9] and the generalized non-renormalization theorem of the coupling constant [23]. In addition, the renormalization constants of softly broken parameters are related to the renormalization constants of supersymmetric parameters.

For constant coupling it is possible to express the invariant counterterms as field and parameter renormalizations. For the z-factors of the supersymmetric parameters, the coupling and the fermion mass parameter, one obtains in loop order l:

$$\begin{aligned} z_{g^2}^{(1)} &= -g^2 z_{\mathrm{YM}}^{(1)} \quad \text{and} \quad z_{g^2}^{(l)} = 0 \quad \text{for} \quad l \geq 2 \ , \\ z_m^{(l)} &= -2g^{2l} z_{\mathrm{matter}}^{(l)} \ . \end{aligned} \tag{20}$$

The z-factors of the soft mass parameters are determined as functions of z_{g^2} and z_m:

$$\begin{aligned} z_{M_\lambda}^{(1)} &= z_{g^2}^{(1)} \quad \text{and} \quad z_{M_\lambda}^{(l)} = 0 \quad \text{for} \quad l \geq 2, \\ z_b^{(l)} &= \Big(2l\frac{M_\lambda m}{b} + 1\Big) z_m^{(l)} \ , \quad z_M^{(l)} = \frac{1}{2} l(l+1) \frac{M_\lambda^2}{M^2} z_m^{(l)} \ . \end{aligned} \tag{21}$$

The renormalization constants to the soft-breaking scalar masses match the relations derived first by [4] and later on by [5] in superspace. The renormalization constants to the gaugino mass are related to the ones of the gauge coupling and are further restricted by the non-renormalization theorem of the gauge coupling in a similar way as proposed in ref. [18], where holomorphicity was imposed as an additional constraint on chiral invariant counterterms. In the present approach the constraint is a consequence of the identity (17).

The results for the counterterms reflect the improved renormalization-behaviour and the special divergency structure of SYM theories. However, unlike in many other models, β functions cannot be inferred from the z-factors (21) and (20). As we will see later, in the course of renormalization the symmetries are broken by two anomalies: Axial symmetry is broken by the Adler-Bardeen anomaly [15] and supersymmetry by a supersymmetry anomaly [13]. For this reason an invariant regularization scheme for the extended model cannot exist, and a discussion of the β functions based on invariant counterterms cannot be performed. A derivation of the β functions requires an algebraic construction based on the anomalous symmetries of the model.

4 Quantization

4.1 The classical action and its symmetries

For quantizing supersymmetric gauge theories in the Wess-Zumino gauge the symmetry transformations of the model, gauge transformations, $U(1)$ axial transformations, supersymmetry and translations are summed up in the BRS transformations [24, 25, 26, 11, 13]:

$$\mathbf{s}\phi = (\delta^{\text{gauge}}_{c(x)} + \delta^{\text{axial}}_{\tilde{c}(x)} + \epsilon^\alpha \delta_\alpha + \bar{\delta}_{\dot{\alpha}} \overline{\epsilon}^{\dot{\alpha}} - i\omega^\mu \partial_\mu)\phi \ . \tag{22}$$

The ghost fields $c(x), \tilde{c}(x)$ replace the local transformation parameters of gauge transformations and axial transformations, and the constant ghosts ϵ^α, $\overline{\epsilon}^{\dot{\alpha}}$ and ω^μ are the constant supersymmetry and translational ghosts, respectively. BRS transformations of the ghosts are determined by the structure constants of the algebra and the algebra of symmetry transformations is expressed in the on-shell nilpotency of the BRS operator. The BRS transformations of the fields (with auxiliary fields being eliminated) are collected in appendix A. They differ from the BRS transformations in the symmetric model by shifts in the scalar components of external fields according to (14).

By means of BRS transformations it is possible to add a BRS invariant gauge-fixing and ghost part to the action

$$\Gamma_{\text{g.f.}} = \mathbf{s}\text{Tr} \int d^4x \, \left(\frac{1}{2}\xi \bar{c} B + \bar{c}\mathcal{F}\right) = \text{Tr} \int d^4x \, \left(\frac{1}{2}\xi B^2 + B\mathcal{F}\right) + \Gamma_{\text{ghost}} \ . \tag{23}$$

The fields B are the Lagrange multiplier fields, and the function $\mathcal{F}$ describes an appropriate linear gauge fixing function for the longitudinal part of the gauge fields: $\mathcal{F} = \partial A + \ldots$.

Finally, the BRS transformations $\mathbf{s}\phi$ that are non-linear in the propagating fields are coupled to external fields Y_ϕ, and the external field part $\Gamma_{\text{ext.f.}} = \int Y_\phi \mathbf{s}\phi$ is added to the classical action, so that

$$\Gamma_{\text{cl}} = \Gamma_{\text{susy}} + \Gamma_{\text{soft}} + \Gamma_{\text{g.f.}} + \Gamma_{\text{ext.f.}} \ . \tag{24}$$

From this complete classical action the auxiliary fields are eliminated using their equations of motion. In this procedure, bilinear expressions in external fields are induced that compensate the equations-of-motion terms in the supersymmetry algebra.

The BRS invariance of Γ_{cl} can be rewritten in the form of the Slavnov-Taylor identity

$$\mathcal{S}(\Gamma_{\text{cl}}) = 0 \ , \tag{25}$$

with the usual bilinear Slavnov-Taylor operator (see eq. (A.7)).

The dependence of $\Gamma_{\rm cl}$ on the field multiplets $\boldsymbol{\eta}$ and $\overline{\boldsymbol{\eta}}$ is constrained by the identity (17)

$$\int d^4x \, \Big(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\Big)\Gamma_{\rm cl} = 0 \ , \tag{26}$$

and by identifying the real part of the lowest components $\eta, \overline{\eta}$ with the local gauge coupling (1). Direct inspection of the diagrams shows that this identification leads to the following topological formula for one-particle irreducible (1PI) diagrams in loop order l:

$$N_{g(x)} = N_{\rm amp.legs} + N_Y + 2N_f + 2N_\chi + 2N_{\eta-\overline{\eta}} + 2(l-1) \ , \tag{27}$$

Here $N_{\rm amp.legs}$ counts the number of external amputated legs with propagating fields ($A^\mu, \lambda, \varphi_A, \psi_A, c, \overline{c}$ and the respective complex conjugate fields), N_Y gives the number of BRS insertions, counted by the number of differentiations with respect to the external fields Y_ϕ. N_f, N_χ and $N_{\eta-\overline{\eta}}$ give the number of insertions corresponding to the respective external fields. The validity of the topological formula in higher orders ensures that the limit to constant coupling results in the 1PI Green functions of ordinary SYM theories with soft breaking.

Furthermore, the classical action is restricted by a softly broken R-symmetry, which is defined in the same way as in SQED [12]. We impose the corresponding Ward identity not only for $\Gamma_{\rm cl}$ but also for the full vertex functional Γ:

$$\mathcal{W}^R\Gamma = 0 \tag{28}$$

with

$$\begin{aligned}\mathcal{W}^R = i\int d^4x \, \Bigg(& \sum_{A=L,R}\Big(\varphi_A\frac{\delta}{\delta\varphi_A} - Y_{\varphi_A}\frac{\delta}{\delta Y_{\varphi_A}}\Big) + \lambda^\alpha\frac{\delta}{\delta\lambda^\alpha} - Y_\lambda^\alpha\frac{\delta}{\delta Y_\lambda^\alpha} \\ & + \tilde{\lambda}^\alpha\frac{\delta}{\delta\tilde{\lambda}^\alpha} - q^\alpha\frac{\delta}{\delta q^\alpha} - \chi^\alpha\frac{\delta}{\delta\chi^\alpha} \\ & - 2(q_F - b)\frac{\delta}{\delta q_F} - 2\Big(f + \frac{M_\lambda}{g^2}\Big)\frac{\delta}{\delta f} - \text{c.c.}\Bigg) \\ & + i\Big(\epsilon^\alpha\frac{\partial}{\partial\epsilon^\alpha} - \overline{\epsilon}^{\dot\alpha}\frac{\partial}{\partial\overline{\epsilon}^{\dot\alpha}}\Big) \ .\end{aligned} \tag{29}$$

It implies invariance under R-parity, where all superpartner-fields are transformed to their negative.

4.2 Renormalization and anomalies

In one-loop order, axial symmetry is broken by the Adler-Bardeen anomaly [15, 27]. And when we impose the identity (26) to all orders[2], i.e.,

$$\int d^4x \, \Big(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\Big)\Gamma = 0 \ , \tag{30}$$

then supersymmetry is broken by a supersymmetry anomaly [13]. Therefore the Slavnov-Taylor identity is anomalous in one-loop order:

$$\mathcal{S}(\Gamma) = r^{(1)}\Delta^{\text{axial}} + r_\eta^{(1)}\Delta^{\text{susy}} + \mathcal{O}(\hbar^2) \ . \tag{31}$$

The anomalous field monomials are given by the following expressions:

$$\Delta^{\text{axial}} = \text{Tr}\int d^4x \ \tilde{c}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}(gA)G_{\rho\sigma}(gA) + \mathcal{O}(\epsilon,\overline{\epsilon}) \tag{32}$$

$$\begin{aligned}
\Delta^{\text{susy}} &= \mathbf{s}\int d^4x \ \ln g(x)(L_{\text{YM}} + \bar{L}_{\text{YM}}) \\
&= (\epsilon^\alpha\delta_\alpha + \overline{\epsilon}^{\dot\alpha}\delta_{\dot\alpha})\int d^4x \ \ln g(x)(L_{\text{YM}} + \bar{L}_{\text{YM}}) \\
&= \int d^4x \ \Big(\ln g(x) i\big(\partial_\mu\Lambda^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{\epsilon}^{\dot\alpha} - \epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\Lambda^{\dot\alpha}\big) \\
&\qquad - \frac{1}{2}g^2(x)(\epsilon\chi + \overline{\chi\epsilon})(L_{\text{YM}} + \bar{L}_{\text{YM}})\Big) \ .
\end{aligned} \tag{33}$$

Here L_{YM} and Λ^α are the F- and the spinor-component of the chiral multiplet $\mathcal{L}_{\text{YM}}$ (11), respectively:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2}\lambda_a\lambda_a + \theta^\alpha\Lambda_\alpha + \theta^2 L_{\text{YM}} \ . \tag{34}$$

The supersymmetry anomaly is different from the Adler-Bardeen anomaly in the following respect: If gauge invariance is imposed, Δ^{axial} cannot be given as a BRS variation of a renormalizable field monomial. In contrast, the supersymmetry anomaly is a BRS variation, but it cannot be absorbed into the counterterm action, since it depends on the logarithm of the coupling. The perturbative loop expansion is a power series in the coupling and, thus, the coefficient of the anomaly is determined by regularization scheme- and gauge-independent one-loop integrals [16].

[2] The supersymmetry anomaly could be shifted to the identity (30) but implies there the unwanted feature of the renormalization of the Θ angle of QCD ($2\eta \equiv \frac{1}{g^2} + i\Theta$) (see [13]).

For Super-Yang-Mills theories the anomaly coefficients $r^{(1)}$ and $r_\eta^{(1)}$ are determined by

$$r^{(1)} = -\frac{1}{16\pi^2}T(R) \ , \qquad r_\eta^{(1)} = \frac{1}{8\pi^2}C(G) \ , \tag{35}$$

where $T(R)\mathbf{1} = T_a T_a$ is the Dynkin index of the matter representation and $C(G)\delta_{ab} = f_{acd}f_{bcd}$ is the quadratic Casimir of the adjoint representation.

For the construction of higher orders it is crucial that both anomalies can be absorbed into a redefined Slavnov-Taylor identity, which defines the 1PI Green functions of the extended model to all orders of perturbation theory:

$$\mathcal{S}^{r_\eta}(\Gamma) + r^{(1)}\delta\mathcal{S}\Gamma = 0 \ . \tag{36}$$

Here $\delta\mathcal{S}$ is the operator of the Adler-Bardeen anomaly:

$$\begin{aligned}\delta\mathcal{S}\Gamma \equiv -4i\int d^4x \ \Big(\tilde{c}\Big(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\Big) + 2i(\epsilon\sigma^\mu)^{\dot\alpha}V_\mu\frac{\delta}{\delta\overline{\chi}^{\dot\alpha}} - 2i(\sigma^\mu\overline{\epsilon})^\alpha V_\mu\frac{\delta}{\delta\chi^\alpha} \\ + 2\overline{\epsilon}_{\dot\alpha}\overline{\tilde{\lambda}}^{\dot\alpha}\frac{\delta}{\delta f} - 2\tilde{\lambda}^\alpha\epsilon_\alpha\frac{\delta}{\delta\overline{f}}\Big)\Gamma = -\Delta^{\text{axial}} + \mathcal{O}(\hbar) \ ,\end{aligned} \tag{37}$$

and in $\mathcal{S}^{r_\eta}(\Gamma)$ the classical Slavnov-Taylor operator (A.7) is supplemented by a differential operator that expresses the supersymmetry anomaly

$$\begin{aligned}\mathcal{S}^{r_\eta}(\Gamma) &\equiv \mathcal{S}(\Gamma) - \int d^4x \ \Big(\delta F(g^2)(\epsilon\chi + \overline{\chi\epsilon})\Big(g^4\frac{\delta}{\delta g^2} + M_\lambda\frac{\delta}{\delta f} + M_\lambda\frac{\delta}{\delta\overline{f}}\Big) \\ &\qquad -i\frac{\delta F}{1+\delta F}\partial_\mu g^2\Big((\sigma^\mu\overline{\epsilon})^\alpha\frac{\delta}{\delta\chi^\alpha} + (\epsilon\sigma^\mu)^{\dot\alpha}\frac{\delta}{\delta\overline{\chi}^{\dot\alpha}}\Big)\Big) \ , \\ &= \mathcal{S}(\Gamma) - r_\eta^{(1)}\Delta^{\text{susy}} + \mathcal{O}(\hbar^2).\end{aligned} \tag{38}$$

The function δF is a power series in g^2. Its lowest order is renormalization-scheme independent and unambiguously determined by the anomaly coefficient (35):

$$\delta F(g^2) = r_\eta^{(1)}g^2 + \mathcal{O}(g^4) \ . \tag{39}$$

Higher orders are scheme-dependent and depend on the normalization conditions for the coupling, or, vice versa, once a specific form for $\delta F(g^2)$ is chosen, the normalization of the coupling is determined. We want to mention already here that the choice

$$\delta F = \frac{r_\eta^{(1)}g^2}{1 - r_\eta^{(1)}g^2} \ , \tag{40}$$

yields the NSVZ expression for the gauge β function [17], whereas

$$\delta F = g^2 r_\eta^{(1)} \tag{41}$$

results in a strictly two-loop β function in Super-Yang-Mills without matter (cf. (63)).

The operator $\mathcal{S}^{r_\eta}(\Gamma)$ can be considered as a modification of the supersymmetry transformation of the local gauge coupling ($F(g^2) = 1 + \delta F(g^2)$):

$$\begin{aligned}
\mathbf{s}^{r_\eta} g^2 &= -g^4(\epsilon^\alpha \chi_\alpha + \overline{\chi}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha})F(g^2) - i\omega^\nu \partial_\nu g^2 \ , \\
\mathbf{s}^{r_\eta}(\eta - \overline{\eta}) &= (\epsilon^\alpha \chi_\alpha - \overline{\chi}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha}) - i\omega^\nu \partial_\nu (\eta - \overline{\eta}) \ , \\
\mathbf{s}^{r_\eta} \chi^\alpha &= 2\epsilon^\alpha \left(f + \frac{M_\lambda}{g^2} \right) + i(\sigma^\mu \overline{\epsilon})^\alpha \left(\partial_\mu g^{-2} \frac{1}{F(g^2)} + \partial_\mu (\eta - \overline{\eta}) \right) - i\omega^\nu \partial_\nu \chi^\alpha \ , \\
\mathbf{s}^{r_\eta} f &= M_\lambda (\epsilon^\alpha \chi_\alpha + \overline{\chi}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha})F(g^2) + i\partial_\mu \chi \sigma^\mu \overline{\epsilon} - i\omega^\mu \partial_\mu f \ .
\end{aligned} \tag{42}$$

These modifications are in agreement with the supersymmetry algebra, and the anomalous Slavnov-Taylor operator (36) and its linearized version have the same nilpotency properties as the classical one:

$$\begin{aligned}
&\left(\mathbf{s}_\Gamma^{r_\eta} + r^{(1)}\delta\mathcal{S}\right)\left(\mathbf{s}_\Gamma^{r_\eta} + r^{(1)}\delta\mathcal{S}\right) = 0 \quad \text{if} \quad \mathcal{S}^{r_\eta}(\Gamma) + r^{(1)}\delta\mathcal{S}\Gamma = 0 \ , \\
&\left(\mathbf{s}_\Gamma^{r_\eta} + r^{(1)}\delta\mathcal{S}\right)\left(\mathcal{S}^{r_\eta} + r^{(1)}\delta\mathcal{S}\right)(\Gamma) = 0 \quad \text{for any functional } \Gamma.
\end{aligned} \tag{43}$$

The anomalous Slavnov-Taylor identity (36) and the identity (30) are the defining symmetry identities for higher-order Green functions. Due to their nilpotency properties (43), algebraic renormalization can be performed to all orders as usually. In contrast, a construction based on the usage of invariant regularization schemes will fail in higher orders. In particular, it is apparent that a local renormalizable action solving the anomalous Slavnov-Taylor identity cannot exist.

From the renormalized Green functions of the extended model one obtains the Green functions of softly broken SYM theory by taking the limit of constant coupling. Denoting the vertex functional of the theory with constant coupling by Γ^{sSYM}, we have

$$\lim_{G\to g} \Gamma\Big|_{\phi^V,\, \mathbf{q},\, \overline{\mathbf{q}}=0} = \Gamma^{\mathrm{sSYM}} \ . \tag{44}$$

4.3 Invariant counterterms

As opposed to the spurion field formalism used in [2], the present construction yields all restrictions on the divergency structure of softly broken SYM theories. Invariant counterterms of the quantized model are invariant with respect to the classical Slavnov-Taylor operator, the Ward operator $\mathcal{W}^R$ and satisfy the identity (30):

$$\mathbf{s}_{\Gamma_{\rm cl}}\Gamma_{\rm ct,inv} = 0 \ , \quad \mathcal{W}^R\Gamma_{\rm ct,inv} = 0 \ , \quad \int d^4x \ \Big(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\Big)\Gamma_{\rm ct,inv} = 0 \ . \tag{45}$$

Furthermore, owing to the identification of $\eta + \overline{\eta}$ with the gauge coupling the formula (27) holds. For the physical, gauge-independent counterterms, these restrictions are nothing but invariance with respect to the classical symmetries. They result therefore in the counterterm action (18) with the constraints (19) and imply the non-renormalization theorems and the relations between the renormalization constants of soft-breaking parameters and supersymmetry parameters stated already after eq. (19).

However, owing to the non-linearity of the Slavnov-Taylor operator the constraints (45) give rise in addition to field redefinitions for the individual propagating fields of the theory:

$$\begin{aligned} A &\to z_A^{(l)} g^{2l} A \ , \qquad \lambda \to z_\lambda^{(l)} g^{2l} \lambda \ , \qquad c \to z_c^{(l)} g^{2l} c \ , \\ \varphi_L &\to z_\varphi^{(l)} g^{2l} \varphi_L \ , \qquad \varphi_R \to z_\varphi^{(l)} g^{2l} \varphi_R \ , \\ \psi_L &\to z_\psi^{(l)} g^{2l} \psi_L \ , \qquad \psi_R \to z_\psi^{(l)} g^{2l} \psi_R \ . \end{aligned} \tag{46}$$

These field redefinitions are supplemented by external field redefinitions to $\mathbf{s}_\Gamma$-invariant expressions (see [16] for explicit expressions).

For the softly broken model also the following generalized field redefinitions are relevant:

$$\begin{aligned} &\lambda_\alpha \to \lambda_\alpha + z_{\lambda A}^{(l)} i g^{2(l+1)} \sigma^\mu_{\alpha\dot\alpha} \overline{\chi}^{\dot\alpha} A_\mu \\ &\psi_{X,\alpha} \to \psi_{X,\alpha} + z_{\psi\varphi}^{(l)} g^{2(l+1)} \chi_\alpha \varphi_A \ , \quad X = L, R \ . \end{aligned} \tag{47}$$

The generalized field redefinitions vanish in the limit of constant coupling, but when they are extended to $\mathbf{s}_\Gamma$-invariant expressions one obtains two non-vanishing contributions depending on the gaugino mass:

$$\begin{aligned} &z_{\lambda A}^{(l)} M_\lambda g^{2l} \int d^4x \ \left(i Y_\lambda \sigma^\mu \overline{\epsilon} A_\mu + \text{c.c.} \right) \ , \\ &z_{\psi\varphi}^{(l)} g^{2l} M_\lambda \int d^4x \ \left(\epsilon^\alpha Y_{\psi_L \alpha} \varphi_L + \epsilon^\alpha Y_{\psi_R \alpha} \varphi_R + \text{c.c.} \right) \ . \end{aligned} \tag{48}$$

Their appearance indicates the appearance of corresponding additional gauge-dependent divergences in the external field part. However, as they stem from the generalized field redefinitions, their renormalization is irrelevant for Green functions of physical fields (see also [2]).

The invariant counterterms are the only contributions to Γ that are not determined by the Slavnov-Taylor identity (36) and that have to be fixed by appropriate normalization conditions. In order to fix the counterterms to the physical parameters $z_{g^2}^{(1)}$ and $z_m^{(l)}$, a normalization condition for the coupling in one-loop order and a normalization condition for the fermion mass have to be used.

The normalization of the coupling is fixed via the symmetries for $l > 1$, and the normalization of the soft-breaking parameters is fixed for all l . However, the specific form of the Slavnov-Taylor identity depends on the choice of the function $\delta F(g^2)$ and of the shifts (14). Instead of choosing $\delta F(g^2)$ and the shifts, it is possible to require independent normalization conditions also for the coupling in higher orders and for the soft parameters. In this case, the coupling normalization determines the terms of order higher than g^4 in the function $\delta F(g^2)$ (39), and normalization conditions for soft susy-breaking terms determine higher-order corrections to the classical shifts:

$$
\begin{aligned}
f(x) &\to f(x) + \frac{1}{g^2}\Big(M_\lambda + \sum_{l=1}^{\infty} v_\lambda^{(l)} g^{2l}\Big) \;, \\
q_F &\to q_F(x) - \Big(b + \sum_{l=1}^{\infty} v_b^{(l)} g^{2l}\Big) \;, \\
\tilde{D} &\to \tilde{D} - 2\Big(M + \sum_{l=1}^{\infty} v_M^{(l)} g^{2l}\Big)^2 \;,
\end{aligned}
\tag{49}
$$

Hence, the present model for the renormalization of soft breakings gives a consistent description of all renormalization properties of softly broken supersymmetric Yang-Mills theories. For concrete calculations the simpler version of spurion fields may be preferred, but the results on the divergency structure of the softly broken SYM theory remain valid and can be used for a consistency check and as a guideline for the classification of divergences in concrete diagrams.

5 The β functions of soft-breaking terms

Using the present construction of softly broken SYM theories, it is possible to determine the all-order expressions for the renormalization group β functions of

the soft mass parameters. As mentioned at the end of sec. 3, due to the two anomalies the β functions cannot be inferred from the invariant counterterms but require an algebraic construction. The derivation we use is analogous to the one carried out in softly broken SQED [12], but it differs in the explicit expressions by the appearance of the supersymmetry anomaly in the supersymmetry transformations of the local gauge coupling. In the present paper we skip the construction of the Callan-Symanzik equation. For a more detailed discussion of the different partial differential equations and their relations we refer to [12].

The renormalization group (RG) β functions are uniquely defined only if one specifies the normalization conditions for the parameters of the theory. In common usage, the RG β functions are identified with the β functions of mass-independent schemes. As shown in [28], such schemes correspond to asymptotic normalization conditions, where the (Euclidian) normalization point κ^2 is considered as being much larger than the mass parameters of the theory, and in these schemes the differentiation with respect to the mass parameters is soft to all orders. This property implies the classical scaling equations for the individual mass differentiations to all orders:

$$\begin{aligned} m\partial_m\Gamma &= m\int d^4x\,\Big(\frac{\delta}{\delta q}+\frac{\delta}{\delta\overline{q}}\Big)\Gamma\ , \\ M_\lambda\partial_{M_\lambda}\Gamma &= \int d^4x\,\frac{M_\lambda}{g^2}\Big(\frac{\delta}{\delta f}+\frac{\delta}{\delta\overline{f}}\Big)\Gamma\ , \\ M\partial_M\Gamma &= -4M^2\int d^4x\,\frac{\delta}{\delta\tilde{D}}\Gamma\ , \\ b\partial_b\Gamma &= -b\int d^4x\,\Big(\frac{\delta}{\delta q_F}+\frac{\delta}{\delta\overline{q}_F}\Big)\Gamma\ . \end{aligned} \tag{50}$$

The RG equation, which describes the transformation of the 1PI Green functions under infinitesimal variations of the normalization point κ, can in general be expressed as a linear combination of all invariant operators of the theory. The general basis for linear operators has been constructed in [13] and consists of two gauge-independent operators $\mathcal{D}_{\mathrm{kin}}$ and $\mathcal{D}_{Vv}$, and gauge-dependent field operators $\mathcal{N}_\phi$.

The gauge-independent operators correspond in lowest order to the one-loop coupling renormalization and the mass renormalization given in (20); the gauge dependent operators represent the field redefinitions of eqs. (46) and (47). While $\mathcal{D}_{\mathrm{kin}}$, $\mathcal{D}^{\mathrm{sym}}_{V_v}$ are invariant operators in the strict sense and commute with the anomalous Slavnov-Taylor operator for any functional Γ,

$$\big(\mathbf{s}^{r\eta}_\Gamma + r^{(1)}\delta\mathcal{S}\big)\mathcal{D}\Gamma = \mathcal{D}\big(\mathcal{S}^{r\eta}(\Gamma) + r^{(1)}\delta\mathcal{S}\Gamma\big)\ , \tag{51}$$

the gauge-dependent field operators commute only up to field monomials linear in the propagating fields.

Using these operators, the RG equation can be written as

$$\Big(\kappa\partial_\kappa + \hat\beta^{(1)}_{g^2}\mathcal{D}_{\rm kin} - \sum_{l\geq 1}\Big(\hat\gamma^{(l)}\mathcal{D}^{{\rm sym}(l)}_{Vv} + \hat\gamma^{(l)}_\varphi\mathcal{N}^{(l)}_\varphi + \hat\gamma^{(l)}_\psi\mathcal{N}^{(l)}_\psi + \hat\gamma^{(l)}_{\psi\varphi}\mathcal{N}^{(l)}_{\psi\varphi} + \hat\gamma^{(l)}_A\mathcal{N}^{(l)}_A + \hat\gamma^{(l)}_\lambda\mathcal{N}^{(l)}_\lambda + \hat\gamma^{(l)}_{A\lambda}\mathcal{N}^{(l)}_{A\lambda}\Big)\Big)\Gamma = \Delta_Y \ , \tag{52}$$

where Δ_Y summarizes the terms linear in the propagating fields. Its most important contributions are the field monomials of eq. (48), which belong to the generalized field redefinitions and which are non-vanishing in the limit of constant coupling.

For the β functions of soft mass parameters only the physical operators $\mathcal{D}_{\rm kin}$ and $\mathcal{D}^{\rm sym}_{Vv}$ are relevant. The explicit form of $\mathcal{D}_{\rm kin}$ can be obtained from the result presented in [13] by taking into account the gaugino-mass shift in the F component of the supercoupling. It takes the following form:

$$\mathcal{D}_{\rm kin} = \int d^4x\ F(g^2)\Big(g^4\frac{\delta}{\delta g^2} + M_\lambda\Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta\overline{f}}\Big)\Big) \ . \tag{53}$$

The function $F(g^2) = 1 + \delta F(g^2) = 1 + r^{(1)}_\eta g^2 + \mathcal{O}(g^4)$ is the same function that appears in the anomalous Slavnov-Taylor operator (36) and that absorbs the anomalous supersymmetry-breaking (see (38)).

The explicit form of the invariant operator $D^{{\rm sym}(l)}_{Vv}$ can be found in [13]. It has the decomposition

$$\mathcal{D}^{{\rm sym}(l)}_{Vv} = \mathcal{D}^{(l)}_{Vv} - 8r^{(1)}\big(\mathcal{D}^{(l+1)}_{g^2} + l\big(\mathcal{N}^{(l+1)}_V - 8(l+1)r^{(1)}\delta\mathcal{N}^{(l+2)}_V\big)\big). \tag{54}$$

Here the operator $\mathcal{D}^{(l)}_{Vv}$ is invariant with respect to $\mathrm{s}^{r_\eta}_\Gamma$, i.e,

$$\mathrm{s}^{r_\eta}_\Gamma\mathcal{D}_{Vv}\Gamma = \mathcal{D}_{Vv}\,\mathcal{S}^{r_\eta}(\Gamma) \ , \tag{55}$$

and the additional terms continue $\mathcal{D}_{Vv}$ to an invariant operator with respect to the complete anomalous Slavnov-Taylor operator as defined in eq. (51).

It turns out that the operators composing $D^{\rm sym}_{Vv}$ depend on the components of the anomalous multiplet $\tilde G$ of the gauge coupling $g(x)$,

$$\tilde G(x,\theta,\overline\theta) = g(x) + \mathcal{O}(\theta,\overline\theta) \ , \tag{56}$$

which is defined as the vector multiplet with respect to the anomalous supersymmetry transformations of the coupling (42). Its components are defined by the equation

$$\delta_\alpha^{r\eta}\tilde{G} = \Big(\frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\overline{\theta})_\alpha\partial_\mu\Big)\tilde{G}\ , \qquad \bar{\delta}_{\dot\alpha}^{r\eta}\tilde{G} = \Big(\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\Big)\tilde{G}\ . \tag{57}$$

For the purpose of the present paper we only want to study the limit to constant coupling in the presence of soft mass shifts. In this limit only the F and D components of $\tilde{G}$ are non-vanishing,

$$\lim_{G\to g}\tilde{G}^{2l} = g^{2l}\Big(1 + \theta^2 f^{(l)} + \overline{\theta}^2 f^{(l)} + \frac{1}{4}\theta^2\overline{\theta}^2 d^{(l)}\Big)\ , \tag{58}$$

and we obtain as contributions in loop order l:

$$\begin{aligned} f^{(l)} &= -lM_\lambda F(g^2)\ , \\ d^{(l)} &= 4l(l+1)M_\lambda^2 F^2(g^2) + 4lM_\lambda^2 g^2\partial_{g^2}F^2(g^2)\ . \end{aligned} \tag{59}$$

For constant coupling and for vanishing external fields ϕ^V, $\mathbf{q}$, $\overline{\mathbf{q}}$, the general expressions of ref. [13] yield the following contributions to the individual operators of $D_{Vv}^{\rm sym}$ (54):

$$\begin{aligned} \mathcal{D}_{Vv}^{(l)} &\to g^{2l}\int d^4x\,\Big(d^{(l)}\frac{\delta}{\delta\tilde{D}} - 2m\Big(\frac{\delta}{\delta q} + \frac{\delta}{\delta\overline{q}}\Big) + 2(b - 2mf^{(l)})\Big(\frac{\delta}{\delta q_F} + \frac{\delta}{\delta\overline{q}_F}\Big)\Big)\ , \\ \mathcal{N}_V^{(l)} &\to -2M^2F(g^2)g^{2l}\int d^4x\,\frac{\delta}{\delta\tilde{D}}\ , \\ \mathcal{D}_{g^2}^{(l+1)} &\to g^{2(l+2)}F(g^2)\partial_{g^2} - g^{2l}\big(f^{(l)} - M_\lambda F(g^2)\big)\int d^4x\,\Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta\overline{f}}\Big)\ , \\ \delta\mathcal{N}_V^{(l+2)} &\to 0\ . \end{aligned} \tag{60}$$

For constant coupling it is possible to simplify the operators in (60) and $\mathcal{D}_{\rm kin}$ in (53) further. Using the mass equations (50) we can eliminate the field differentiations appearing in the expressions of (60) and (53) in favor of mass differentiations. When the emerging results for the invariant operators are put into the RG equation (52), we obtain the usual form of a RG equation for the 1PI Green functions of softly broken Super-Yang-Mills theories $\Gamma^{\rm sSYM}$:

$$\begin{aligned} &\Big(\kappa\partial_\kappa + \beta_{g^2}\partial_{g^2} + \beta_{M_\lambda}\partial_{M_\lambda} + \beta_M\partial_M + \beta_b\partial_b + \beta_m\partial_m \\ &\quad - \sum_{\text{field red.}}\gamma_\phi\mathcal{N}_\phi\Big)\,\Gamma^{\rm sSYM} = \Delta_Y\ . \end{aligned} \tag{61}$$

The construction determines the β functions appearing here completely: The β function of the supersymmetric mass parameter m is determined by the gauge-independent coefficients $\hat{\gamma}$ of $\mathcal{D}_{Vv}$ in (52),

$$\beta_m = 2\gamma \ , \quad \text{with} \quad \gamma \equiv \sum_l g^{2l} \hat{\gamma}^{(l)} \ , \tag{62}$$

and the β function of the gauge coupling is given as an all-order expression:

$$\beta_{g^2} = g^4 \Big(\hat{\beta}^{(1)}_{g^2} + 8 r^{(1)} \gamma \Big) F(g^2) \ . \tag{63}$$

In order to obtain the familiar all-order expressions for the β functions of soft supersymmetry-breaking parameters we rewrite the explicit dependence on the loop order l in the operators of eq. (60) using a differentiation ∂_{g^2} acting on γ (62) and find

$$\begin{aligned}
\beta_{M_\lambda} &= M_\lambda F(g^2) g^2 \partial_{g^2} \Big(g^2 \big(\hat{\beta}^{(1)}_{g^2} + 8 r^{(1)} \gamma \big) \Big), \\
\beta_M &= M \Bigg(\frac{M_\lambda^2 F^2(g^2)}{M^2} g^2 \partial_{g^2} \big(g^2 \partial_{g^2} \gamma \big) + \frac{M_\lambda^2 F^2(g^2)}{M^2} g^2 \partial_{g^2} \gamma \big(1 + g^2 \partial_{g^2} \ln F(g^2) \big) \\
&\qquad + 4 r^{(1)} g^2 F(g^2) g^2 \partial_{g^2} \gamma \Bigg) \ , \\
\beta_b &= 2b \Bigg(\gamma + 2 \frac{M_\lambda F(g^2) m}{b} g^2 \partial_{g^2} \gamma \Bigg) \ .
\end{aligned} \tag{64}$$

In this way, all β functions are determined in a closed form. They depend on the one-loop β function $\hat{\beta}^{(1)}_{g^2}$, the anomalous dimension of the supersymmetric mass γ, the anomaly coefficient of the Adler-Bardeen anomaly $r^{(1)}$ and the function $F(g^2)$, which includes in order g^2 the scheme-independent coefficient of the supersymmetry anomaly $r^{(1)}_\eta$ (35).

In higher orders the soft β functions (64) are not equivalent to the ones quoted in the literature [4, 18, 19, 5]. But it is well-known that β functions are scheme-dependent[3]. Indeed, the usual β functions are obtained by a finite redefinition of the gaugino mass parameter. When the shift in the F component of the coupling is modified according to $M_\lambda \to \frac{1}{F(g^2)} M_\lambda$ (cf. eq. (49)), the gaugino-mass equation takes the form

$$M_\lambda \partial_{M_\lambda} \Gamma = \int d^4x \ \frac{M_\lambda}{F(g^2) g^2} \Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta \overline{f}} \Big) \Gamma = 0 \tag{65}$$

[3]For scheme-dependence of soft β functions in supersymmetric theories see e.g. ref. [29].

instead of the form in (50), and the redefined shift with its soft mass equation defines just another mass-independent scheme.

In the redefined scheme, the β functions change according to $M_\lambda \to M_\lambda/F(g^2)$ and hence take the form

$$\begin{aligned}
\beta_{M_\lambda} &= M_\lambda g^2\partial_{g^2}\Big(g^2F(g^2)(\hat\beta^{(1)}_{g^2}+8r^{(1)}\gamma)\Big) = M_\lambda g^2\partial_{g^2}\left(\frac{\beta_{g^2}}{g^2}\right),\\
\beta_M &= M\bigg(\frac{M_\lambda^2}{M^2}g^2\partial_{g^2}\big(g^2\partial_{g^2}\gamma\big)+\frac{M_\lambda^2}{M^2}g^2\partial_{g^2}\gamma\Big(1+g^2\partial_{g^2}\ln F(g^2)\Big)\\
&\qquad +4r^{(1)}g^2F(g^2)g^2\partial_{g^2}\gamma\bigg),\\
\beta_b &= 2b\bigg(\gamma+2\frac{M_\lambda m}{b}g^2\partial_{g^2}\gamma\bigg).
\end{aligned}\tag{66}$$

The β functions of the gaugino mass and the b parameter correspond now exactly to the expressions quoted in the literature (see [4, 18, 19, 5]). For the β function of the scalar mass M we find a closed expression in terms of γ and $F(g^2)$ generalizing the scalar-mass β functions of [21, 22] to arbitrary mass-independent normalization conditions of the gauge coupling.

The scalar-mass β function is particularly interesting. Naive supergraph arguments fail to produce the correct form of β_M, since a certain contribution called X term [20] is missing. The X term is defined as the additional non-invariant contribution to the scalar mass β function in the parametrization

$$\beta_M = M\frac{M_\lambda^2}{M^2}\Big(g^2\partial_{g^2}(g^2\partial_{g^2}\gamma)+(1+X)g^2\partial_{g^2}\gamma\Big),\tag{67}$$

and the present construction determines its value as

$$X = g^2\partial_{g^2}\ln F(g^2)+4\frac{M^2}{M_\lambda^2}r^{(1)}g^2F(g^2) = g^2\left(r^{(1)}_\eta+4r^{(1)}\frac{M^2}{M_\lambda^2}\right)+\mathcal{O}(g^4)\,.\tag{68}$$

with $r^{(1)}_\eta = C(G)/8\pi^2$ and $r^{(1)} = -T(R)/16\pi^2$ (see (35)). The result derived here clarifies why the X term escapes simple superspace arguments: Since its lowest-order coefficients are identified with the two anomaly coefficients of SYM theories, it is evident that the X term itself cannot be obtained by arguments based on invariant schemes.

The results of the present construction make the notion of the all-order expressions for β functions more precise by connecting the all-order expressions to the conditions on the parameters of the theory. These conditions are encoded in the

transformation properties and mass shifts of external fields which appear in their explicit form in the anomalous Slavnov–Taylor identity of the extended model (36). In particular, choosing the function $F(g^2)$ in the Slavnov-Taylor identity as the NSVZ function

$$F(g^2) = \frac{1}{1 - r_\eta^{(1)} g^2} \ , \tag{69}$$

and choosing a mass-independent scheme where (65) holds, one obtains the well-known NSVZ expressions for the gauge-β function (63) and for the X term:

$$\beta_{g^2}^{NSVZ} = \frac{g^4(\hat{\beta}_{g^2}^{(1)} + 8r^{(1)}\gamma)}{1 - r_\eta^{(1)} g^2} \ , \tag{70}$$

$$X^{NSVZ} = g^2 \frac{r_\eta^{(1)} + 4r^{(1)}\frac{M^2}{M_\lambda^2}}{1 - r_\eta^{(1)} g^2} \ . \tag{71}$$

6 Conclusions

The construction of the present paper gives a consistent description of softly broken Super-Yang-Mills theories including all specific renormalization properties of supersymmetric field theories. These are the non-renormalization theorems and the relations between the renormalization constants of soft-breaking and supersymmetric parameters. Both are expressed as relations for the invariant counterterms and in terms of the β functions. All renormalization properties have been derived from symmetries of the classical action of the extended model and their extension to the 1PI Green functions. Since the classical symmetries are broken by two anomalies – the Adler-Bardeen anomaly and a supersymmetric anomaly – the 1PI Green functions are characterized by an anomalous Slavnov-Taylor identity. It is shown that the two-loop order of the gauge-β function as well as the two-loop contribution of the X term are both induced by the anomalies.

While the present construction embeds SYM theories into an extended model, where the supersymmetric structure is more appropriately characterized, it is possible to use the simpler version of the spurion fields for practical calculations. The results of this paper remain valid irrespective of the specific scheme and model one uses for the inclusion of soft breakings. Thus, they can be used for a systematic classification of divergences and counterterms in explicit one- and two-loop calculations, and in particular seemingly accidental cancellations of divergences can be proven via the non-renormalization theorems.

In the present paper we have restricted ourselves to a theory with a simple non-abelian gauge group and one matter multiplet of charged Dirac fermions in an

irreducible representation of the gauge group. The most important example for such a theory is supersymmetric QCD. Generalizations to models with matter multiplets in reducible representations including chiral trilinear interactions of supersymmetry with their soft-breaking interactions are straightforward since no further anomalies will show up. Thus, the peculiarities of renormalization with local couplings are completely accounted for in the simple model considered here. However, since the present construction is based on the different symmetries of the model, a careful specification of axial symmetries and their axial vector multiplets has to be done for any specific model under consideration.

In the present construction we have excluded parity-violating masses for soft breaking. Introducing them would lead to a non-trivial change of the extended model. In the spirit of the present construction they could be introduced if we introduce also a photon multiplet with a shifted D-component. Since the photon should be treated as a propagating field, it differs from the external axial vectors that are coupled to the parity-even combination of scalar mass terms. Hence, in the end one has to to consider the renormalization of the Fayet-Iliopoulos D-terms [30], which might induce other properties as the ones derived in the present paper (cf. [31, 32]). Closely related is the extension to theories with spontaneous breaking of gauge symmetry like the MSSM. Also in this case the construction of the present paper has to be generalized.

Finally we are aiming to the renormalization of the complete MSSM with its non-renormalization theorems. In such a study the construction of the present paper is an important ingredient and we are convinced that a construction along its lines will give important new insights into the symmetry structure as well as the renormalization structure of the MSSM.

Acknowledgments

We thank R. Flume and W. Hollik for useful discussions.

A The BRS transformations and the Slavnov-Taylor identity

In this appendix we list the BRS transformations of the propagating and external fields. Compared to the BRS transformations of [13] they include the soft mass shifts in the F- and D-components of external fields.

- BRS transformations of the gauge vector multiplet and of the Faddeev-Popov ghost.

$$\begin{aligned}
\mathrm{s}A_\mu &= \frac{1}{g}\partial_\mu g c + ig\big[A_\mu, c\big] + i\epsilon\sigma_\mu\overline{\lambda} - i\lambda\sigma_\mu\overline{\epsilon} \\
&\quad + \frac{1}{2}g^2(\epsilon\chi + \overline{\chi\epsilon})A_\mu - i\omega^\nu\partial_\nu A_\mu \ , \\
\mathrm{s}\lambda^\alpha &= -ig\big\{\lambda, c\big\} + \frac{i}{2g}(\epsilon\sigma^{\rho\sigma})^\alpha G_{\rho\sigma}(gA) + i\epsilon^\alpha\, g(\overline{\varphi}_L T_a\varphi_L - \varphi_R T_a\overline{\varphi}_R)\tau_a \\
&\quad + \frac{1}{2}\epsilon^\alpha g^2(\chi\lambda - \overline{\chi}\overline{\lambda}) + \frac{1}{2}g^2(\epsilon\chi + \overline{\chi\epsilon})\lambda^\alpha - i\omega^\nu\partial_\nu\lambda^\alpha \ , \\
\mathrm{s}c &= -igc\,c + 2i\epsilon\sigma^\nu\overline{\epsilon}A_\nu + \frac{1}{2}g^2(\epsilon\chi + \overline{\chi\epsilon})c - i\omega^\nu\partial_\nu c \ .
\end{aligned} \tag{A.1}$$

- BRS transformations of matter fields

$$\begin{aligned}
\mathrm{s}\varphi_L &= -i(gc_aT_a - \tilde{c})\varphi_L + \sqrt{2}\,\epsilon\psi_L - i\omega^\nu\partial_\nu\varphi_L \ , \\
\mathrm{s}\psi_L^\alpha &= -i(gc_aT_a - \tilde{c})\,\psi_L^\alpha \\
&\quad - \sqrt{2}\,\epsilon^\alpha\,(\overline{q} + m)\overline{\varphi}_R - \sqrt{2}\,i(\overline{\epsilon\sigma}^\mu)^\alpha D_\mu\varphi_L - i\omega^\nu\partial_\nu\psi_L^\alpha \ , \\
\mathrm{s}\varphi_R &= i\varphi_R(gc_aT_a + \tilde{c}) + \sqrt{2}\,\epsilon\psi_R - i\omega^\nu\partial_\nu\varphi_R \ , \\
\mathrm{s}\psi_R^\alpha &= i\psi_R^\alpha(gc_aT_a + \tilde{c}) \\
&\quad - \sqrt{2}\,\epsilon^\alpha\,(\overline{q} + m)\overline{\varphi}_L - \sqrt{2}\,i(\overline{\epsilon\sigma}^\mu)^\alpha D_\mu\varphi_R - i\omega^\nu\partial_\nu\psi_R^\alpha \ .
\end{aligned} \tag{A.2}$$

- BRS transformations of the axial vector multiplet and of the axial ghost

$$\begin{aligned}
\mathrm{s}V_\mu &= \partial_\mu\tilde{c} + i\epsilon\sigma_\mu\overline{\tilde{\lambda}} - i\tilde{\lambda}\sigma_\mu\overline{\epsilon} - i\omega^\nu\partial_\nu V_\mu \ , \\
\mathrm{s}\tilde{\lambda}^\alpha &= \frac{i}{2}(\epsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma}(V) + \frac{i}{2}\epsilon^\alpha\,(\tilde{D} - 2M^2) - i\omega^\nu\partial_\nu\tilde{\lambda}^\alpha \ , \\
\mathrm{s}\tilde{D} &= 2\epsilon\sigma^\mu\partial_\mu\overline{\tilde{\lambda}} + 2\partial_\mu\tilde{\lambda}\sigma^\mu\overline{\epsilon} - i\omega^\nu\partial_\nu\tilde{D} \ , \\
\mathrm{s}\tilde{c} &= 2i\epsilon\sigma^\nu\overline{\epsilon}V_\nu - i\omega^\nu\partial_\nu\tilde{c} \ .
\end{aligned} \tag{A.3}$$

- BRS transformations of the local coupling and its superpartners (1)

$$\begin{aligned}
\mathrm{s}\eta &= \epsilon^\alpha \chi_\alpha - i\omega^\nu \partial_\nu \eta \ , \\
\mathrm{s}\chi_\alpha &= 2i(\sigma^\mu \overline{\epsilon})_\alpha \partial_\mu \eta + 2\epsilon_\alpha (f + \frac{M_\lambda}{g^2}) - i\omega^\mu \partial_\mu \chi_\alpha \ , \\
\mathrm{s}f &= -M_\lambda(\epsilon\chi + \overline{\chi\epsilon}) + i\partial_\mu \chi \sigma^\mu \overline{\epsilon} - i\omega^\mu \partial_\mu f \ .
\end{aligned} \tag{A.4}$$

- BRS transformations of q-multiplets (4)

$$\begin{aligned}
\mathrm{s}q &= +2i\tilde{c}(q+m) + \epsilon^\alpha q_\alpha - i\omega^\nu \partial_\nu q \ , \\
\mathrm{s}q_\alpha &= +2i\tilde{c}q_\alpha + 2i(\sigma^\mu \overline{\epsilon})_\alpha D_\mu q + 2\epsilon_\alpha (q_F - b) - i\omega^\mu \partial_\mu q_\alpha \ , \\
\mathrm{s}q_F &= +2i\tilde{c}(q_F - b) + iD_\mu q^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \overline{\epsilon}^{\dot{\alpha}} - 4i\overline{\tilde{\lambda}}_{\dot{\alpha}} \overline{\epsilon}^{\dot{\alpha}} (q+m) - i\omega^\mu \partial_\mu q_F \ .
\end{aligned} \tag{A.5}$$

  The covariant derivative is defined by

$$D_\mu q^i = (\partial_\mu - 2iV_\mu)(q^i + (m, 0, -b)) \tag{A.6}$$

The classical Slavnov-Taylor identity (25) expresses in functional form BRS invariance of the classical action and on-shell nilpotency of BRS transformations. The Slavnov-Taylor operator acting on a general functional $\mathcal{F}$ is defined as

$$\begin{aligned}
\mathcal{S}(\mathcal{F}) = \int d^4x \ \Big\{ & \frac{\delta \mathcal{F}}{\delta \rho_\mu} \frac{\delta \mathcal{F}}{\delta A^\mu} + \frac{\delta \mathcal{F}}{\delta Y_{\lambda\alpha}} \frac{\delta \mathcal{F}}{\delta \lambda^\alpha} + \frac{\delta \mathcal{F}}{\delta Y^{\dot{\alpha}}_{\overline{\lambda}}} \frac{\delta \mathcal{F}}{\delta \overline{\lambda}_{\dot{\alpha}}} + \frac{\delta \mathcal{F}}{\delta Y_c} \frac{\delta \mathcal{F}}{\delta c} \\
& + sB \frac{\delta \mathcal{F}}{\delta B} + s\bar{c} \frac{\delta \mathcal{F}}{\delta \bar{c}} \\
& + \Big(\frac{\delta \mathcal{F}}{\delta Y_{\varphi_L}} \frac{\delta \mathcal{F}}{\delta \varphi_L} + \frac{\delta \mathcal{F}}{\delta Y_{\overline{\varphi}_L}} \frac{\delta \mathcal{F}}{\delta \overline{\varphi}_L} + \frac{\delta \mathcal{F}}{\delta Y_{\psi_{L\alpha}}} \frac{\delta \mathcal{F}}{\delta \psi^\alpha_L} + \frac{\delta \mathcal{F}}{\delta Y^{\dot{\alpha}}_{\overline{\psi}_L}} \frac{\delta \mathcal{F}}{\delta \overline{\psi}_{L\dot{\alpha}}} + (_{L \to R}) \Big) \\
& + s\eta^i \frac{\delta \mathcal{F}}{\delta \eta^i} + s\overline{\eta}^i \frac{\delta \mathcal{F}}{\delta \overline{\eta}^i} + sV^i \frac{\delta \mathcal{F}}{\delta V^i} + sq^i \frac{\delta \mathcal{F}}{\delta q^i} + s\overline{q}^i \frac{\delta \mathcal{F}}{\delta \overline{q}^i} \Big\} + s\omega^\nu \frac{\partial \mathcal{F}}{\partial \omega^\nu} \ .
\end{aligned} \tag{A.7}$$

Here we have defined $\eta^i = (\eta, \chi^\alpha, f)$, $V^i = (V^\mu, \tilde{\lambda}, \overline{\tilde{\lambda}}, \tilde{D})$ and $q^i = (q, q^\alpha, q_F)$.

References

[1] N. Maggiore, O. Piguet and S. Wolf, *Nucl. Phys.* **B476** (1996) 329.

[2] W. Hollik, E. Kraus and D. Stöckinger, hep-ph/0007134.

[3] L. Girardello and M.T. Grisaru, *Nucl. Phys.* **B194** (1982) 65.

[4] Y. Yamada, *Phys. Rev.* **D50** (1994) 3537.

[5] L.A. Avdeev, D.I. Kazakov and I.N. Kondrashuk, *Nucl. Phys.* **B510** (1998) 289; D.I. Kazakov, *Phys. Lett.* **B448** (1999) 201.

[6] D.I. Kazakov and V.N. Velizhanin, *Phys. Lett.* **B485** (2000) 383.

[7] I.N. Kondrashuk, *J. Phys.* **A33** (2000) 6399.

[8] K. Fujikawa and W. Lang, *Nucl. Phys.* **B88** (1975) 61.

[9] M.T. Grisaru, W. Siegel and M. Rocek, *Nucl. Phys.* **B159** (1979) 429; M.T. Grisaru and W. Siegel, *Nucl. Phys.* **B201** (1982) 292.

[10] R. Flume and E. Kraus, *Nucl. Phys.* **B569** (2000) 625.

[11] E. Kraus and D. Stöckinger, hep-th/0105028, to be published in *Nucl. Phys.* **B**.

[12] E. Kraus and D.Stöckinger, *Phys. Rev.* **D64** (2001) 115012.

[13] E. Kraus, *Nucl. Phys.* **B620** (2002) 55.

[14] N. Arkani-Hamed, G.F. Giudice, M.A. Luty and R.Rattazzi, *Phys. Rev.* **D58** (1998) 115005.

[15] S.L. Adler, *Phys. Rev.* **177** (1969) 2426.

[16] E. Kraus, hep-ph/0110323.

[17] V. Novikov, M. Shifman, A. Vainshtein and V. Zakharov, *Nucl. Phys.* **B229** (1983) 381; *Phys. Lett.* **B166** (1986) 329.

[18] J. Hisano and M. Shifman, *Phys. Rev.* **D56** (1997) 5475.

[19] J. Jack and D.R.T. Jones, *Phys. Lett.* **B415** (1997) 383.

[20] J. Jack, D.R.T. Jones and A. Pickering, *Phys. Lett.* **B426** (1998) 73.

[21] T. Kobayashi, J. Kubo, and G. Zoupanos, *Phys. Lett.* **B427** (1998) 291.

[22] J. Jack, D.R.T. Jones and A. Pickering, *Phys. Lett.* **B432** (1998) 114.

[23] M.A. Shifman and A.I. Vainshtein, *Nucl. Phys.* **B277** (1986) 456.

[24] P.L. White, *Class. Quantum Grav.* **9** (1992) 1663.

[25] N. Maggiore, O. Piguet and S. Wolf, *Nucl. Phys.* **B458** (1996) 403.

[26] W. Hollik, E. Kraus and D. Stöckinger, *Eur. Phys. J.***C11** (1999) 365.

[27] S.L. Adler and W.A. Bardeen, *Phys. Rev.* **182** (1969) 1517.

[28] E. Kraus, *Helv. Phys. Acta* **67** (1994) 424.

[29] S.P. Martin and M. Vaughn, *Phys. Lett.* **B318** (1993) 331.

[30] P. Fayet and J. Iliopoulos, *Phys. Lett.* **B51** (1974) 461.

[31] I. Jack, D.R.T. Jones and S. Parson, *Phys. Rev.* **D62** (2000) 125022;
I. Jack and D.R.T. Jones *Phys. Rev.* **D63** (2001) 075010.

[32] D.I. Kazakov and V.N. Velizhanin, hep-ph/0110144.