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Joint Angular Distribution of $B_s \to J/\psi\phi$ with Subsequent $\phi \to \rho\pi$ and $J/\psi \to \ell^+\ell^-$ Decays

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The covariant density matrix formalism is used to obtain the joint angular distribution of the decay $B_s \to J/\psi\phi(1020)$ with subsequent $\phi \to \rho\pi$ and $J/\psi \to \ell^+\ell^-$ decays. The result is shown algebraically to be a special case of our previous work on the decay distribution of $B_d \to J/\psi K_1(1270)$ with $K_1 \to K\pi$ and $J/\psi \to \ell^+\ell^-$.

In the upgraded Tevatron Run II experiment, substantial numbers of $B_s \to J/\psi\phi(1020)$ events are expected in the future. Previous experimental works have been reported by the CDF Collaboration [1], and is well known that the decay process is described by three Lorentz invariant terms, *i.e.*, two CP–even terms and one CP–odd term. The joint angular distribution of $B_s \to J/\psi\phi$ with subsequent $\phi \to K^+K^-$ and $J/\psi \to \ell^+\ell^-$ decays can be described [2] in the same way as that of $B_d \to J/\psi K^*$ with subsequent $K^* \to$

$K\pi$ and $J/\psi \to \ell^+\ell^-$ decays [3], and a recent experimental investigation of the latter case has been reported by the Barbar [4] and the Belle [5] groups. Therefore, the joint angular distribution of $B_s \to J/\psi\phi$ will be determined with high precision in the future.

In the $B_s \to J/\psi\phi$ case, one expects $\phi \to \rho\pi$ and $\phi \to \pi\pi\pi$ in addition to the dominant $\phi \to K^+K^-$ decay. It will be difficult, if not impossible, to identify the $\phi \to \rho\pi$ events experimentally at Tevatron even though several hundred events are produced in the future. However, there is some possibility to construct a B_s factory in the future where the $\phi \to \rho\pi$ decay can be investigated more cleanly, and that would give additional information on the $B_s \to J/\psi\phi$ process.

Recently, the covariant density matrix formalism was used to obtain the joint angular distributions of $B_d \to J/\psi K^*(890)$, as well as $B_d \to J/\psi K_1(1270)$, followed by their subsequent decays [6]. The purpose of this brief report is to use this method to obtain the joint angular distribution of $B_s \to J/\psi\phi$ followed by $\phi \to \rho\pi$ and $J/\psi \to \ell^+\ell^-$ in the transversity basis [2, 3]. Since $B_d \to J/\psi K^*(890)$ and $B_s \to J/\psi\phi(1020)$ can be treated in the same way for $K^* \to K\pi$ and $\phi \to K^+K^-$, most of the results in Ref. 6 can be used to obtain the density matrix of J/ψ after the $\phi \to \rho\pi$ decay in the process $B_s \to J/\psi\phi(1020)$.

As shown in Ref. 6, the angular distribution of the decay $J/\psi \to \ell^+\ell^-$ is described in terms of the density matrix $\rho_{\lambda\lambda'}$ in the transversity basis, defined as in Fig. 1 in the J/ψ rest frame, as follows:

$$\begin{aligned} d\Gamma \;\sim\; \Big[\; & 1 + \rho_{00} + (1 - 3\rho_{00})\sin^2\theta_{tr}\cos^2\phi_{tr} \\ & + 2\mathrm{Re}(\rho_{1\text{-}1})(\sin^2\theta_{tr}\sin^2\phi_{tr} - \cos^2\theta_{tr}) \\ & - 2\mathrm{Im}(\rho_{1\text{-}1})\sin 2\theta_{tr}\sin\phi_{tr} \\ & + \sqrt{2}\mathrm{Re}(\rho_{10} - \rho_{\text{-}10})\sin^2\theta_{tr}\sin 2\phi_{tr} \\ & - \sqrt{2}\mathrm{Im}(\rho_{10} + \rho_{\text{-}10})\sin 2\theta_{tr}\cos\phi_{tr} \;\Big], \end{aligned} \tag{1}$$

where θ_{tr} and ϕ_{tr} are the polar and azimuthal angles of the outgoing lepton ℓ^+ in the transversity basis as defined in Refs. 2 and 3, where the unit vectors $(\hat{n}_3, \hat{n}_1, \hat{n}_2)$ are along the $(\hat{x}, \hat{y}, \hat{z})$ axis.

The explicit values of the density matrix elements $\rho_{\lambda\lambda'}$ are calculated from the amplitude of the J/ψ production process. The matrix element of the $B_s \to J/\psi\phi$ decay is given in terms of three independent Lorentz scalars,

A, B and C, as [7, 8]

$$\mathcal{M} = A\epsilon_1^*\cdot\epsilon_2^* + \frac{B}{m_1 m_2}\epsilon_1^*\cdot k\,\epsilon_2^*\cdot q + \frac{iC}{m_1 m_2}\epsilon^{\mu\nu\sigma\tau}\epsilon_{1\mu}^*\epsilon_{2\nu}^* q_\sigma k_\tau, \tag{2}$$

where $\epsilon_i (i = 1, 2)$ are the polarization vectors of J/ψ and ϕ, respectively, and m_1, m_2 and q^μ, k^μ are their masses and momenta. Instead of scalar amplitudes A, B and C it is more convenient to introduce three transversity amplitudes $A_0, A_\parallel$ and $A_\perp$ to specify the CP property of the decay process. The latter are related to the former by [2]

$$\begin{aligned} A_0 &= -xA - (x^2-1)B, \\ A_\parallel &= \sqrt{2}\,A, \\ A_\perp &= \sqrt{2(x^2-1)}\,C, \end{aligned} \tag{3}$$

where x is defined as

$$x = \frac{k\cdot q}{m_1 m_2} = \frac{m_B^2 - m_1^2 - m_2^2}{2m_1 m_2}. \tag{4}$$

When the decay of the B_s into J/ψ and ϕ is specified by the polarization vectors $\epsilon_1(q, \lambda_1)$ and $\epsilon_2(k, \lambda_2)$, respectively, the joint density matrix elements of J/ψ and ϕ can be obtained from the square of Eq. (2) and by using the following relation [9] for each $\epsilon_i (i = 1, 2)$:

$$\epsilon^\mu(q,\lambda)\epsilon^{\nu *}(q,\lambda') = \frac{1}{3}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_1^2}\right)\delta_{\lambda'\lambda} - \frac{i}{2m_1}\epsilon^{\mu\nu\sigma\tau} q_\sigma n_\tau^i (S^i)_{\lambda'\lambda} - \frac{1}{2} n_i^\mu n_j^\nu (S^{ij})_{\lambda'\lambda}, \tag{5}$$

where the (S^i) are the standard matrix representations of the spin-1 angular momentum operators and (S^{ij}) are traceless and symmetric matrices.

Then, one obtains

$$\begin{aligned} \rho_{\lambda_1\lambda_1',\lambda_2\lambda_2'} \;\sim\; & \frac{1}{9}(|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2)\delta_{\lambda_1\lambda_1',\lambda_2\lambda_2'} \\ & - \frac{2}{3\sqrt{\Delta}}\mathrm{Re}(A_\parallel^* A_\perp)\{m_1 n_1^i\cdot k\,(S^i)_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} + m_2 n_2^k\cdot q\,\delta_{\lambda_1\lambda_1'}(S^k)_{\lambda_2\lambda_2'}\} \\ & + \frac{1}{\Delta}\Big\{\;\; (|A_\parallel|^2 + |A_\perp|^2) m_1 m_2\; n_1^i\cdot k n_2^k\cdot q \end{aligned}$$

$$
\begin{aligned}
&+\sqrt{\frac{\Delta}{2}}\mathrm{Im}(A_0^* A_\perp)\langle qkn_1^i n_2^k\rangle \\
&-\frac{\sqrt{2}}{4}\Delta\mathrm{Re}(A_0^* A_\parallel)(n_1^i{\cdot}n_2^k-\frac{4q{\cdot}k}{\Delta}n_1^i{\cdot}kn_2^k{\cdot}q)\Big\}(S^i)_{\lambda_1\lambda_1'}(S^k)_{\lambda_2\lambda_2'} \\
&-\frac{1}{3\Delta}(2|A_0|^2-|A_\parallel|^2-|A_\perp|^2)\Big\{\ m_1^2\ n_1^i{\cdot}k\ n_1^j{\cdot}k\ (S^{ij})_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} \\
&\qquad\qquad +m_2^2\ n_2^k{\cdot}q\ n_2^l{\cdot}q\ \delta_{\lambda_1\lambda_1'}(S^{kl})_{\lambda_2\lambda_2'}\Big\} \\
&-\frac{1}{\Delta}\Big\{\ \sqrt{2}\mathrm{Im}(A_0^* A_\parallel)\langle qkn_1^i n_2^k\rangle \\
&\quad+\frac{2}{\sqrt{\Delta}}\mathrm{Re}(A_\parallel^* A_\perp)m_1m_2n_1^i{\cdot}kn_2^k{\cdot}q \\
&\quad-\sqrt{\frac{\Delta}{2}}\mathrm{Re}(A_0^* A_\perp)\Big(n_1^i{\cdot}n_2^k-\frac{4q{\cdot}k}{\Delta}n_1^i{\cdot}kn_2^k{\cdot}q\Big)\Big\} \\
&\quad\times\{m_2n_2^l{\cdot}q\ (S^i)_{\lambda_1\lambda_1'}(S^{kl})_{\lambda_2\lambda_2'}+m_1n_1^j{\cdot}q\ (S^{ij})_{\lambda_1\lambda_1'}(S^k)_{\lambda_2\lambda_2'}\} \\
&+\Big\{\frac{4}{\Delta^2}|A^0|^2m_1^2m_2^2n_1^i{\cdot}kn_1^j{\cdot}kn_2^k{\cdot}qn_2^l{\cdot}q \\
&\quad+\frac{1}{8}|A_\parallel|^2(n_1^i{\cdot}n_2^k-\frac{4q{\cdot}k}{\Delta}n_1^i{\cdot}kn_2^k{\cdot}q)(n_1^j{\cdot}n_2^l-\frac{4q{\cdot}k}{\Delta}n_1^j{\cdot}kn_2^l{\cdot}q) \\
&\quad+\frac{1}{2\Delta}|A_\perp|^2\langle qkn_1^i n_2^k\rangle\langle qkn_1^j n_2^l\rangle \\
&\quad-\frac{\sqrt{2}}{2\Delta}\mathrm{Re}(A_0^* A_\parallel)m_1m_2n_1^i{\cdot}kn_2^k{\cdot}q(n_1^j{\cdot}n_2^l-\frac{4q{\cdot}k}{\Delta}n_1^j{\cdot}kn_2^l{\cdot}q) \\
&\quad+\frac{2\sqrt{2}}{\Delta\sqrt{\Delta}}\mathrm{Im}(A_0^* A_\perp)m_1m_2n_1^i{\cdot}kn_2^k{\cdot}q\langle qkn_1^j n_2^l\rangle \\
&\quad-\frac{1}{2\sqrt{\Delta}}\mathrm{Im}(A_\parallel^* A_\perp)(n_1^i{\cdot}n_2^k-\frac{4q{\cdot}k}{\Delta}n_1^i{\cdot}kn_2^k{\cdot}q)\langle qkn_1^j n_2^l\rangle\Big\} \\
&\quad\times(S^{ij})_{\lambda_1\lambda_1'}(S^{kl})_{\lambda_2\lambda_2'}. \qquad (6)
\end{aligned}
$$

Here, the density matrix is multiplied by $(|A_0|^2+|A_\parallel|^2+|A_\perp|^2)$ for convenience, and Δ and $\langle qkn_1^j n_2^l\rangle$ are defined as

$$
\begin{aligned}
\Delta &= (m_B^4+m_1^4+m_2^4-2m_B^2m_1^2-2m_B^2m_2^2-2m_1^2m_2^2) \\
&= 4[(q.k)^2-m_1^2m_2^2]=4m_1^2m_2^2(x^2-1), \qquad (7) \\
\langle qkn_1^j n_2^l\rangle &= \epsilon^{\mu\nu\sigma\tau}q_\mu k_\nu(n_\sigma^j)_1(n_\tau^l)_2. \qquad (8)
\end{aligned}
$$

The amplitude of the decay process $\phi(k,\lambda_2) \to \rho(\tilde{k}')\pi(\tilde{k})$ is described by

$$\mathcal{M}_2 = if\epsilon^{\mu\nu\lambda\tau}\epsilon_{2\mu}(k)\epsilon_{3\nu}{}^*(\tilde{k}')k_\lambda\tilde{k}_\tau, \tag{9}$$

where ϵ_3^μ and $\tilde{k}'$ are the polarization vector and the momentum of ρ, respectively. After the polarization of ρ is summed over, one obtains

$$\rho^D_{\lambda_2'\lambda_2}(\phi) = \frac{1}{3}\delta_{\lambda_2'\lambda_2} + \frac{m_\phi^2}{\bar{\Delta}}\tilde{k}\cdot n_i\tilde{k}\cdot n_j(S^{ij})_{\lambda_2'\lambda_2}, \tag{10}$$

where $\bar{\Delta}$ is defined as

$$\bar{\Delta} = m_\phi^4 + m_\rho^4 + m_\pi^4 - 2m_\phi^2 - 2m_\rho^2 - 2m_\pi^2. \tag{11}$$

The result of Eq. (10) can be obtained from Eq. (6). Then, the density matrix of J/ψ can be obtained from the product

$$\rho_{\lambda_1\lambda_1',\lambda_2\lambda_2'}\rho^D_{\lambda_2'\lambda_2}(\phi). \tag{12}$$

The joint angular distribution for the decay process $B_s \to J/\psi\phi$ ($J/\psi \to \ell^+\ell^-, \phi \to \rho\pi$) in the transversity basis becomes

$$\begin{aligned}
\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_{tr} d\phi_{tr}} &= \frac{9}{64\pi}\Big[\, 2|A_0|^2\Big\{\sin^2\theta_1(1-\sin^2\theta_{tr}\cos^2\phi_{tr})\Big\} \\
&+ |A_\parallel|^2\Big\{\cos^2\theta_1(1-\sin^2\theta_{tr}\sin^2\phi_{tr}) + \sin^2\theta_{tr}\Big\} \\
&+ |A_\perp|^2\Big\{1-\sin^2\theta_{tr}\sin^2\phi_{tr} + \cos^2\theta_1\sin^2\theta_{tr}\Big\} \\
&- \mathrm{Im}\Big(A_\parallel^* A_\perp\Big)\sin^2\theta_1\sin 2\theta_{tr}\sin\phi_{tr} \\
&+ \frac{1}{\sqrt{2}}\mathrm{Re}\Big(A_0^* A_\parallel\Big)\sin 2\theta_1\sin^2\theta_{tr}\sin 2\phi_{tr} \\
&- \frac{1}{\sqrt{2}}\mathrm{Im}\Big(A_0^* A_\perp\Big)\sin 2\theta_1\sin 2\theta_{tr}\cos\phi_{tr}\,\Big],
\end{aligned} \tag{13}$$

where θ_1 is the angle between the ϕ direction and the π direction in the ϕ rest frame, and the three amplitudes are normalized as $|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1$. It is noted that in Ref. 6 we have considered the process $B \to J/\psi K_1(1270)$ and $J/\psi \to \ell^+\ell^-$, $K_1 \to K\pi$ and Eq. (13) is a special case obtained by putting $\alpha = -1$ in Eq. (28) or $\xi = 3/2$ in Eq. (29) of Ref. 6. This method can be used to discuss the polarization effects in W boson productions at e^+e^- linear collider [10].

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References

[1] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **75**, 1451 (1995); Phys. Rev. D **53**, 3496 (1996).

[2] I. Dunietz, H. J. Lipkin, H. Quinn, A. Snider, and W. Toki, Phys. Rev. D **43**, 2193 (1991).

[3] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B **369**, 144 (1996).

[4] Babar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 241801 (2001).

[5] Belle Collaboration, K. Abe *et al.*, BELLE-CONF-0105 (2001).

[6] G. Kramer, H. S. Song, and Chaehyun Yu, Phys. Rev. D **65**, 091501(R) (2002); *ibid.* **66**, 095901(E) (2002).

[7] G. Kramer and W. F. Palmer, Phys. Rev. D **45**, 193 (1992).

[8] G. Valencia, Phys. Rev. D **39**, 3339 (1989).

[9] S. Y. Choi, Taeyeon Lee, and H. S. Song, Phys. Rev. D **40**, 2477 (1989).

[10] D. W. Jung, K. Y. Lee, H. S. Song, and C. Yu, J. Korean Phys. Soc. **36**, 258 (2000); Czech. J. Phys. **51**, A173 (2001).

Figure Captions

Fig. 1. Definitions of the angles in the transversity basis. θ_{tr} and ϕ_{tr} are the polar and the azimuthal angles of ℓ^+ in the J/ψ rest frame. θ_1 is the angle between the ϕ direction and the π direction in the ϕ rest frame.

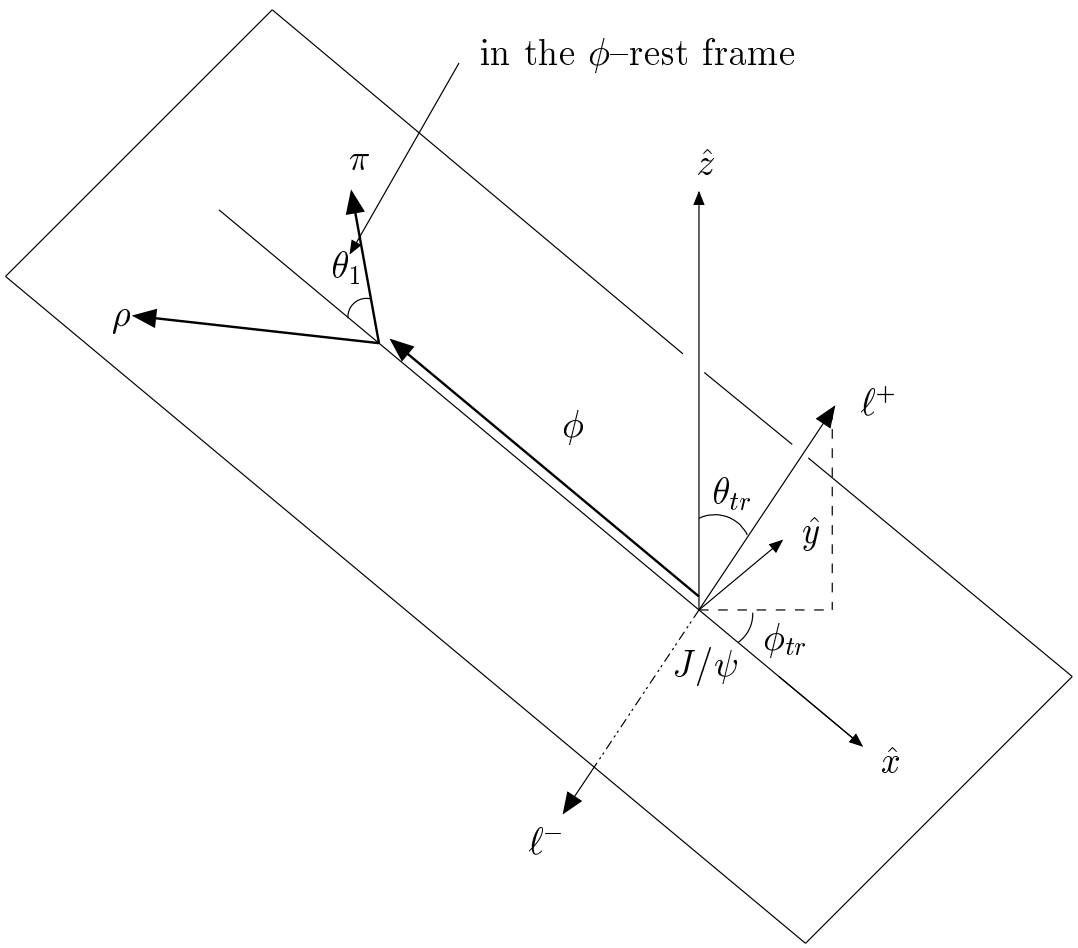
in the ϕ–rest frame
π
$\hat{z}$
θ_1
ρ
ϕ
ℓ^+
θ_{tr}
$\hat{y}$
ϕ_{tr}
J/ψ
$\hat{x}$
ℓ^-

Figure 1: