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Renormalization of the Minimal Supersymmetric Standard Model[1]

Wolfgang Hollik[a,b], Elisabeth Kraus[c], Markus Roth[a],
Christian Rupp[d], Klaus Sibold[e], Dominik Stöckinger[f]

[a] Institut für Theoretische Physik, Universität Karlsruhe, Germany
[b] Max-Planck-Institut für Physik, München, Germany
[c] Institut für Theoretische Physik, Universität Bonn, Germany
[d] Institut für Theoretische Physik, Universität Bern, Switzerland
[e] Institut für Theoretische Physik, Universität Leipzig, Germany
[f] Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Abstract

The renormalization of the Minimal Supersymmetric Standard Model of electroweak interactions is presented to all orders of perturbation theory using the algebraic method. Special attention is directed to the issues of soft supersymmetry breaking, gauge fixing, and infrared finiteness. We discuss the implications of $\hbar$-dependent field parametrizations on the counterterm structure and provide a complete set of on-shell normalization conditions.

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1 Introduction

Independent of the success of the Standard Model (SM) [1], manifested in more than a decade of precision tests [2], the quest for a more comprehensive description of the fundamental interactions has brought forward the concept of supersymmetry as the favored extension of the SM. The Minimal Supersymmetric Standard Model (MSSM) [3] is the theoretically best motivated and conceptually most elaborated and predictive framework beyond the SM. The possible existence of a light Higgs boson, in a mass range consistent with the electroweak precision analysis [2] and with the direct searches [4], would find a natural explanation within the MSSM. Precision analyses of the MSSM [5], performed in analogy to those of the SM, show a similar quality for the overall fits as in the SM. They are presently based on one-loop calculations [6, 7, 8] with specific two-loop improvements [9] on the ρ parameter.

With the increase of the experimental precision at future colliders, decisive precision tests of both the SM and the MSSM will become possible (see e.g. the reports [10, 11]). On the theoretical side, a thorough control of the quantization and the renormalization of the MSSM as a supersymmetric gauge theory, with spontaneously broken gauge symmetry and softly broken supersymmetry, is required. This is not only a theoretical question for establishing a solid and consistent basis but also a matter of practical importance for concrete higher-order calculations, where the quantum contributions to the Green functions have to fulfil the symmetry properties of the underlying theory. An increasing number of phenomenological applications has been carried out in the Wess-Zumino gauge where the number of unphysical degrees of freedom is minimal, but where supersymmetry is no longer manifest. Moreover, a manifestly supersymmetric and gauge-invariant regularization for divergent loop integrals is missing. Hence, renormalization and the structure of counterterms have to be adapted by exploiting the basic symmetries expressed in terms of the supersymmetric BRS transformations [12, 13]. An additional complication in the conventional approach assuming an invariant regularization scheme, however, arises from the modification of the symmetry transformations themselves by higher-order terms.

The method of algebraic renormalization, applied in [14, 15] to the electroweak SM, avoids the difficulties of the conventional approach. The theory is defined at the classical as well as the quantum level by the particle content and by the basic symmetries. The essential features of the algebraic method are the combination of all symmetries into the BRS transformations leading to the Slavnov-Taylor (ST) identity. In this way, the theory is defined by symmetry requirements that have to be satisfied after renormalization in all orders of perturbation theory. In the case of symmetry violation during the explicit calculation of vertex functions in a given order, additional non-invariant counterterms would be uniquely determined to restore the symmetry, besides the invariant counterterms needed for absorbing the divergences and for the normalization of fields and parameters. Examples are given in [16, 17] for supersymmetric QED and QCD and in [18] for the SM case.

In this article we adopt the algebraic method to define the (electroweak part of the) MSSM as a quantum theory to all orders, including a complete discussion of renormalization. The fundamental symmetries are established in a functional way introducing the ST and Ward operators, and the invariant and non-invariant counterterms for the renormalization are discussed, together with a proof of the infrared (IR) finiteness of the renormalized theory. In section 2 we define the foundations of the MSSM specifying the field multiplets, the classical action, and the gauge and soft symmetry breakings. Section 3 is devoted to the discussion of quantization and renormalization based on the symmetry requirements, and of the IR finiteness. In section 4 the on-shell renormalization is formulated and it is shown that the complete set of on-shell conditions can be fulfilled.

The rather involved structure of the MSSM leads to a series of specific difficulties which had to be overcome and which will be addressed in the paper:

- In the MSSM supersymmetry is softly broken. Since our approach of algebraic renormalization relies substantially on symmetries, we couple the soft breaking terms to external spurion fields such that the combined system is again supersymmetric. Two different methods which accomplish this have been discussed [13, 19]; here we use a synthesis of both methods which avoids unphysical counterterms (section 2.2).

- For gauge fixing, we use a variant of the R_ξ gauge, which breaks rigid gauge invariance. In order to have a rigid Ward identity, external fields have to be introduced that restore the invariance. The particular structure of the MSSM Higgs sector requires two external tensor fields $\Phi_1^{a\,i}$, $\Phi_2^{a\,i}$ transforming as $\mathrm{ad} \otimes T$, where T is the representation of the Higgs fields (section 2.6).

- Since the U(1) gauge field transforms linearly as opposed to its superpartner, algebraic consistency conditions have to be established. Nilpotency of the linearized Slavnov-Taylor (ST) operator holds only on a suitably defined subspace (section 3.1). In particular, it has to be clarified to what extent the usual cohomology structure is affected (section 3.3).

- The model has to be formulated in terms of physical fields describing mass eigenstates. However, the relation of physical and symmetric fields is modified by loop graphs which implies that the Ward and ST operators — formulated in terms of physical fields — become $\hbar$ dependent. The field reparametrization corresponds to an additional type of invariant counterterms (section 3.4).

- Several field monomials would give rise to IR-singular insertions and must not be used for the cancellation of breaking terms. We have to ensure that such counterterms can indeed be avoided (section 3.6).

- Since the model possesses many symmetries, there are not enough free parameters to establish all relevant on-shell normalization conditions. Instead it has to be shown that some of the normalization conditions are fulfilled automatically due to the ST and Ward identities (section 4.3).

- The interpretation of the on-shell conditions for the Higgs fields is complicated by the mixing of the physical A^0 boson and the unphysical longitudinal gauge-boson degrees of freedom. It has to be clarified whether the on-shell condition for the mass M_{A^0} really corresponds to the pole of the A^0 propagator (section 4.3.1).

2 Foundations of the MSSM

The MSSM of electroweak interactions is a supersymmetric SU(2)×U(1) gauge theory where supersymmetry is softly broken by Girardello-Grisaru terms. Its field content comprises the following superfields

$\hat{V}$	real SU(2) gauge superfield, $\hat{V} = \hat{V}^a T^a$,
$\hat{V}'$	real U(1) gauge superfield,
$\hat{H}_1$, $\hat{H}_2$	two SU(2) doublets of chiral Higgs multiplets,
$\hat{L}$	SU(2) doublet of the chiral left-handed lepton multiplet,
$\hat{R}$	singlet of the chiral right-handed electron multiplet,
$\hat{Q}$	SU(2) doublet of the chiral left-handed quark multiplet,
$\hat{U}$, $\hat{D}$	singlets of chiral right-handed up and down quark multiplets.

We restrict our analysis to one generation of matter fields. As required by anomaly cancellation, the quark fields appear in three colours, but colour indices are suppressed throughout. To be close to phenomenological applications we work in the Wess-Zumino gauge, where the superfield expansions in terms of component fields read

$$\hat{V} = \theta\sigma^\mu\bar{\theta}V_\mu + \bar{\theta}^2\theta^\alpha\lambda_\alpha + \theta^2\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} + \frac{1}{2}\theta^2\bar{\theta}^2 D_V, \tag{2.1a}$$

$$\hat{V}' = \theta\sigma^\mu\bar{\theta}V'_\mu + \bar{\theta}^2\theta^\alpha\lambda'_\alpha + \theta^2\bar{\theta}_{\dot{\alpha}}\bar{\lambda}'^{\dot{\alpha}} + \frac{1}{2}\theta^2\bar{\theta}^2 D'_V, \tag{2.1b}$$

$$\hat{H}_i = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(H_i + \sqrt{2}\theta^\alpha h_{i\alpha} + \theta^2 F_i\right), \qquad i = 1,2, \tag{2.1c}$$

$$\hat{L} = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(L + \sqrt{2}\theta^\alpha l_\alpha + \theta^2 F_L\right), \tag{2.1d}$$

$$\hat{R} = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(R + \sqrt{2}\theta^\alpha r_\alpha + \theta^2 F_R\right), \tag{2.1e}$$

$$\hat{Q} = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(Q + \sqrt{2}\theta^\alpha q_\alpha + \theta^2 F_Q\right), \tag{2.1f}$$

$$\hat{U} = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(U + \sqrt{2}\theta^\alpha u_\alpha + \theta^2 F_U\right), \tag{2.1g}$$

$$\hat{D} = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(D + \sqrt{2}\theta^\alpha d_\alpha + \theta^2 F_D\right). \tag{2.1h}$$

An overview of the component fields is given in table 2.1, and our conventions for superspace and gauge-group generators can be found in appendix A.

	Bosonic fields	Fermionic fields	Isospin	Hypercharge
Gauge	V^a	λ^a	triplet	0
multiplets	V'	λ'	singlet	0
Higgs fields	$H_1 = (H_1^1, H_1^2)^T$	$h_1 = (h_1^1, h_1^2)^T$	doublet	-1
	$H_2 = (H_2^1, H_2^2)^T$	$h_2 = (h_2^1, h_2^2)^T$	doublet	1
Leptons	$L = (L^1, L^2)^T$	$l = (l^1, l^2)^T$	doublet	-1
	R	r	singlet	2
Quarks	$Q = (Q^1, Q^2)^T$	$q = (q^1, q^2)^T$	doublet	1/3
	U	u	singlet	$-4/3$
	D	d	singlet	2/3

Table 2.1: Multiplets and field content of the theory

The defining symmetries of the MSSM are SU(2)×U(1) gauge invariance and supersymmetry. The transformation laws of both symmetries are formulated in the BRS form [20] with ghosts $c = c^a T^a$ for local SU(2) gauge symmetry (weak isospin), c' for local U(1) gauge symmetry (hypercharge), ε^α for rigid supersymmetry, and ξ^μ for rigid translational transformations. ε^α is a constant complex bosonic and ξ^μ a constant imaginary fermionic ghost (see [16, 21, 12, 22] for details on the BRS formulation of supersymmetry). The BRS transformation of a field ϕ out of (2.1) reads

$$\mathrm{s}\,\phi = (\delta_c + \delta_{c'} + \varepsilon^\alpha Q_\alpha + \bar{Q}_{\dot\alpha}\bar{\varepsilon}^{\dot\alpha} - \mathrm{i}\xi^\mu\partial_\mu)\phi, \tag{2.2}$$

where δ_c, $\delta_{c'}$ are local gauge transformations with ghosts c, c' used as transformation parameters, and where Q_α, $\bar{Q}_{\dot\alpha}$ are the supersymmetry generators. The explicit form of all BRS transformations can be found in appendix B. Unless we eliminate the auxiliary D and F fields, the BRS transformations are nilpotent:

$$\mathrm{s}^{\,2}\phi = 0 \qquad\qquad \text{for all fields } \phi. \tag{2.3}$$

Although BRS invariance includes a substitute for local gauge invariance, rigid gauge invariance does not necessarily follow from BRS invariance in higher orders and has to be established in addition. The SU(2)×U(1) gauge transformations δ_ω, $\delta_{\omega'}$ read

$$\delta_\omega\phi = -\mathrm{i}g\omega\phi \qquad \text{for isospin doublets,} \tag{2.4a}$$
$$\delta_\omega\phi = 0 \qquad \text{for isospin singlets,} \tag{2.4b}$$
$$\delta_\omega\phi = -\mathrm{i}g[\omega,\phi] \qquad \text{for fields in the adjoint representation,} \tag{2.4c}$$
$$\delta_{\omega'}\phi = -\mathrm{i}g'\omega'\frac{Y}{2}\phi \qquad \text{for all fields} \tag{2.4d}$$

with constant transformation parameters $\omega = \omega^a T^a$, ω'. The hypercharges Y of the fields are given in table 2.1 and are related to the electric charge by the Gell-Mann–Nishijima

Type	CP transformation	Fields
Scalar	$\phi \xrightarrow{\text{CP}} \bar{\phi}$	$H_1^i, H_2^i, L^i, R, Q^i, U, D, A, G^\pm, H^\pm, H, h, A_1, A_2$
Real pseudo-scalar	$\phi \xrightarrow{\text{CP}} -\phi$	$c, c', \bar{c}, \bar{c}', B, B', G^0, A^0$
Vector boson	$\phi^\mu \xrightarrow{\text{CP}} -\bar{\phi}_\mu$	$V^\mu, V'^\mu, A^\mu, Z^\mu, W^{\pm\mu}$
Weyl spinor	$\phi_\alpha \xrightarrow{\text{CP}} -\mathrm{e}^{-\mathrm{i}\omega_{\text{CP}}}\bar{\phi}^{\dot{\alpha}}$	$\lambda_\alpha, \lambda'_\alpha, h^i_{1\alpha}, h^i_{2\alpha}, l^i_\alpha, r_\alpha, q^i_\alpha, u_\alpha, d_\alpha, a_\alpha, \chi^\pm_\alpha, \chi^0_\alpha$
	$\bar{\phi}^{\dot{\alpha}} \xrightarrow{\text{CP}} \mathrm{e}^{-\mathrm{i}\omega_{\text{CP}}}\phi_\alpha$	

Table 2.2: CP transformations (ω_{CP} is an arbitrary phase)

relation

$$Q_{\text{em}} = T^3 + \frac{Y}{2}. \tag{2.5}$$

Furthermore, we assume lepton- and baryon-number conservation, CP invariance, and continuous R symmetry. The CP transformations are given in table 2.2. The continuous R transformation with transformation parameter α reads ($\hat{\phi}$ stands for an arbitrary superfield):

$$\hat{\phi}(x,\theta,\bar{\theta}) \to \mathrm{e}^{2\mathrm{i}n\alpha}\hat{\phi}(x, \mathrm{e}^{-\mathrm{i}\alpha}\theta, \mathrm{e}^{\mathrm{i}\alpha}\bar{\theta}). \tag{2.6}$$

For the R weights n of chiral multiplets we choose $n = 1/2$ for $\hat{H}_1, \hat{H}_2$, $n = 1/4$ for $\hat{L}, \hat{R}, \hat{Q}, \hat{U}, \hat{D}$. The R weights of the other multiplets are zero.

2.1 Classical action in superfield formulation

To construct the most general classical action, it is convenient to start from the superspace formulation, which is more condensed and incorporates supersymmetry manifestly. The supersymmetric kinetic and interaction part of the classical action reads

$$\begin{aligned}
\Gamma_{\text{susy}} \;=\; & \frac{1}{16}\sum_{i=1,2}\int \mathrm{d}^8z\, \bar{\hat{H}}_i e^{2g\hat{V}+g'Y\hat{V}'}\hat{H}_i + \frac{1}{16}\int \mathrm{d}^8z\, \bar{\hat{L}} e^{2g\hat{V}+g'Y\hat{V}'}\hat{L} \\
& + \frac{1}{16}\int \mathrm{d}^8z\, \bar{\hat{R}} e^{g'Y\hat{V}'}\hat{R} + \frac{1}{16}\int \mathrm{d}^8z\, \bar{\hat{Q}} e^{2g\hat{V}+g'Y\hat{V}'}\hat{Q} \\
& + \frac{1}{16}\int \mathrm{d}^8z\, \bar{\hat{U}} e^{g'Y\hat{V}'}\hat{U} + \frac{1}{16}\int \mathrm{d}^8z\, \bar{\hat{D}} e^{g'Y\hat{V}'}\hat{D} \\
& - \frac{1}{512g^2}\int \mathrm{d}^6z\, 2\mathrm{tr}[\hat{F}^\alpha\hat{F}_\alpha] - \frac{1}{128g'^2}\int \mathrm{d}^6z\, \hat{F}'^\alpha\hat{F}'_\alpha - \frac{v'}{8}\int \mathrm{d}^8z\, \hat{V}' \\
& + \Bigg[-\frac{f_R}{4}\int \mathrm{d}^6z\, \hat{H}_1^T(\mathrm{i}\sigma_2)\hat{L}\hat{R} - \frac{f_U}{4}\int \mathrm{d}^6z\, \hat{H}_2^T(\mathrm{i}\sigma_2)\hat{Q}\hat{U} \\
& \qquad - \frac{f_D}{4}\int \mathrm{d}^6z\, \hat{H}_1^T(\mathrm{i}\sigma_2)\hat{Q}\hat{D} + \frac{\mu}{4}\int \mathrm{d}^6z\, \hat{H}_1^T(\mathrm{i}\sigma_2)\hat{H}_2 + \text{c.c.}\Bigg]
\end{aligned} \tag{2.7}$$

with

$$\hat{F}^\alpha = \bar{\mathscr{D}}^2 \mathrm{e}^{-2g\hat{V}} \mathscr{D}^\alpha \mathrm{e}^{2g\hat{V}}, \qquad \hat{F}'^\alpha = \bar{\mathscr{D}}^2 \mathrm{e}^{-g'\hat{V}'} \mathscr{D}^\alpha \mathrm{e}^{g'\hat{V}'}. \tag{2.8}$$

Note that we added a Fayet-Iliopoulos term involving v' to the action. Explicit results of the classical action in terms of component fields can be found in appendix C.

2.2 Soft supersymmetry breaking

Since so far no supersymmetric partners of the Standard Model particles have been experimentally observed, supersymmetry has to be broken in such a way that the superpartners acquire a higher mass. Explicit supersymmetry breaking is also necessary to trigger the spontaneous breakdown of gauge symmetry by the Higgs mechanism. The MSSM restricts the breaking terms to the soft breaking terms found by Girardello and Grisaru [23]. These comprise mass terms for scalar fields and gauginos and up to trilinear interactions between the scalar components of either purely chiral or purely antichiral multiplets. They are generally believed to be a sufficient starting point for supersymmetry phenomenology.

In order to motivate our strategy how soft supersymmetry breaking can be introduced in the MSSM, we consider slepton and gaugino mass terms as examples. They can be written as

$$\int \mathrm{d}^8 z\, \theta^2 \bar{\theta}^2 \bar{\hat{L}} \mathrm{e}^{2g\hat{V}+g'Y\hat{V}'} \hat{L} \sim \int \mathrm{d}^4 x\, \bar{L} L, \tag{2.9}$$

$$\int \mathrm{d}^6 z\, \theta^2 \hat{F}'^\alpha \hat{F}'_\alpha \sim \int \mathrm{d}^4 x\, \lambda'^\alpha \lambda'_\alpha, \tag{2.10}$$

A customary tool to control symmetry-breaking terms is to couple them to external fields in such a way that the total action is again invariant. These external fields must have inhomogeneous transformation laws such that the breaking terms reappear for vanishing external fields. For this purpose we choose a chiral scalar spurion field $\hat{A}$ as proposed in [23]:

$$\hat{A} = \mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu} \left(A + \sqrt{2}\theta^\alpha a_\alpha + \theta^2 F_A \right). \tag{2.11}$$

By shifting the θ^2 component of this field, $F_A \to F_A + v_A$, the desired inhomogeneity is introduced into the transformation law, and for $\hat{A} \to \theta^2 v_A$ the above breaking terms are produced when the field is coupled as

$$\int \mathrm{d}^8 z\, \hat{A}\bar{\hat{A}} \bar{\hat{L}} \mathrm{e}^{2g\hat{V}+g'Y\hat{V}'} \hat{L} \quad \overset{\hat{A}\to\theta^2 v_A}{\longrightarrow} \quad v_A^2 \int \mathrm{d}^8 z\, \theta^2 \bar{\theta}^2 \bar{\hat{L}} \mathrm{e}^{2g\hat{V}+g'Y\hat{V}'} \hat{L}, \tag{2.12}$$

$$\int \mathrm{d}^6 z\, \hat{A} \hat{F}'^\alpha \hat{F}'_\alpha \quad \overset{\hat{A}\to\theta^2 v_A}{\longrightarrow} \quad v_A \int \mathrm{d}^6 z\, \theta^2 F^\alpha F_\alpha. \tag{2.13}$$

The BRS transformations of the spurion multiplet are given by

$$\mathrm{s}\, A = \sqrt{2}\varepsilon^\alpha a_\alpha - \mathrm{i}\xi^\mu \partial_\mu A, \tag{2.14a}$$

$$\mathrm{s}\, a_\alpha = \sqrt{2}\varepsilon_\alpha (F_A + v_A) + \mathrm{i}\sqrt{2}\sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\partial_\mu A - \mathrm{i}\xi^\mu \partial_\mu a_\alpha, \tag{2.14b}$$

$$\mathrm{s}\, F_A = \mathrm{i}\sqrt{2}\partial_\mu a^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} - \mathrm{i}\xi^\mu \partial_\mu F_A. \tag{2.14c}$$

The spurion fields provide an adequate characterization of soft supersymmetry breaking versus soft breaking of gauge invariance. However, since the spurion field is dimensionless, it can in principle appear in arbitrary powers in the action. In this way infinitely many new renormalization parameters could appear in higher orders of perturbation theory, but it has been shown in [19] that the infinite towers of terms in the action are irrelevant for physical processes and it suffices to include terms up to first order in each $\hat A$, $\bar{\hat A}$ and $\hat A\bar{\hat A}$.

An alternative approach to soft supersymmetry breaking has been presented in [13], using no superfield but two ordinary scalar fields of dimension 1 which form a BRS doublet. Because of their dimension, only finitely many terms can be formed, and since BRS doublets cannot contribute to the cohomology, it is clear that no new anomaly can emerge. We can pass over from the chiral spurion field $\hat A$ to the BRS doublet A_1, A_2 (originally named u, v) of [13] by identifying

$$A = 0, \qquad a^\alpha = \sqrt{2}\varepsilon^\alpha A_1, \qquad F_A = A_2. \tag{2.15}$$

The transformation law for the BRS doublet[2] A_1, A_2 is

$$\mathrm{s}\, A_1 = A_2 + v_A - \mathrm{i}\xi^\mu \partial_\mu A_1, \tag{2.16a}$$

$$\mathrm{s}\, A_2 = \mathrm{i}2\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\partial_\mu A_1 - \mathrm{i}\xi^\mu \partial_\mu A_2. \tag{2.16b}$$

From now on we consider a restricted classical action (and later on the vertex functional) where the spurion multiplet is always parametrized as in (2.15). Equivalently, it depends on A_1 only through the combination $a^\alpha = \sqrt{2}\varepsilon^\alpha A_1$.

Since we have put $A = 0$ in (2.15), powers of $\hat A$ vanish and the most general real and chiral polynomials in $\hat A$, $\bar{\hat A}$ are given by

$$\mathscr{P}_{\mathrm{real}} = c_{\mathrm{real},1}(\hat A + \bar{\hat A}) + c_{\mathrm{real},2}\hat A\bar{\hat A}, \qquad \mathscr{P}_{\mathrm{chiral}} = c_{\mathrm{chiral}}\hat A. \tag{2.17}$$

[2]Strictly speaking, the BRS doublet is given by A_1 and $A_2' \equiv A_2 + v_A - \mathrm{i}\xi^\mu\partial_\mu A_1$, since $\mathrm{s}\, A_1 = A_2'$, $\mathrm{s}\, A_2' = 0$.

By inserting polynomials of the form (2.17) in Γ wherever possible, we obtain

$$\begin{aligned}
\Gamma_{\rm soft} &= \frac{1}{16}\sum_{i=1,2}\int \mathrm{d}^8z\, \mathscr{P}_{H_i}\bar{\hat{H}}_i e^{2g\hat{V}+g'Y\hat{V}'}\hat{H}_i + \frac{1}{16}\int \mathrm{d}^8z\, \mathscr{P}_L \bar{\hat{L}} e^{2g\hat{V}+g'Y\hat{V}'}\hat{L} \\
&\quad + \frac{1}{16}\int \mathrm{d}^8z\, \mathscr{P}_R \bar{\hat{R}} e^{g'Y\hat{V}'}\hat{R} + \frac{1}{16}\int \mathrm{d}^8z\, \mathscr{P}_Q \bar{\hat{Q}} e^{2g\hat{V}+g'Y\hat{V}'}\hat{Q} \\
&\quad + \frac{1}{16}\int \mathrm{d}^8z\, \mathscr{P}_U \bar{\hat{U}} e^{g'Y\hat{V}'}\hat{U} + \frac{1}{16}\int \mathrm{d}^8z\, \mathscr{P}_D \bar{\hat{D}} e^{g'Y\hat{V}'}\hat{D} \\
&\quad + \Bigg[-\frac{c_V}{512g^2}\int \mathrm{d}^6z\, \hat{A}2\mathrm{tr}[\hat{F}^\alpha\hat{F}_\alpha] - \frac{c_{V'}}{128g'^2}\int \mathrm{d}^6z\, \hat{A}\hat{F}'^\alpha\hat{F}'_\alpha \\
&\qquad - \frac{c_{HLR}}{4}\int \mathrm{d}^6z\, \hat{A}\hat{H}_1^T(\mathrm{i}\sigma_2)\hat{L}\hat{R} - \frac{c_{HQU}}{4}\int \mathrm{d}^6z\, \hat{A}\hat{H}_2^T(\mathrm{i}\sigma_2)\hat{Q}\hat{U} \\
&\qquad - \frac{c_{HQD}}{4}\int \mathrm{d}^6z\, \hat{A}\hat{H}_1^T(\mathrm{i}\sigma_2)\hat{Q}\hat{D} + \frac{c_{HH}}{4}\int \mathrm{d}^6z\, \hat{A}\hat{H}_1^T(\mathrm{i}\sigma_2)\hat{H}_2 + \mathrm{c.c.}\Bigg] \qquad (2.18)
\end{aligned}$$

with

$$\mathscr{P}_\phi = c_{\phi 1}(\hat{A}+\bar{\hat{A}}) + c_{\phi 2}\bar{\hat{A}}\hat{A}, \qquad \phi = H_1, H_2, L, R, Q, U, D \qquad (2.19)$$

and $c_{\phi 1}$, $c_{\phi 2}$ real.

2.3 External fields and elimination of auxiliary fields

The BRS transformations of most of the fields [see eqs. (B.97a)–(B.99i)] are non-linear in the propagating fields, i.e. they correspond to composite operators in the quantum theory. Therefore, we couple the non-linear BRS transformations to external source fields Y, y in such a way that differentiation of the action with respect to these external fields yields the desired non-linear field expressions. This yields the external field part of the classical action:

$$\begin{aligned}
\Gamma_{\rm ext} &= \int \mathrm{d}^4x \Big[2\mathrm{tr}\left(Y_V^\mu \mathrm{s}\, V_\mu + y_\lambda^\alpha \mathrm{s}\, \lambda_\alpha + \bar{y}_{\lambda\dot\alpha}\mathrm{s}\, \bar\lambda^{\dot\alpha} + Y_c \mathrm{s}\, c\right) + y_{\lambda'}^\alpha \mathrm{s}\, \lambda'_\alpha + \bar{y}_{\lambda'\dot\alpha}\mathrm{s}\, \bar\lambda'^{\dot\alpha} \\
&\quad + Y_L^T \mathrm{s}\, L + \bar{Y}_L \mathrm{s}\, \bar{L}^T + y_l^{\alpha T}\mathrm{s}\, l_\alpha + \bar{y}_{l\dot\alpha}\mathrm{s}\, \bar{l}^{\dot\alpha T} + Y_R \mathrm{s}\, R + \bar{Y}_R \mathrm{s}\, \bar{R} + y_r^\alpha \mathrm{s}\, r_\alpha + \bar{y}_{r\dot\alpha}\mathrm{s}\, \bar{r}^{\dot\alpha}\Big] \\
&\quad + \text{analogous quark and Higgs terms.} \qquad (2.20)
\end{aligned}$$

We do not introduce Y fields corresponding to the D and F fields as these fields will be eliminated later. The Y fields are invariant under BRS transformations:

$$\mathrm{s}\, Y = 0, \qquad \mathrm{s}\, y = 0. \qquad (2.21)$$

Since $\mathrm{s}^{\,2} = 0$, $\Gamma_{\rm ext}$ is BRS invariant, and we obtain

$$\mathrm{s}\,(\Gamma_{\rm susy} + \Gamma_{\rm soft} + \Gamma_{\rm ext}) = 0. \qquad (2.22)$$

The definition of the Y fields is such that $\Gamma_{\rm ext}$ is bosonic and real.[3] The gauge transformations of the external fields Y are such that $\Gamma_{\rm ext}$ is gauge invariant.

The auxiliary fields can be eliminated from the action and the BRS transformations by using the equations of motion. The resulting replacements are

$$\begin{aligned}
D_V &= -g\mathrm{T}^a\left[\bar{H}_1\mathrm{T}^a H_1 + \bar{H}_2\mathrm{T}^a H_2 + \bar{L}\mathrm{T}^a L + \bar{Q}\mathrm{T}^a Q\right] - y^\alpha_\lambda \varepsilon_\alpha + \bar{y}_{\lambda\dot\alpha}\bar\varepsilon^{\dot\alpha} \\
&\quad + c_V A_1 \varepsilon^\alpha \lambda_\alpha + c_V \bar{A}_1 \bar\varepsilon_{\dot\alpha}\bar\lambda^{\dot\alpha}, && \text{(2.23a)} \\
D'_V &= v' - \frac{g'}{2}\left[\bar{H}_1 Y H_1 + \bar{H}_2 Y H_2 + \bar{L} Y L + \bar{R} Y R + \bar{Q} Y Q + \bar{U} Y U + \bar{D} Y D\right] \\
&\quad - y^\alpha_{\lambda'}\varepsilon_\alpha + \bar{y}_{\lambda'\dot\alpha}\bar\varepsilon^{\dot\alpha} + c_{V'}A_1\varepsilon^\alpha\lambda'_\alpha + c_{V'}\bar{A}_1\bar\varepsilon_{\dot\alpha}\bar\lambda'^{\dot\alpha}, && \text{(2.23b)} \\
F_1 &= -f_R(\mathrm{i}\sigma_2)L^*\bar{R} - f_D(\mathrm{i}\sigma_2)Q^*\bar{D} + \mu(\mathrm{i}\sigma_2)H_2^* + \sqrt{2}\bar{y}_{h_1\dot\alpha}\bar\varepsilon^{\dot\alpha} \\
&\quad - c_{H1}\left[(A_2+v_A)H_1 + \sqrt{2}A_1\varepsilon^\alpha h_{1\alpha}\right], && \text{(2.23c)} \\
F_2 &= -f_U(\mathrm{i}\sigma_2)Q^*\bar{U} - \mu(\mathrm{i}\sigma_2)H_1^* + \sqrt{2}\bar{y}_{h_2\dot\alpha}\bar\varepsilon^{\dot\alpha} \\
&\quad - c_{H2}\left[(A_2+v_A)H_2 + \sqrt{2}A_1\varepsilon^\alpha h_{2\alpha}\right], && \text{(2.23d)} \\
F_L &= f_R(\mathrm{i}\sigma_2)H_1^*\bar{R} + \sqrt{2}\bar{y}_{l\dot\alpha}\bar\varepsilon^{\dot\alpha} - c_{L1}\left[(A_2+v_A)L + \sqrt{2}A_1\varepsilon^\alpha l_\alpha\right], && \text{(2.23e)} \\
F_R &= -f_R\bar{H}_1(\mathrm{i}\sigma_2)L^* + \sqrt{2}\bar{y}_{r\dot\alpha}\bar\varepsilon^{\dot\alpha} - c_{R1}\left[(A_2+v_A)R + \sqrt{2}A_1\varepsilon^\alpha r_\alpha\right], && \text{(2.23f)} \\
F_Q &= f_D(\mathrm{i}\sigma_2)H_1^*\bar{D} + f_U(\mathrm{i}\sigma_2)H_2^*\bar{U} + \sqrt{2}\bar{y}_{q\dot\alpha}\bar\varepsilon^{\dot\alpha} \\
&\quad - c_{Q1}\left[(A_2+v_A)Q + \sqrt{2}A_1\varepsilon^\alpha q_\alpha\right], && \text{(2.23g)} \\
F_U &= -f_U\bar{H}_2(\mathrm{i}\sigma_2)Q^* + \sqrt{2}\bar{y}_{u\dot\alpha}\bar\varepsilon^{\dot\alpha} - c_{U1}\left[(A_2+v_A)U + \sqrt{2}A_1\varepsilon^\alpha u_\alpha\right], && \text{(2.23h)} \\
F_D &= -f_D\bar{H}_1(\mathrm{i}\sigma_2)Q^* + \sqrt{2}\bar{y}_{d\dot\alpha}\bar\varepsilon^{\dot\alpha} - c_{D1}\left[(A_2+v_A)D + \sqrt{2}A_1\varepsilon^\alpha d_\alpha\right]. && \text{(2.23i)}
\end{aligned}$$

After eliminating the auxiliary fields, the nilpotency is respected only on shell, i.e.

$$\mathrm{s}^2\phi \;=\; 0 + \text{equations of motion.} \tag{2.24}$$

Since the action is stationary with respect to the auxiliary fields after their elimination, the invariance $\mathrm{s}\,(\Gamma_{\rm susy}+\Gamma_{\rm soft}+\Gamma_{\rm ext})=0$ still holds if the results (2.23) are used in the BRS transformations (B.97a)–(B.99i). Furthermore, the bilinear terms in the external fields introduced in [13, 16, 19, 21, 12] are automatically included in $\Gamma_{\rm ext}$ after inserting (2.23).

2.4 Soft supersymmetry-breaking parameters

After elimination of the auxiliary fields, we can evaluate the physical terms induced by the shift in the spurion field. We obtain the following contributions in the limit $\hat{A}=\theta^2 v_A$

[3] In our conventions y_λ is a bosonic spinor and its complex conjugate is $\bar{y}_\lambda$. Conversely, Y_L is a fermionic scalar and its complex conjugate is $Y_L^* = -\bar{Y}_L$.

and $Y = 0$:

$$\begin{aligned}\Gamma_{\text{soft}}|_{\hat{A}=Y=0} &= -\int \mathrm{d}^4x \, \left(m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_{\tilde{l}}^2|L|^2 + m_{\tilde{e}}^2|R|^2\right) \\ &\quad -\int \mathrm{d}^4x \, \tfrac{1}{2}\left[M_2 2\mathrm{tr}\,(\lambda^\alpha\lambda_\alpha) + M_1\lambda'^\alpha\lambda'_\alpha + \text{c.c.}\right] \\ &\quad -\int \mathrm{d}^4x \, \left[m_3^2 H_1^T(\mathrm{i}\sigma_2)H_2 + f_R A_e H_1^T(\mathrm{i}\sigma_2)LR + \text{c.c.}\right] \\ &\quad + \text{ analogous quark terms.} \end{aligned} \tag{2.25}$$

with the explicit results

$$m_i^2 = v_A^2\left(c_{H_i1}^2 - c_{H_i2}\right) + Y\tfrac{g'}{2}v', \qquad i = 1,2, \tag{2.26a}$$
$$m_{\tilde{l}}^2 = v_A^2\left(c_{L1}^2 - c_{L2}\right) + Y\tfrac{g'}{2}v', \tag{2.26b}$$
$$m_{\tilde{r}}^2 = v_A^2\left(c_{R1}^2 - c_{R2}\right) + Y\tfrac{g'}{2}v', \tag{2.26c}$$
$$m_3^2 = c_{HH}v_A - \mu v_A(c_{H_11} + c_{H_21}), \tag{2.26d}$$
$$M_1^2 = -2v_A c'_V, \tag{2.26e}$$
$$M_2^2 = -2v_A c_V, \tag{2.26f}$$
$$f_R A_e = v_A(c_{L1}f_R + c_{R1}v_A + c_{H_11}f_R - c_{HLR}), \tag{2.26g}$$

where Y symbolizes the hypercharge of the corresponding field as defined in table 2.1. The equations (2.26) show that only certain combinations of the spurion parameters are physical parameters of the MSSM, namely m_1^2, m_2^2, $m_{\tilde{l}}^2$, $m_{\tilde{e}}^2$, M_1, M_2, m_3^2, A_e, and similar parameters $m_{\tilde{q}}^2$, $m_{\tilde{u}}^2$, $m_{\tilde{d}}^2$, A_u, A_d for the quark fields. The remaining parameters are irrelevant for physical observables [19].

2.5 Field parametrization

The question of field parametrization is a central aspect of spontaneously broken gauge theories. Up to now we have introduced fields which are multiplets under the SU(2)×U(1) gauge group (*symmetric* fields). They do not correspond directly to mass eigenstates. However, we are still free to choose a basis of independent *physical* fields in terms of which the vertex functional and the symmetry operators are parametrized.

In this section we introduce a set of fields that correspond to mass eigenstates. Schematically, we define the physical fields as general linear combinations of the symmetric fields with $\hbar$-dependent coefficients:

$$\phi_i^{\text{sym}} = R_{ij}\phi_j^{\text{phys}}, \qquad R_{ij} = \sum_{n=0}^{\infty} R_{ij}^{(n)}\hbar^n. \tag{2.27}$$

Later in section 4, it will be shown that by adjusting the $R_{ij}^{(n)}$ the mass matrix can be diagonalized[4] to all orders in $\hbar$, i.e.

$$\mathrm{Re}\Gamma_{\phi_i^{\mathrm{phys}}\phi_j^{\mathrm{phys}}}\Big|_{p^2=m_i^2} = 0, \tag{2.28}$$

where m_i is the mass of the field ϕ_i^{phys}. In this way, the ϕ_i^{phys} can be identified with mass eigenstates up to effects due to non-vanishing imaginary parts of self energies.

From the phenomenological point of view it is certainly desirable to work with physical fields and on-shell normalization conditions. In some cases, we are also forced to use a specific parametrization due to IR finiteness as we will see in section 3.6. Classically, $R^{(0)}$ can be chosen as a rotation matrix. Loop corrections lead in general to remixing of the classically diagonal fields so that the matrix R has to be a general non-singular matrix. Since mixing is only possible between fields with matching quantum numbers, the general demixing matrix R can be decomposed into several submatrices R_ϕ for the various kinds of fields.

The mass eigenstates of the gauge bosons are the Z boson, $W^\pm$ boson, and the photon. We write

$$\begin{pmatrix} V_\mu' \\ V_\mu^3 \end{pmatrix} = R_V \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}, \qquad \frac{1}{\sqrt{2}}(V_\mu^1 \mp \mathrm{i}V_\mu^2) = W_\mu^\pm. \tag{2.29}$$

Here, R_V is a real 2×2 matrix. Similarly, we write for ghosts and antighosts

$$\begin{pmatrix} c' \\ c^3 \end{pmatrix} = R_c \begin{pmatrix} c_A \\ c_Z \end{pmatrix}, \qquad \frac{1}{\sqrt{2}}(c^1 \mp \mathrm{i}c^2) = c^\pm, \tag{2.30a}$$

$$\begin{pmatrix} \bar{c}' \\ \bar{c}^3 \end{pmatrix} = R_{\bar{c}} \begin{pmatrix} \bar{c}_A \\ \bar{c}_Z \end{pmatrix}, \qquad \frac{1}{\sqrt{2}}(\bar{c}^1 \mp \mathrm{i}\bar{c}^2) = \bar{c}^\pm \tag{2.30b}$$

with real 2×2 matrices R_c, $R_{\bar{c}}$. As usual, we will couple the auxiliary fields $B^{(\prime)}$ to linear gauge-fixing functions $\mathcal{F}^{(\prime)} = \partial^\mu V_\mu^{(\prime)} + \cdots$ in the gauge-fixing term (see section 2.6). For linear gauge fixing the B fields have no interaction vertices, and they are auxiliary fields. Thus, independent mixing matrices for B fields are not required and we choose

$$\begin{pmatrix} B' \\ B_3 \end{pmatrix} = R_V^{-1T} \begin{pmatrix} B_A \\ B_Z \end{pmatrix}, \qquad \frac{1}{\sqrt{2}}(B^1 \mp \mathrm{i}B^2) = B^\pm. \tag{2.31}$$

For the scalar Higgs fields, we have three real 2×2 matrices R_{H^0}, $R_{H^\pm}$, and R_{A^0} for the neutral CP-even, CP-odd, and charged sector, respectively. With the help of the parametrization

$$\bar{H}_1(\mathrm{i}\sigma_2) = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(\phi_1^0 + \mathrm{i}\rho_1^0) \end{pmatrix}, \qquad H_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(\phi_2^0 + \mathrm{i}\rho_2^0) \end{pmatrix} \tag{2.32}$$

[4]To be precise, this diagonalization is not possible for the mixings between Higgs fields and the unphysical longitudinal gauge bosons and B fields.

we introduce the matrices R_{H^0}, R_{A^0}, $R_{H^\pm}$ as follows:

$$\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} = R_{H^0} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}, \quad \begin{pmatrix} \rho_1^0 \\ \rho_2^0 \end{pmatrix} = R_{A^0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \quad \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = R_{H^\pm} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}. \quad (2.33)$$

The electric charges of the fields are indicated in the superscript. The neutral fields are real and the charged ones complex with $G^- = \bar{G}^+$, $H^- = \bar{H}^+$. Furthermore, the fields A^0 and G^0 are CP odd. The notation anticipates that the normalization conditions will be such that the $G^{0,\pm}$ correspond to the unphysical Goldstone degrees of freedom.

Gauginos and Higgsinos also mix. This requires two complex 2×2 matrices for the charged sector and one complex 4×4 matrix for the neutral sector:

$$\begin{pmatrix} \lambda_\alpha^+ \\ h_{2\alpha}^1 \end{pmatrix} = R_{\chi^+} \begin{pmatrix} \chi_{1\alpha}^+ \\ \chi_{2\alpha}^+ \end{pmatrix}, \quad \begin{pmatrix} \lambda_\alpha^- \\ h_{1\alpha}^2 \end{pmatrix} = R_{\chi^-} \begin{pmatrix} \chi_{1\alpha}^- \\ \chi_{2\alpha}^- \end{pmatrix}, \quad \begin{pmatrix} \lambda'_\alpha \\ \lambda_\alpha^3 \\ h_{1\alpha}^1 \\ h_{2\alpha}^2 \end{pmatrix} = R_{\chi^0} \begin{pmatrix} \chi_{1\alpha}^0 \\ \chi_{2\alpha}^0 \\ \chi_{3\alpha}^0 \\ \chi_{4\alpha}^0 \end{pmatrix}. \quad (2.34)$$

In order to respect all normalization conditions in the matter sector of the MSSM (see section 4), we introduce normalization factors for the left-handed fermions:

$$q_\alpha^1 = R_{u_L} u_{L\alpha}, \qquad q_\alpha^2 = R_{d_L} d_{L\alpha}, \quad (2.35)$$

$$l_\alpha^1 = R_{\nu_L} \nu_{L\alpha}, \qquad l_\alpha^2 = R_{e_L} e_{L\alpha}. \quad (2.36)$$

Since we consider only one generation, there is no mixing within quarks and leptons. However, there is in general a mixing of the superpartners of the left- and right-handed sfermions:

$$\begin{pmatrix} L^2 \\ \bar{R} \end{pmatrix} = R_{\tilde{e}} \begin{pmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{pmatrix}, \qquad L^1 = \tilde{\nu}_1, \quad (2.37)$$

$$\begin{pmatrix} Q^1 \\ \bar{U} \end{pmatrix} = R_{\tilde{u}} \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{pmatrix}, \qquad \begin{pmatrix} Q^2 \\ \bar{D} \end{pmatrix} = R_{\tilde{d}} \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix}. \quad (2.38)$$

2.6 Gauge fixing

To quantize the theory, gauge fixing is necessary. We use a gauge-fixing and ghost term of the general form

$$\begin{aligned} \Gamma_{\text{g.f.}} &= \int \mathrm{d}^4x\, \mathrm{s} \left[2\mathrm{tr}\left(\bar{c}\mathscr{F} + \tfrac{1}{2}\zeta \bar{c} B\right) + \bar{c}'\mathscr{F}' + \tfrac{1}{2}\zeta' \bar{c}' B' \right] \\ &= \int \mathrm{d}^4x\, 2\mathrm{tr}\left(B\mathscr{F} + \tfrac{1}{2}\zeta B^2 - \bar{c}\mathrm{s}\,\mathscr{F} - \mathrm{i}\xi^\mu \partial_\mu \bar{c}\mathscr{F} - \mathrm{i}\zeta \varepsilon^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\varepsilon}^{\dot{\alpha}} \bar{c} \partial_\mu \bar{c}\right) \\ &\quad + \int \mathrm{d}^4x \left(B'\mathscr{F}' + \tfrac{1}{2}\zeta' B'^2 - \bar{c}'\mathrm{s}\,\mathscr{F}' - \mathrm{i}\xi^\mu \partial_\mu \bar{c}'\mathscr{F}' - \mathrm{i}\zeta' \varepsilon^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\varepsilon}^{\dot{\alpha}} \bar{c}' \partial_\mu \bar{c}' \right) \quad (2.39) \end{aligned}$$

with gauge-fixing functions $\mathscr{F} = \mathscr{F}^a T^a$, $\mathscr{F}'$. The auxiliary fields B, B' are introduced as the BRS transformations of the antighosts $\bar{c}$, $\bar{c}'$ (see appendix B). In this way, $\mathrm{s}^2 = 0$ holds also on these fields and the gauge-fixing and ghost action (2.39) is BRS invariant, $\mathrm{s}\,\Gamma_{\mathrm{g.f.}} = 0$. For perturbative calculations, the B fields can be readily eliminated using their equations of motion, yielding the usual form of the gauge-fixing term $-2\mathrm{tr}(\mathscr{F}^2)/(2\zeta) - (\mathscr{F}')^2/(2\zeta')$.

Analogously to the decomposition of the symmetric gauge-boson fields into mass eigenstates (2.29) we rewrite the gauge-fixing functions as

$$\begin{pmatrix} \mathscr{F}' \\ \mathscr{F}^3 \end{pmatrix} = R_V \begin{pmatrix} \mathscr{F}_A \\ \mathscr{F}_Z \end{pmatrix}, \qquad \frac{1}{2}(\mathscr{F}^1 \mp \mathrm{i}\mathscr{F}^2) = \mathscr{F}^\pm. \tag{2.40}$$

For the gauge-fixing functions $\mathscr{F}^\pm$, $\mathscr{F}_A$, and $\mathscr{F}_Z$ we choose a linear function in propagating fields. Especially useful for phenomenological calculations are the gauge-fixing functions

$$\mathscr{F}^\pm = \partial^\mu W_\mu^\pm \pm \mathrm{i} M_W (\zeta^{G^\pm} G^\pm + \zeta^{H^\pm} H^\pm), \tag{2.41a}$$

$$\begin{pmatrix} \mathscr{F}_A \\ \mathscr{F}_Z \end{pmatrix} = \begin{pmatrix} \partial^\mu A_\mu \\ \partial^\mu Z_\mu \end{pmatrix} + M_Z \begin{pmatrix} \zeta^{AG^0} & \zeta^{AA^0} \\ \zeta^{ZG^0} & \zeta^{ZA^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \tag{2.41b}$$

where M_W and M_Z are the (real) masses of the $W^\pm$ and Z bosons.

Such a gauge-fixing condition gives masses to the unphysical Goldstone bosons, and it allows to eliminate mixing terms between Goldstone bosons and longitudinal gauge bosons from the action by a suitable choice of the gauge parameters $\zeta^{G^\pm}$, ζ^{AG^0}, and ζ^{ZG^0}. Furthermore, the physical Higgs bosons $H^\pm$ and A^0 can be excluded from the gauge-fixing functions by defining $\zeta^{H^\pm} = \zeta^{AA^0} = \zeta^{ZA^0} = 0$. In the actual renormalization analysis of section 3 we stay general and treat all ζ parameters as independent.

The gauge-fixing functions (2.41) exhibit a severe problem: they are not gauge covariant, and the gauge-fixing term (2.39) is, therefore, not invariant with respect to *rigid* gauge transformations. Since rigid invariance is an important ingredient of the definition of the MSSM, it has to be restored.

The violation of rigid gauge invariance is particularly apparent if $\mathscr{F}'$, $\mathscr{F}^a$ are rewritten in terms of the original Higgs fields

$$\mathscr{F}^a = \partial^\mu V_\mu^a - (\mathrm{i}\bar{\mathbf{v}}_1^a H_1 + \mathrm{i}\bar{\mathbf{v}}_2^a H_2 + \mathrm{h.c.}), \tag{2.42a}$$

$$\mathscr{F}' = \partial^\mu V_\mu' - (\mathrm{i}\bar{\mathbf{v}}_1' H_1 + \mathrm{i}\bar{\mathbf{v}}_2' H_2 + \mathrm{h.c.}) \tag{2.42b}$$

with fixed isospin doublets $\mathbf{v}'_{1,2}$, $\mathbf{v}^a_{1,2}$. From comparing (2.42) with (2.41) we get

$$\mathbf{v}_1^1 = \begin{pmatrix} 0 \\ (\mathbf{v}_1^1)^2 \end{pmatrix}, \quad \mathbf{v}_1^2 = \begin{pmatrix} 0 \\ \mathrm{i}(\mathbf{v}_1^1)^2 \end{pmatrix}, \quad \mathbf{v}_1^3 = \begin{pmatrix} (\mathbf{v}_1^3)^1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_1' = \begin{pmatrix} (\mathbf{v}_1')^1 \\ 0 \end{pmatrix}, \tag{2.43a}$$

$$\mathbf{v}_2^1 = \begin{pmatrix} (\mathbf{v}_2^1)^1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2^2 = \begin{pmatrix} -\mathrm{i}(\mathbf{v}_2^1)^1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2^3 = \begin{pmatrix} 0 \\ (\mathbf{v}_2^3)^2 \end{pmatrix}, \quad \mathbf{v}_2' = \begin{pmatrix} 0 \\ (\mathbf{v}_2')^2 \end{pmatrix}. \tag{2.43b}$$

where the vanishing components are in agreement with electric charge conservation. The six non-vanishing components are given by

$$\begin{pmatrix} (\mathbf{v}_1^1)^2 \\ (\mathbf{v}_2^1)^1 \end{pmatrix} = \frac{M_W}{\sqrt{2}} R_{H^\pm}^{-1T} \begin{pmatrix} \zeta^{G^\pm} \\ \zeta^{H^\pm} \end{pmatrix}, \tag{2.44a}$$

$$\begin{pmatrix} -(\mathbf{v}_1')^1 & (\mathbf{v}_2')^2 \\ -(\mathbf{v}_1^3)^1 & (\mathbf{v}_2^3)^2 \end{pmatrix} = \frac{M_Z}{\sqrt{2}} R_{A^0}^{-1} \begin{pmatrix} \zeta^{AG^0} & \zeta^{AA^0} \\ \zeta^{ZG^0} & \zeta^{ZA^0} \end{pmatrix} R_V \,. \tag{2.44b}$$

The solution of the non-covariance problem is to make the isovectors $\mathbf{v}_i$ *local*, i.e. to introduce eight external isospin doublets $(\Phi_i^a + \mathbf{v}_i^a)$, $(\Phi_i' + \mathbf{v}_i')$ where the $\mathbf{v}_i$ appear as constant shifts. Using these external fields, we redefine the gauge-fixing functions into the final form,

$$\mathscr{F}^a = \partial^\mu V_\mu^a - [\mathrm{i}(\bar{\Phi}_1^a + \bar{\mathbf{v}}_1^a)H_1 + \mathrm{i}(\bar{\Phi}_2^a + \bar{\mathbf{v}}_2^a)H_2 + \mathrm{h.c.}], \tag{2.45a}$$

$$\mathscr{F}' = \partial^\mu V_\mu' - [\mathrm{i}(\bar{\Phi}_1' + \bar{\mathbf{v}}_1')H_1 + \mathrm{i}(\bar{\Phi}_2' + \bar{\mathbf{v}}_2')H_2 + \mathrm{h.c.}], \tag{2.45b}$$

which is used from now on. The point in the replacement $\mathbf{v}_i \to (\Phi_i + \mathbf{v}_i)$ is that non-trivial gauge transformation laws can be assigned to the external fields Φ_i. With the choice

$$\delta_{\omega'}\Phi_i^a = -\mathrm{i}g'\omega'\frac{Y}{2}(\Phi_i^a + \mathbf{v}_i^a), \qquad \delta_{\omega'}\Phi_i' = -\mathrm{i}g'\omega'\frac{Y}{2}(\Phi_i' + \mathbf{v}_i'), \tag{2.46a}$$

$$\delta_\omega\Phi_i^a = -\mathrm{i}g\omega(\Phi_i^a + \mathbf{v}_i^a) - \mathrm{i}g(\mathrm{i}\epsilon^{bca})\omega^b(\Phi_i^c + \mathbf{v}_i^c), \qquad \delta_\omega\Phi_i' = -\mathrm{i}g\,\omega(\Phi_i' + \mathbf{v}_i'), \tag{2.46b}$$

the two 8-component objects $(\Phi_i + \mathbf{v}_i)$ for $i = 1, 2$ transform according to the product of the adjoint and the fundamental representation of SU(2)×U(1), where the hypercharges in (2.46) are defined as those of the corresponding Higgs fields (see table 2.1). As a consequence, the gauge-fixing term (2.39) becomes invariant under rigid gauge transformations. For $\Phi_i = 0$ the gauge-fixing functions reduce to (2.41).

For the BRS transformations of the Φ_i, we introduce further external fields Ψ_i^a, Ψ_i' with $Q_{\Phi\Pi} = 1$ which form BRS doublets[5] together with Φ_i^a, Φ_i'. The BRS transformations of the fields Φ_i, Ψ_i are defined as follows

$$\mathrm{s}\,\Phi_i^a = \Psi_i^a - \mathrm{i}\xi^\mu\partial_\mu\Phi_i^a, \qquad \mathrm{s}\,\Psi_i^a = 2\mathrm{i}\varepsilon^\alpha\sigma_{\alpha\dot{\alpha}}^\mu\bar{\varepsilon}^{\dot{\alpha}}\partial_\mu\Phi_i^a - \mathrm{i}\xi^\mu\partial_\mu\Psi_i^a, \tag{2.47a}$$

$$\mathrm{s}\,\Phi_i' = \Psi_i' - \mathrm{i}\xi^\mu\partial_\mu\Phi_i', \qquad \mathrm{s}\,\Psi_i' = 2\mathrm{i}\varepsilon^\alpha\sigma_{\alpha\dot{\alpha}}^\mu\bar{\varepsilon}^{\dot{\alpha}}\partial_\mu\Phi_i' - \mathrm{i}\xi^\mu\partial_\mu\Psi_i'. \tag{2.47b}$$

The rigid gauge transformations and hypercharges of Ψ_i are defined analogously to the ones of Φ_i but without $\mathbf{v}_i$.

2.7 Classical action

With the gauge fixing introduced in the above section 2.6 we can complete the discussion of the classical approximation of the MSSM. Its building blocks are the manifestly

[5] Footnote 2 applies also here.

supersymmetric part, Γ_{susy} (2.7), the soft-supersymmetry breaking part, Γ_{soft} (2.18), the external-field part, Γ_{ext} (2.20), and the gauge-fixing term, $\Gamma_{\text{g.f.}}$ (2.39). Hence, a classical action satisfying all symmetries reads

$$\Gamma_{\text{cl}} = \Gamma_{\text{susy}} + \Gamma_{\text{soft}} + \Gamma_{\text{ext}} + \Gamma_{\text{g.f.}}. \tag{2.48}$$

Although spontaneous symmetry breaking has already been taken into account in the definition of the gauge-fixing term, the finite vacuum expectation values of the Higgs bosons have not been introduced so far. Therefore, no physical particle masses are generated in (2.48). In the present framework, where rigid gauge invariance is ensured by using the external fields Φ_i in the gauge-fixing term, the vacuum expectation values arise along with a new class of free parameters. The situation is similar to the one in [14, 24], where, however, the multiplet structure of the external fields Φ_i is simpler.

From the tensor fields Φ_i^a, Ψ_i^a we can form SU(2) doublets by contraction with the generators

$$\hat{\Phi}_i + \hat{\mathbf{v}}_i = 2T^a(\Phi_i^a + \mathbf{v}_i^a), \qquad \hat{\Psi} = 2T^a\Psi_i^a. \tag{2.49}$$

The external fields $(\hat{\Phi}_i + \hat{\mathbf{v}}_i)$, $(\Phi_i' + \mathbf{v}_i')$ have the same quantum numbers and rigid gauge-transformation laws as the physical Higgs fields H_i. However, the BRS transformations of the Φ fields prevent them from appearing arbitrarily in the classical action. A possibility to introduce them into the classical action consists in redefining the BRS transformation laws of the Higgs fields

$$\mathrm{s}\, H_i \to (\mathrm{s}\, H_i)|_{H_i \to H_i^{\text{eff}}} - x_i(\hat{\Psi}_i - \mathrm{i}\xi^\mu\partial_\mu\hat{\Phi}_i) - x_i'(\Psi_i' - \mathrm{i}\xi^\mu\partial_\mu\Phi_i') \tag{2.50}$$

and simultaneously substituting

$$H_i \to H_i^{\text{eff}} = H_i + x_i(\hat{\Phi}_i + \hat{\mathbf{v}}_i) + x_i'(\Phi_i' + \mathbf{v}') \tag{2.51}$$

everywhere in the classical action apart from the gauge-fixing term. The new parameters x_i, x_i' are arbitrary and have to be fixed by normalization conditions. The H_i^{eff} obey the BRS transformation law of the original Higgs field. In this way, the complete vacuum expectation value of the effective Higgs fields H_i^{eff} is generated via the fields $\hat{\Phi}_i + \hat{\mathbf{v}}_i$, $\Phi_i' + \mathbf{v}_i'$:

$$\langle H_i^{\text{eff}}\rangle = H_i^{\text{eff}}|_{\phi\to 0} = 2x_iT^a\mathbf{v}_i^a + x_i'\mathbf{v}_i' \equiv \vec{v}_i. \tag{2.52}$$

According to the form of $\mathbf{v}_i^a$, $\mathbf{v}_i'$ in (2.44), the vacuum expectation values of the effective Higgs bosons read

$$\vec{v}_1 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \vec{v}_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{2.53}$$

in agreement with electric charge conservation. CP invariance restricts v_1, v_2 to be real. When we perform the replacements (2.50) and (2.51) in Γ_{cl}, we obtain another solution

of the classical action obeying all symmetries. It turns out that this solution contains all parameters needed for the renormalization. Hence, we adopt this solution and define the classical action Γ_{cl} in this way. x_1, x_2, x_1', x_2' are additional free parameters. Owing to the decomposition (2.52) of $\vec{v}_1$, $\vec{v}_2$, it is equivalent to eliminate x_1', x_2' in favour of v_1, v_2 and choose

$$v_1, \quad v_2, \quad x_1, \quad x_2 \tag{2.54}$$

instead of x_1', x_2', x_1, x_2 as free parameters of the theory.

The appearance of $\vec{v}_i$ in H_i^{eff} accounts for the spontaneous breaking of gauge invariance. In the following, we sketch the resulting bilinear part of Γ_{cl}. The corresponding calculations are well-known in the literature (see e.g. [3, 25]). Hence we just quote the results. The minimization conditions yield

$$-\frac{\delta\Gamma_{\text{cl}}}{\delta\text{Re}H_1^1}\bigg|_{\phi=0} = 2\left[(m_1^2+\mu^2)v_1 + m_3^2 v_2 + \frac{g^2+g'^2}{4}v_1(v_1^2-v_2^2)\right] \equiv \sqrt{2}\,t_1 \stackrel{!}{=} 0, \tag{2.55a}$$

$$-\frac{\delta\Gamma_{\text{cl}}}{\delta\text{Re}H_2^2}\bigg|_{\phi=0} = 2\left[(m_2^2+\mu^2)v_2 + m_3^2 v_1 - \frac{g^2+g'^2}{4}v_2(v_1^2-v_2^2)\right] \equiv \sqrt{2}\,t_2 \stackrel{!}{=} 0. \tag{2.55b}$$

The minimization of the Higgs potential is equivalent to vanishing tadpole contributions t_1, t_2.

Higgs and gauge-boson sector

The bilinear part of Γ_{cl} contains the mass matrices for the symmetric fields. These mass matrices have to be diagonalized by the demixing matrices R_ϕ. In the Higgs and gauge-boson sector, the classical demixing matrices are rotation matrices with three angles θ_W, β, α defined by

$$\tan\beta = \frac{v_2}{v_1}, \qquad \tan 2\alpha = \tan 2\beta \frac{M_{A^0}^2 + M_Z^2}{M_{A^0}^2 - M_Z^2}, \qquad \cos\theta_W = \frac{M_W}{M_Z} \tag{2.56a}$$

with $-\pi/2 < \alpha < 0$. The shorthands $s_\alpha = \sin\alpha$, $s_\beta = \sin\beta$, $s_W = \sin\theta_W$ (and similar for cosine) are used in the following. Classically, the demixing matrices read

$$R^{(0)}_{A^0,H^\pm} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}, \quad R^{(0)}_{H^0} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}, \quad R^{(0)}_{V} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}. \tag{2.57a}$$

After diagonalizing the mass matrices, the masses of the $W^\pm$ boson, Z boson, and the A^0 field result in

$$M_W^2 = \frac{1}{2}g^2(v_1^2+v_2^2), \tag{2.58a}$$

$$M_Z^2 = \frac{1}{2}(g^2+g'^2)(v_1^2+v_2^2), \tag{2.58b}$$

$$M_{A^0}^2 = \sin^2\beta\frac{t_1}{\sqrt{2}v_1} + \cos^2\beta\frac{t_2}{\sqrt{2}v_2} - m_3^2(\tan\beta+\cot\beta). \tag{2.58c}$$

We consider M_W^2, M_Z^2, and $M_{A^0}^2$ as independent parameters in the following. Then the masses of the remaining Higgs bosons are dependent parameters and are determined in lowest order by

$$M_{H^\pm}^{2\,(0)} = M_{A^0}^2 + M_W^2, \tag{2.59a}$$

$$M_{H^0,h^0}^{2\,(0)} = \frac{1}{2}\left(M_{A^0}^2 + M_Z^2 \pm \sqrt{(M_{A^0}^2 + M_Z^2)^2 - 4M_Z^2 M_{A^0}^2 \cos^2(2\beta)}\right). \tag{2.59b}$$

As indicated by the superscript $^{(0)}$, these mass relations are only valid in lowest order. In general, the mass relations (2.59) are corrected by loop contributions.

In the chargino and neutralino sector, the lowest-order demixing matrices are conventionally denoted by

$$R_{\chi^+}^{(0)} = \mathcal{V}^{-1}, \qquad R_{\chi^-}^{(0)} = \mathcal{U}^{-1}, \qquad R_{\chi^0}^{(0)} = \mathcal{N}^{-1}, \tag{2.60}$$

where $\mathcal{V}, \mathcal{U}, \mathcal{N}$ are unitary matrices and satisfy

$$\mathcal{N}^* \mathcal{Y} \mathcal{N}^\dagger = \mathrm{diag}(m_{\chi_1^0}, \ldots, m_{\chi_4^0}), \qquad \mathcal{U}^* \mathcal{X} \mathcal{V}^\dagger = \mathrm{diag}(m_{\chi_1^+}, m_{\chi_2^+}) \tag{2.61}$$

with the neutralino and chargino mass matrices

$$\mathcal{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}, \tag{2.62a}$$

$$\mathcal{X} = \begin{pmatrix} M_2 & M_W\sqrt{2}s_\beta \\ M_W\sqrt{2}c_\beta & \mu \end{pmatrix}. \tag{2.62b}$$

After defining the Majorana spinors of the neutralinos and the Dirac spinors of the charginos (with $\bar{\chi}^\pm = \overline{\chi^\mp}$)

$$\chi_i^0 = \begin{pmatrix} \chi_{i\alpha}^0 \\ \bar{\chi}_i^{0\dot{\alpha}} \end{pmatrix}, \qquad \chi_i^\pm = \begin{pmatrix} \chi_{i\alpha}^\pm \\ \bar{\chi}_i^{\pm\dot{\alpha}} \end{pmatrix}, \tag{2.63}$$

the bilinear part of the classical action contributing to the Higgs and gauge-boson sector reads

$$\begin{aligned}
\Gamma_{\mathrm{cl}}|_{\mathrm{bil,Higgs,gauge}} = \int \mathrm{d}^4x \Bigg\{ & |\partial^\mu H^+|^2 + |\partial^\mu G^+|^2 - M_{H^\pm}^{2(0)} |H^+|^2 \\
& + \frac{1}{2}\left[(\partial^\mu H^0)^2 + (\partial^\mu h^0)^2 + (\partial^\mu G^0)^2 + (\partial^\mu A^0)^2 - M_{H^0}^{2(0)}(H^0)^2 - M_{h^0}^{2(0)}(h^0)^2 - M_{A^0}^2 (A^0)^2\right] \\
& - \frac{1}{4}\left(A^{\mu\nu}A_{\mu\nu} + Z^{\mu\nu}Z_{\mu\nu} + 2W^{+\mu\nu}W^-_{\mu\nu}\right) + \frac{1}{2}M_Z^2 Z^\mu Z_\mu + M_W^2 W^{+\mu}W^-_\mu \\
& - M_Z Z^\mu \partial_\mu G^0 + \mathrm{i}M_W\left(W^{+\mu}\partial_\mu G^- - W^{-\mu}\partial_\mu G^+\right) \\
& + \frac{1}{2}\sum_{i=1}^{4}\left(\bar{\chi}_i^0 \mathrm{i}\gamma^\mu \partial_\mu \chi_i^0 - m_{\chi_i^0}\bar{\chi}_i^0 \chi_i^0\right) + \sum_{i=1}^{2}\left(\bar{\chi}_i^- \mathrm{i}\gamma^\mu \partial_\mu \chi_i^+ - m_{\chi_i^+}\bar{\chi}_i^- \chi_i^+\right)\Bigg\},
\end{aligned} \tag{2.64}$$

where the linear parts of the field strength tensors are given by $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$ with $V = A, Z, W^\pm$. The masses $M^{(0)}_{H^0}$, $M^{(0)}_{h^0}$, and $M^{(0)}_{H^\pm}$ and three of the neutralino and chargino masses are not considered as independent parameters of the MSSM, but they are functions of the other masses and couplings.

Matter sector

In the matter sector, the quark and lepton masses are given by

$$m_u = -f_U v_2, \qquad m_d = f_D v_1, \qquad m_e = f_R v_1. \tag{2.65}$$

The mass matrices of the up- and down-type squarks in the basis $(Q^1, \bar{U})$, $(Q^2, \bar{D})$ have the following form

$$M^2_{\tilde{u}} = \begin{pmatrix} m_u^2 + m_{\tilde{q}}^2 + M_Z^2 c_{2\beta}(T_3 - Q_{\rm em} s_W^2) & m_u\,(-\mu/\tan\beta + A_u) \\ m_u\,(-\mu/\tan\beta + A_u) & m_u^2 + m_{\tilde{u}}^2 + M_Z^2 c_{2\beta} Q_{\rm em} s_W^2 \end{pmatrix}, \tag{2.66a}$$

$$M^2_{\tilde{d}} = \begin{pmatrix} m_d^2 + m_{\tilde{q}}^2 + M_Z^2 c_{2\beta}(T_3 - Q_{\rm em} s_W^2) & m_d\,(-\mu\tan\beta + A_d) \\ m_d\,(-\mu\tan\beta + A_d) & m_d^2 + m_{\tilde{d}}^2 + M_Z^2 c_{2\beta} Q_{\rm em} s_W^2 \end{pmatrix} \tag{2.66b}$$

with the electric charge $Q_{\rm em}$ and $c_{2\beta} = \cos(2\beta)$. The corresponding mass matrix for the selectron in the basis $(L^2, \bar{R})$ can be derived from (2.66b) by replacing $d \to e$ and $\tilde{d} \to \tilde{e}$. The demixing matrices $R_{\tilde{f}}$ have to be chosen in such a way that they satisfy

$$R^{(0)-1}_{\tilde{f}} M^2_{\tilde{f}} R^{(0)}_{\tilde{f}} = \mathrm{diag}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}) \tag{2.67}$$

with $\tilde{f} = \tilde{u}, \tilde{d}, \tilde{e}$. Since there is no sneutrino mixing the sneutrino mass is given by

$$m^2_{\tilde{\nu}} = m^2_{\tilde{l}} + \frac{1}{2} M_Z^2 \cos(2\beta). \tag{2.68}$$

With the definition of the Dirac spinors

$$e = \begin{pmatrix} l^1_\alpha \\ \bar{r}^{\dot{\alpha}} \end{pmatrix}, \qquad \nu = \begin{pmatrix} l^2_\alpha \\ 0 \end{pmatrix}, \qquad u = \begin{pmatrix} q^1_\alpha \\ \bar{u}^{\dot{\alpha}} \end{pmatrix}, \qquad d = \begin{pmatrix} q^2_\alpha \\ \bar{d}^{\dot{\alpha}} \end{pmatrix}, \tag{2.69}$$

and the lowest-order values $R^{(0)}_{\nu_L} = R^{(0)}_{e_L} = R^{(0)}_{u_L} = R^{(0)}_{d_L} = 1$, the bilinear part of the matter sector can be written as

$$\Gamma_{\rm cl}|_{\rm bil,matter} = \int \mathrm{d}^4x \left[\bar{f}\,(\mathrm{i}\gamma^\mu \partial_\mu - m_f)\, f + \sum_{i=1,2} |\partial^\mu \tilde{f}_i|^2 - m^2_{\tilde{f}_i} |\tilde{f}_i|^2 \right], \tag{2.70}$$

where f, $\tilde{f}$ run over all matter fields, i.e. $f = e, \nu, u, d$ and $\tilde{f}_i = \tilde{\nu}_i, \tilde{e}_i, \tilde{u}_i, \tilde{d}_i$.

Gauge-fixing sector

We come back to the gauge-fixing and ghost terms already discussed in section 2.6 in order to evaluate the mass eigenstates of ghosts. For vanishing Φ fields the gauge-fixing functions of (2.45) reduce to the functions $\mathscr{F}^{\pm}$, $\mathscr{F}_A$, and $\mathscr{F}_Z$ defined in (2.41). Classically, evaluating the bilinear ghost terms in (2.39) yields

$$\begin{aligned}\Gamma_{\mathrm{cl}}|_{\mathrm{bil,gh}} \;=\; \int \mathrm{d}^4x \Bigg\{ & -(\bar{c}_A, \bar{c}_Z) \left[\Box + M_Z^2 \begin{pmatrix} 0 & \zeta^{AG^0} \\ 0 & \zeta^{ZG^0} \end{pmatrix} \right] \begin{pmatrix} c_A \\ c_Z \end{pmatrix} \\ & - (\bar{c}^-, \bar{c}^+) \left[\Box + M_W^2 \zeta^{G^\pm} \right] \begin{pmatrix} c^+ \\ c^- \end{pmatrix} \Bigg\} \end{aligned} \tag{2.71}$$

with $\Box = \partial^\mu \partial_\mu$ and where the lowest-order ghost and antighost demixing matrices are given by

$$R_c^{(0)} = R_{\bar{c}}^{(0)} = R_V^{(0)}. \tag{2.72}$$

An especially convenient choice is the R_ξ gauge which corresponds to

$$\zeta^{G^\pm} = \zeta^{ZG^0} = \zeta' = \zeta \equiv \xi, \qquad \zeta^{H^\pm} = \zeta^{ZA^0} = \zeta^{AG^0} = \zeta^{AA^0} = 0. \tag{2.73}$$

In the R_ξ gauge the gauge-boson–Higgs mixing in (2.64) cancels when the auxiliary B fields are eliminated by their equations of motion. Furthermore, the ghost and Goldstone-boson masses are proportional to the masses of the corresponding gauge bosons:

$$M_{c_A}^{2\ (0)} = 0, \tag{2.74a}$$

$$M_{c_Z}^{2\ (0)} = M_{G^0}^{2\ (0)} = \xi M_Z^2, \tag{2.74b}$$

$$M_{c^\pm}^{2\ (0)} = M_{G^\pm}^{2\ (0)} = \xi M_W^2. \tag{2.74c}$$

2.8 Free parameters of the MSSM

The free parameters of the MSSM can be summarized as follows:

$$\text{coupling constants} \qquad g, g', f_R, f_U, f_D, \mu, \tag{2.75a}$$

$$\text{soft parameters} \qquad M_1, M_2, m_1, m_2, m_3, m_{\tilde{l}}, m_{\tilde{e}}, m_{\tilde{q}}, m_{\tilde{u}}, m_{\tilde{d}}, A_e, A_u, A_d, \tag{2.75b}$$

$$\text{Higgs/}\Phi\text{ parameters} \qquad v_1, v_2, x_1, x_2, \tag{2.75c}$$

$$\text{gauge parameters} \qquad \zeta, \zeta', \zeta^{G^\pm}, \zeta^{H^\pm}, \zeta^{ZG^0}, \zeta^{ZA^0}, \zeta^{AG^0}, \zeta^{AA^0}. \tag{2.75d}$$

In section 2.7 some of these parameters were related to a set of parameters with a more direct physical interpretation. Schematically, the relations between the two sets of pa-

rameters are

$$
\begin{aligned}
g, g', v_1, v_2 &\leftrightarrow e, M_Z, M_W, \tan\beta, && (2.76)\\
m_1, m_2, m_3 &\leftrightarrow M_{A^0}, t_1, t_2, && (2.77)\\
\mu, M_1, M_2 &\leftrightarrow m_{\chi_1^\pm}, m_{\chi_2^\pm}, m_{\chi_1^0}, && (2.78)\\
f_R, f_U, f_D &\leftrightarrow m_e, m_u, m_d, && (2.79)\\
m_{\tilde{l}}, m_{\tilde{e}}, m_{\tilde{q}}, m_{\tilde{d}}, m_{\tilde{u}} &\leftrightarrow m_{\tilde{\nu}_1}, m_{\tilde{e}_1}, m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_{\tilde{d}_1}, && (2.80)
\end{aligned}
$$

where the electric charge is defined by

$$
e = g\sin\theta_W = g'\cos\theta_W. \qquad (2.81)
$$

Of course, the choice of the independent parameters in the neutralino and sfermion sectors is not unique.

We close with a remark on the mass relations present in the MSSM. In the MSSM not all masses are independent. The masses of $H^\pm$, H^0, h^0, $\chi^0_{2,3,4}$, and of $\tilde{e}_2$, $\tilde{d}_2$ are functions of the free parameters of the MSSM. Accordingly, there is a distinction between the notions ${M^2_{H^\pm}}^{(0)}$ and $M^2_{H^\pm}$ (and similar for the other dependent masses). ${M^2_{H^\pm}}^{(0)}$ denotes the result of the lowest-order formula (2.59a) and is thus always to be treated as an abbreviation for $M^2_{A^0} + M^2_W$. In particular, in the renormalization transformation ${M^2_{H^\pm}}^{(0)} \to {M^2_{H^\pm}}^{(0)} + \delta M^2_{H^\pm}$, used in section 4, $\delta M^2_{H^\pm}$ is a substitute of $\delta M^2_{A^0} + \delta M^2_W$. On the other hand, $M^2_{H^\pm}$ denotes the physical mass of the charged Higgs boson, which can be calculated order by order as a function of the independent parameters of the MSSM.

3 Renormalization

In this section general aspects of the renormalization of the MSSM are presented. After establishing the symmetries in a functional way by introducing the ST and Ward operators, all defining symmetries of the MSSM are summarized, and all invariant and symmetry-restoring counterterms required for the renormalization are discussed. The section is completed with a proof of the IR finiteness of the theory.

3.1 Functional characterization of the symmetries

In order to characterize the MSSM in a regularization-scheme independent way, it is convenient to formulate the fundamental symmetries in terms of a set of functional equations comprising the ST identity for BRS invariance and the Ward identities for rigid gauge and R invariance.

ST identity

BRS invariance is expressed in form of a ST identity

$$\mathscr{S}(\Gamma) = 0 \tag{3.1}$$

with the ST operator

$$\begin{aligned}
\mathscr{S}(\Gamma) \;=\; & \int \mathrm{d}^4x \left\{ 2\mathrm{tr}\left[\frac{\delta\Gamma}{\delta Y_V^\mu}\frac{\delta\Gamma}{\delta V_\mu} + \frac{\delta\Gamma}{\delta y_{\lambda\alpha}}\frac{\delta\Gamma}{\delta\lambda^\alpha} + \frac{\delta\Gamma}{\delta\bar{y}_\lambda^{\dot\alpha}}\frac{\delta\Gamma}{\delta\bar\lambda_{\dot\alpha}} + \frac{\delta\Gamma}{\delta Y_c}\frac{\delta\Gamma}{\delta c} + \mathrm{s}\,\bar c\frac{\delta\Gamma}{\delta\bar c} + \mathrm{s}\,B\frac{\delta\Gamma}{\delta B}\right] \right. \\
& + \mathrm{s}\,V'_\mu\frac{\delta\Gamma}{\delta V'_\mu} + \frac{\delta\Gamma}{\delta y_{\lambda'\alpha}}\frac{\delta\Gamma}{\delta\lambda'^\alpha} + \frac{\delta\Gamma}{\delta\bar{y}_{\lambda'}^{\dot\alpha}}\frac{\delta\Gamma}{\delta\bar\lambda'_{\dot\alpha}} + \mathrm{s}\,c'\frac{\delta\Gamma}{\delta c'} + \mathrm{s}\,\bar c'\frac{\delta\Gamma}{\delta\bar c'} + \mathrm{s}\,B'\frac{\delta\Gamma}{\delta B'} \\
& + \mathrm{s}\,A_1\frac{\delta\Gamma}{\delta A_1} + \mathrm{s}\,\bar A_1\frac{\delta\Gamma}{\delta\bar A_1} + \mathrm{s}\,A_2\frac{\delta\Gamma}{\delta A_2} + \mathrm{s}\,\bar A_2\frac{\delta\Gamma}{\delta\bar A_2} \\
& + \mathrm{s}\,\Phi_i^{aT}\frac{\delta\Gamma}{\delta\Phi_i^{aT}} + \mathrm{s}\,\Phi_i'^T\frac{\delta\Gamma}{\delta\Phi_i'^T} + \mathrm{s}\,\Psi_i^{aT}\frac{\delta\Gamma}{\delta\Psi_i^{aT}} + \mathrm{s}\,\Psi_i'^T\frac{\delta\Gamma}{\delta\Psi_i'^T} \\
& + \mathrm{s}\,\bar\Phi_i^a\frac{\delta\Gamma}{\delta\bar\Phi_i^a} + \mathrm{s}\,\bar\Phi'_i\frac{\delta\Gamma}{\delta\bar\Phi'_i} + \mathrm{s}\,\bar\Psi_i^a\frac{\delta\Gamma}{\delta\bar\Psi_i^a} + \mathrm{s}\,\bar\Psi'_i\frac{\delta\Gamma}{\delta\bar\Psi'_i} \\
& + \frac{\delta\Gamma}{\delta Y_L}\frac{\delta\Gamma}{\delta L^T} + \frac{\delta\Gamma}{\delta\bar Y_L^T}\frac{\delta\Gamma}{\delta\bar L} + \frac{\delta\Gamma}{\delta y_{l\alpha}}\frac{\delta\Gamma}{\delta l^{\alpha T}} + \frac{\delta\Gamma}{\delta\bar{y}_l^{\dot\alpha T}}\frac{\delta\Gamma}{\delta\bar l_{\dot\alpha}} \\
& \left. + \frac{\delta\Gamma}{\delta Y_R}\frac{\delta\Gamma}{\delta R} + \frac{\delta\Gamma}{\delta\bar Y_R}\frac{\delta\Gamma}{\delta\bar R} + \frac{\delta\Gamma}{\delta y_{r\alpha}}\frac{\delta\Gamma}{\delta r^\alpha} + \frac{\delta\Gamma}{\delta\bar{y}_r^{\dot\alpha}}\frac{\delta\Gamma}{\delta\bar r_{\dot\alpha}} \right\} + \mathrm{s}\,\xi^\mu\frac{\partial\Gamma}{\partial\xi^\mu} \\
& + \text{analogous quark and Higgs terms.}
\end{aligned} \tag{3.2}$$

For subsequent discussions we introduce the condensed notation

$$\mathscr{S}(\Gamma) = \sum_a \frac{\delta\Gamma}{\delta Y_a}\frac{\delta\Gamma}{\delta\phi_a} + \sum_b \mathrm{s}\,\phi'_b\frac{\delta\Gamma}{\delta\phi'_b}, \tag{3.3}$$

where ϕ_a runs over all non-linearly transforming fields and ϕ'_b over all linearly transforming ones. Owing to the construction of Γ_{cl}, the ST identity $\mathscr{S}(\Gamma_{\mathrm{cl}}) = 0$ holds at the classical level. At higher orders, $\mathscr{S}(\Gamma) = 0$ will be imposed as a condition on the vertex functional Γ.

In addition to the ST operator, we define the linearized ST operator

$$\mathscr{S}_\gamma = \sum_a \left(\frac{\delta\gamma}{\delta Y_a}\frac{\delta}{\delta\phi_a} + \frac{\delta\gamma}{\delta\phi_a}\frac{\delta}{\delta Y_a} \right) + \sum_b \mathrm{s}\,\phi'_b\frac{\delta}{\delta\phi'_b}, \tag{3.4}$$

which obeys the relation

$$\mathscr{S}(\gamma+\Delta) = \mathscr{S}(\gamma) + \mathscr{S}_\gamma\Delta + \mathcal{O}(\Delta^2), \tag{3.5}$$

where γ, Δ are arbitrary field polynomials.

For the algebraic characterization of possible anomalies the nilpotency relations of the ST operator

$$\mathscr{S}_\gamma \mathscr{S}(\gamma) = 0 \qquad \text{for all } \gamma, \tag{3.6a}$$

$$\mathscr{S}_{\Gamma_{\rm cl}} \mathscr{S}_{\Gamma_{\rm cl}} = 0, \tag{3.6b}$$

have to be realized. These nilpotency relations are important ingredients for the algebraic proof of the renormalizability discussed later in section 3.3.

However, a short calculation shows that (3.6a) is in general not true:

$$\mathscr{S}_\gamma \mathscr{S}(\gamma) = \sum_b (\mathscr{S}_\gamma \mathrm{s}\, \phi'_b) \frac{\delta \gamma}{\delta \phi'_b} = (\mathscr{S}_\gamma \mathrm{s}\, V'_\mu) \frac{\delta \gamma}{\delta V'_\mu}. \tag{3.7}$$

The non-vanishing r.h.s. of (3.7) originates from the different treatment of linear and non-linear BRS transformations in the ST operator (3.2). BRS transformations of the linearly transforming fields are "hard coded" in $\mathscr{S}$ while those of the non-linearly transforming ones are coupled to source fields Y, y and, therefore, obtain higher-order corrections. We ensure (3.6a) for $\gamma = \Gamma$ by imposing the requirement

$$\mathscr{S}_\Gamma \mathrm{s}\, V'_\mu = 0, \tag{3.8}$$

which holds in particular for $\Gamma = \Gamma_{\rm cl}$. In terms of the linear functional differential operator

$$w_\mu^{\lambda'} = \varepsilon^\alpha \sigma_{\mu\alpha\dot\alpha} \frac{\delta}{\delta \bar y_{\lambda'\dot\alpha}} - \frac{\delta}{\delta y_{\lambda'\alpha}} \sigma_{\mu\alpha\dot\alpha} \bar\varepsilon^{\dot\alpha}, \tag{3.9}$$

the requirement (3.8) can be rewritten as the linear equation

$$w_\mu^{\lambda'} \Gamma = 2\mathrm{i} \varepsilon^\alpha \sigma^\nu_{\alpha\dot\alpha} \bar\varepsilon^{\dot\alpha} F'_{\mu\nu} + \mathrm{i} \xi^\nu \partial_\nu \left(\varepsilon^\alpha \sigma_{\mu\alpha\dot\alpha} \bar\lambda'^{\dot\alpha} + \lambda'^\alpha \sigma_{\mu\alpha\dot\alpha} \bar\varepsilon^{\dot\alpha} \right). \tag{3.10}$$

Also the second nilpotency condition is in general violated. Using $\mathscr{S}(\Gamma_{\rm cl}) = 0$ and (3.8), the square of $\mathscr{S}_{\Gamma_{\rm cl}}$ yields the non-vanishing result

$$\mathscr{S}_{\Gamma_{\rm cl}} \mathscr{S}_{\Gamma_{\rm cl}} = -\frac{\delta(\mathrm{s}\, V'_\mu)}{\delta \lambda'^\alpha} \frac{\delta \Gamma_{\rm cl}}{\delta V'_\mu} \frac{\delta}{\delta y_{\lambda'^\alpha}} + \frac{\delta(\mathrm{s}\, V'_\mu)}{\delta \bar\lambda'^{\dot\alpha}} \frac{\delta \Gamma_{\rm cl}}{\delta V'_\mu} \frac{\delta}{\delta \bar y_{\lambda'\dot\alpha}}. \tag{3.11}$$

However, it is possible to define a space of constrained functionals on which the linearized ST operator is nilpotent. Since the r.h.s. of (3.11) is proportional to $w_\mu^{\lambda'}$, the operator $\mathscr{S}_{\Gamma_{\rm cl}}$ is nilpotent on the kernel of $w_\mu^{\lambda'}$, i.e.

$$\mathscr{S}^2_{\Gamma_{\rm cl}} \gamma = 0 \qquad \text{if } w_\mu^{\lambda'} \gamma = 0. \tag{3.12}$$

This is a weaker but sufficient form of (3.6b). The constraint $w_\mu^{\lambda'} \gamma = 0$ means simply that γ depends not arbitrarily on y'_λ, $\bar y'_\lambda$ but only on a certain combination,

$$\gamma(y_{\lambda'}, \bar y_{\lambda'}) = \gamma(\Upsilon), \qquad \Upsilon = \varepsilon^\alpha y'_{\lambda\alpha} - \bar\varepsilon_{\dot\alpha} \bar y'^{\dot\alpha}_\lambda. \tag{3.13}$$

Ward identities

Rigid gauge invariance is formulated in the form of Ward identities

$$\mathscr{W}(\omega)\Gamma = 0, \qquad \mathscr{W}'(\omega')\Gamma = 0, \tag{3.14}$$

where the Ward operators for rigid SU(2), $\mathscr{W}$, and for rigid U(1) gauge invariance, $\mathscr{W}'$, are defined by

$$\mathscr{W}(\omega) = \int \mathrm{d}^4x \left[\sum_a \delta_\omega \phi'_a \frac{\delta}{\delta \phi'_a} + \sum_b \left(\delta_\omega \phi_b \frac{\delta}{\delta \phi_b} + \delta_\omega Y_b \frac{\delta}{\delta Y_b} \right) \right], \tag{3.15a}$$

$$\mathscr{W}'(\omega') = \int \mathrm{d}^4x \left[\sum_a \delta_{\omega'} \phi'_a \frac{\delta}{\delta \phi'_a} + \sum_b \left(\delta_{\omega'} \phi_b \frac{\delta}{\delta \phi_b} + \delta_{\omega'} Y_b \frac{\delta}{\delta Y_b} \right) \right]. \tag{3.15b}$$

The Ward operators for lepton-number conservation, $\mathscr{W}_L$, for quark-number conservation, $\mathscr{W}_Q$, and for R symmetry, $\mathscr{W}^R$ are defined in an obvious way. Explicit expressions for the Ward operators are given in appendix D.

Since the gauge group of the MSSM is non-semisimple not all couplings of the classical action are fixed by the algebra of the gauge group. The abelian couplings, i.e. the hypercharges of matter couplings to V'_μ, are not determined by (3.15b) but have to be determined by an additional symmetry, either by the local U(1) Ward identity or by the corresponding ghost equation [14, 26, 27]. For the classical action the hypercharges of the MSSM are determined by the Gell-Mann–Nishijima relation (2.5). At higher orders, the functional analog of the Gell-Mann–Nishijima relation holds for the Ward operators:

$$\frac{1}{e}\mathscr{W}_{\mathrm{em}}(\omega_{\mathrm{em}}) = \left[\frac{1}{g}\mathscr{W}(T^3\omega_{\mathrm{em}}) + \frac{1}{g'}\mathscr{W}'(\omega_{\mathrm{em}}) \right], \tag{3.16}$$

where the Ward operator of electromagnetic symmetry $\mathscr{W}_{\mathrm{em}}$ is defined similar to (3.15) with $\delta_{\omega_{\mathrm{em}}} = -\mathrm{i}e\omega_{\mathrm{em}}Q_{\mathrm{em}}$.

We choose to determine the abelian couplings by the local Ward identity corresponding to the abelian operator $\mathscr{W}'$, which can be derived also in supersymmetric models [16]:

$$\mathscr{W}'_{\mathrm{local}}\Gamma = \Box(B' + \mathrm{i}\xi^\mu \partial_\mu \bar{c}') \tag{3.17}$$

with the Ward operator

$$\mathscr{W}'_{\mathrm{local}} = \frac{1}{g'} \left[\sum_a \delta_{\omega'} \phi'_a \frac{\delta}{\delta \phi'_a} + \sum_b \left(\delta_{\omega'} \phi_b \frac{\delta}{\delta \phi_b} + \delta_{\omega'} Y_b \frac{\delta}{\delta Y_b} \right) \right] - \partial^\mu \frac{\delta}{\delta V'^\mu}. \tag{3.18}$$

If (3.17) is established to all orders, the hypercharges are fixed in such a way that electromagnetic symmetry is gauged and that the Adler-Bardeen anomaly is cancelled to all orders.

Due to parity non-conservation and due to supersymmetry, invariant regularization schemes of the MSSM probably do not exist. Hence one has to construct the MSSM by algebraic renormalization restoring the defining symmetries by (scheme-dependent) non-invariant counterterms order by order in perturbation theory. For this purpose one needs a complete algebraic characterization of invariant counterterms and breaking terms, which is governed by the algebra of defining symmetry operators. Hence, we conclude this section by listing the commutation relations of the ST and Ward operators:

$$[\mathscr{W}(\omega_1), \mathscr{W}(\omega_2)] = \mathscr{W}([\omega_1, \omega_2]), \tag{3.19a}$$

$$[\mathscr{W}_1, \mathscr{W}_2] = 0, \tag{3.19b}$$

$$[\mathscr{W}(\omega), \mathscr{W}_1] = 0, \tag{3.19c}$$

$$[\mathscr{W}'_{\text{local}}, \mathscr{W}_1] = 0, \tag{3.19d}$$

$$\mathscr{W}(\omega)\mathscr{S}(\Gamma) - \mathscr{S}_\Gamma \mathscr{W}(\omega)\Gamma = 0, \tag{3.19e}$$

$$\mathscr{W}_1\mathscr{S}(\Gamma) - \mathscr{S}_\Gamma \mathscr{W}_1\Gamma = 0, \tag{3.19f}$$

$$\mathscr{W}'_{\text{local}}\mathscr{S}(\Gamma) - \mathscr{S}_\Gamma \mathscr{W}'_{\text{local}}\Gamma = -2\mathrm{i}\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\partial_\mu \frac{\delta\Gamma}{\delta c'} + \mathrm{i}\xi^\nu \partial_\nu \partial^\mu \frac{\delta\Gamma}{\delta V'^\mu}, \tag{3.19g}$$

where $\mathscr{W}_1$, $\mathscr{W}_2$ stand for the Ward operators $\mathscr{W}'(\omega')$, $\mathscr{W}_L$, $\mathscr{W}_Q$, $\mathscr{W}^R$.

3.2 Defining symmetries

To define a quantum theory in the framework of algebraic renormalization, a suitable set of requirements has to be imposed on the vertex functional Γ. Γ is calculated order by order in perturbation theory starting from the classical action. In higher orders, intermediate results for regularized loop diagrams and counterterms are regularization-scheme dependent, but the final results for observables are regularization-scheme independent and unique if the requirements are chosen appropriately.

A complete set of conditions for the MSSM is given in the following, where symmetric fields are understood to be expressed by the physical fields. We will see that this set fixes Γ uniquely up to the usual freedom of mass, coupling-constant, and wave-function renormalizations.

The conditions express the symmetries, the minimization of the Higgs potential as well as constraints for the gauge fixing and the ξ^μ dependence.

- CP invariance (see table 2.2).
- Γ depends on A_1 only through the combination $\varepsilon_\alpha A_1$.
- ST identity with the ST operator (3.2):

$$\mathscr{S}(\Gamma) = 0. \tag{3.20}$$

- Nilpotency condition for the ST operator:

$$w_\mu^{\lambda'}\Gamma = 2\mathrm{i}\varepsilon^\alpha\sigma^\nu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}F'_{\mu\nu} + \mathrm{i}\xi^\nu\partial_\nu\left(\varepsilon^\alpha\sigma_{\mu\alpha\dot\alpha}\bar\lambda'^{\dot\alpha} + \lambda'^\alpha\sigma_{\mu\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\right), \tag{3.21}$$

  where $w_\mu^{\lambda'}$ is defined in (3.9).

- Rigid SU(2)×U(1) gauge invariance:

$$\mathscr{W}\Gamma = 0, \qquad \mathscr{W}'\Gamma = 0. \tag{3.22}$$

- Lepton- and quark-number conservation:

$$\mathscr{W}_L\Gamma = 0, \qquad \mathscr{W}_Q\Gamma = 0. \tag{3.23}$$

- Continuous R symmetry:

$$\mathscr{W}^R\Gamma = 0. \tag{3.24}$$

- Local U(1) gauge invariance:

$$\mathscr{W}'_{\mathrm{local}}\Gamma = \Box(B' + \mathrm{i}\xi^\mu\partial_\mu\bar c'). \tag{3.25}$$

- Gauge-fixing conditions:

$$\frac{\delta\Gamma}{\delta B^a} = \mathscr{F}^a + \zeta B^a, \qquad \frac{\delta\Gamma}{\delta B'} = \mathscr{F}' + \zeta' B', \tag{3.26}$$

  where $\mathscr{F}^a$, $\mathscr{F}'$ are defined in (2.45).

- Translational-ghost equation:

$$\frac{\delta\Gamma}{\delta\xi_\mu} = -\mathrm{i}\sum_a Y_a\partial^\mu\phi_a\,. \tag{3.27}$$

- Minimum requirement for the Higgs potential:

$$\int\mathrm{d}^4x\ \left.\frac{\delta\Gamma}{\delta H_i(x)}\right|_{\phi\to 0} = 0. \tag{3.28}$$

3.3 Cohomology and symmetry-restoring counterterms

The general strategy of calculating radiative corrections to a given order in perturbation theory for a renormalizable model consists of three steps:

- In the first step all loop diagrams of order $\hbar^n$ are calculated within a chosen regularization scheme.
- In general the result does not satisfy the defining symmetry properties. To restore the symmetry identities symmetry-restoring counterterms $\Gamma^{(n)}_{\text{ct,restore}}$ have to be added. One contribution to $\Gamma_{\text{ct,restore}}$ originates from the non-invariance of the regularization scheme. Another contribution to $\Gamma_{\text{ct,restore}}$ is given by products of lower-order counterterms involving for instance $\delta g^{(n-m)}\delta z^{(m)}$ with $0 < m < n$.
- Finally, invariant counterterms $\Gamma^{(n)}_{\text{ct,inv}}$ are added to cancel UV singularities and to establish the normalization conditions.

The classical action, the invariant counterterms $\Gamma_{\text{ct,inv}}$ (see section 3.4), and the symmetry-restoring counterterms $\Gamma_{\text{ct,restore}}$ add up to

$$\Gamma_{\text{eff}} = \Gamma_{\text{cl}} + \sum_{n=1}^{\infty} \left(\Gamma^{(n)}_{\text{ct,inv}} + \Gamma^{(n)}_{\text{ct,restore}} \right) \tag{3.29}$$

from which the Feynman rules are derived. The full vertex functional Γ differs from Γ_{eff} by non-local loop contributions.

In the following, we give a few comments to the symmetry-restoring counterterms and possible anomalies. The third step is treated in detail in section 3.4.

We consider the ST identity (3.20) and the nilpotency requirement (3.21) as an example. Assuming that all symmetry identities have already been established up to order $\hbar^{n-1}$ the symmetry identities yield

$$w^{\lambda'}_{\mu}\Gamma - \text{r.h.s. of (3.21)} = \hbar^n \Delta_{w^{\lambda'}\mu} + \mathcal{O}(\hbar^{n+1}), \tag{3.30}$$

$$\mathscr{S}(\Gamma) = \hbar^n \Delta_{\text{ST}} + \mathcal{O}(\hbar^{n+1}). \tag{3.31}$$

Owing to the quantum action principle [28], $\Delta_{w^{\lambda'}}$, Δ_{ST} are local functionals. They have to be absorbed by adding symmetry-restoring counterterms of order $\hbar^n$ to Γ_{eff}. In this way, the symmetry properties can be restored in the given order.

Since the nilpotency condition (3.21) is the basis for the restoration of the ST identity, (3.30) has to be recovered as a first step. It can be shown that $\Delta_{w^{\lambda'}}$ can be written as a variation

$$\Delta_{w^{\lambda'}\mu} = w^{\lambda'}_{\mu}\hat{\Delta}_{w^{\lambda'}} \tag{3.32}$$

with a local, power-counting renormalizable functional $\hat{\Delta}_{w^{\lambda'}}$, we can add $-\hat{\Delta}_{w^{\lambda'}}$ to Γ and recover (3.30) in order $\hbar^n$.

Similarly, the breaking of the ST identity can be absorbed, if Δ_{ST} can be written as

$$\Delta_{\text{ST}} = \mathscr{S}_{\Gamma_{\text{cl}}}\hat{\Delta}_{\text{ST}}. \tag{3.33}$$

The question whether any $\Delta_{\rm ST}$ can be written in the form of (3.33) can be reduced to the algebraic problem to find the cohomology of $\mathscr{S}_{\Gamma_{\rm cl}}$. In our case, the nilpotency of the ST operator $\mathscr{S}_{\Gamma_{\rm cl}}$ has to be established by restricting $\Delta_{\rm ST}$ and $\hat{\Delta}_{\rm ST}$ to the kernel of $w_\mu^{\lambda'}$ or equivalently to functionals depending on $y_{\lambda'}$, $\bar{y}_{\lambda'}$ only through Υ defined in (3.13). Indeed, if the nilpotency condition (3.21) holds we find

$$0 = w_\mu^{\lambda'} \mathscr{S}(\Gamma) = w_\mu^{\lambda'} \Delta_{\rm ST} + \mathcal{O}(\hbar^{n+1}), \tag{3.34}$$

$$0 = \mathscr{S}_\Gamma \mathscr{S}(\Gamma) = \mathscr{S}_{\Gamma_{\rm cl}} \Delta_{\rm ST} + \mathcal{O}(\hbar^{n+1}), \tag{3.35}$$

which implies that $\Delta_{\rm ST}$ lies in the kernel of $w_\mu^{\lambda'}$ and satisfies the consistency condition for possible anomalies:

$$\mathscr{S}_{\Gamma_{\rm cl}} \Delta_{\rm ST} = 0. \tag{3.36}$$

Since $\mathscr{S}^2_{\Gamma_{\rm cl}} \gamma = 0$ for $w_\mu^{\lambda'} \gamma = 0$ as shown in section 3.1, the most general solution of the consistency condition can be written as follows

$$\Delta_{\rm ST} = \sum_i r_i \mathscr{A}_i + \mathscr{S}_{\Gamma_{\rm cl}} \hat{\Delta}_{\rm ST} \qquad \text{with } w_\mu^{\lambda'} \hat{\Delta}_{\rm ST} = 0, \tag{3.37}$$

where $\mathscr{S}_{\Gamma_{\rm cl}} \hat{\Delta}_{\rm ST}$ is a trivial solution of (3.36). The breaking terms $\mathscr{A}_i$ satisfy $\mathscr{S}_{\Gamma_{\rm cl}} \mathscr{A}_i = 0$, but cannot be written as $\mathscr{S}_{\Gamma_{\rm cl}} \hat{\mathscr{A}}_i$, i.e. they span the cohomology of $\mathscr{S}_{\Gamma_{\rm cl}}$ on the kernel of $w_\mu^{\lambda'}$. Hence, $\mathscr{A}_i$ constitute possible anomalies.

We assume that the only possible anomaly is the Adler-Bardeen anomaly in its supersymmetric version, whose coefficient r_i vanishes in the MSSM owing to the choice of the matter field representations. So we are left with the breaking terms $\mathscr{S}_{\Gamma_{\rm cl}} \hat{\Delta}_{\rm ST}$. $\Delta_{\rm ST}$ can be cancelled by adding the counterterms $-\hat{\Delta}_{\rm ST}$ to Γ. Because of (3.37) they do not destroy the nilpotency relation and both symmetry requirements can be satisfied simultaneously. The remaining symmetries can be restored in a similar manner.

3.4 Invariant counterterms

Having established the defining symmetries of the model by adding suitable symmetry-restoring counterterms, we are still free to add invariant counterterms which are in agreement with all symmetry properties. There are two fundamental types of invariant counterterms: the first type is required for the cancellation of UV singularities, while the second type comprises finite field reparametrizations as discussed in section 2.5.

Invariant counterterms of the first type have to respect all symmetry properties of the MSSM (see section 3.2). In particular, the counterterms of order $\hbar^n$ are restricted by

$$\mathscr{S}_{\Gamma_{\rm cl}} \Gamma^{(n)}_{\rm ct,inv} = 0, \qquad w_\mu^{\lambda'} \Gamma^{(n)}_{\rm ct,inv} = 0, \qquad W \Gamma^{(n)}_{\rm ct,inv} = 0, \tag{3.38a}$$

$$\frac{\delta \Gamma^{(n)}_{\rm ct,inv}}{\delta B} = 0, \qquad \frac{\delta \Gamma^{(n)}_{\rm ct,inv}}{\delta B'} = 0, \qquad \frac{\delta \Gamma^{(n)}_{\rm ct,inv}}{\delta \xi^\mu} = 0, \tag{3.38b}$$

where the Ward operator W runs over $\mathscr{W}, \mathscr{W}', \mathscr{W}_L, \mathscr{W}_Q, \mathscr{W}^R$. If the vertex functional Γ satisfies all symmetry identities, $\Gamma + \Gamma_{\rm ct,inv}$ satisfy the symmetries as well. Since $\mathscr{S}_{\Gamma_{\rm cl}}$ is nilpotent on the kernel of $w_\mu^{\lambda'}$, the counterterms $\Gamma^{(n)}_{\rm ct,inv}$ can be rewritten into the form

$$\Gamma^{(n)}_{\rm ct,inv} = \Gamma^{(n)}_{\rm ct,inv,1a} + \mathscr{S}_{\Gamma_{\rm cl}} \hat{\Gamma}^{(n)}_{\rm ct,inv,1b}, \tag{3.39}$$

where $\mathscr{S}_{\Gamma_{\rm cl}} \Gamma^{(n)}_{\rm ct,inv,1a} = 0$ but $\Gamma^{(n)}_{\rm ct,inv,1a}$ cannot be written as a $\mathscr{S}_{\Gamma_{\rm cl}}$ variation.

The strategy to find all invariant counterterms of the first type is the following: we first neglect the soft and spontaneous symmetry breaking and determine the most general contribution to $\Gamma_{\rm ct,inv,1a}$. Then it is shown that the soft and spontaneous symmetry breaking contribute only to $\Gamma_{\rm ct,inv,1b}$. In a second step, the most general form of $\Gamma_{\rm ct,inv,1b}$ is constructed.

The structure of $\Gamma^{(n)}_{\rm ct,inv,1a}$ can be derived from the supersymmetric SU(2)×U(1) gauge theory without spontaneous and soft-symmetry breaking, where the spurion fields and the Φ fields are absent. The invariant counterterms $\Gamma^{(n)}_{\rm ct,inv,1a}$ correspond to the free parameters of $\Gamma_{\rm susy}$ (2.7) [21]. In the MSSM, $\Gamma^{(n)}_{\rm ct,inv,1a}$ can be written as a combination of parameter renormalizations of the supersymmetric and gauge-invariant couplings $g, f_{R,U,D}, \mu, v'$ and of a combined field and parameter renormalization of $V'^\mu, \lambda', y_{\lambda'}, c', \bar{c}', B', \zeta', g'$ as in [16]. The counterterm for g' is not independent but governed by the field renormalization of V'^μ owing to the requirement of the local U(1) Ward identity (3.25). Note that V'^μ is the only propagating, linearly transforming field of the MSSM. The field renormalizations of the propagating, non-linearly transforming fields are included in $\Gamma_{\rm ct,inv,1b}$ due to the presence of $\Gamma_{\rm ext}$. Hence, $\Gamma^{(n)}_{\rm ct,inv,1a}$ takes the form

$$\begin{aligned}
\Gamma^{(n)}_{\rm ct,inv,1a} \;=\; & \left(\delta^{(n)} g\frac{\partial}{\partial g} + \delta^{(n)} f_R\frac{\partial}{\partial f_R} + \delta^{(n)} f_U\frac{\partial}{\partial f_U} + \delta^{(n)} f_D\frac{\partial}{\partial f_D} + \delta^{(n)}\mu\frac{\partial}{\partial \mu} + \delta^{(n)} v'\frac{\partial}{\partial v'}\right)\Gamma_{\rm cl} \\
& + \frac{1}{2}\delta^{(n)} z_{V'}\left[\int {\rm d}^4x \left(V'^\mu\frac{\delta}{\delta V'^\mu} + \lambda'^\alpha\frac{\delta}{\delta\lambda'^\alpha} - y^\alpha_{\lambda'}\frac{\delta}{\delta y^\alpha_{\lambda'}} + \bar{\lambda}'_{\dot\alpha}\frac{\delta}{\delta\bar{\lambda}'_{\dot\alpha}} - \bar{y}_{\lambda'\dot\alpha}\frac{\delta}{\delta\bar{y}_{\lambda'\dot\alpha}}\right.\right. \\
& \left.\left. + c'\frac{\delta}{\delta c'} - \bar{c}'\frac{\delta}{\delta\bar{c}'} - B'\frac{\delta}{\delta B'}\right) - g'\frac{\partial}{\partial g'} + 2\zeta'\frac{\partial}{\partial\zeta'}\right]\Gamma_{\rm cl}.
\end{aligned} \tag{3.40}$$

Special attention has to be drawn to the spurion parameters. These parameters are introduced in section 2.2 with the help of the spurion superfield, and the spurion superfield is replaced later by the BRS doublet $(A_1, A_2 + v_A)$ [c.f. (2.15)]. This replacement does not invalidate the conditions (3.38), but it excludes certain counterterms involving A, a_α. As a special property of BRS doublets, all counterterms involving BRS doublets are $\mathscr{S}_{\Gamma_{\rm cl}}$ variations and thus included in $\Gamma^{(n)}_{\rm ct,inv,1b}$, which follows from the well-known theorems (see e.g. section 5.2 of [29]) implying that the cohomology of $\mathscr{S}_{\Gamma_{\rm cl}}$ is isomorphic to a subspace of the cohomology of $\mathscr{S}_{\Gamma_{\rm cl}}|_{\rm doublets=0}$ in the space where the doublets are set to zero.

Similar arguments hold also for the invariant counterterms involving the other BRS

doublets $(\Phi_i+\mathbf{v}_i,\Psi)$, $(\Phi_i'+\mathbf{v}_i',\Psi')$, $(\bar{c},B)$, $(\bar{c}',B')$. In particular, the invariant counterterms including these fields contribute to $\Gamma_{\text{ct,inv,1b}}$.

In a second step, the most general form of $\Gamma^{(n)}_{\text{ct,inv,1b}} = \mathscr{S}_{\Gamma_{\text{cl}}}\hat{\Gamma}^{(n)}_{\text{ct,inv,1b}}$ is obtained by writing down all possible local field polynomials with dimension 4 and $Q_{\Phi\Pi}=-1$ whose $\mathscr{S}_{\Gamma_{\text{cl}}}$ variation satisfies all symmetry requirements. Apart from the global symmetries, the requirement that $\Gamma^{(n)}_{\text{ct,inv}}$ depends on A_1 only through the combination $a_\alpha = \sqrt{2}\epsilon_\alpha A_1$ is important and reduces the number of possible spurion counterterms strongly.

As shown in appendix E, one part of the resulting invariant counterterms can be identified with the renormalization of parameters and fields in Γ_{cl}. This part of $\Gamma_{\text{ct,inv,1b}}$ comprises the field renormalization of all non-linearly transforming fields, the counterterms to all soft-breaking parameters, and the renormalization of x_1, x_2, v_1, v_2. Due to the presence of Γ_{ext} and $\Gamma_{\text{g.f.}}$, the field renormalizations must be compensated by similar renormalizations for the Y fields, B fields, and gauge parameters ζ,ζ'. The remaining part of the counterterms in $\Gamma_{\text{ct,inv,1b}}$ is irrelevant for physical quantities and discarded in the following.

Combining the results for $\Gamma^{(n)}_{\text{ct,inv,1a}}$ and $\Gamma^{(n)}_{\text{ct,inv,1b}}$ shows that the relevant counterterms of the first type are generated by renormalization of the fields and parameters in Γ_{cl}. The parameter renormalization in $\Gamma_{\text{ct,inv}}$ can be obtained by applying the following renormalization transformations on Γ_{cl}

$$g_i \to g_i + \sum_n \delta^{(n)} g_i \tag{3.41}$$

for the free parameters g_i of the MSSM listed in (2.75a)–(2.75c). The general renormalization transformations for the fields are given by

$$\begin{aligned}
\{V^\mu, Y_V^\mu, B, \bar{c}, \zeta\} &\to \{\sqrt{z_V}V^\mu, \sqrt{z_V}^{-1}Y_V, \sqrt{z_V}^{-1}B, \sqrt{z_V}^{-1}\bar{c}, z_V\zeta\},\\
\{\lambda, y_\lambda\} &\to \{\sqrt{z_\lambda}\lambda, \sqrt{z_\lambda}^{-1}y_\lambda\},\\
\{c, Y_c\} &\to \{\sqrt{z_c}c, \sqrt{z_c}^{-1}Y_c\},\\
\{V'^\mu, \lambda', y_{\lambda'}, c', B', \bar{c}', \zeta', g'\} &\to \left\{\begin{array}{l} \sqrt{z_{V'}}V'^\mu, \sqrt{z_{V'}}\lambda', \sqrt{z_{V'}}^{-1}y_{\lambda'},\\ \sqrt{z_{V'}}c', \sqrt{z_{V'}}^{-1}B', \sqrt{z_{V'}}^{-1}\bar{c}', z_{V'}\zeta', \sqrt{z_{V'}}^{-1}g' \end{array}\right\},\\
\{H_i, Y_{H_i}\} &\to \{\sqrt{z_{H_i}}H_i, \sqrt{z_{H_i}}^{-1}Y_{H_i}\},\\
\{h_i, y_{h_i}\} &\to \{\sqrt{z_{h_i}}h_i, \sqrt{z_{h_i}}^{-1}y_{h_i}\},\\
(\Phi_i^a + \mathbf{v}_i^a, \Psi_i^a) &\to \sqrt{z_V}\sqrt{z_{H_i}}^{-1}(\Phi_i^a + \mathbf{v}_i^a, \Psi_i^a),\\
(\Phi_i' + \mathbf{v}_i', \Psi_i') &\to \sqrt{z_{V'}}\sqrt{z_{H_i}}^{-1}(\Phi_i' + \mathbf{v}_i', \Psi_i'),\\
\{L, Y_L\} &\to \{\sqrt{z_L}L, \sqrt{z_L}^{-1}Y_L\},\\
&\quad \text{(and analogously for the sfermions } R, Q, U, D),\\
\{l, y_l\} &\to \{\sqrt{z_l}l, \sqrt{z_l}^{-1}Y_l\},\\
&\quad \text{(and analogously for the fermions } r, q, u, d).
\end{aligned} \tag{3.42}$$

The second type of invariant counterterms differs substantially from the first type. From the algebraic point of view, $\Gamma_{\text{ct,inv}}$ is sufficient to cancel all UV singularities. However, we are still free to choose a basis of independent *physical* fields in terms of which the vertex functional is parametrized.

This parametrization is defined by the choice of finite demixing matrices R. Indeed, changing these matrices $R = R^{(0)}(1 + \sum_n \delta^{(n)} R)$ simultaneously in the vertex functional Γ and the ST and Ward identities preserves all symmetries. Since the validity of the symmetries is not affected, these new contributions satisfy

$$\delta^{(n)}\mathscr{S}(\Gamma_{\text{cl}}) + \mathscr{S}_{\Gamma_{\text{cl}}}\ \Gamma^{(n)}_{\text{ct,inv,finite}} = 0, \tag{3.43a}$$

$$\delta^{(n)} W\Gamma_{\text{cl}} + W^{(0)}\ \Gamma^{(n)}_{\text{ct,inv,finite}} = 0. \tag{3.43b}$$

In accordance with (2.27), the invariant counterterms $\Gamma^{(n)}_{\text{ct,inv,finite}}$ and the counterterms of the symmetry operators $\delta^{(n)}\mathscr{S}$, $\delta^{(n)}W$ are equivalently generated by the replacement

$$\phi_j^{\text{phys}} \to \delta^{(n)} R_{ji} \phi_i^{\text{phys}}. \tag{3.44}$$

Symbolically, $\Gamma^{(n)}_{\text{ct,inv,finite}}$ is given by

$$\Gamma^{(n)}_{\text{ct,inv,finite}} = \delta^{(n)} R_{ji} \left(\int \mathrm{d}^4x\, \phi_i^{\text{phys}} \frac{\delta}{\delta \phi_j^{\text{phys}}} \right) \Gamma_{\text{cl}}, \tag{3.45}$$

where ϕ_i^{phys} runs over all propagating fields. The matrix R has a block structure since mixing is only possible between fields with equal quantum numbers. The different submatrices $R^{(0)}_\phi$ are already introduced in section 2.5.

The net result for the invariant counterterms is very simple. The invariant counterterms of order $\hbar^n$ correspond to the renormalization of all free parameters and fields of the classical action. They can be generated if the following transformations and replacements are performed in the classical action:

$$g_i \to g_i + \sum_n \delta^{(n)} g_i, \tag{3.46a}$$

$$\phi^{\text{sym}} \to \sqrt{z}\, \phi^{\text{sym}}, \tag{3.46b}$$

$$\phi^{\text{sym}} = R\, \phi^{\text{phys}}, \tag{3.46c}$$

and by extracting the pure $\delta^{(n)}$ part. This shows that the classical action we started with was complete and that the theory is multiplicatively renormalizable.

3.5 Masslessness of the photon

The MSSM contains exactly one unbroken gauge symmetry, and this implies that there is exactly one massless gauge boson. For completeness, we will give a simple proof of this

well-known statement, relying only on the ST identity. In addition, we will show that there is one massless Faddeev-Popov ghost and that there are three massless directions of the Higgs potential.

The results of this section hold in all orders of perturbation theory and are important for the physical interpretation as well as for the theoretical considerations of the following sections. They will be of particular importance for the discussion of the IR finiteness and of the renormalization scheme.

For convenience, we use the condensed notation $V^a = (V^1, V^2, V^3, V')$ and similarly for c^a, $\bar{c}^a$, B^a. Field derivatives are denoted by subscripts, i.e. $\Gamma_\phi = \delta\Gamma/\delta\phi$ etc.

Using $\delta^2\mathscr{S}(\Gamma)/(\delta c^a \delta H_l^j) = 0$ we obtain

$$0 = \Gamma_{c^a Y_i^k}\Gamma_{H_l^j H_i^k} + \Gamma_{c^a \bar{Y}_i^k}\Gamma_{H_l^j \bar{H}_i^k} + \Gamma_{c^a Y_V^{b\mu}}\Gamma_{H_l^j V_\mu^b}, \tag{3.47}$$

where the last term vanishes for $p = 0$ since both factors are proportional to p^μ. Combining (3.47) with a similar identity involving $\bar{H}_l^j$ we derive the following identity at $p = 0$:

$$0 = (\Gamma_{c^a Y_i^k}, \Gamma_{c^a \bar{Y}_i^k}) \begin{pmatrix} \Gamma_{H_l^j H_i^k} & \Gamma_{\bar{H}_l^j H_i^k} \\ \Gamma_{H_l^j \bar{H}_i^k} & \Gamma_{\bar{H}_l^j \bar{H}_i^k} \end{pmatrix}. \tag{3.48}$$

If the vector $(\Gamma_{c^a Y_i^k}, \Gamma_{c^a \bar{Y}_i^k})$ is non-vanishing for fixed a, it constitutes an eigenvector of the Higgs self-energy matrix to the eigenvalue zero. Thus, to each non-vanishing $(\Gamma_{c^a Y_i^k}, \Gamma_{c^a \bar{Y}_i^k})$ there is a massless direction of the Higgs self-energy matrix.

In the case of gauge bosons, the ST identity $\delta^2\mathscr{S}(\Gamma)/(\delta c^a \delta V^{b\mu})$ reads

$$0 = \Gamma_{c^a Y_i^k}\Gamma_{V^{b\mu} H_i^k} + \Gamma_{c^a \bar{Y}_i^k}\Gamma_{V^{b\mu} \bar{H}_i^k} + \Gamma_{c^a Y_{V\nu}^c}\Gamma_{V^{b\mu} V^{c\nu}}. \tag{3.49}$$

Suppose there is a non-trivial linear combination of the four vectors $(\Gamma_{c^a Y_i^k}, \Gamma_{c^a \bar{Y}_i^k})$ that satisfies

$$\sum_{a=1,2,3,'} b_a (\Gamma_{c^a Y_i^k}, \Gamma_{c^a \bar{Y}_i^k}) = 0. \tag{3.50}$$

After contracting with b_a and performing the derivative $\partial/\partial p^\mu$ at $p = 0$, (3.49) reduces to

$$0 = \sum_{a,c=1,2,3,'} b_a \, D_{ac} \, \Gamma^1_{V^b V^c}, \tag{3.51}$$

where the Lorentz decompositions $\Gamma_{c^a Y_{V\nu}^c} = p_\nu D_{ac}$ and $\Gamma_{V^{b\mu} V^{c\nu}} = g_{\mu\nu}\Gamma^1_{V^b V^c} + p_\mu p_\nu \Gamma^2_{V^b V^c}$ are used. Since D_{ac} is invertible, $\Gamma^1_{V^b V^c} \propto p^2$. After defining the transverse and longitudinal parts of the gauge-boson self energy

$$\Gamma_{V^{b\mu} V^{c\nu}} = -\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Gamma^T_{V^b V^c} - \frac{p_\mu p_\nu}{p^2}\Gamma^L_{V^b V^c}, \tag{3.52}$$

we find $\Gamma^T_{V^bV^c} = \Gamma^1_{V^bV^c} \propto p^2$. This proves that there is a massless gauge boson corresponding to the linear combination (3.50).

Similarly, contracting the identity $\delta^2\mathscr{S}(\Gamma)/(\delta c^a \delta B^b)$ with b_a yields

$$0 = \sum_{a=1,2,3,\prime} b_a \, \Gamma_{c^a \bar{c}^b}, \tag{3.53}$$

proving that there is a massless ghost.

As a result, each non-vanishing vector $(\Gamma_{c^a Y^i_k}, \Gamma_{c^a \bar{Y}^i_k})$ corresponds to a broken symmetry and implies the existence of a massless Goldstone direction of the Higgs potential. Furthermore, a vanishing linear combination (3.50) corresponds to an unbroken symmetry and implies the existence of a massless gauge boson and a massless ghost.

In the MSSM, the five physical Higgs bosons and the $W^\pm$ and Z bosons are massive in lowest order and thus also in higher orders. Therefore, the four eigenvectors $(\Gamma_{c^a Y^i_k}, \Gamma_{c^a \bar{Y}^i_k})$, $a = 1, 2, 3, \prime$ of the Higgs self-energy matrix are linearly dependent, and there exists at least one vanishing, non-trivial linear combination (3.50) and at least one unbroken symmetry. On the other hand, there is at most one massless gauge boson and thus at most one unbroken symmetry in higher orders.

Combining both results we find that to all orders there is exactly one unbroken and three broken symmetries, and there are one massless gauge boson, one massless ghost, and three massless would-be Goldstone modes.

3.6 Off-shell infrared finiteness

Since our model contains massless fields, infrared (IR) singularities can appear in loop diagrams. Partly, these IR singularities are of physical origin and well understood [30]. They originate from soft photons and cancel in combination with the corresponding contributions from the bremsstrahlung processes. On the other hand, additional artificial IR singularities without any physical meaning can appear in principle. Such IR singularities would arise already in off-shell amplitudes and would destroy the mathematical consistency of the model. The purpose of the present section is to prove the absence of such unphysical IR singularities to all orders. In the following, we restrict our discussion only to these artificial IR singularities.

While the appearance or non-appearance of IR singularities is not specific to any regularization scheme, we adopt Lowenstein's generalization of the BPHZ subtraction scheme [31] because it provides a very systematic way to deal with IR singularities. This BPHZL scheme assigns to massless fields an auxiliary mass $M(s-1)$ that regularizes all IR singularities in intermediate steps. At the end the limit $s \to 1$ is taken. The Bogoliubov R

operation for a renormalization part γ reads

$$R\gamma = \left(1 - t^{\rho(\gamma)-1}_{p,s-1}\right)\left(1 - t^{\delta(\gamma)}_{p,s}\right)\gamma, \tag{3.54}$$

where t^d_x is the Taylor expansion operator in the variable x up to degree d. The subtractions are performed in the external momenta p as well as in the parameter s (resp. $s-1$). $\rho(\gamma)$, $\delta(\gamma)$ are the IR resp. UV degrees of divergence of the diagram γ, given by the usual power-counting law in terms of the IR and UV dimensions of the fields. While we use the canonical dimensions for the UV dimensions, the freedom in assigning IR degrees has to be used to meet some basic requirements.

The two subtractions in (3.54) are compatible if the following inequalities are satisfied:

$$\dim_{\mathrm{IR}}\phi_i \geq \dim_{\mathrm{UV}}\phi_i, \tag{3.55a}$$

$$\dim_{\mathrm{IR}}\phi_i + \dim_{\mathrm{IR}}\phi_j \leq 4 + r_{ij}, \tag{3.55b}$$

where r_{ij} is the scaling exponent of the propagator $\Delta_{ij} = \langle\phi_i\phi_j\rangle$,

$$\Delta_{ij}(\lambda p, \lambda(s-1)) \sim \lambda^{r_{ij}} \qquad \text{for } \lambda \to 0,\ s \to 1. \tag{3.56}$$

The BPHZL procedure then yields finite results at non-exceptional momenta for all diagrams that contain an arbitrary number of integrated vertices of IR dimension $\dim_{\mathrm{IR}} \geq 4$. In addition, a diagram is allowed to contain one single integrated vertex of dimension $4 > \dim_{\mathrm{IR}} \geq 5/2$. As a consequence, if all interaction vertices in Γ_{eff} — the classical action plus counterterms — have $\dim_{\mathrm{IR}} \geq 4$, then all diagrams contributing to Γ are IR finite. There is an exception: the IR degree of terms that are linear in propagating fields is irrelevant as such terms cannot occur in one-particle-irreducible (1PI) loop diagrams.

In addition to the IR finiteness of Γ, there is a more subtle requirement. The application of Ward and ST operators on the vertex functional yields an insertion which has to be off-shell IR finite, since otherwise the notion of "symmetry transformation" would be meaningless. If Γ_{eff} has $\dim_{\mathrm{IR}} = 4$, an operator $\int \mathrm{d}^4x\, \phi_i \delta/\delta\phi_j$ leads to an insertion with $\dim_{\mathrm{IR}} = 4 + \dim_{\mathrm{IR}}\phi_i - \dim_{\mathrm{IR}}\phi_j$. The insertions resulting from rigid symmetry operators must have $\dim_{\mathrm{IR}} \geq 5/2$. Again, there is the exception that linear contributions to an insertion are allowed to have arbitrarily low IR dimension. Moreover, local Ward operators yield non-integrated insertions and can have arbitrary IR dimensions.

In order to prove the IR finiteness, we have to find a choice of IR dimensions that meets the requirements (3.55). Then it has to be shown that only the IR-safe terms listed above appear in Γ_{eff} and the Ward and ST operators. For this proof it is important to exploit the freedom in assigning the IR dimensions in such a way that all IR dimensions are as high as possible, otherwise forbidden terms in Γ_{eff} or the symmetry operators cannot be avoided.

Field	A^μ	c_A	B_A	$\bar{c}_A$	c_Z	B_Z	$\bar{c}_Z$	$c_\pm$	$B_\pm$	$\bar{c}_\pm$	l^1_α
$\dim_{\rm IR}$	1	0	2	2	1	2	3	1	2	3	3/2

Table 3.3: Exceptional IR dimensions of propagating fields

A possible choice of IR dimensions is as follows. The dynamical fields

$$W^{\pm\mu}, Z^\mu, G^0, G^\pm, A^0, H^\pm, H^0, h^0, \chi^\pm_\alpha, \chi^0_\alpha, L, R, l^2_\alpha, r_\alpha, Q, U, D, q_\alpha, u_\alpha, d_\alpha \tag{3.57}$$

have $\dim_{\rm IR} = 2$ as usual for massive fields. The assignments for the remaining propagating fields are collected in table 3.3. For the constant ghosts $\xi^\mu, \varepsilon_\alpha$ we assign the UV and IR dimension zero, and we set the ultraviolet dimensions of all non-constant external fields to their canonical dimension. To the external source fields Y, y we assign IR dimensions that equal their canonical dimensions. For the external fields Φ, A_2 we distinguish between the components appearing with a shift in some Ward or ST operator and the remaining ones. In the SU(2) Ward identity there are three shifted components, which can be identified in the spirit of external Goldstone modes as

$$\sum_{i=1,2} -\mathrm{i}g\frac{\delta}{\delta\Phi^a_i}\left(T^b\mathbf{v}^a_i + \mathrm{i}\epsilon^{bca}\mathbf{v}^c_i\right) - \mathrm{i}g'\frac{\delta}{\delta\Phi'_i}T^b\mathbf{v}'_i + \text{c.c.} \sim \frac{\delta}{\delta\mathscr{G}^b}. \tag{3.58}$$

In the same way we define an external Goldstone mode $\mathscr{G}^R$ corresponding to R symmetry by requiring that this field carries the whole shift contribution of the R Ward identity,

$$\sum_{i=1,2} \mathrm{i}\frac{\delta}{\delta\Phi^a_i}\mathbf{v}^a_i + \mathrm{i}\frac{\delta}{\delta\Phi'_i}\mathbf{v}'_i - 2\mathrm{i}\frac{\delta}{\delta A_2}v_A + \text{c.c.} \sim \frac{\delta}{\delta\mathscr{G}^R}. \tag{3.59}$$

We consider $\mathscr{G}^1$, $\mathscr{G}^2$, $\mathscr{G}^3$, $\mathscr{G}^R$, A_2 and 28 independent components of Φ, Φ' as our basis fields. The $\mathscr{G}$ modes have $\dim_{\rm IR} = 1$, the remaining fields have $\dim_{\rm IR} = 2$. These exceptional assignments as well as the IR dimensions for the constant ghosts can be found in table 3.4.

Using these assignments we will show in the following that the vertex functional as well as the insertions produced by Ward and ST operators are off-shell IR finite. We begin by decomposing $\Gamma_{\rm eff}$ into a part $\Gamma^{\rm dyn}_{\rm eff}$ that is at least quadratic in propagating fields and an at most linear part $\Gamma^{\rm triv}_{\rm eff}$. By requiring $\dim_{\rm IR}\Gamma^{\rm dyn}_{\rm eff} = 4$ we ensure IR finiteness of all Green functions. $\Gamma^{\rm dyn}_{\rm eff}$ leads to a vertex functional $\Gamma^{\rm dyn}$, and the full vertex functional is given by

$$\Gamma = \Gamma^{\rm dyn} + \Gamma^{\rm triv}_{\rm eff}, \tag{3.60}$$

since the terms in $\Gamma^{\rm triv}_{\rm eff}$ cannot contribute to non-trivial 1PI diagrams. Applying a Ward operator yields

$$\mathscr{W}\Gamma = \mathscr{W}\Gamma^{\rm dyn} + \mathscr{W}\Gamma^{\rm triv}_{\rm eff} = [\mathscr{W}\Gamma_{\rm eff}]\cdot\Gamma^{\rm dyn} + \mathscr{W}\Gamma^{\rm triv}_{\rm eff} = [\mathscr{W}\Gamma_{\rm eff}]\cdot\Gamma + \mathscr{W}\Gamma^{\rm triv}_{\rm eff}. \tag{3.61}$$

Field	ε^α	ξ^μ	A_1	A_2	$\mathscr{G}^b, \mathscr{G}^R$	Φ, Φ' (remnant)	Ψ, Ψ'
$\dim_{\rm IR}$	0	0	1	2	1	2	2

Table 3.4: IR dimensions of non-propagating fields

Clearly, the IR dimension of $\mathscr{W}\Gamma_{\rm eff}$ must be $\dim_{\rm IR} \geq 5/2$ while the IR dimension of $\mathscr{W}\Gamma^{\rm triv}_{\rm eff}$ is irrelevant. Dangerous contributions to the Ward operator are e.g.

$$A^\mu \frac{\delta}{\delta Z^\mu}, \qquad c_A \frac{\delta}{\delta c_Z}, \qquad \frac{\delta}{\delta \mathscr{G}^b}. \tag{3.62}$$

Owing to the choices of the IR dimensions, the operators in (3.62) lead to insertions with $\dim_{\rm IR} = 3$, which are allowed. This explains in particular the choices for the $\mathscr{G}$ modes.

Indeed, it is easy to check that our choice of IR dimensions guarantees that all Ward operators lead to IR-finite insertions. In the same way we can show the IR finiteness of the ST identity. Our decomposition of Γ yields three contributions

$$\mathscr{S}(\Gamma) = \mathscr{S}(\Gamma^{\rm dyn}) + \mathscr{S}_{\Gamma^{\rm dyn}}\Gamma^{\rm triv}_{\rm eff} + \int \mathrm{d}^4x \, \frac{\delta\Gamma^{\rm triv}_{\rm eff}}{\delta Y_a}\frac{\delta\Gamma^{\rm triv}_{\rm eff}}{\delta\phi_a}, \tag{3.63}$$

the last of which is IR harmless. The first contribution $\mathscr{S}(\Gamma^{\rm dyn})$ corresponds to an insertion[6] whose IR dimension is determined by the quantum action principle [28], which assigns a degree of $8-\dim_{\rm IR}Y-\dim_{\rm IR}\phi$ to the term $(\delta\Gamma^{\rm dyn}/\delta Y)(\delta\Gamma^{\rm dyn}/\delta\phi)$ and $\dim_{\rm IR}(\mathbf{s}\,\phi')+4-\dim_{\rm IR}\phi'$ to $\mathbf{s}\,\phi'(\delta\Gamma/\delta\phi')$. Owing to the choice of the IR dimensions, $\dim_{\rm IR}(\mathscr{S}(\Gamma^{\rm dyn})) \geq 5/2$.

These simple arguments are sufficient to show the IR finiteness of $\mathscr{S}(\Gamma^{\rm dyn})$ because $\Gamma^{\rm dyn}_{\rm eff}$ contains only contributions with $\dim_{\rm IR} \geq 4$. They cannot be extended to the complete vertex functional, since we have allowed for terms with $\dim_{\rm IR} < 4$ in $\Gamma^{\rm triv}_{\rm eff}$. Instead, we discuss the second contribution in (3.63) separately. It reads explicitly

$$\mathscr{S}_{\Gamma^{\rm dyn}}\Gamma^{\rm triv}_{\rm eff} = \int \mathrm{d}^4x \, \frac{\delta\Gamma^{\rm dyn}}{\delta Y_a}\frac{\delta\Gamma^{\rm triv}_{\rm eff}}{\delta\phi_a} + \int \mathrm{d}^4x \, \frac{\delta\Gamma^{\rm triv}_{\rm eff}}{\delta Y_a}\frac{\delta\Gamma^{\rm dyn}}{\delta\phi_a} + \int \mathrm{d}^4x \, \mathbf{s}\,\phi'_b \frac{\delta\Gamma^{\rm triv}_{\rm eff}}{\delta\phi'_b}. \tag{3.65}$$

The third term in (3.65) is trivial, since it involves no loop contributions. The first two terms have the form of a Ward identity with local, at most linear field transformations

[6] This can be seen by temporarily introducing the generating functional Z of general Green functions by Legendre transformation introducing sources j_a, j'_b and exponentiation. In contrast to the ST identity on Γ, the insertions of the ST operator on Z have the explicit form

$$\mathscr{S}(Z) = \left[\frac{\delta\Gamma_{\rm eff}}{\delta Y_a} j_a + \mathbf{s}\,\phi'_b j'_b\right]^\rho_\delta \cdot Z, \tag{3.64}$$

where the required degree of subtraction can be read off.

$\tilde{\delta}Y_a = \delta\Gamma^{\rm triv}_{\rm eff}/\delta\phi_a$, $\tilde{\delta}\phi_a = \delta\Gamma^{\rm triv}_{\rm eff}/\delta Y_a$. By explicitly inspecting all possible terms in $\Gamma^{\rm triv}_{\rm eff}$ we find that this leads to an insertion with $\dim_{\rm IR} \geq 5/2$. Thus we have shown that our choice of IR dimensions of the fields indeed leads to finite Ward and ST identities.

It remains one problem to be solved: for the restoration of the Ward and ST identities we might be forced to use IR-forbidden symmetry-restoring counterterms in $\Gamma_{\rm eff}$. The complete list of such IR-dangerous counterterms involving at least two propagating fields reads

$$A^\mu A_\mu, \quad A^\mu Z_\mu, \quad c_A\bar{c}_A, \quad c_A\bar{c}_Z, \quad c_Z\bar{c}_A. \tag{3.66}$$

All other IR-dangerous terms are excluded from $\Gamma_{\rm eff}$ by lepton-number conservation for terms involving neutrinos, and due to CP conservation for terms involving $\mathscr{G}^b$, $\mathscr{G}^R$, like $A^\mu A_\mu \mathscr{G}^b$ and $c_A\bar{c}_A\mathscr{G}^b$.

None of the terms in (3.66) appears in the classical action. If these terms can also be excluded from the counterterm part of $\Gamma_{\rm eff}$, the BPHZL scheme guarantees the IR finiteness of the vertex functional Γ. This is a special property of the BPHZL scheme since loop corrections corresponding to terms in (3.66) vanish automatically at $p = 0$ due to the special form of the subtraction operator (3.54). In general, IR finiteness of Γ follows from the conditions

$$\Gamma_{AA}|_{p=0} = \Gamma_{AZ}|_{p=0} = \Gamma_{c_A\bar{c}_A}|_{p=0} = \Gamma_{c_A\bar{c}_Z}|_{p=0} = \Gamma_{c_Z\bar{c}_A}|_{p=0} = 0. \tag{3.67}$$

It remains to be shown that the IR-dangerous counterterms are not required for the restoration of the symmetry identities.

Firstly we discuss the term $A^\mu Z_\mu$. Using the results of section 2.5 we find that among the invariant counterterms of the first type there are the parameters from field reparametrization

$$\left[(\delta^{(n)}R_V)_{ZA}A_\mu\frac{\delta}{\delta Z_\mu} + (\delta^{(n)}R_V)_{ZZ}Z_\mu\frac{\delta}{\delta Z_\mu}\right]\frac{1}{2}\left(M_Z^2 Z^\mu Z_\mu\right)$$
$$= (\delta^{(n)}R_V)_{ZA}M_Z^2 A^\mu Z_\mu + (\delta^{(n)}R_V)_{ZZ}M_Z^2 Z^\mu Z_\mu \tag{3.68}$$

at our disposal. Clearly, $(\delta^{(n)}R_V)_{ZA}$ can be adjusted such that $\Gamma_{A^\mu Z^\nu}|_{p=0}$. Similarly, the constants $(\delta^{(n)}R_c)_{ZA}$ and $(\delta^{(n)}R_{\bar{c}})_{ZA}$, corresponding to the field reparametrization of c, $\bar{c}$, can be used to cancel the counterterms $c_A\bar{c}_Z$, $c_Z\bar{c}_A$ and equivalently to establish $\Gamma_{c_A\bar{c}_Z}|_{p=0} = \Gamma_{c_Z\bar{c}_A}|_{p=0} = 0$.

The remaining conditions in (3.67) cannot be established by a field reparametrization. Instead, we show that they are satisfied owing to the result of section 3.5 that there is exactly one unbroken gauge symmetry in the MSSM.

We turn to the term $A^\mu A_\mu$. As shown in section 3.5, the determinant of the gauge-boson self energy vanishes at $p = 0$. Since $\Gamma_{ZZ}|_{p=0}, \Gamma_{W^+W^-}|_{p=0} \neq 0$, and since the off-diagonal

elements $\Gamma_{AZ}|_{p=0}$ vanish, the photon is massless, i.e. $\Gamma_{AA}|_{p=0} = 0$. In the same way it is shown that $\Gamma_{c_A\bar{c}_A}|_{p=0} = 0$.

This completes our proof that the conditions (3.67) can be satisfied without destroying the symmetries. Hence, in the BPHZL scheme the counterterm contributions to the forbidden terms in (3.66) vanish, and the IR finiteness is proven.

In schemes that are different from the BPHZL scheme, the counterterm contributions to Γ_{eff} might be different, but since Γ is a regularization-scheme independent quantity, the infrared finiteness of Γ holds for all schemes. For example, in dimensional regularization the loop and counterterm contributions to the IR-dangerous terms need not vanish individually but the net result for Γ is the same as in the BPHZL scheme.

4 Renormalization scheme

In this section we specify a renormalization scheme for the MSSM. We use an on-shell scheme where all normalization conditions are formulated at the physical particle masses, and where transitions between different fields are avoided for on-shell momenta as far as possible. As a consequence, up to effects due to the finite widths, every field corresponds either to a mass eigenstate or to an unphysical degree of freedom with simple formal properties.

Before we start with the discussion of the renormalization scheme, we introduce some notations. The complex mass $\mathscr{M}_\phi$ of an unstable particle ϕ corresponds to the location of the particle pole in the complex invariant-mass plane and is decomposed into a real mass M_ϕ and a finite width Γ_ϕ as follows

$$\mathscr{M}_\phi^2 = M_\phi^2 - \mathrm{i}\Gamma_\phi M_\phi. \tag{4.1}$$

Since the complex particle pole is gauge independent, it is a proper quantity for the definition of normalization conditions.

4.1 Normalization conditions

The on-shell normalization conditions are listed separately for the different sectors of the MSSM.

Gauge-boson and Higgs sector

Since the $W^\pm$ and Z bosons are unstable particles, the poles of their propagators are complex. Since we require that all tadpole contributions vanish, the poles $\mathscr{M}_{W,Z}^2$ are

determined by the normalization conditions

$$\det \begin{pmatrix} \Gamma^T_{AA} & \Gamma^T_{AZ} \\ \Gamma^T_{ZA} & \Gamma^T_{ZZ} \end{pmatrix}\Big|_{p^2=\mathscr{M}^2_Z} = 0, \qquad \Gamma^T_{W^+W^-}|_{p^2=\mathscr{M}^2_W} = 0, \tag{4.2}$$

where Γ^T is the transverse part of the gauge-boson self energy defined in (3.52). The real masses M_W, M_Z are adopted as the two free parameters in the gauge-boson sector. The imaginary parts of $\mathscr{M}^2_{W,Z}$ are determined as functions of $M_{W,Z}$. To establish the masslessness of the photon and to avoid mixing of gauge-bosons as far as possible, we require the conditions at the real masses:

$$\Gamma^T_{AA}|_{p^2=0} = 0, \qquad \Gamma^T_{AZ}|_{p^2=0} = 0, \tag{4.3a}$$

$$\mathrm{Re}\Gamma^T_{ZA}|_{p^2=M^2_Z} = 0. \tag{4.3b}$$

The normalization conditions for the residua of the full gauge-boson propagators are given by

$$\partial_{p^2}\Gamma^T_{AA}|_{p^2=0} = 1, \qquad \mathrm{Re}\partial_{p^2}\Gamma^T_{ZZ}|_{p^2=M^2_Z} = 1, \tag{4.4a}$$

$$\mathrm{Re}\partial_{p^2}\Gamma^T_{W^+W^-}|_{p^2=M^2_W} = 1. \tag{4.4b}$$

In the Higgs sector, the real mass M_{A^0} of the neutral CP-odd Higgs boson A_0 is chosen as the only free parameter. Accordingly, we require the normalization condition

$$\det \begin{pmatrix} \Gamma_{G^0G^0} & \Gamma_{G^0A^0} \\ \Gamma_{A^0G^0} & \Gamma_{A^0A^0} \end{pmatrix}\Big|_{p^2=\mathscr{M}^2_{A^0}} = 0. \tag{4.5}$$

with $\mathrm{Re}\mathscr{M}^2_{A^0} = M^2_{A^0}$. The masses of the Higgs bosons H^0, h^0, $H^\pm$ are no free parameters of the MSSM. These masses $\mathscr{M}_{H^0}$, $\mathscr{M}_{h^0}$, $\mathscr{M}_{H^\pm}$ are defined by

$$\det \begin{pmatrix} \Gamma_{H^0H^0} & \Gamma_{H^0h^0} \\ \Gamma_{h^0H^0} & \Gamma_{h^0h^0} \end{pmatrix}\Big|_{p^2=\mathscr{M}^2_{H^0},\mathscr{M}^2_{h^0}} = 0, \quad \det \begin{pmatrix} \Gamma_{G^+G^-} & \Gamma_{G^+H^-} \\ \Gamma_{H^+G^-} & \Gamma_{H^+H^-} \end{pmatrix}\Big|_{p^2=\mathscr{M}^2_{H^\pm}} = 0. \tag{4.6}$$

In this way, the complex masses of the Higgs bosons H^0, h^0, $H^\pm$ can be calculated order by order as a function of M_{A^0} and the other MSSM parameters (c.f. section 2.8). The demixing conditions in the Higgs sector are given by

$$\mathrm{Re}\Gamma_{A^0B_A}|_{p^2=M^2_{A^0}} = 0, \qquad \mathrm{Re}\Gamma_{A^0B_Z}|_{p^2=M^2_{A^0}} = 0, \tag{4.7a}$$

$$\mathrm{Re}\Gamma_{A^0G^0}|_{p^2=M^2_{A^0}} = 0, \qquad \mathrm{Re}\Gamma_{H^+H^-}|_{p^2=M^2_{H^\pm}} = 0, \tag{4.7b}$$

$$\mathrm{Re}\Gamma_{H^+G^-}|_{p^2=M^2_{H^\pm}} = 0, \qquad \mathrm{Re}\Gamma_{H^+B^-}|_{p^2=M^2_{H^\pm}} = 0, \tag{4.7c}$$

$$\mathrm{Re}\Gamma_{H^0h^0}|_{p^2=M^2_{H^0}} = 0, \qquad \mathrm{Re}\Gamma_{h^0H^0}|_{p^2=M^2_{h^0}} = 0. \tag{4.7d}$$

The demixing between Higgs bosons and longitudinal gauge bosons is not realized in (4.7). The effects of this mixing are controlled by the ST identity as shown later in section 4.3.

The residua of the Higgs propagators are fixed by

$$\mathrm{Re}\partial_{p^2}\Gamma_{A^0A^0}|_{p^2=M^2_{A^0}} = 1, \qquad \mathrm{Re}\partial_{p^2}\Gamma_{H^0H^0}|_{p^2=M^2_{H^0}} = 1, \tag{4.8a}$$

$$\mathrm{Re}\partial_{p^2}\Gamma_{h^0h^0}|_{p^2=M^2_{h^0}} = 1, \qquad \mathrm{Re}\partial_{p^2}\Gamma_{H^+H^-}|_{p^2=M^2_{H^\pm}} = 1, \tag{4.8b}$$

$$\partial_{p^2}\Gamma_{G^0G^0}|_{p^2=0} = 1, \qquad \partial_{p^2}\Gamma_{G^+G^-}|_{p^2=0} = 1. \tag{4.8c}$$

Chargino and neutralino sector

In the chargino and neutralino sector, we adopt the three real masses $m_{\chi_1^\pm}$, $m_{\chi_2^\pm}$, $m_{\chi_1^0}$ as the free parameters and require

$$\det\left(\Gamma_{f_i\bar{f}_j}\right)|_{p^2=\mathscr{M}^2_{f_k}} = 0, \tag{4.9}$$

where the Dirac spinors f_i run either over the charginos $\chi_i^\pm$ with $i = 1,2$ or over the neutralinos χ_i^0 with $i = 1,2,3,4$. By this equation also the imaginary parts of $\mathscr{M}^2_{\chi_1^\pm,\chi_2^\pm,\chi_1^0}$ and the complex masses $\mathscr{M}^2_{\chi^0_{2,3,4}}$ are defined as functions of the MSSM input parameters. At the real masses, we require the demixing conditions ($i \neq j$)

$$\widetilde{\mathrm{Re}}\Gamma_{f_i\bar{f}_j}\, u_{f_j}|_{p^2=m^2_{f_j}} = 0 \tag{4.10}$$

and normalization conditions for the residua

$$\lim_{p^2\to m^2_{f_i}} \frac{1}{\not{p} - m_{f_i}} \widetilde{\mathrm{Re}}\Gamma_{f_i\bar{f}_i}\, u_{f_i} = u_{f_i}, \tag{4.11}$$

where the spinors u_{f_i} are solutions of the Dirac equation $(\not{p} - m_{f_i})u_{f_i} = 0$ for $p^2 = m^2_{f_i}$, and $\widetilde{\mathrm{Re}}$ takes the real part of the loop integrals but not of the couplings and the γ matrices.

Matter sector

Since our matter representation is restricted to one generation, we choose the three real masses m_e, m_u, m_d as the free parameter of the quark and lepton sector. As in the chargino and neutralino sector, we require the following on-shell conditions:

$$\det\left(\Gamma_{f\bar{f}}\right)|_{p^2=\mathscr{M}^2_f} = 0, \tag{4.12}$$

where f runs over the massive fermions e, u, d. The residua of the quark and lepton propagators are fixed by

$$\lim_{p^2\to m^2_f} \frac{1}{\not{p} - m_f} \widetilde{\mathrm{Re}}\Gamma_{f\bar{f}} u_f = u_f, \tag{4.13}$$

where f runs over all Dirac spinors ν, e, u, d, and the spinors u_f satisfy $(\not{p} - m_f)u_f = 0$ for $p^2 = m_f^2$.

In the squark and slepton sector, we choose the real masses of the sfermions $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{d}_1$, $\tilde{\nu}_1$, $\tilde{e}_1$ as input parameters. The on-shell conditions for the squark and slepton sector read

$$\det\left(\Gamma_{\tilde{f}_i\tilde{f}_j^\dagger}\right)\Big|_{p^2=\mathscr{M}_{\tilde{f}_k}} = 0 \tag{4.14}$$

with $\tilde{f}_i = \tilde{u}_i$, $\tilde{d}_i$, $\tilde{\nu}_i$, $\tilde{e}_i$ and $i, j, k = 1, 2$. The masses of the sfermions $\tilde{d}_2$, $\tilde{e}_2$ are dependent parameters. The normalization conditions of the demixing and the residua of the propagators are given by $(i \neq j)$

$$\mathrm{Re}\Gamma_{\tilde{f}_i\tilde{f}_j^\dagger}|_{p^2=M^2_{\tilde{f}_j}} = 0, \qquad \mathrm{Re}\partial_{p^2}\Gamma_{\tilde{f}_i\tilde{f}_i^\dagger}|_{p^2=M^2_{\tilde{f}_i}} = 1. \tag{4.15}$$

Gauge-fixing sector

The unphysical Goldstone bosons are characterized as the three massless directions of the scalar potential:

$$\Gamma_{G^0G^0}|_{p^2=0} = 0, \qquad \Gamma_{A^0G^0}|_{p^2=0} = 0, \tag{4.16a}$$

$$\Gamma_{G^+G^-}|_{p^2=0} = 0, \qquad \Gamma_{G^+H^-}|_{p^2=0} = 0. \tag{4.16b}$$

The on-shell conditions in the ghost sector read

$$\det\begin{pmatrix} \Gamma_{c_A\bar{c}_A} & \Gamma_{c_A\bar{c}_Z} \\ \Gamma_{c_Z\bar{c}_A} & \Gamma_{c_Z\bar{c}_Z} \end{pmatrix}\Bigg|_{p^2=\mathscr{M}^2_{c_Z}} = 0, \qquad \Gamma_{c^+\bar{c}^-}|_{p^2=\mathscr{M}^2_{c^\pm}} = 0. \tag{4.17}$$

The demixing conditions are split into normalization conditions required for the IR finiteness of the MSSM (see section 3.6)

$$\Gamma_{c_A\bar{c}_A}|_{p^2=0} = 0, \qquad \Gamma_{c_A\bar{c}_Z}|_{p^2=0} = 0, \tag{4.18a}$$

$$\Gamma_{c_Z\bar{c}_A}|_{p^2=0} = 0, \tag{4.18b}$$

and the remaining demixing conditions

$$\mathrm{Re}\Gamma_{c_A\bar{c}_Z}|_{p^2=M^2_{c_Z}} = 0, \qquad \mathrm{Re}\Gamma_{c_Z\bar{c}_A}|_{p^2=M^2_{c_Z}} = 0. \tag{4.19a}$$

$M^2_{c_Z}$, $M^2_{c^\pm}$ are no free parameters of the MSSM and depend on the gauge-fixing parameters. The conditions for the residua of the ghost propagators are given by

$$\partial_{p^2}\Gamma_{c_A\bar{c}_A}|_{p^2=0} = 1, \qquad \mathrm{Re}\partial_{p^2}\Gamma_{c_Z\bar{c}_Z}|_{p^2=M^2_{c_Z}} = 1, \tag{4.20a}$$

$$\mathrm{Re}\partial_{p^2}\Gamma_{c^+\bar{c}^-}|_{p^2=M^2_{c^\pm}} = 1. \tag{4.20b}$$

Comments on the normalization conditions

The normalization conditions (4.3a) and (4.18a) are necessary for the IR finiteness of the MSSM [c.f. (3.66)] as discussed in detail in section 3.6.

Since not all masses of the MSSM are independent parameters, there is a conceptual difference between the on-shell conditions concerning real masses that are free parameters of the MSSM, like M_{A^0} in (4.5), and masses that are dependent parameters, like $M_{H^\pm}$ in (4.6). The former serve as normalization conditions for the respective self energies, while the latter are simply definitions for the masses $\mathscr{M}_{H^\pm}$ etc.

We have not established the demixing between the physical Higgs fields A^0, $H^\pm$, the Goldstone bosons G^0, $G^\pm$, the longitudinal gauge bosons $\partial^\mu A_\mu$, $\partial^\mu Z_\mu$, $\partial^\mu W_\mu^\pm$, and the auxiliary B fields. Therefore, the propagators in this sector have to be determined by inverting the full matrix of the 2-point functions between the Higgs, gauge-boson, and B fields. It is not obvious whether $\mathscr{M}^2_{A^0,H^\pm}$ as defined by (4.5), (4.6) coincide with the poles of the propagators. That they actually do is shown in section 4.3 using the ST identity.

For unstable particles, a complete demixing is generally not possible due to the non-vanishing of the imaginary parts of the self-energy corrections. The demixing is realized only up to terms of the order of the finite widths of unstable particles. On the other hand, unstable particles appear, in a strict sense, only as resonances in scattering processes and do not contribute to asymptotic states.

4.2 Generating invariant counterterms

In this section, the consequences of section 3.4 for generating the invariant counterterms in practice are drawn. In section 3.4, we have seen that all invariant counterterms are generated by applying three kinds of renormalization transformations to the classical action corresponding to the renormalization of the free parameters of the model, the field renormalization of symmetric fields, and the finite field reparametrizations of symmetric fields by physical fields. Schematically, the transformations read

$$g_i \to g_{i\text{bare}} = g_i + \delta g_i, \tag{4.21a}$$

$$\phi^{\text{sym}} \to \sqrt{z}\phi^{\text{sym}}, \tag{4.21b}$$

$$\phi^{\text{sym}} = R\phi^{\text{phys}}. \tag{4.21c}$$

where g_i stands for an arbitrary invariant counterterm. Clearly, there is an overlap between the renormalization of the symmetric fields by the counterterm $\sqrt{z}$ and the reparametrization in terms of physical fields involving the demixing matrices R. In Γ and Γ_{eff} the counterterms $\sqrt{z}$ and R appear only in the combination $\sqrt{z}R$. Both counterterms $\sqrt{z}$ and R can be simply combined to one general matrix valued field renormalization.

Applying the transformations (4.21) on the classical action yields a bare action,

$$\Gamma_{\rm cl} \to \Gamma_{\rm bare}, \tag{4.22}$$

$\Gamma_{\rm bare}$ is introduced only as an auxiliary quantity to generate invariant counterterms and does not necessarily coincide with $\Gamma_{\rm eff}$ (3.29) from which the Feynman rules are derived. The invariant counterterms of order $\hbar^n$ can be obtained by expanding all counterterms in $\Gamma_{\rm bare}$, i.e. $g_{i\rm bare} = g_i + \sum_n \delta^{(n)} g_i$, omitting all contributions involving products of counterterms, and taking only the pure $\delta^{(n)}$ contributions:

$$\Gamma^{(n)}_{\rm ct,inv} \equiv \delta^{(n)}\Gamma = \Gamma_{\rm bare}|_{\text{1st order in } \delta^{(n)}}. \tag{4.23}$$

The structure of $\Gamma^{(n)}_{\rm ct,inv}$ is identical for all n and, in particular, coincides with the one of $\Gamma^{(1)}_{\rm ct,inv}$.

Two possibilities exist how the classical action can be parametrized. Either $\Gamma_{\rm cl}$ can be formulated in terms of the symmetric fields or in terms of physical fields:

$$\Gamma_{\rm cl} = \Gamma^{\rm sym}_{\rm cl}(g_i, \phi^{\rm sym}) = \Gamma^{\rm phys}_{\rm cl}(g_i, \phi^{\rm phys}). \tag{4.24}$$

Both parametrizations are related by

$$\Gamma^{\rm phys}_{\rm cl}(g_i, \phi^{\rm phys}) = \Gamma^{\rm sym}_{\rm cl}(g_i, R^{(0)}\phi^{\rm phys}), \tag{4.25}$$

where $R^{(0)}$ denotes the lowest-order demixing matrix. Depending on whether $\Gamma^{\rm sym}_{\rm cl}$ or $\Gamma^{\rm phys}_{\rm cl}$ is used as a starting point, different but equivalent parametrizations of $\Gamma_{\rm ct,inv}$ are obtained. We sketch in the following both possibilities in the general case before applying them to the individual sectors of the MSSM.

It is closer to section 3.4 to start with $\Gamma^{\rm sym}_{\rm cl}(g_i, \phi^{\rm sym})$. The transformations (4.21) yield

$$\Gamma_{\rm bare} = \Gamma^{\rm sym}_{\rm cl}(g_{i\rm bare}, \sqrt{z}R\phi^{\rm phys}). \tag{4.26}$$

As a result only the combination $\sqrt{z}R$ appears in $\Gamma^{\rm sym}_{\rm cl}$. It will be useful to split off the lowest-order part and define

$$\sqrt{Z}_{\text{1st param.}} = (R^{(0)})^{-1}\sqrt{z}R, \tag{4.27}$$

where $\sqrt{Z}$ is a general field renormalization matrix. This kind of parametrization for the Z factors will be used in the sfermion sector and in the neutralino-chargino sector.

An equivalent but different parametrization can be obtained by using $\Gamma^{\rm phys}_{\rm cl}$ as the starting point. For the fields $\phi^{\rm phys}$ the renormalization transformation (4.21) yields

$$\phi^{\rm phys} = (R^{(0)})^{-1}\phi^{\rm sym} \to (R^{(0)}_{\rm bare})^{-1}\sqrt{z}R\phi^{\rm phys}, \tag{4.28}$$

where $R^{(0)}_{\text{bare}}$ is obtained from $R^{(0)}$, which is a function of the parameters g_i, by replacing $g_i \to g_{i\text{bare}}$. In agreement with (4.25) and (4.26), the bare action can be written as

$$\Gamma_{\text{bare}} = \Gamma^{\text{phys}}_{\text{cl}} \left(g_{i\text{bare}}, (R^{(0)}_{\text{bare}})^{-1} \sqrt{z} R \phi^{\text{phys}} \right). \tag{4.29}$$

In this case, the combination

$$\sqrt{Z}_{\text{2nd param.}} = (R^{(0)}_{\text{bare}})^{-1} \sqrt{z} R \tag{4.30}$$

appears naturally, and thus the renormalization transformations amount simply to

$$g_i \to g_i + \delta g_i, \tag{4.31a}$$

$$\phi^{\text{phys}} \to \sqrt{Z} \phi^{\text{phys}}. \tag{4.31b}$$

The renormalization constants $\sqrt{Z}$ introduced in both parametrizations differ. The two parametrizations can be related by

$$\sqrt{Z}_{\text{2nd param.}} = (R^{(0)}_{\text{bare}})^{-1} R^{(0)} \sqrt{Z}_{\text{1st param.}}. \tag{4.32}$$

The second parametrization is most useful in cases in which the analytic form of $R^{(0)}$ is explicitly known, and where the form of $\Gamma^{\text{phys}}_{\text{cl}}$ has a simple structure. We will use it in the gauge-boson, Higgs, and ghost sector.

4.3 Evaluation of the normalization conditions

The general structure of the invariant counterterms is discussed in detail in sections 3.4 and 4.2. As the net result, there are counterterms to all independent parameters as well as general matrix valued field renormalizations.

Assuming that the vertex functional is already renormalized up to order $\hbar^{n-1}$, the normalization condition have to be fulfilled by a proper choice of the genuine invariant counterterms of order $\hbar^n$, since the lower-order counterterms are already fixed. In this section, we want to verify whether the normalization conditions can be satisfied.

Gauge-boson sector

The invariant counterterms of the gauge-boson sector are obtained from the classical action by performing the following renormalization transformations:

$$M_W \to M_{W\,\text{bare}} = M_W + \delta M_W, \tag{4.33}$$

$$M_Z \to M_{Z\,\text{bare}} = M_Z + \delta M_Z, \tag{4.34}$$

$$W^{\pm}_{\mu} \to \sqrt{z_V} W^{\pm}_{\mu}, \tag{4.35}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \to \sqrt{Z_V} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}. \tag{4.36}$$

The field-renormalization matrix $\sqrt{Z_V}$ reads

$$\sqrt{Z_V} = \begin{pmatrix} \cos\theta_{W\,\mathrm{bare}} & \sin\theta_{W\,\mathrm{bare}} \\ -\sin\theta_{W\,\mathrm{bare}} & \cos\theta_{W\,\mathrm{bare}} \end{pmatrix} \begin{pmatrix} \sqrt{z_{V'}} & 0 \\ 0 & \sqrt{z_V} \end{pmatrix} R_V, \tag{4.37}$$

where the second parametrization (4.30) is used. According to (2.56a), the bare weak mixing angle is defined by

$$\cos\theta_{W\,\mathrm{bare}} = \frac{M_{W\,\mathrm{bare}}}{M_{Z\,\mathrm{bare}}}. \tag{4.38}$$

Working out the pure $\delta^{(n)}$ part of Γ_{bare}, we obtain the invariant counterterms of order $\hbar^n$:

$$\begin{aligned} \Gamma^{(n)}_{\mathrm{ct,inv}}|_{\mathrm{gauge}} \;=\; & \int \mathrm{d}^4x \left\{ -\frac{1}{4}(A^{\mu\nu}, Z^{\mu\nu})\delta^{(n)}Z_V \begin{pmatrix} A_{\mu\nu} \\ Z_{\mu\nu} \end{pmatrix} - \frac{1}{2}\delta^{(n)} z_V W^{+\mu\nu} W^-_{\mu\nu} \right. \\ & + \frac{1}{2}(A^\mu, Z^\mu) \left[\begin{pmatrix} 0 & 0 \\ 0 & M_Z^2 \end{pmatrix} \delta^{(n)}Z_V + \begin{pmatrix} 0 & 0 \\ 0 & \delta^{(n)}M_Z^2 \end{pmatrix} \right] \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \\ & \left. + \left(\delta^{(n)} z_V M_W^2 + \delta^{(n)} M_W^2\right) W^{+\mu} W^-_\mu \right\}, \end{aligned} \tag{4.39}$$

where $A^{\mu\nu}, Z^{\mu\nu}, W^{\pm\mu\nu}$ are defined as in (2.64). This yields the following contributions to the vertex functions appearing in the normalization conditions:

$$\delta^{(n)}\Gamma^T_{AA}|_{p^2=0} = 0, \tag{4.40a}$$
$$\delta^{(n)}\Gamma^T_{AZ}|_{p^2=0} = -\tfrac{1}{2}(\delta^{(n)}Z_V)_{ZA} M_Z^2, \tag{4.40b}$$
$$\delta^{(n)}\Gamma^T_{ZZ}|_{p^2=M_Z^2} = -\delta^{(n)}M_Z^2, \tag{4.40c}$$
$$\delta^{(n)}\Gamma^T_{ZA}|_{p^2=M_Z^2} = \tfrac{1}{2}(\delta^{(n)}Z_V)_{AZ} M_Z^2, \tag{4.40d}$$
$$\delta^{(n)}\Gamma^T_{W^+W^-}|_{p^2=M_W^2} = -\delta^{(n)}M_W^2, \tag{4.40e}$$
$$\partial_{p^2}\delta^{(n)}\Gamma^T_{AA}|_{p^2=0} = (\delta^{(n)}Z_V)_{AA}, \tag{4.40f}$$
$$\partial_{p^2}\delta^{(n)}\Gamma^T_{ZZ}|_{p^2=M_Z^2} = (\delta^{(n)}Z_V)_{ZZ}, \tag{4.40g}$$
$$\partial_{p^2}\delta^{(n)}\Gamma^T_{W^+W^-}|_{p^2=M_W^2} = \delta^{(n)}z_V. \tag{4.40h}$$

The notation $\delta^{(n)}\Gamma$ is used for the contributions of the invariant counterterms of order $\hbar^n$. The counterterm contribution to the determinant appearing in (4.2) is not given explicitly, but it is obvious that (4.2) can be satisfied by adjusting $\delta^{(n)}M_{W,Z}$. The structure of (4.40) shows that the demixing and residua conditions fix the counterterms $\delta^{(n)}z_V$, $\delta^{(n)}Z_V$. However, there is no counterterm available for the on-shell condition of the photon. As already discussed in section 3.6, (4.40a) is a direct consequence of the ST identity and of the demixing normalization condition (4.40b).

As a result, all normalization conditions in the gauge-boson sector can be satisfied.

Ghost sector

The complete field-renormalization transformations in the ghost sector read

$$\begin{pmatrix} c_A \\ c_Z \end{pmatrix} \to \sqrt{Z_c} \begin{pmatrix} c_A \\ c_Z \end{pmatrix}, \qquad \begin{pmatrix} \bar{c}_A \\ \bar{c}_Z \end{pmatrix} \to \sqrt{Z_{\bar{c}}} \begin{pmatrix} \bar{c}_A \\ \bar{c}_Z \end{pmatrix}, \tag{4.41}$$

$$c^\pm \to \sqrt{z_c} c^\pm, \qquad \bar{c}^\pm \to \sqrt{z_V}^{-1} \bar{c}^\pm \tag{4.42}$$

and the renormalization of the gauge-fixing parameters are given by

$$\zeta^{\phi_i \phi_j} \to \zeta^{\phi_i \phi_j} + \delta^{(n)} \zeta^{\phi_i \phi_j}. \tag{4.43}$$

As in the gauge-boson sector, the second parametrization of section 4.2 is used:

$$\sqrt{Z_c} = \begin{pmatrix} \cos\theta_{W\,\mathrm{bare}} & \sin\theta_{W\,\mathrm{bare}} \\ -\sin\theta_{W\,\mathrm{bare}} & \cos\theta_{W\,\mathrm{bare}} \end{pmatrix} \begin{pmatrix} \sqrt{z_{V'}} & 0 \\ 0 & \sqrt{z_c} \end{pmatrix} R_c, \tag{4.44}$$

$$\sqrt{Z_{\bar{c}}} = \begin{pmatrix} \cos\theta_{W\,\mathrm{bare}} & \sin\theta_{W\,\mathrm{bare}} \\ -\sin\theta_{W\,\mathrm{bare}} & \cos\theta_{W\,\mathrm{bare}} \end{pmatrix} \begin{pmatrix} \sqrt{z_{V'}}^{-1} & 0 \\ 0 & \sqrt{z_V}^{-1} \end{pmatrix} R_{\bar{c}}. \tag{4.45}$$

The result for the invariant counterterms of order $\hbar^n$ reads

$$\begin{aligned} \Gamma^{(n)}_{\mathrm{ct,inv}}|_{\mathrm{gh}} = \int \mathrm{d}^4x \Bigg\{ & -(\bar{c}_A, \bar{c}_Z) \Bigg[\Box \frac{\delta^{(n)} Z_{\bar{c}}{}^T + \delta^{(n)} Z_c}{2} + \frac{1}{2} \delta^{(n)} Z_{\bar{c}} \begin{pmatrix} 0 & \zeta^{AG^0} \\ 0 & \zeta^{ZG^0} \end{pmatrix} M_Z^2 \\ & + \begin{pmatrix} 0 & \zeta^{AG^0} \\ 0 & \zeta^{ZG^0} \end{pmatrix} \frac{1}{2} \delta^{(n)} Z_c M_Z^2 + \begin{pmatrix} 0 & \zeta^{AG^0} \\ 0 & \zeta^{ZG^0} \end{pmatrix} \delta^{(n)} M_Z^2 + M_Z^2 \begin{pmatrix} 0 & \delta^{(n)} \zeta^{AG^0} \\ 0 & \delta^{(n)} \zeta^{ZG^0} \end{pmatrix} \Bigg] \begin{pmatrix} c_A \\ c_Z \end{pmatrix} \\ & - (\bar{c}^-, \bar{c}^+) \Bigg[\left(\Box + \zeta^{G^\pm} M_W^2 \right) \frac{\delta^{(n)} z_c - \delta^{(n)} z_V}{2} + \delta^{(n)} \zeta^{G^\pm} M_W^2 + \zeta^{G^\pm} \delta^{(n)} M_W^2 \Bigg] \begin{pmatrix} c^+ \\ c^- \end{pmatrix} \Bigg\}. \end{aligned} \tag{4.46}$$

Using the classical value $\zeta^{AG^0} = 0$ corresponding to the R_ξ gauge (see section 2.7) the contributions to the self energies appearing in the normalization conditions read

$$\delta^{(n)} \Gamma_{c_A \bar{c}_A}|_{p^2=0} = 0, \tag{4.47a}$$

$$\delta^{(n)} \Gamma_{c_A \bar{c}_Z}|_{p^2=0} = -\tfrac{1}{2} \zeta^{ZG^0} M_Z^2 (\delta^{(n)} Z_c)_{ZA}, \tag{4.47b}$$

$$\delta^{(n)} \Gamma_{c_Z \bar{c}_A}|_{p^2=0} = -\tfrac{1}{2} \zeta^{ZG^0} M_Z^2 (\delta^{(n)} Z_{\bar{c}})_{ZA} - \delta^{(n)} \zeta^{AG^0} M_Z^2, \tag{4.47c}$$

$$\delta^{(n)} \Gamma_{c_Z \bar{c}_Z}|_{p^2=M_{c_Z}^2} = -\zeta^{ZG^0} \delta^{(n)} M_Z^2 - \delta^{(n)} \zeta^{ZG^0} M_Z^2, \tag{4.47d}$$

$$\delta^{(n)} \Gamma_{c_A \bar{c}_Z}|_{p^2=M_{c_Z}^2} = \tfrac{1}{2} (\delta^{(n)} Z_{\bar{c}})_{AZ} \zeta^{ZG^0} M_Z^2, \tag{4.47e}$$

$$\delta^{(n)} \Gamma_{c_Z \bar{c}_A}|_{p^2=M_{c_Z}^2} = \tfrac{1}{2} (\delta^{(n)} Z_c)_{AZ} \zeta^{ZG^0} M_Z^2 - \delta^{(n)} \zeta^{AG^0} M_Z^2, \tag{4.47f}$$

$$\delta^{(n)} \Gamma_{c^+ \bar{c}^-}|_{p^2=M_{c^\pm}^2} = -\delta^{(n)} \zeta^{G^\pm} M_W^2 - \zeta^{G^\pm} \delta^{(n)} M_W^2, \tag{4.47g}$$

$$\partial_{p^2} \delta^{(n)} \Gamma_{c_A \bar{c}_A}|_{p^2=0} = \tfrac{1}{2} \left[(\delta^{(n)} Z_c)_{AA} + (\delta^{(n)} Z_{\bar{c}})_{AA} \right], \tag{4.47h}$$

$$\partial_{p^2} \delta^{(n)} \Gamma_{c_Z \bar{c}_Z}|_{p^2=M_{c_Z}^2} = \tfrac{1}{2} \left[(\delta^{(n)} Z_c)_{ZZ} + (\delta^{(n)} Z_{\bar{c}})_{ZZ} \right], \tag{4.47i}$$

$$\partial_{p^2} \delta^{(n)} \Gamma_{c^+ \bar{c}^-}|_{p^2=M_{c^\pm}^2} = \tfrac{1}{2} \left(\delta^{(n)} z_c - \delta^{(n)} z_V \right). \tag{4.47j}$$

As in the case of the photon, there is no invariant counterterm that can be adjusted in order to satisfy the normalization condition (4.47a). However, $\Gamma_{c_A\bar{c}_A}|_{p^2=0} = 0$ holds automatically as a consequence of the ST identity if $\Gamma_{c_A\bar{c}_Z}|_{p^2=0} = 0$ and $\Gamma_{c_Z\bar{c}_A}|_{p^2=0} = 0$.

The gauge parameters ζ^{AG^0}, ζ^{ZG^0}, $\zeta^{G^\pm}$ and their counterterms are fixed by the gauge-fixing conditions (3.26). Since there are no free counterterms available in (4.47d) and (4.47g), the values of $\mathscr{M}^2_{c_Z}$, $\mathscr{M}^2_{c^\pm}$ are dependent parameters. They are determined via the conditions $\Gamma_{c_Z\bar{c}_Z}|_{p^2=\mathscr{M}^2_{c_Z}} = 0$ and $\Gamma_{c^+\bar{c}^-}|_{p^2=\mathscr{M}^2_{c^\pm}} = 0$.

In this way all desired normalization conditions in the ghost sector can be satisfied.

4.3.1 Scalar and longitudinal gauge-boson sector

Furthermore, we consider the normalization conditions in the sector of neutral, CP-odd, and charged Higgs bosons, of longitudinal gauge bosons, and of B fields.

Using the renormalization transformations

$$M^2_{A^0} \to M^2_{A^0} + \delta M^2_{A^0}, \tag{4.48}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \to \sqrt{Z_{A^0}} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \tag{4.49}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \to \sqrt{Z_{H^\pm}} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \tag{4.50}$$

in the classical action, we obtain the invariant counterterms of order $\hbar^n$. We use the second parametrization (4.30):

$$\sqrt{Z_{A^0,H^\pm}} = \begin{pmatrix} \cos\beta_{\text{bare}} & \sin\beta_{\text{bare}} \\ -\sin\beta_{\text{bare}} & \cos\beta_{\text{bare}} \end{pmatrix} \begin{pmatrix} \sqrt{z_{H_1}} & 0 \\ 0 & \sqrt{z_{H_2}} \end{pmatrix} R_{A^0,H^\pm}. \tag{4.51}$$

Dividing the present sector into a pure Higgs-boson part $\Gamma_{\text{ct,inv}}|_{HH}$, a Higgs–gauge-boson part $\Gamma_{\text{ct,inv}}|_{HV}$, and a Higgs–$B$-field part $\Gamma_{\text{ct,inv}}|_{HB}$, we obtain for the respective counterterm contributions:

$$\begin{aligned}
\Gamma^{(n)}_{\text{ct,inv}}|_{HH} = \int \mathrm{d}^4x \Bigg\{ & -\frac{1}{2}(G^0, A^0)\Box\delta^{(n)} Z_{A^0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \\
& - (G^0, A^0)\left[\begin{pmatrix} 0 & 0 \\ 0 & M^2_{A^0} \end{pmatrix} \delta^{(n)} Z_{A^0} + \begin{pmatrix} 0 & 0 \\ 0 & \delta^{(n)} M^2_{A^0} \end{pmatrix} \right] \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \\
& + (G^-,\ H^-)\Bigg[-\Box \frac{\delta^{(n)} Z^T_{H^\pm} + \delta^{(n)} Z_{H^\pm}}{2} - \begin{pmatrix} 0 & 0 \\ 0 & \delta^{(n)} M^2_{H^\pm} \end{pmatrix} \\
& - \frac{1}{2}\left(\delta^{(n)} Z^T_{H^\pm} \begin{pmatrix} 0 & 0 \\ 0 & M^{2(0)}_{H^\pm} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & M^{2(0)}_{H^\pm} \end{pmatrix} \delta^{(n)} Z_{H^\pm} \right) \Bigg] \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \Bigg\} \\
& + \text{tadpoles},
\end{aligned} \tag{4.52}$$

$$
\begin{aligned}
\Gamma^{(n)}_{\mathrm{ct,inv}}|_{HV} = \int \mathrm{d}^4x \Bigg\{ & -(\partial_\mu G^0, \partial_\mu A^0) \Bigg[\frac{\delta^{(n)} Z^T_{A^0}}{2} \begin{pmatrix} 0 & M_Z \\ 0 & 0 \end{pmatrix} \\
& + \begin{pmatrix} 0 & M_Z \\ 0 & 0 \end{pmatrix} \frac{\delta^{(n)} Z_V}{2} + \begin{pmatrix} 0 & \delta^{(n)} M_Z \\ 0 & 0 \end{pmatrix} \Bigg] \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} \\
& + \Bigg[-\mathrm{i}(\partial_\mu G^+, \partial_\mu H^+) \Bigg[\frac{\delta^{(n)} Z^T_{H^\pm} + \delta^{(n)} z_V}{2} \begin{pmatrix} M_W \\ 0 \end{pmatrix} + \begin{pmatrix} \delta^{(n)} M_W \\ 0 \end{pmatrix} \Bigg] W^{-\mu} + \mathrm{c.c.} \Bigg] \Bigg\} \\
& + \text{tadpoles}.
\end{aligned} \tag{4.53}
$$

The part $\Gamma_{\mathrm{ct,inv}}|_{HB}$ involving B fields will be discussed separately later.

The tadpole contributions contain the counterterms $\delta^{(n)} t_1$, $\delta^{(n)} t_2$, which are fixed by the conditions $\delta\Gamma/\delta H^0 = 0$ and $\delta\Gamma/\delta h^0 = 0$. Since tadpole contributions contain no other adjustable counterterms, they are not written explicitly.

The constant $\delta^{(n)} M^{2\,(0)}_{H^\pm}$ is not a free parameter, but it is determined by the lowest-order relation $M^{2\,(0)}_{H^\pm} = M^2_{A^0} + M^2_W$ (2.59a) and hence $\delta^{(n)} M^{2\,(0)}_{H^\pm} = \delta^{(n)} M^2_{A^0} + \delta^{(n)} M^2_W$ (see section 2.8).

The counterterms contribute in the following way to the vertex functions involving the physical Higgs bosons A^0, $H^\pm$ (up to tadpoles):

$$\delta^{(n)}\Gamma_{A^0A^0}|_{p^2=M^2_{A^0}} = -\delta^{(n)} M^2_{A^0}, \tag{4.54a}$$

$$\delta^{(n)}\Gamma_{A^0G^0}|_{p^2=M^2_{A^0}} = \tfrac{1}{2} M^2_{A^0} (\delta^{(n)} Z_{A^0})_{AG}, \tag{4.54b}$$

$$\delta^{(n)}\Gamma_{A^0A_\mu}|_{p^2=M^2_{A^0}} = 0, \tag{4.54c}$$

$$\delta^{(n)}\Gamma_{A^0Z_\mu}|_{p^2=M^2_{A^0}} = -\mathrm{i}p^\mu \tfrac{1}{2} (\delta^{(n)} Z_{A^0})_{AG} M_Z, \tag{4.54d}$$

$$\delta^{(n)}\Gamma_{H^+H^-}|_{p^2=M^2_{H^\pm}} = -\delta^{(n)} M^2_{H^\pm}, \tag{4.54e}$$

$$\delta^{(n)}\Gamma_{H^+G^-}|_{p^2=M^2_{H^+}} = \tfrac{1}{2} (\delta^{(n)} Z_{H^\pm})_{HG} M^{2\,(0)}_{H^\pm}, \tag{4.54f}$$

$$\delta^{(n)}\Gamma_{H^+W^-_\mu}|_{p^2=M^2_{H^\pm}} = p^\mu \tfrac{1}{2} (\delta^{(n)} Z_{H^\pm})_{HG} M_W. \tag{4.54g}$$

The contributions to the vertex functions involving would-be Goldstone bosons read

$$\delta^{(n)}\Gamma_{G^0G^0}|_{p^2=0} = 0, \tag{4.55a}$$

$$\delta^{(n)}\Gamma_{A^0G^0}|_{p^2=0} = -M^2_{A^0} (\delta^{(n)} Z_{A^0})_{GH}, \tag{4.55b}$$

$$\delta^{(n)}\Gamma_{G^+G^-}|_{p^2=0} = 0, \tag{4.55c}$$

$$\delta^{(n)}\Gamma_{G^+H^-}|_{p^2=0} = -M^{2\,(0)}_{H^\pm} (\delta^{(n)} Z_{H^\pm})_{GH}. \tag{4.55d}$$

Clearly, the demixing conditions for $\Gamma_{A^0G^0}$, $\Gamma_{H^+G^-}$ can be satisfied by a suitable choice of the non-diagonal elements of $\delta^{(n)} Z_{A^0,H^\pm}$, and the on-shell condition for $\Gamma_{A^0A^0}$ can be satisfied by choosing $\delta^{(n)} M^2_{A^0}$ appropriately.

However, in the sector of the charged scalars $H^\pm$, $G^\pm$, the counterterm $\delta^{(n)}M^2_{H^\pm}$ is not independent as discussed above, and hence $M^2_{H^\pm}$ is not a free parameter.

We turn to an important question mentioned already at the end of section 4.1. It is posed by the mixings between the scalars and the longitudinal gauge bosons: between A^0 and A^μ, A^0 and Z^μ, and $H^\pm$ and $W^{\pm\mu}$. No counterterms are available to cancel these mixings. Therefore, the Higgs propagators are not obtained from inverting only the Higgs self-energy matrix but by inverting the larger matrix

$$\Gamma_{HVB} \;=\; \left(\begin{array}{cc|c} \Gamma_{h_ih_j} & \Gamma_{h_iV^b} & \Gamma_{h_iB^b} \\ \Gamma_{V^ah_j} & \Gamma_{V^aV^b} & \Gamma_{V^aB^b} \\ \hline \Gamma_{B^ah_j} & \Gamma_{B^aV^b} & \Gamma_{B^aB^b} \end{array}\right), \tag{4.56}$$

where for simplicity h_i, V^a, B^a have been written for all Higgs, gauge-boson, and B-field degrees of freedom. It is not obvious whether $\mathscr{M}^2_{A^0}$, $\mathscr{M}^2_{H^\pm}$ as defined by (4.5), (4.6) coincide with the poles of the propagators, i.e. with the zeros of $\det(\Gamma_{HVB})$. The ST identity answers this question (c.f. [32]). We need the ingredients

$$0 \;=\; \Gamma_{c^cY^b_V}\Gamma_{V^aV^b} + \Gamma_{c^cY_{h_j}}\Gamma_{V^ah_j}, \tag{4.57}$$

$$0 \;=\; \Gamma_{c^cY^b_V}\Gamma_{h_iV^b} + \Gamma_{c^cY_{h_j}}\Gamma_{h_ih_j}, \tag{4.58}$$

and the fact that $\Gamma_{c^cY_{V^b}}$ is invertible for $p \neq 0$ since $\Gamma_{c^cY^{b\mu}_V}(p,-p) = -\mathrm{i}p_\mu\delta_{cb} + \mathcal{O}(\hbar)$. The relations (4.57), (4.58) follow from the ST identities $0 = \delta^2\mathscr{S}(\Gamma)/(\delta V^a\delta c^c)$ and $0 = \delta^2\mathscr{S}(\Gamma)/(\delta h_i\delta c^c)$, respectively.

Firstly we consider the restrictions of gauge invariance on the submatrix Γ_{HV}, defined in analogy to (4.56). From (4.57), (4.58) we obtain

$$\Gamma_{HV}d^{(c)} \;=\; 0 \quad\text{with}\quad d^{(c)} = \left(\begin{array}{c} \Gamma_{c^cY_{h_j}} \\ \Gamma_{c^cY^b_V} \end{array}\right), \tag{4.59}$$

so for every generator of the gauge group there is a zero mode of Γ_{HV} for any p^2. However, if we suppose p^2 is chosen such that $\det(\Gamma_{h_ih_j}) = 0$, then there are coefficients $d^{(0)}_j$ with $\Gamma_{h_ih_j}d^{(0)}_j = 0$. Using (4.58), $\Gamma_{h_ih_j}d^{(0)}_j = 0$ implies also $\Gamma_{V^ah_j}d^{(0)}_j = 0$. Thus

$$\Gamma_{HV}d^{(0)} \;=\; 0 \quad\text{with}\quad d^{(0)} = \left(\begin{array}{c} d^{(0)}_j \\ 0 \end{array}\right), \tag{4.60}$$

i.e. there is an additional zero mode of Γ_{HV}. Owing to the invertibility of $\Gamma_{c^cY_{V^b}}$, the $d^{(c)}$ with $c = 0,\ldots,n$ constitute $n+1$ linearly independent vectors, where n is the number of gauge-group generators.

For Γ_{HVB} this implies that there are $n+1$ linearly independent vectors with

$$\Gamma_{HVB}\left(\frac{d^{(c)}}{0}\right) = \left(\frac{0}{x^a}\right), \tag{4.61}$$

where the non-zero entries x^a are confined to the lower n rows corresponding to the B^a [c.f. (4.56)]. Hence, there must be a non-trivial linear combination of the $d^{(c)}$ which is annihilated by Γ_{HVB}, so that $\det(\Gamma_{HVB}) = 0$ holds as was to be shown.

Thus, $\det(\Gamma_{h_i h_j}) = 0$ for $p^2 \neq 0$ implies $\det(\Gamma_{HVB}) = 0$. Therefore, the conditions (4.5), (4.6) really characterize $\mathscr{M}^2_{A^0,H^\pm}$ as poles of the Higgs propagators. It is noteworthy that in this argument the form of the gauge fixing and thus the B-dependent terms are irrelevant.

In the Goldstone sector there are also no free renormalization constants available to satisfy the diagonal normalization conditions $\Gamma_{G^0G^0} = \Gamma_{G^+G^-} = 0$ at $p = 0$. Again these conditions hold automatically as a consequence of the ST identity. As shown in section 3.5, the eigenvalue zero of the Higgs self-energy matrix at $p = 0$ is threefold degenerate, corresponding to the three spontaneously broken gauge symmetries. Since also $\Gamma_{G^0A^0} = \Gamma_{G^+H^-} = 0$ at $p = 0$, this implies that $\Gamma_{G^0G^0} = \Gamma_{G^+G^-} = 0$ at $p = 0$. The three fields $G^{0,\pm}$ thus correspond to the three massless directions of the scalar potential. In appendix F the characterization of the Goldstone modes is discussed in more detail, and explicit results for the derivatives $\delta/\delta G^a$ are given.

The conditions for the residues of the propagators can be satisfied by a suitable choice of the diagonal field-renormalization constants:

$$\delta^{(n)}\partial_{p^2}\Gamma_{A^0A^0}|_{p^2=M^2_{A^0}} = (\delta^{(n)}Z_{A^0})_{GG}, \tag{4.62a}$$

$$\delta^{(n)}\partial_{p^2}\Gamma_{H^+H^-}|_{p^2=M^2_{H^\pm}} = (\delta^{(n)}Z_{H^\pm})_{GG}, \tag{4.62b}$$

$$\delta^{(n)}\partial_{p^2}\Gamma_{G^0G^0}|_{p^2=0} = (\delta^{(n)}Z_{A^0})_{AA}, \tag{4.62c}$$

$$\delta^{(n)}\partial_{p^2}\Gamma_{G^+G^-}|_{p^2=0} = (\delta^{(n)}Z_{H^\pm})_{HH}. \tag{4.62d}$$

We come back to the part of the vertex functional involving the B fields Γ_{HB}. There are no loop contributions to this part of the vertex functional, and this part is completely governed by the gauge-fixing conditions (3.26) and the form of the gauge-fixing functions (2.45). Clearly, the demixing requirements involving B fields, $\Gamma_{H^+B^-} = \Gamma_{A^0B_A} = \Gamma_{A^0B_A} = 0$, have to be compatible with the gauge-fixing functions.

In fact, we have anticipated these normalization conditions in section 2.6 by requiring that $\mathscr{F}^{a,\prime}|_{\Phi=0}$ depend only on the Goldstone fields but not on the physical Higgs fields $H^\pm$, A^0. This requirement forced us to introduce the background Higgs multiplets $\Phi_i^{a,\prime}$ and the construction of section 2.6. On the other hand, it guarantees the demixing conditions $\Gamma_{H^+B^-} = \Gamma_{A^0B_A} = \Gamma_{A^0B_A} = 0$ so that the $H^\pm$, A^0 fields are as close to mass eigenstates as possible.

CP-even Higgs bosons

In the former sectors the ST identity is always required for the proof that all normalization conditions can be satisfied. In the remaining sectors, there are enough counterterms at our disposal, and the ST identity need not be considered. Therefore, we discuss this sections only briefly and emphasize the common structure of all sectors.

We continue with the CP-even part of the Higgs sector. The renormalization transformation reads

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} \to \sqrt{Z_{H^0}} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} \tag{4.63}$$

with

$$\sqrt{Z_{H^0}} = \begin{pmatrix} \cos\alpha_{\text{bare}} & \sin\alpha_{\text{bare}} \\ -\sin\alpha_{\text{bare}} & \cos\alpha_{\text{bare}} \end{pmatrix} \begin{pmatrix} \sqrt{z_{H_1}} & 0 \\ 0 & \sqrt{z_{H_2}} \end{pmatrix} R_{H^0}. \tag{4.64}$$

The generated invariant counterterms are

$$\begin{aligned} \Gamma^{(n)}_{\text{ct,inv}}|_{Hh} \;=\; & \int \mathrm{d}^4x \, \frac{1}{2}(H^0, h^0) \Bigg[-\Box \delta^{(n)} Z_{H^0} - \begin{pmatrix} \delta^{(n)} M^2_{H^0} & 0 \\ 0 & \delta^{(n)} M^2_{h^0} \end{pmatrix} \\ & + \begin{pmatrix} {M^2_{H^0}}^{(0)} & 0 \\ 0 & {M^2_{h^0}}^{(0)} \end{pmatrix} \delta^{(n)} Z_{H^0} \Bigg] \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} \\ & + \text{tadpoles}. \end{aligned} \tag{4.65}$$

The mass parameters appearing in (4.65) are not free parameter of the MSSM. Thus the on-shell conditions $\mathrm{Re}\Gamma_{H^0H^0}|_{p^2=\mathscr{M}^2_{H^0}} = 0$ and $\mathrm{Re}\Gamma_{h^0h^0}|_{p^2=\mathscr{M}^2_{h^0}} = 0$ determine the values of $\mathscr{M}^2_{H^0}$, $\mathscr{M}^2_{h^0}$. The remaining on-shell conditions in this sector can be satisfied by adjusting the field-renormalization matrix $\delta^{(n)} Z_{H^0}$.

Charginos and neutralinos

In the chargino-neutralino sector, it is more convenient to apply the first kind of field renormalization as discussed in section 4.2. Thus, we use the parametrization of the classical action in terms of symmetric fields and parameters and perform the following renormalization transformations:

$$\mathcal{X} \to \mathcal{X} + \delta\mathcal{X}, \qquad \mathcal{Y} \to \mathcal{Y} + \delta\mathcal{Y}, \tag{4.66}$$

$$\begin{pmatrix} \lambda^+_\alpha \\ h^1_{2\alpha} \end{pmatrix} \to \begin{pmatrix} \sqrt{z_\lambda}\lambda^+_\alpha \\ \sqrt{z_{h_2}} h^1_{2\alpha} \end{pmatrix}, \qquad \begin{pmatrix} \lambda^+_\alpha \\ h^1_{2\alpha} \end{pmatrix} = R_{\chi^+} \begin{pmatrix} \chi^+_{1\alpha} \\ \chi^+_{2\alpha} \end{pmatrix}, \tag{4.67}$$

$$\begin{pmatrix} \lambda^-_\alpha \\ h^2_{1\alpha} \end{pmatrix} \to \begin{pmatrix} \sqrt{z_\lambda}\lambda^-_\alpha \\ \sqrt{z_{h_1}} h^2_{1\alpha} \end{pmatrix}, \qquad \begin{pmatrix} \lambda^-_\alpha \\ h^2_{1\alpha} \end{pmatrix} = R_{\chi^-} \begin{pmatrix} \chi^-_{1\alpha} \\ \chi^-_{2\alpha} \end{pmatrix}, \tag{4.68}$$

$$\begin{pmatrix} \lambda'_\alpha \\ \lambda^3_\alpha \\ h^1_{1\alpha} \\ h^2_{2\alpha} \end{pmatrix} \to \begin{pmatrix} \sqrt{z_{V'}}\lambda'_\alpha \\ \sqrt{z_\lambda}\lambda^3_\alpha \\ \sqrt{z_{h_1}}h^1_{1\alpha} \\ \sqrt{z_{h_2}}h^2_{2\alpha} \end{pmatrix}, \qquad \begin{pmatrix} \lambda'_\alpha \\ \lambda^3_\alpha \\ h^1_{1\alpha} \\ h^2_{2\alpha} \end{pmatrix} = R_{\chi^0} \begin{pmatrix} \chi^0_{1\alpha} \\ \chi^0_{2\alpha} \\ \chi^0_{3\alpha} \\ \chi^0_{4\alpha} \end{pmatrix}. \tag{4.69}$$

This transformations reflect directly the results of section 3.4. The matrices $\mathcal{X}$, $\mathcal{Y}$ are defined in (2.62a) and (2.62b). Combining both field transformations, we can write in agreement with (4.27)

$$\begin{pmatrix} \lambda^+_\alpha \\ h^1_{2\alpha} \end{pmatrix} \to R^{(0)}_{\chi^+} \sqrt{Z_{\chi^+}} \begin{pmatrix} \chi^+_{1\alpha} \\ \chi^+_{2\alpha} \end{pmatrix}, \tag{4.70}$$

$$\begin{pmatrix} \lambda^-_\alpha \\ h^2_{1\alpha} \end{pmatrix} \to R^{(0)}_{\chi^-} \sqrt{Z_{\chi^-}} \begin{pmatrix} \chi^-_{1\alpha} \\ \chi^-_{2\alpha} \end{pmatrix}, \tag{4.71}$$

$$\begin{pmatrix} \lambda'_\alpha \\ \lambda^3_\alpha \\ h^1_{1\alpha} \\ h^2_{2\alpha} \end{pmatrix} \to R^{(0)}_{\chi^0} \sqrt{Z_{\chi^0}} \begin{pmatrix} \chi^0_{1\alpha} \\ \chi^0_{2\alpha} \\ \chi^0_{3\alpha} \\ \chi^0_{4\alpha} \end{pmatrix}. \tag{4.72}$$

In the following $\mathcal{U}$, $\mathcal{V}$, $\mathcal{N}$ denote the inverse lowest-order demixing matrices $(R^{(0)}_{\chi^{+,-,0}})^{-1}$ [c.f. (2.60)]. Applying the transformations to the classical action yield the following invariant counterterms in the chargino-neutralino sector:

$$\begin{aligned} \Gamma^{(n)}_{\text{ct,inv}}|_{\chi^\pm} \;=\; & \int \mathrm{d}^4x \Bigg\{ \sum_{i,j=1}^{2} \bar\chi^-_i \mathrm{i}\gamma^\mu \partial_\mu \frac{1}{2} \left(\delta^{(n)} Z_{\chi^+} + \delta^{(n)} Z^T_{\chi^+}\right)_{ij} P_L \chi^+_j \\ & + \bar\chi^+_i \mathrm{i}\gamma^\mu \partial_\mu \frac{1}{2} \left(\delta^{(n)} Z_{\chi^-} + \delta^{(n)} Z^T_{\chi^-}\right)_{ij} P_R \chi^-_j \\ & - \left(\bar\chi^-_i \left[\mathcal{U}^* \delta^{(n)} \mathcal{X} \mathcal{V}^\dagger + \frac{1}{2} \left(\delta^{(n)} Z^T_{\chi^-} M_{\chi^\pm} + M_{\chi^\pm} \delta^{(n)} Z_{\chi^+} \right) \right]_{ij} P_L \chi^+_j + \text{c.c.} \right) \Bigg\}, \end{aligned} \tag{4.73}$$

$$\begin{aligned} \Gamma^{(n)}_{\text{ct,inv}}|_{\chi^0} \;=\; & \int \mathrm{d}^4x \Bigg\{ \frac{1}{2} \bar\chi^0_i \mathrm{i}\gamma^\mu \partial_\mu \frac{1}{2} \left(\delta^{(n)} Z_{\chi^0} + \delta^{(n)} Z^T_{\chi^0}\right)_{ij} \chi^0_j \\ & - \bar\chi^0_i \left[\mathcal{N}^* \delta^{(n)} \mathcal{Y} \mathcal{N}^\dagger + \frac{1}{2} \left(\delta^{(n)} Z^T_{\chi^0} M_{\chi^0} + M_{\chi^0} \delta^{(n)} Z_{\chi^0} \right) \right]_{ij} P_L \chi^0_j + \text{c.c.} \Bigg\}. \end{aligned} \tag{4.74}$$

The left- and right-handed projectors are denoted by $P_{L,R} = (1 \pm \gamma_5)/2$. The diagonal mass matrices are $M_{\chi^0} = \mathcal{N}^* \mathcal{Y} \mathcal{N}^\dagger = \mathrm{diag}(m_{\chi^0_1}, \ldots, m_{\chi^0_4})$, $M_{\chi^\pm} = \mathcal{U}^* \mathcal{X} \mathcal{V}^\dagger = \mathrm{diag}(m_{\chi^\pm_1}, m_{\chi^\pm_2})$.

In $\delta^{(n)}\mathcal{X}$ and $\delta^{(n)}\mathcal{Y}$ there appear on the one hand the three independent renormalization constants $\delta^{(n)}\mu$, $\delta^{(n)}M_1$, $\delta^{(n)}M_2$. On the other hand, $\delta^{(n)}\mathcal{X}$ and $\delta^{(n)}\mathcal{Y}$ depend on further renormalization constants, namely $\delta^{(n)}M_{W,Z}$ and $\delta^{(n)}\tan\beta$. The latter constants are fixed in other sectors — $\delta^{(n)}M_{W,Z}$ have been fixed in the gauge-boson sector, and $\delta^{(n)}\tan\beta$ has

not been fixed yet, but as it is a physical parameter we choose not to dispose of it via an on-shell or demixing condition.

Hence, the three on-shell conditions for the poles at $\mathscr{M}^2_{\chi^+_{1,2},\chi^0_1}$ can be satisfied by choosing $\delta^{(n)}\mu$, $\delta^{(n)}M_1$, $\delta^{(n)}M_2$ appropriately. In this way, all masses in the chargino-neutralino sector are determined.

The remaining normalization conditions in this sector are the demixing conditions and the conditions for the residua. It is easy to see that these conditions can all be satisfied by one unique choice of the $\delta^{(n)}Z_{\chi^{+,-,0}}$.

4.3.2 Quarks and leptons

The renormalization of the quarks and the charged leptons is identical, so we consider the up-quark u as an example. The relevant renormalization transformations are

$$\begin{aligned} m_u &\to m_u + \delta m_u, && (4.75)\\ u &\to \sqrt{Z_{u_L}}P_L u + \sqrt{Z_{u_R}}P_R u, && (4.76)\\ \sqrt{Z_{u_L}} &= \sqrt{z_q}R_{u_L}, && (4.77)\\ \sqrt{Z_{u_R}} &= \sqrt{z_u}. && (4.78) \end{aligned}$$

These transformations result in the following counterterms of order $\hbar^n$:

$$\begin{aligned} \Gamma^{(n)}_{\mathrm{ct,inv}}|_u &= \int \mathrm{d}^4x\, \bar{u}\Big[\mathrm{i}\gamma^\mu\partial_\mu(\delta^{(n)}Z_{u_L}P_L + \delta^{(n)}Z_{u_R}P_R) \\ &\quad - \delta^{(n)}m_u - m_u\frac{\delta^{(n)}Z_{u_L} + \delta^{(n)}Z_{u_R}}{2}\Big]u. \qquad (4.79) \end{aligned}$$

Obviously, the on-shell condition for the mass and for the residue can be satisfied by adjusting the renormalization constants $\delta^{(n)}m_u$ and $\delta^{(n)}Z_{u_{L,R}}$.

4.3.3 Squarks and sleptons

In the squark-slepton sector we apply the first kind of parametrization discussed in section 4.2, like in the chargino-neutralino sector. The renormalization transformations read

$$\begin{aligned} M^2_{\tilde{f}} &\to M^2_{\tilde{f}} + \delta M^2_{\tilde{f}}, \qquad \tilde{f} = \tilde{e}, \tilde{d}, \tilde{u}, && (4.80)\\ \begin{pmatrix} Q^1 \\ U^\dagger \end{pmatrix} &\to R^{(0)}_{\tilde{u}}\sqrt{Z_{\tilde{u}}}\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{pmatrix}, && (4.81)\\ \begin{pmatrix} Q^2 \\ D^\dagger \end{pmatrix} &\to R^{(0)}_{\tilde{d}}\sqrt{Z_{\tilde{d}}}\begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix}, && (4.82) \end{aligned}$$

$$\begin{pmatrix} L^2 \\ R^\dagger \end{pmatrix} \to R^{(0)}_{\tilde{e}} \sqrt{Z_{\tilde{e}}} \begin{pmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{pmatrix}, \tag{4.83}$$

$$L^1 = \sqrt{z_L}\tilde{\nu}_1, \tag{4.84}$$

where

$$R^{(0)}_{\tilde{u}} \sqrt{Z_{\tilde{u}}} = \begin{pmatrix} \sqrt{z_Q} & 0 \\ 0 & \sqrt{z_U} \end{pmatrix} R_{\tilde{u}}, \tag{4.85}$$

$$R^{(0)}_{\tilde{d}} \sqrt{Z_{\tilde{d}}} = \begin{pmatrix} \sqrt{z_Q} & 0 \\ 0 & \sqrt{z_D} \end{pmatrix} R_{\tilde{d}}, \tag{4.86}$$

$$R^{(0)}_{\tilde{e}} \sqrt{Z_{\tilde{e}}} = \begin{pmatrix} \sqrt{z_L} & 0 \\ 0 & \sqrt{z_R} \end{pmatrix} R_{\tilde{e}} \tag{4.87}$$

in agreement with the general formula (4.30). Applying these transformations to the classical action, the following counterterm contributions are obtained:

$$\begin{aligned} \Gamma^{(n)}_{\text{ct,inv}}|_{\tilde{f}} \;=\; & \int \mathrm{d}^4x \sum_{\tilde{f}=\tilde{u},\tilde{d},\tilde{e},\tilde{\nu}} -(\tilde{f}_1^\dagger, \tilde{f}_2^\dagger) \Big[\frac{1}{2}(\delta^{(n)} Z^T_{\tilde{f}} + \delta^{(n)} Z_{\tilde{f}})\Box \\ & + R^{(0)T}_{\tilde{f}} \delta^{(n)} M^2_{\tilde{f}} R^{(0)}_{\tilde{f}} + \frac{1}{2}(\delta^{(n)} Z^T_{\tilde{f}} M^2_{\tilde{f}} + M^2_{\tilde{f}} \delta^{(n)} Z_{\tilde{f}}) \Big] \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}, \end{aligned} \tag{4.88}$$

where in the case of the sneutrino $\tilde{\nu}$ the matrix $\delta^{(n)} Z_{\tilde{f}}$ degenerates to $\delta^{(n)} Z_{\tilde{\nu}} = \delta^{(n)} z_L$ and $\delta^{(n)} M^2_{\tilde{\nu}}$ has to be replaced by $\delta^{(n)} m^2_{\tilde{\nu}}$.

The counterterms, which are already determined in other sectors, are $\delta^{(n)}\mu$, $\delta^{(n)} m_{e,d,u}$, $\delta^{(n)} M_{W,Z}$, $\delta^{(n)} \tan\beta$, and the A parameters $\delta^{(n)} A_{e,d,u}$. The remaining five counterterms appearing in $\delta^{(n)} M^2_{\tilde{f}}$ are $\delta^{(n)} m^2_{\tilde{l},\tilde{e},\tilde{q},\tilde{d},\tilde{u}}$. These free mass counterterms are sufficient to satisfy the five on-shell conditions (4.14) for the sfermions $\tilde{f} = \tilde{u}_1, \tilde{u}_2, \tilde{d}_1, \tilde{\nu}, \tilde{e}_1$. After fixing these five counterterms, all masses in this sector are determined. The on-shell conditions for $\tilde{f} = \tilde{d}_2, \tilde{e}_2$ can only be fulfilled by choosing $\mathscr{M}^2_{\tilde{d}_2}$, $\mathscr{M}^2_{\tilde{e}_2}$ appropriately.

The demixing conditions and the conditions for the residua fix uniquely the values of the field renormalization constants $\delta^{(n)} Z_{\tilde{u},\tilde{d},\tilde{e}}$, $\delta^{(n)} z_L$.

4.3.4 Summary

The evaluation of the normalization conditions shows that all normalization conditions of section 4.1 can be satisfied. However, two non-trivial aspects of the normalization conditions are remarkable.

For the IR and Goldstone normalization conditions $\Gamma^T_{AA} = \Gamma_{c_A \bar{c}_A} = \Gamma_{G^0 G^0} = \Gamma_{G^+ G^-} = 0$ at $p = 0$, there are no free invariant counterterms available. Rather, these conditions

hold automatically as a consequence of the ST identity and the corresponding demixing conditions at $p = 0$. Furthermore, owing to the scalar-vector mixing, the definition of $\mathscr{M}^2_{A^0,H^\pm}$ by itself does not guarantee that $\mathscr{M}^2_{A^0,H^\pm}$ correspond to the poles of the Higgs propagators. Again the ST identity has been used to prove this correspondence.

As a result of the normalization conditions, the counterterms to the following quantities are determined: $M^2_{W,Z}$, $M^2_{A^0}$, $t_{1,2}$, μ, $M_{1,2}$, $m_{e,d,u}$, $m_{\tilde{l},\tilde{e},\tilde{q},\tilde{d},\tilde{u}}$, all field renormalization constants $\sqrt{Z_\phi}$, $\sqrt{z_\phi}$ (see section 3.4) except for certain combinations of $\sqrt{Z_c}$, $\sqrt{Z_{\bar{c}}}$, $\sqrt{z_c}$.

The normalization conditions of section 4.1 do not determine the counterterms to the parameters e, $\tan\beta$, $A_{e,d,u}$. These physical parameters can be fixed via five additional normalization conditions.

Discussion and outlook

In this paper we have presented the renormalization of the MSSM. The main results are the formulation of the Ward and ST identities, the R_ξ gauge conditions and the on-shell normalization conditions. We have shown that all these conditions can be satisfied simultaneously and that the MSSM is multiplicatively renormalizable and infrared finite.

More in detail, the elements of the construction are the following. The basic structure of the symmetry identities is similar to the one in the Standard Model [14], supersymmetry being treated as in [13, 16, 19, 21]. Supersymmetry and gauge invariance are formulated in terms of a ST identity that involves supersymmetry and translational ghosts in addition to the usual Faddeev-Popov ghosts; the Abelian subgroup is specified by a local Ward identity, and soft breaking is introduced via an external chiral supermultiplet. In order to account for the spontaneous breaking of gauge invariance, we reparametrize all fields in terms of physical fields with a more direct correspondence to mass eigenstates. The vacuum expectation values of the Higgs fields themselves are extended to external fields $(\Phi + \mathbf{v})$, since otherwise the R_ξ gauge fixing would explicitly break global gauge invariance.

However, unlike in the simpler models studied in [14, 24], it is impossible in the MSSM to use external fields with the multiplet structure of the dynamical Higgs fields, i.e. two isospin doublets $(\Phi + \mathbf{v})_{1,2}$. The reason is the mixing of the Goldstone fields $G^{0,\pm}$ with the physical Higgs fields $A^0, H^\pm$, which would inevitably lead to the appearance of $A^0, H^\pm$ in the gauge-fixing term. Instead, we choose two 8-component external fields transforming according to the product of the adjoint and the fundamental representation.

Further complications arise in the proof of the infrared finiteness and the discussion of the normalization conditions. In order to prove the infrared finiteness, we use on the one hand the ST identity like in [14] to constrain the appearance of the massless fields in the Green functions. On the other hand, in the symmetry identities also the external

fields could cause infrared problems, and we have to use a special reparametrization to obtain the optimal assignment of the infrared dimensions. In the case of the normalization conditions, the ST identity is necessary to show that the on-shell conditions at $p^2 = 0$ can be satisfied. Owing to the mixing of the physical Higgs fields with unphysical Goldstone and longitudinal gauge fields, it is not directly obvious that the on-shell condition for $M^2_{A^0}$ really corresponds to the pole of the A^0 propagator. This correspondence can again be proven via the ST identity.

There is an interesting difference between the on-shell conditions in the SM and the ones in the MSSM. In the SM all particle masses can be chosen as free input parameters and, conversely, all parameters except for the electric charge are fixed via the on-shell conditions. In the MSSM, however, the masses of H^0, h^0, $H^\pm$, $\chi^0_{2,3,4}$, and of $\tilde{d}_2$, $\tilde{e}_2$ are not free. They are fixed via the symmetries as functions of the free input parameters. Moreover, the free parameters $\tan\beta$ and $A_{u,d,e}$ are usually not fixed via on-shell conditions but via other conditions, e.g. in the interaction sector.

On the practical side, the results of the present paper yield the basis for the determination of the counterterms in the MSSM. Since no consistent symmetry-preserving regularization is known, in general symmetry-restoring counterterms are needed. They are determined by the symmetry identities and can be calculated using the same methods as in [16, 17] in the case of supersymmetric QED and QCD. The invariant counterterms are determined by the normalization conditions presented in section 4.

In the present approach to the renormalization of the MSSM the non-renormalization theorems [33] and the relations between the renormalization constants of soft-breaking and supersymmetric parameters [34] escaped the algebraic treatment since they are — as it is well known — not a direct consequence of supersymmetry. In order to derive these improved renormalization properties in the framework of algebraic renormalization one has to use the methods of [35, 36] with an extended version of the classical action. The non-renormalization theorems of the MSSM would yield stringent tests of concrete loop calculations, and they would make visible the improved renormalization behaviour — one of the main motivations to study supersymmetry at all — in terms of explicit equations.

A Notations and conventions

We use the following conventions for the metric tensor $g^{\mu\nu}$ and the generators of the SU(2) gauge group T^a:

$$g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1), \qquad \mathrm{T}^a = \frac{1}{2}\sigma^a, \tag{A.89}$$

where the Pauli matrices are denoted by σ^a, $a = 1,2,3$. Hence, the generators of the SU(2) gauge group obey the following relations:

$$[\mathrm{T}^a, \mathrm{T}^b] = \mathrm{i}\epsilon^{abc}\mathrm{T}^c, \qquad 2\mathrm{tr}(\mathrm{T}^a\mathrm{T}^b) = \delta^{ab} \tag{A.90}$$

with $\epsilon^{123} = +1$. Spinorial derivatives are defined as follows

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha{}^\beta, \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta^{\dot\beta} = \delta_{\dot\alpha}{}^{\dot\beta} \tag{A.91}$$

and the superspace integrations read

$$\int \mathrm{d}^8 z \;= \int \mathrm{d}^4 x \, \mathscr{D}^2 \bar{\mathscr{D}}^2, \qquad \int \mathrm{d}^6 z \;= \int \mathrm{d}^4 x \, \mathscr{D}^2 \tag{A.92}$$

with

$$\mathscr{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \qquad \bar{\mathscr{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu. \tag{A.93}$$

Furthermore, we use the following notations:

$$\sigma^{\mu\nu}_{\alpha\beta} = \frac{\mathrm{i}}{2}(\sigma^\mu{}_{\alpha\dot\alpha}\bar\sigma^{\nu\dot\alpha}{}_\beta - \sigma^\nu{}_{\alpha\dot\alpha}\bar\sigma^{\mu\dot\alpha}{}_\beta), \qquad \bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} = \frac{\mathrm{i}}{2}(\bar\sigma^{\mu\alpha}_{\dot\alpha}\sigma^\nu{}_{\alpha\dot\beta} - \bar\sigma^{\nu\alpha}_{\dot\alpha}\sigma^\mu{}_{\alpha\dot\beta}). \tag{A.94}$$

The field-strength tensors and covariant derivatives are defined by

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + \mathrm{i}g[V_\mu, V_\nu], \tag{A.95a}$$

$$F'_{\mu\nu} = \partial_\mu V'_\nu - \partial_\nu V'_\mu, \tag{A.95b}$$

$$\mathscr{D}_\mu\phi = \begin{cases} \partial_\mu\phi + \mathrm{i}g[V_\mu, \phi] & \text{adjoint representation,} \\ \partial_\mu\phi + \mathrm{i}gV_\mu\phi + \mathrm{i}g'V'_\mu\frac{Y}{2}\phi & \text{isospin doublet,} \\ \partial_\mu\phi + \mathrm{i}g'V'_\mu\frac{Y}{2}\phi & \text{isospin singlet.} \end{cases} \tag{A.95c}$$

In our conventions, the rules for complex conjugation are

$$\overline{\mathrm{s}\,\phi} = \mathrm{s}\,\overline{\phi} \qquad \text{for bosonic } \phi, \tag{A.96a}$$

$$\overline{\mathrm{s}\,\phi} = -\mathrm{s}\,\overline{\phi} \qquad \text{for fermionic } \phi. \tag{A.96b}$$

B BRS transformations

The BRS transformations of the field components of the MSSM read:

Gauge multiplets:

$$\mathrm{s}\, V_\mu = \mathscr{D}_\mu c + \varepsilon^\alpha \sigma_{\mu\alpha\dot\alpha}\bar\lambda^{\dot\alpha} + \lambda^\alpha \sigma_{\mu\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} - \mathrm{i}\xi^\nu \partial_\nu V_\mu, \tag{B.97a}$$

$$\mathrm{s}\, \lambda_\alpha = \frac{1}{2}\varepsilon^\beta \sigma^{\mu\nu}_{\beta\alpha} F_{\mu\nu} - \mathrm{i}g\{\lambda_\alpha, c\} + \varepsilon_\alpha D_V - \mathrm{i}\xi^\mu \partial_\mu \lambda_\alpha, \tag{B.97b}$$

$$\mathrm{s}\, \bar\lambda_{\dot\alpha} = -\frac{1}{2}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}\bar\varepsilon^{\dot\beta} F_{\mu\nu} - \mathrm{i}g\{\bar\lambda_{\dot\alpha}, c\} - \bar\varepsilon_{\dot\alpha} D_V - \mathrm{i}\xi^\mu \partial_\mu \bar\lambda_{\dot\alpha}, \tag{B.97c}$$

$$\mathrm{s}\, D_V = \mathrm{i}\mathscr{D}_\mu \lambda^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} - \mathrm{i}\mathscr{D}_\mu \varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha} + \mathrm{i}g[D_V, c] - \mathrm{i}\xi^\mu \partial_\mu D_V, \tag{B.97d}$$

$$\mathrm{s}\, V'_\mu = \partial_\mu c' + \varepsilon^\alpha \sigma_{\mu\alpha\dot\alpha}\bar\lambda'^{\dot\alpha} + \lambda'^\alpha \sigma_{\mu\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} - \mathrm{i}\xi^\nu \partial_\nu V'_\mu, \tag{B.97e}$$

$$\mathrm{s}\, \lambda'_\alpha = \frac{1}{2}\varepsilon^\beta \sigma^{\mu\nu}_{\beta\alpha} F'_{\mu\nu} + \varepsilon_\alpha D'_V - \mathrm{i}\xi^\mu \partial_\mu \lambda'_\alpha, \tag{B.97f}$$

$$\mathrm{s}\, \bar\lambda'_{\dot\alpha} = -\frac{1}{2}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}\bar\varepsilon^{\dot\beta} F'_{\mu\nu} - \bar\varepsilon_{\dot\alpha} D'_V - \mathrm{i}\xi^\mu \partial_\mu \bar\lambda'_{\dot\alpha}, \tag{B.97g}$$

$$\mathrm{s}\, D'_V = \mathrm{i}\partial_\mu \lambda'^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} - \mathrm{i}\partial_\mu \varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\lambda'^{\dot\alpha} - \mathrm{i}\xi^\mu \partial_\mu D'_V. \tag{B.97h}$$

Matter Fields (The Higgs and quark fields transform analogously to L, l, F_L):

$$\mathrm{s}\, L = -\mathrm{i}gcL - \mathrm{i}g'\frac{Y}{2}c'L + \sqrt{2}\varepsilon^\alpha l_\alpha - \mathrm{i}\xi^\mu \partial_\mu L, \tag{B.98a}$$

$$\mathrm{s}\, l_\alpha = -\mathrm{i}gcl_\alpha - \mathrm{i}g'\frac{Y}{2}c' l_\alpha + \sqrt{2}\varepsilon_\alpha F_L + \mathrm{i}\sqrt{2}\sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\mathscr{D}_\mu L - \mathrm{i}\xi^\mu \partial_\mu l_\alpha, \tag{B.98b}$$

$$\begin{aligned}\mathrm{s}\, F_L = {} & -\mathrm{i}gcF_L - \mathrm{i}g'\frac{Y}{2}c'F_L - 2g\bar\varepsilon_{\dot\alpha}\bar\lambda^{\dot\alpha} L - g'Y\bar\varepsilon_{\dot\alpha}\bar\lambda'^{\dot\alpha} L + \mathrm{i}\sqrt{2}\mathscr{D}_\mu l^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} \\ & - \mathrm{i}\xi^\mu \partial_\mu F_L,\end{aligned} \tag{B.98c}$$

$$\mathrm{s}\, R = -\mathrm{i}g'\frac{Y}{2}c'R + \sqrt{2}\varepsilon^\alpha r_\alpha - \mathrm{i}\xi^\mu \partial_\mu R, \tag{B.98d}$$

$$\mathrm{s}\, r_\alpha = -\mathrm{i}g'\frac{Y}{2}c' r_\alpha + \sqrt{2}\varepsilon_\alpha F_R + \mathrm{i}\sqrt{2}\sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\mathscr{D}_\mu R - \mathrm{i}\xi^\mu \partial_\mu r_\alpha, \tag{B.98e}$$

$$\mathrm{s}\, F_R = -\mathrm{i}g'\frac{Y}{2}c'F_R - g'Y\bar\varepsilon_{\dot\alpha}\bar\lambda'^{\dot\alpha} R + \mathrm{i}\sqrt{2}\mathscr{D}_\mu r^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} - \mathrm{i}\xi^\mu \partial_\mu F_R. \tag{B.98f}$$

Ghosts, Antighosts, and Lagrange multipliers:

$$\mathrm{s}\, c = -\mathrm{i}gcc + 2\mathrm{i}\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} V_\mu - \mathrm{i}\xi^\mu \partial_\mu c, \tag{B.99a}$$

$$\mathrm{s}\, \bar c = B - \mathrm{i}\xi^\mu \partial_\mu \bar c, \tag{B.99b}$$

$$\mathrm{s}\, B = 2\mathrm{i}\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\partial_\mu \bar c - \mathrm{i}\xi^\mu \partial_\mu B, \tag{B.99c}$$

$$\mathrm{s}\, c' = 2\mathrm{i}\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha} V'_\mu - \mathrm{i}\xi^\mu \partial_\mu c', \tag{B.99d}$$

$$\mathrm{s}\, \bar c' = B' - \mathrm{i}\xi^\mu \partial_\mu \bar c', \tag{B.99e}$$

$$\mathrm{s}\, B' = 2\mathrm{i}\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}\partial_\mu \bar c' - \mathrm{i}\xi^\mu \partial_\mu B', \tag{B.99f}$$

$$\mathrm{s}\, \xi^\mu = 2\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha}\bar\varepsilon^{\dot\alpha}, \tag{B.99g}$$

$$\mathrm{s}\, \varepsilon_\alpha = 0, \tag{B.99h}$$

$$\mathrm{s}\, \bar\varepsilon_{\dot\alpha} = 0. \tag{B.99i}$$

Y represents the hypercharge of the respective field as given in table 2.1.

The commutation relations of the generators are encoded in the transformations of the ghosts in (B.99a)–(B.99i): cc in $\mathbf{s}\,c$ contains the commutator $[T^a, T^b]$ of SU(2) generators due to the anticommuting nature of c. The additional term $\varepsilon^\alpha \sigma^\mu_{\alpha\dot\alpha} \bar\varepsilon^{\dot\alpha} V_\mu$ in $\mathbf{s}\,c$ corresponds to the supersymmetry anticommutator $\{Q_\alpha, \bar Q_{\dot\alpha}\}$ and reflects the fact that in the Wess-Zumino gauge supersymmetry transformations are accompanied by field dependent gauge transformations. The usual well-known part of the supersymmetry anticommutator is found in $\mathbf{s}\,\xi^\mu$.

C Explicit results for the classical action

The results of the classical action in terms of component fields are listed in the following:

$$\frac{1}{16}\int \mathrm{d}^8 z\, \bar{\hat L} \mathrm{e}^{2g\hat V + g' Y \hat V'} \hat L = \int \mathrm{d}^4 x \left(\overline{\mathscr{D}_\mu L} \mathscr{D}^\mu L + \mathrm{i} l^\alpha \sigma^\mu_{\alpha\dot\alpha} \overline{\mathscr{D}_\mu l}^{\dot\alpha} + \bar F_L F_L + g \bar L D_V L \right.$$
$$\left. - \sqrt{2} g \bar l_{\dot\alpha} \bar\lambda^{\dot\alpha} L - \sqrt{2} g \bar L \lambda^\alpha l_\alpha + g' \frac{Y}{2} \bar L D'_V L - g' \frac{Y}{2} \sqrt{2} \bar l_{\dot\alpha} \bar\lambda'^{\dot\alpha} L - g' \frac{Y}{2} \sqrt{2} \bar L \lambda'^\alpha l_\alpha \right), \quad \text{(C.1a)}$$

$$\frac{1}{16}\int \mathrm{d}^8 z\, \bar{\hat R} \mathrm{e}^{g' Y \hat V'} \hat R = \int \mathrm{d}^4 x \left(\overline{\mathscr{D}_\mu R} \mathscr{D}^\mu R + \mathrm{i} r^\alpha \sigma^\mu_{\alpha\dot\alpha} \overline{\mathscr{D}_\mu r}^{\dot\alpha} + \bar F_R F_R + g' \frac{Y}{2} \bar R D'_V R \right.$$
$$\left. - g' \frac{Y}{2} \sqrt{2} \bar r_{\dot\alpha} \bar\lambda'^{\dot\alpha} R - g' \frac{Y}{2} \sqrt{2} \bar R \lambda'^\alpha r_\alpha \right), \quad \text{(C.1b)}$$

$$-\frac{1}{512 g^2}\int \mathrm{d}^6 z\, 2\mathrm{tr}[\hat F^\alpha \hat F_\alpha] = \int \mathrm{d}^4 x\, 2\mathrm{tr}\left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathrm{i} \lambda^\alpha \sigma^\mu_{\alpha\dot\alpha} \mathscr{D}_\mu \bar\lambda^{\dot\alpha} + \frac{1}{2} D_V^2 \right], \quad \text{(C.1c)}$$

$$-\frac{1}{512 g^2}\int \mathrm{d}^6 z\, \hat A 2\mathrm{tr}[\hat F^\alpha \hat F_\alpha] = \int \mathrm{d}^4 x\, 2\mathrm{tr}\left[-A_1 \varepsilon^\alpha \lambda_\alpha D_V + \frac{1}{2} A_1 \lambda^\alpha \sigma^{\mu\nu}{}_\alpha{}^\beta \varepsilon_\beta F_{\mu\nu} \right.$$
$$\left. + \frac{1}{2}(A_2 + v_A) \lambda^\alpha \lambda_\alpha \right], \quad \text{(C.1d)}$$

$$-\frac{1}{128 g'^2}\int \mathrm{d}^6 z\, \hat F'^\alpha \hat F'_\alpha = \int \mathrm{d}^4 x \left(-\frac{1}{4} \hat F'_{\mu\nu} \hat F'^{\mu\nu} + \mathrm{i} \lambda'^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\lambda'^{\dot\alpha} + \frac{1}{2} D_V'^2 \right), \quad \text{(C.1e)}$$

$$-\frac{1}{128 g'^2}\int \mathrm{d}^6 z\, \hat A \hat F'^\alpha \hat F'_\alpha = \int \mathrm{d}^4 x \left[-A_1 \varepsilon^\alpha \lambda'_\alpha D'_V + \frac{1}{2} A_1 \lambda'^\alpha \sigma^{\mu\nu}{}_\alpha{}^\beta \varepsilon_\beta F'_{\mu\nu} \right.$$
$$\left. + \frac{1}{2}(A_2 + v_A) \lambda'^\alpha \lambda'_\alpha \right], \quad \text{(C.1f)}$$

$$-\frac{1}{4}\int \mathrm{d}^6 z\, \hat H_1^T (\mathrm{i}\sigma_2) \hat L \hat R = \int \mathrm{d}^4 x \left[-H_1^T (\mathrm{i}\sigma_2) l r + H_1^T (\mathrm{i}\sigma_2) F_L R + H_1^T (\mathrm{i}\sigma_2) L F_R \right.$$
$$\left. - h_1^{T\alpha} (\mathrm{i}\sigma_2) l_\alpha R - h_1^{T\alpha} (\mathrm{i}\sigma_2) L r_\alpha + F_1^T (\mathrm{i}\sigma_2) L R \right], \quad \text{(C.1g)}$$

$$\frac{1}{4}\int \mathrm{d}^6 z\, \hat{H}_1^T(\mathrm{i}\sigma_2)\hat{H}_2 = \int \mathrm{d}^4 x \left[-H_1^T(\mathrm{i}\sigma_2)F_2 - F_1^T(\mathrm{i}\sigma_2)H_2 + h_1^{T\alpha}(\mathrm{i}\sigma_2)h_{2\alpha}\right] \tag{C.1h}$$

with

$$\hat{F}^{\alpha} = 4g\mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(-2\lambda^\alpha - 2\theta^\alpha D_V + \sigma^{\mu\nu\alpha\beta}\theta_\beta F_{\mu\nu} - 2\mathrm{i}\sigma^{\mu\alpha}{}_{\dot{\alpha}}\mathscr{D}_\mu\bar{\lambda}^{\dot{\alpha}}\theta^2\right), \tag{C.2a}$$

$$\hat{F}'^{\alpha} = 2g'\mathrm{e}^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(-2\lambda'^\alpha - 2\theta^\alpha D'_V + \sigma^{\mu\nu\alpha\beta}\theta_\beta F'_{\mu\nu} - 2\mathrm{i}\sigma^{\mu\alpha}{}_{\dot{\alpha}}\mathscr{D}'_\mu\bar{\lambda}'^{\dot{\alpha}}\theta^2\right). \tag{C.2b}$$

The remaining terms of the classical action can be derived by obvious substitutions or by the fact, that a product of chiral superfields is again a chiral superfield. For instance, the results involving a chiral superfield $\hat{A}\hat{L}$ can be simply obtained from the corresponding result with a chiral field $\hat{L}$ if we substitute

$$L \to 0, \qquad l_\alpha \to \sqrt{2}\varepsilon_\alpha A_1 L, \qquad F_L \to (A_2 + v_A)L - \sqrt{2}A_1\varepsilon^\alpha l_\alpha. \tag{C.3}$$

D Explicit form of Ward operators

$$\begin{aligned}
\mathscr{W}(\omega) = \int \mathrm{d}^4 x\ \Big\{ & 2\mathrm{tr}\Big[-\mathrm{i}g[\omega,V^\mu]\frac{\delta}{\delta V^\mu} - \mathrm{i}g[\omega,\lambda^\alpha]\frac{\delta}{\delta\lambda^\alpha} - \mathrm{i}g[\omega,\bar{\lambda}_{\dot{\alpha}}]\frac{\delta}{\delta\bar{\lambda}_{\dot{\alpha}}} - \mathrm{i}g[\omega,c]\frac{\delta}{\delta c} \\
& + \mathrm{i}g[\omega,Y_V^\mu]\frac{\delta}{\delta Y_V^\mu} + \mathrm{i}g[\omega,y_\lambda^\alpha]\frac{\delta}{\delta y_\lambda^\alpha} + \mathrm{i}g[\omega,\bar{y}_{\lambda\dot{\alpha}}]\frac{\delta}{\delta\bar{y}_{\lambda\dot{\alpha}}} + \mathrm{i}g[\omega,Y_c]\frac{\delta}{\delta Y_c} \\
& - \mathrm{i}g[\omega,B]\frac{\delta}{\delta B} - \mathrm{i}g[\omega,\bar{c}]\frac{\delta}{\delta\bar{c}}\Big] \\
& - \mathrm{i}g\left[\omega(\Phi_i^a + \mathbf{v}_i^a) + (\mathrm{i}\epsilon^{bca})\omega^b(\Phi_i^c + \mathbf{v}_i^c)\right]^T \frac{\delta}{\delta\Phi_i^{aT}} - \mathrm{i}g\,\omega(\Phi'_i + \mathbf{v}'_i)^T\frac{\delta}{\delta\Phi_i'^T} \\
& + \mathrm{i}g\left[\omega(\bar{\Phi}_i^a + \bar{\mathbf{v}}_i^a) + (\mathrm{i}\epsilon^{bca})\omega^b(\bar{\Phi}_i^c + \bar{\mathbf{v}}_i^c)\right]\frac{\delta}{\delta\bar{\Phi}_i^a} + \mathrm{i}g\,\omega(\bar{\Phi}'_i + \bar{\mathbf{v}}'_i)\frac{\delta}{\delta\bar{\Phi}'_i} \\
& - \mathrm{i}g\left[\omega\Psi_i^a + (\mathrm{i}\epsilon^{bca})\omega^b\Psi_i^c\right]^T\frac{\delta}{\delta\Psi_i^{aT}} - \mathrm{i}g\,\omega\Psi_i'^T\frac{\delta}{\delta\Psi_i'^T} \\
& + \mathrm{i}g\left[\omega\bar{\Psi}_i^a + (\mathrm{i}\epsilon^{bca})\omega^b\bar{\Psi}_i^c\right]\frac{\delta}{\delta\bar{\Psi}_i^a} + \mathrm{i}g\,\omega\bar{\Psi}'_i\frac{\delta}{\delta\bar{\Psi}'_i} \\
& - \mathrm{i}g(\omega L)^T\frac{\delta}{\delta L^T} + \mathrm{i}g\omega\bar{L}\frac{\delta}{\delta\bar{L}} - \mathrm{i}g(\omega l^\alpha)^T\frac{\delta}{\delta l^{\alpha T}} - \mathrm{i}g\omega\bar{l}_{\dot{\alpha}}\frac{\delta}{\delta\bar{l}_{\dot{\alpha}}} \\
& + \mathrm{i}g(\omega Y_L)^T\frac{\delta}{\delta Y_L^T} - \mathrm{i}g\omega\bar{Y}_L\frac{\delta}{\delta\bar{Y}_L} + \mathrm{i}g(\omega y_l^\alpha)^T\frac{\delta}{\delta y_l^{\alpha T}} + \mathrm{i}g\omega\bar{y}_{l\dot{\alpha}}\frac{\delta}{\delta\bar{y}_{l\dot{\alpha}}} \\
& - \mathrm{i}g\omega R\frac{\delta}{\delta R} + \mathrm{i}g\omega\bar{R}\frac{\delta}{\delta\bar{R}} - \mathrm{i}g\omega r^\alpha\frac{\delta}{\delta r^\alpha} - \mathrm{i}g\omega\bar{r}_{\dot{\alpha}}\frac{\delta}{\delta\bar{r}_{\dot{\alpha}}} \\
& + \mathrm{i}g\omega Y_R\frac{\delta}{\delta Y_R} - \mathrm{i}g\omega\bar{Y}_R\frac{\delta}{\delta\bar{Y}_R} + \mathrm{i}g\omega y_r^\alpha\frac{\delta}{\delta y_r^\alpha} + \mathrm{i}g\omega\bar{y}_{r\dot{\alpha}}\frac{\delta}{\delta\bar{y}_{r\dot{\alpha}}}\Big\} \\
& + \text{analogous quark and Higgs terms,}
\end{aligned} \tag{D.1a}$$

$$
\begin{aligned}
\mathscr{W}'(\omega') = \int \mathrm{d}^4x \; \Bigg[& -\mathrm{i}g'\omega'\frac{Y}{2}(\Phi_i^a + \mathbf{v}_i^a)^T \frac{\delta}{\delta \Phi_i^{aT}} - \mathrm{i}g'\omega'\frac{Y}{2}(\Phi_i' + \mathbf{v}_i')^T \frac{\delta}{\delta \Phi_i'^T} \\
& + \mathrm{i}g'\omega'\frac{Y}{2}(\bar{\Phi}_i^a + \bar{\mathbf{v}}_i^a)\frac{\delta}{\delta \bar{\Phi}_i^a} + \mathrm{i}g'\omega'\frac{Y}{2}(\bar{\Phi}_i' + \bar{\mathbf{v}}_i')\frac{\delta}{\delta \bar{\Phi}_i'} \\
& - \mathrm{i}g'\omega'\frac{Y}{2}\Psi_i^{aT}\frac{\delta}{\delta \Psi_i^{aT}} - \mathrm{i}g'\omega'\frac{Y}{2}\Psi_i'^T\frac{\delta}{\delta \Psi_i'^T} + \mathrm{i}g'\omega'\frac{Y}{2}\bar{\Psi}_i^a\frac{\delta}{\delta \bar{\Psi}_i^a} + \mathrm{i}g'\omega'\frac{Y}{2}\bar{\Psi}_i'\frac{\delta}{\delta \bar{\Psi}_i'} \\
& - \mathrm{i}g'\frac{Y}{2}\omega' L^T\frac{\delta}{\delta L^T} + \mathrm{i}g'\frac{Y}{2}\omega'\bar{L}\frac{\delta}{\delta \bar{L}} - \mathrm{i}g'\frac{Y}{2}\omega' l^{\alpha T}\frac{\delta}{\delta l^{\alpha T}} - \mathrm{i}g'\frac{Y}{2}\omega'\bar{l}_{\dot{\alpha}}\frac{\delta}{\delta \bar{l}_{\dot{\alpha}}} \\
& + \mathrm{i}g'\frac{Y}{2}\omega' Y_L^T\frac{\delta}{\delta Y_L^T} - \mathrm{i}g'\frac{Y}{2}\omega'\bar{Y}_L\frac{\delta}{\delta \bar{Y}_L} + \mathrm{i}g'\frac{Y}{2}\omega' y_l^{\alpha T}\frac{\delta}{\delta y_l^{\alpha T}} + \mathrm{i}g'\frac{Y}{2}\omega'\bar{y}_{l\dot{\alpha}}\frac{\delta}{\delta \bar{y}_{l\dot{\alpha}}} \\
& - \mathrm{i}g'\frac{Y}{2}\omega' R\frac{\delta}{\delta R} + \mathrm{i}g'\frac{Y}{2}\omega'\bar{R}\frac{\delta}{\delta \bar{R}} - \mathrm{i}g'\frac{Y}{2}\omega' r^{\alpha}\frac{\delta}{\delta r^{\alpha}} - \mathrm{i}g'\frac{Y}{2}\omega'\bar{r}_{\dot{\alpha}}\frac{\delta}{\delta \bar{r}_{\dot{\alpha}}} \\
& + \mathrm{i}g'\frac{Y}{2}\omega' Y_R\frac{\delta}{\delta Y_R} - \mathrm{i}g'\frac{Y}{2}\omega'\bar{Y}_R\frac{\delta}{\delta \bar{Y}_R} + \mathrm{i}g'\frac{Y}{2}\omega' y_r^{\alpha}\frac{\delta}{\delta y_r^{\alpha}} + \mathrm{i}g'\frac{Y}{2}\omega'\bar{y}_{r\dot{\alpha}}\frac{\delta}{\delta \bar{y}_{r\dot{\alpha}}} \Bigg] \\
& + \text{analogous quark and Higgs terms,}
\end{aligned} \tag{D.1b}
$$

$$
\begin{aligned}
\mathscr{W}_L = \int \mathrm{d}^4x \; \Bigg[& L^T\frac{\delta}{\delta L^T} - \bar{L}\frac{\delta}{\delta \bar{L}} + l^{\alpha T}\frac{\delta}{\delta l^{\alpha T}} + \bar{l}_{\dot{\alpha}}\frac{\delta}{\delta \bar{l}_{\dot{\alpha}}} \\
& - Y_L^T\frac{\delta}{\delta Y_L^T} + \bar{Y}_L\frac{\delta}{\delta \bar{Y}_L} - y_l^{\alpha T}\frac{\delta}{\delta y_l^{\alpha T}} - \bar{y}_{l\dot{\alpha}}\frac{\delta}{\delta \bar{y}_{l\dot{\alpha}}} \Bigg] \\
& + \text{analogous terms for right-handed leptons,}
\end{aligned} \tag{D.1c}
$$

$$
\begin{aligned}
\mathscr{W}_Q = \int \mathrm{d}^4x \; \Bigg[& Q^T\frac{\delta}{\delta Q^T} - \bar{Q}\frac{\delta}{\delta \bar{Q}} + q^{\alpha T}\frac{\delta}{\delta q^{\alpha T}} + \bar{q}_{\dot{\alpha}}\frac{\delta}{\delta \bar{q}_{\dot{\alpha}}} \\
& - Y_Q^T\frac{\delta}{\delta Y_Q^T} + \bar{Y}_Q\frac{\delta}{\delta \bar{Y}_Q} - y_q^{\alpha T}\frac{\delta}{\delta y_q^{\alpha T}} - \bar{y}_{q\dot{\alpha}}\frac{\delta}{\delta \bar{y}_{q\dot{\alpha}}} \Bigg] \\
& + \text{analogous terms for right-handed quarks,}
\end{aligned} \tag{D.1d}
$$

$$
\begin{aligned}
\mathscr{W}^R = \int \mathrm{d}^4x \; \Bigg\{ & 2\mathrm{tr}\left[-\mathrm{i}\lambda^{\alpha}\frac{\delta}{\delta \lambda^{\alpha}} - \mathrm{i}g\bar{\lambda}_{\dot{\alpha}}\frac{\delta}{\delta \bar{\lambda}_{\dot{\alpha}}} + \mathrm{i}y_{\lambda}^{\alpha}\frac{\delta}{\delta y_{\lambda}^{\alpha}} + \mathrm{i}\bar{y}_{\lambda\dot{\alpha}}\frac{\delta}{\delta \bar{y}_{\lambda\dot{\alpha}}} \right] \\
& - \mathrm{i}\lambda'^{\alpha}\frac{\delta}{\delta \lambda'^{\alpha}} - \mathrm{i}g\bar{\lambda}'_{\dot{\alpha}}\frac{\delta}{\delta \bar{\lambda}'_{\dot{\alpha}}} + \mathrm{i}y_{\lambda'}^{\alpha}\frac{\delta}{\delta y_{\lambda'}^{\alpha}} + \mathrm{i}\bar{y}_{\lambda'\dot{\alpha}}\frac{\delta}{\delta \bar{y}_{\lambda'\dot{\alpha}}} \\
& - 2\mathrm{i}A_2\frac{\delta}{\delta A_2} + 2\mathrm{i}\bar{A}_2\frac{\delta}{\delta \bar{A}_2} \\
& + \mathrm{i}(\Phi_i^a + \mathbf{v}_i^a)\frac{\delta}{\delta \Phi_i^{aT}} + \mathrm{i}(\Phi_i' + \mathbf{v}_i')^T\frac{\delta}{\delta \Phi_i'^T} - \mathrm{i}(\bar{\Phi}_i^a + \bar{\mathbf{v}}_i^a)\frac{\delta}{\delta \bar{\Phi}_i^a} - \mathrm{i}(\bar{\Phi}_i' + \bar{\mathbf{v}}_i')\frac{\delta}{\delta \bar{\Phi}_i'} \\
& + \mathrm{i}\Psi_i^a\frac{\delta}{\delta \Psi_i^{aT}} + \mathrm{i}\Psi_i'^T\frac{\delta}{\delta \Psi_i'^T} - \mathrm{i}\bar{\Psi}_i^a\frac{\delta}{\delta \bar{\Psi}_i^a} - \mathrm{i}\bar{\Psi}_i'\frac{\delta}{\delta \bar{\Psi}_i'} \\
& + \mathrm{i}H_1^T\frac{\delta}{\delta H_1^T} - \mathrm{i}\bar{H}_1\frac{\delta}{\delta \bar{H}_1} + \mathrm{i}H_2^T\frac{\delta}{\delta H_2^T} - \mathrm{i}\bar{H}_2\frac{\delta}{\delta \bar{H}_2} \\
& - \mathrm{i}Y_1^T\frac{\delta}{\delta Y_1^T} + \mathrm{i}\bar{Y}_1\frac{\delta}{\delta \bar{Y}_1} - \mathrm{i}Y_2^T\frac{\delta}{\delta Y_2^T} + \mathrm{i}\bar{Y}_2\frac{\delta}{\delta \bar{Y}_2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\mathrm{i}}{2}L^T\frac{\delta}{\delta L^T}-\frac{\mathrm{i}}{2}\bar{L}\frac{\delta}{\delta\bar{L}}-\frac{\mathrm{i}}{2}l^{\alpha T}\frac{\delta}{\delta l^{\alpha T}}-\frac{\mathrm{i}}{2}\bar{l}_{\dot\alpha}\frac{\delta}{\delta\bar{l}_{\dot\alpha}}\\
&-\frac{\mathrm{i}}{2}Y_L^T\frac{\delta}{\delta Y_L^T}+\frac{\mathrm{i}}{2}\bar{Y}_L\frac{\delta}{\delta\bar{Y}_L}+\frac{\mathrm{i}}{2}y_l^{\alpha T}\frac{\delta}{\delta y_l^{\alpha T}}+\frac{\mathrm{i}}{2}\bar{y}_{l\dot\alpha}\frac{\delta}{\delta\bar{y}_{l\dot\alpha}}\\
&+\frac{\mathrm{i}}{2}R\frac{\delta}{\delta R}-\frac{\mathrm{i}}{2}\bar{R}\frac{\delta}{\delta\bar{R}}-\frac{\mathrm{i}}{2}r^{\alpha}\frac{\delta}{\delta r^{\alpha}}-\frac{\mathrm{i}}{2}\bar{r}_{\dot\alpha}\frac{\delta}{\delta\bar{r}_{\dot\alpha}}\\
&-\frac{\mathrm{i}}{2}Y_R\frac{\delta}{\delta Y_R}+\frac{\mathrm{i}}{2}\bar{Y}_R\frac{\delta}{\delta\bar{Y}_R}+\frac{\mathrm{i}}{2}y_r^{\alpha}\frac{\delta}{\delta y_r^{\alpha}}+\frac{\mathrm{i}}{2}\bar{y}_{r\dot\alpha}\frac{\delta}{\delta\bar{y}_{r\dot\alpha}}\Bigg\}\\
&-\mathrm{i}\varepsilon^{\alpha}\frac{\partial}{\partial\varepsilon^{\alpha}}-\mathrm{i}\bar{\varepsilon}_{\dot\alpha}\frac{\partial}{\partial\bar{\varepsilon}_{\dot\alpha}}\\
&+\text{analogous quark terms.}
\end{aligned}
\tag{D.1e}
$$

E Cohomologically trivial invariant counterterms

In this section we sketch the determination of the most general form of the cohomologically trivial contribution $\mathscr{S}_{\Gamma_{\mathrm{cl}}}\hat{\Gamma}_{\mathrm{ct,inv,1b}}$ to the invariant counterterms. This contribution satisfies the first of the conditions in (3.38) automatically since $\mathscr{S}_{\Gamma_{\mathrm{cl}}}$ is nilpotent. The remaining conditions in (3.38) restrict the form of $\mathscr{S}_{\Gamma_{\mathrm{cl}}}\hat{\Gamma}_{\mathrm{inv2}}$. In addition, the counterterms have to depend on the field A_1 only through the combination $a_\alpha=\sqrt{2}\epsilon_\alpha A_1$.

- Gauge-fixing sector: this sector contains the field monomials depending on $\bar{c}$, $\bar{c}'$, B, and B'. These terms are fixed to all orders by the gauge-fixing condition.
- Field monomials involving the translational ghost ξ^μ: these monomials are fixed to all orders by the translation ghost equation.
- Field renormalization: counterterms corresponding to field renormalization arise as

$$
\mathscr{S}_{\Gamma_{\mathrm{cl}}}\int\mathrm{d}^4x\,Y_{\phi_i}\phi_i=\pm\int\mathrm{d}^4x\,\left(\phi_i\frac{\delta}{\delta\phi_i}-Y_{\phi_i}\frac{\delta}{\delta Y_{\phi_i}}\right)\Gamma_{\mathrm{cl}},
\tag{E.2}
$$

  where ϕ_i, Y_{ϕ_i} run through all pairs of fields and corresponding Y fields. The $\pm$ signs hold for bosonic and fermionic ϕ_i, respectively. In order not to invalidate the gauge-fixing conditions, the field renormalizations of the gauge bosons and Higgs fields have to be supplemented by suitable renormalizations of the gauge-fixing terms.
- Renormalization of x_1, x_2, x_1', x_2': these renormalizations are generated by the following $\mathscr{S}_{\Gamma_{\mathrm{cl}}}$ variations:

$$
\mathscr{S}_{\Gamma_{\mathrm{cl}}}\int\mathrm{d}^4x\,Y_i^T\left(\hat{\Phi}_i+\hat{\mathbf{v}}_i\right)=\int\mathrm{d}^4x\,\left[\left(\hat{\Phi}_i+\mathbf{v}_i\right)^T\frac{\delta}{\delta H_i}-Y_i^T\hat{\Psi}_i\right]\Gamma_{\mathrm{cl}}
\tag{E.3}
$$

  and similar for $(\hat{\Phi}_i+\hat{\mathbf{v}}_i)\to(\Phi_i'+\mathbf{v}_i')$.

- Field monomials including spurion fields: only combinations of the following invariant counterterms obey the requirement that A_1 appears only in the combination $A_1\epsilon_\alpha$.

  - Counterterms related to the soft-breaking parameters c_{HH}, $c_{V'}$, c_V, c_{HLR}, c_{HQD}, c_{HQU}:

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,[A_1[H_1^{{\rm eff}\,T}(\mathrm{i}\sigma_2)H_2^{\rm eff}], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,[A_1\lambda'^\alpha\lambda'_\alpha], \tag{E.4}$$

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,[A_1\mathrm{tr}(\lambda^\alpha\lambda_\alpha)], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,[A_1H_1^{{\rm eff}\,T}(i\sigma_2)LR], \tag{E.5}$$

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,[A_1H_1^{{\rm eff}\,T}(i\sigma_2)QD], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,[A_1H_2^{{\rm eff}\,T}(i\sigma_2)QU]. \tag{E.6}$$

  - Counterterms causing field and BRS redefinitions:

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_l^{T\alpha}\epsilon_\alpha L\right], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_r^{\alpha}\epsilon_\alpha R\right], \tag{E.7}$$

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_q^{T\alpha}\epsilon_\alpha Q\right], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_u^{\alpha}\epsilon_\alpha U\right], \tag{E.8}$$

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_d^{\alpha}\epsilon_\alpha D\right], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_i^{T\alpha}\epsilon_\alpha H_i^{\rm eff}\right], \tag{E.9}$$

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_i^{T\alpha}\epsilon_\alpha(\hat\Phi_i+\hat{\mathbf{v}}_i)\right], \qquad \mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1y_i^{T\alpha}\epsilon_\alpha(\Phi'_i+\mathbf{v}'_i)\right]. \tag{E.10}$$

    These counterterms correspond to the renormalization of c_{H_i1}, c_{L1}, c_{R1}, c_{Q1}, c_{U1}, c_{D1} apart from the counterterms (E.10). The counterterms (E.10) are defined analogous to the counterterm (E.7) with $H_i^{\rm eff}$ replaced by the Φ fields, but do not correspond to parameters in the classical action. All of these counterterms (E.10) are of the type of the u_3 counterterms discussed in [19].

  - Additional unphysical counterterm:

$$\mathscr{S}_{\Gamma_{\rm cl}}\int \mathrm{d}^4x\,\left[A_1\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\mathrm{tr}(V_\mu\bar y_\lambda^{\dot\alpha})\right]. \tag{E.11}$$

    This counterterm corresponds to the v_3 counterterms discussed in [19]. An analogous counterterm involving the abelian gauge field V'_μ is not invariant since it violates the nilpotency condition $w_\mu^{\lambda'}\Gamma_{\rm ct,inv}=0$.

  - Counterterms corresponding to the soft-breaking parameters c_{H_i2}, c_{L2}, c_{R2}, c_{Q2},

c_{U2}, c_{D2}:

$$\mathscr{S}_{\Gamma_{\rm cl}} \int \mathrm{d}^4x \, \left\{ \bar{A}_1 \bar{H}_i^{\rm eff} \left[(A_2 + v_A) H_i^{\rm eff} - A_1 \sqrt{2} \epsilon^\alpha h_{i\alpha} \right] \right\}, \tag{E.12}$$

$$\mathscr{S}_{\Gamma_{\rm cl}} \int \mathrm{d}^4x \, \left\{ \bar{A}_1 \bar{L} \left[(A_2 + v_A) L - A_1 \sqrt{2} \epsilon^\alpha l_\alpha \right] \right\}, \tag{E.13}$$

$$\mathscr{S}_{\Gamma_{\rm cl}} \int \mathrm{d}^4x \, \left\{ \bar{A}_1 \bar{R} \left[(A_2 + v_A) R - A_1 \sqrt{2} \epsilon^\alpha r_\alpha \right] \right\}, \tag{E.14}$$

$$\mathscr{S}_{\Gamma_{\rm cl}} \int \mathrm{d}^4x \, \left\{ \bar{A}_1 \bar{Q} \left[(A_2 + v_A) Q - A_1 \sqrt{2} \epsilon^\alpha q_\alpha \right] \right\}, \tag{E.15}$$

$$\mathscr{S}_{\Gamma_{\rm cl}} \int \mathrm{d}^4x \, \left\{ \bar{A}_1 \bar{U} \left[(A_2 + v_A) U - A_1 \sqrt{2} \epsilon^\alpha u_\alpha \right] \right\}, \tag{E.16}$$

$$\mathscr{S}_{\Gamma_{\rm cl}} \int \mathrm{d}^4x \, \left\{ \bar{A}_1 \bar{D} \left[(A_2 + v_A) D - A_1 \sqrt{2} \epsilon^\alpha d_\alpha \right] \right\}. \tag{E.17}$$

An explicit term-by-term analysis shows that no further invariant counterterm exists that involves at least one dynamical field. The remaining counterterms consist purely of external fields and are therefore unphysical.

F Characterization of the Goldstone Modes

In the physical part of the classical action, only the combination $H_i + x_i \hat{\Phi}_i + x_i' \Phi_i'$ occurs. In this section we investigate the relation between 2-point functions of H_i, $\hat{\Phi}_i$ and Φ_i' in higher orders and demonstrate how the definition of Goldstone fields has to be adjusted.

Classically, the Goldstone modes correspond to the flat directions of the potential and can be algebraically characterized as

$$\sum_{i=1}^{2} \left(-\mathrm{i}g \frac{\delta}{\delta H_i} T^a \vec{v}_i + \text{c.c.} \right) \sim \frac{\delta}{\delta G^a}. \tag{F.18}$$

This characterization implies $\Gamma_{G^a X}|_{p=0} = 0$ for all fields $\phi \neq B$ at the classical level.

On the quantum level, a characterization of the Goldstone modes can be derived from the Ward and ST identities.

The rigid Ward identity, differentiated w.r.t. a field ϕ ($\phi \neq B$), yields at $p = 0$

$$\sum_{a=1,2,3,'} \left(\delta_b \mathbf{v}_i^{aj} \Gamma_{\phi \Phi_i^{aj}} + \delta_b \bar{\mathbf{v}}_i^{aj} \Gamma_{\phi \bar{\Phi}_i^{aj}} \right) = 0, \tag{F.19}$$

where $\delta_\omega \mathbf{v}_i^{aj} = (\delta_\omega \Phi_i^{aj})|_{\Phi=0}$, $\delta_\omega \mathbf{v}_i'^{j} = (\delta_\omega \Phi_i'^{j})|_{\Phi'=0}$. Using the Φ fields defined in eq. (3.58) this results in

$$\Gamma_{\phi \mathscr{G}^b}|_{p=0} = 0, \tag{F.20}$$

i.e. the modes $\mathscr{G}^b$, whose definition (3.58) contains no higher-order contributions, correspond to flat directions of the scalar potential.

(F.19) does not characterize the dynamical Goldstone modes. To obtain such a characterization, it has to be supplemented with a relation of 2-point functions involving Φ fields and the corresponding ones involving the Higgs fields $H_{1,2}$. Such a relation can be derived from the ST identity.

Differentiating the ST identity w.r.t. Ψ^a $(a = 1, 2, 3, ')$ and an arbitrary physical field ϕ $(\phi \neq B)$ we obtain

$$\Gamma_{\Psi_i^{aj} Y_r^s} \Gamma_{\phi H_r^s} + \Gamma_{\Psi_i^{aj} \bar{Y}_r^s} \Gamma_{\phi \bar{H}_r^s} + \Gamma_{\phi \Phi_i^{aj}} = 0. \tag{F.21}$$

A similar identity can be derived for $\Gamma_{\phi \bar{\Phi}_i^{aj}}$.

Using these relations of the 2-point functions in (F.19) yields at $p = 0$

$$\begin{aligned} 0 \;=\; & \sum_{a=1,2,3,'} \Big[\delta_b \mathbf{v}_i^{aj} \left(\Gamma_{\Psi_i^{aj} Y_r^s} \Gamma_{\phi H_r^s} + \Gamma_{\Psi_i^{aj} \bar{Y}_r^s} \Gamma_{\phi \bar{H}_r^s} \right) \\ & + \delta_b \bar{\mathbf{v}}_i^{aj} \left(\Gamma_{\bar{\Psi}_i^{aj} Y_r^s} \Gamma_{\phi H_r^s} + \Gamma_{\bar{\Psi}_i^{aj} \bar{Y}_r^s} \Gamma_{\phi \bar{H}_r^s} \right) \Big] . \end{aligned} \tag{F.22}$$

This identity characterizes the Goldstone modes as

$$\begin{aligned} \frac{\delta}{\delta G^b} \;\sim\; & \sum_{a=1,2,3,'} \Bigg[\left(\delta_b \mathbf{v}_i^{aj} \Gamma_{\Psi_i^{aj} Y_r^s} + \delta_b \bar{\mathbf{v}}_i^{aj} \Gamma_{\bar{\Psi}_i^{aj} Y_r^s} \right) \Big|_{p=0} \frac{\delta}{\delta H_r^s} \\ & + \left(\delta_b \mathbf{v}_i^{aj} \Gamma_{\Psi_i^{aj} \bar{Y}_r^s} + \delta_b \bar{\mathbf{v}}_i^{aj} \Gamma_{\bar{\Psi}_i^{aj} \bar{Y}_r^s} \right) \Big|_{p=0} \frac{\delta}{\delta \bar{H}_r^s} \Bigg] . \end{aligned} \tag{F.23}$$

With this characterization, we obtain

$$\Gamma_{G^b \phi} |_{p=0} = 0, \tag{F.24}$$

so that the G^b indeed correspond to the flat directions of the potential. For $\phi = B$ (F.21) is replaced by

$$\Gamma_{\Psi_i^{aj} Y_r^s} \Gamma_{B^b H_r^s} + \Gamma_{\Psi_i^{aj} \bar{Y}_r^s} \Gamma_{B^b \bar{H}_r^s} + \Gamma_{B^b \Phi_i^{aj}} + \Gamma_{\Psi_i^{aj} \bar{c}^b} = 0, \tag{F.25}$$

which results in

$$\Gamma_{B^b G^a} |_{p=0} \neq 0. \tag{F.26}$$

In section 4.3 we have seen that there are exactly three linearly independent modes $G^b = G^0, G^{\pm}$ which satisfy (F.24). (F.23) supplements this result with an explicit formula for the Goldstone modes and correspondingly with the first column of the matrices R_{A^0}, $R_{H^{\pm}}$.

However, in contrast to the characterization of the Φ fields, (F.23) contains contributions of higher orders.

Finally, we evaluate the characterization (F.23) at the classical level. In lowest-order, we have

$$\Gamma_{\Psi_i^{aj} Y_r^s} = -2x_i \delta_{ir} T^{a\,s}{}_j, \qquad \Gamma_{\Psi_i'^j Y_r^s} = -x_i' \delta_{ir} \delta^{js} \tag{F.27}$$

and thus obtain

$$\frac{\delta}{\delta G^b} \sim \delta_b \left[2x_i (T^a \mathbf{v}_i^a)^s + x_i' \mathbf{v}_i'^s \right] \frac{\delta}{\delta H_i^s} + \text{c.c.} \tag{F.28}$$

Owing to the decomposition (2.52) for $\vec{v}_i$, this is identical to equation (F.18).

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