HU-EP-02/36
DESY 02-125

arXiv:hep-lat/0209025 v1 3 Sep 2002

HU-EP-02/36
DESY 02-125
September 2002


# Running quark mass in two flavor QCD *

F. Knechtli[a], M. Della Morte[b], J. Rolf[a], R. Sommer[b], I. Wetzorke[c], U. Wolff[a] (ALPHA collaboration)

[a]Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstr. 110, 10115 Berlin, Germany

[b]DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany

[c]NIC/DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany

We present first results for the step scaling function $\sigma_{\rm P}$ of the renormalization factor $Z_{\rm P}$ of the pseudoscalar density. The simulations are performed within the framework of the Schrödinger functional with two flavors of O($a$) improved Wilson fermions. The knowledge of $\sigma_{\rm P}$ is required to compute the renormalization group invariant quark masses. We also study the performance of a variant of the HMC algorithm using two pseudofermion fields.

## 1. THE RGI MASS

Lattice QCD is a theory which has as free parameters the bare gauge coupling $g_0$ and the bare current quark masses $m_i$, $i = u, d, s, \ldots$. The hadronic scales like $F_\pi$ or $m_{\rm K}$ are connected to the perturbative high energy regime of QCD via the running of renormalized couplings

$$\overline{g}^2(\mu) = Z_g g_0^2\,, \qquad \overline{m}_i(\mu) = Z_m m_i\,, \tag{1}$$

where $\mu$ is the renormalization scale. At high energies the $\Lambda$ parameter and the renormalization group invariant (RGI) quark masses $M_i$ can be determined.

In the following we assume that the renormalization conditions are independent of the quark masses themselves. The running of the renormalized couplings is described by the renormalization group equations (RGE)

$$\mu\frac{{\rm d}\overline{g}}{{\rm d}\mu} = \beta(\overline{g})\,, \qquad \mu\frac{{\rm d}\overline{m}_i}{{\rm d}\mu} = \tau(\overline{g})\overline{m}_i\,. \tag{2}$$

The $\beta$ and $\tau$ functions are non-perturbatively defined if this is true for $\overline{g}$ and $\overline{m}_i$. Their perturbative expansions are

$$\beta(\overline{g}) \underset{\overline{g}\to 0}{\sim} -\overline{g}^3\{b_0 + b_1\overline{g}^2 + b_2\overline{g}^4 + \ldots\}\,, \tag{3}$$

$$\tau(\overline{g}) \underset{\overline{g}\to 0}{\sim} -\overline{g}^2\{d_0 + d_1\overline{g}^2 + \ldots\}\,. \tag{4}$$

RGI quantities $P$ do not depend on the renormalization scale $\mu$

$$\mu\frac{{\rm d}}{{\rm d}\mu}P(\mu, \overline{g}, \overline{m}_i) \;=\; 0\,. \tag{5}$$

Examples are the $\Lambda$ parameter and the RGI quark masses

$$\Lambda \;=\; \mu(b_0\overline{g}^2)^{-b_1/2b_0^2}\exp\left\{-\frac{1}{2b_0\overline{g}^2}\right\} \times \exp\left\{-\int_0^{\overline{g}}{\rm d}x\left[\frac{1}{\beta(x)} + \frac{1}{b_0x^3} - \frac{b_1}{b_0^2x}\right]\right\} \tag{6}$$

$$M_i \;=\; \overline{m}_i(2b_0\overline{g}^2)^{-d_0/2b_0} \times \exp\left\{-\int_0^{\overline{g}}{\rm d}x\left[\frac{\tau(x)}{\beta(x)} - \frac{d_0}{b_0x}\right]\right\}\,, \tag{7}$$

where $\overline{g} = \overline{g}(\mu)$ and $\overline{m}_i = \overline{m}_i(\mu)$. These RGI parameters of QCD are defined beyond perturbation theory and their connections between different renormalization schemes can be given in a simple and exact way. For these reasons $\Lambda$ and $M_i$ should be taken as the fundamental parameters of QCD.

In this talk we present first results on the non-perturbative computation of the running of $\overline{m}(\mu)$ in the Schrödinger Functional (SF) renormalization scheme with two dynamical flavors of $O(a)$

*Talk presented by F. Knechtli under the original title "Towards the computation of the strange quark mass with dynamical quarks". Work supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2000-00145, Hadrons/Lattice QCD.

improved Wilson fermions. The strategy of our computation closely follows ref. [1,2]. We will give as a result an approximation to the flavor independent ratio $M/\overline{m}(\mu)$ as a function of $\mu/\Lambda$.

## 2. THE RUNNING MASS

In continuum QCD a renormalized mass in the SF scheme is defined through the PCAC relation which involves the renormalized axial current $(A_{\rm R})_\mu(x)$ and the renormalized pseudoscalar density $P_{\rm R}(x)$

$$\partial_\mu (A_{\rm R})_\mu = (\overline{m}_i + \overline{m}_j) P_{\rm R} \quad (8)$$

$$(A_{\rm R})_\mu(x) = Z_{\rm A} \overline{\psi}_i(x) \gamma_\mu \gamma_5 \psi_j(x) \quad (9)$$

$$P_{\rm R}(x) = Z_{\rm P} \overline{\psi}_i(x) \gamma_5 \psi_j(x)\,. \quad (10)$$

The renormalization constant $Z_{\rm A}$ is fixed by Ward identities. The renormalization constant $Z_{\rm P}$ is defined as

$$Z_{\rm P}(L) = \frac{\sqrt{3 f_1}}{f_{\rm P}(L/2)} \text{ at } m_i = 0\,,\ i = u, \ldots\,. \quad (11)$$

For notation and details of the calculation we refer to [1,3]. The correlation functions $f_1$ and $f_P$ are computed at zero bare current quark masses $m_i$. The renormalization scale in the SF scheme is identified with the inverse spatial box size $\mu = 1/L$. Keeping the physical size $L$ constant can be realized by keeping the renormalized coupling $\overline{g}(L)$ fixed.

A rigorous definition of the renormalized mass can be given in the lattice regularization of QCD by

$$\overline{m}_i(\mu) = \lim_{a\to 0} \frac{Z_{\rm A}(g_0)}{Z_{\rm P}(g_0, L/a)} m_i(g_0)\Big|_{u=\overline{g}^2(L)}\,. \quad (12)$$

The running of the mass can be described by the flavor independent step scaling function $\sigma_{\rm P}(u)$

$$\frac{\overline{m}(\mu)}{\overline{m}(\mu/2)} = \lim_{a\to 0} \frac{Z_{\rm P}(g_0, 2L/a)}{Z_{\rm P}(g_0, L/a)}\Big|_{u=\overline{g}^2(L)} = \sigma_{\rm P}(u)\,. \quad (13)$$

Together with the step scaling function for the gauge coupling $\sigma(u) = \overline{g}^2(2L)$ [4] we can solve a system of coupled recursions: $k = 0, 1, 2, \ldots$

$$\begin{cases} u_0 = \overline{g}^2(L_{\rm max}) = 3.3000 \\ \sigma(u_{k+1}) = u_k \end{cases} \quad (14)$$

$$\Rightarrow u_k = \overline{g}^2(2^{-k} L_{\rm max})$$

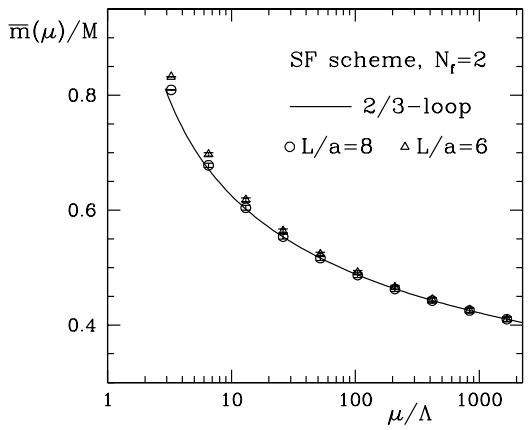


Figure 1. The running of the renormalized quark mass.

$$\begin{cases} v_0 = 1 \\ v_{k+1} = \dfrac{v_k}{\sigma_{\rm P}(u_k)} \end{cases} \quad (15)$$

$$\Rightarrow v_k = \frac{\overline{m}(\mu_{\rm min})}{\overline{m}(2^k \mu_{\rm min})} \text{ at } \mu_{\rm min} = \frac{1}{2L_{\rm max}}\,.$$

The largest coupling $u_0$ defines the reference scale $L_{\rm max}$. At high energies contact with perturbation theory can safely be made and the RGI parameters are determined from eq. (6) and eq. (7) using the perturbative 2–loop $\tau$ function and 3–loop $\beta$ function.

We computed the lattice step scaling function $\Sigma_{\rm P}(u, a/L)$ for 6 couplings in the range $u = 3.33 \ldots 0.98$ on $L/a = 6$ and $L/a = 8$ (requiring also $2L/a$) lattices. We interpolate $\Sigma_{\rm P}$ by the Ansatz

$$\Sigma_{\rm P}(u, a/L) = 1 - \ln(2) d_0 u + p_2 u^2 + p_3 u^3 \quad (16)$$

with fitted parameters $p_2$ and $p_3$. Using it in the recursion eq. (15) we obtain the following approximations for

$$\frac{M}{\overline{m}(\mu_{\rm min})} : \begin{cases} 1.202(13) & L/a = 6 \\ 1.236(15) & L/a = 8 \end{cases} \quad (17)$$

(contact with perturbation theory is made at the scale $2^{-8} L_{\rm max}$). Combining with recursion eq. (14), from which we get in the continuum limit $\ln(\Lambda L_{\rm max}) = -1.85(13)$ [4], we plot in Fig. 1 the

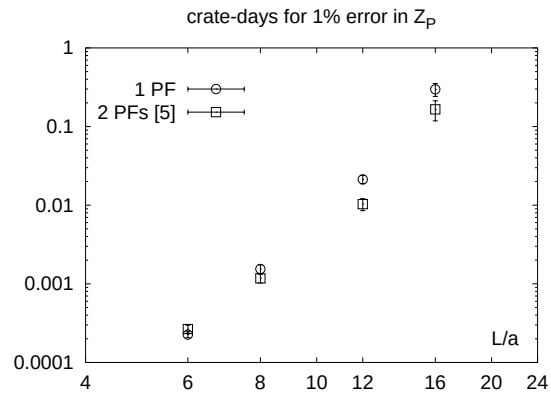


Figure 2. Performance of the HMC algorithm using one or two psedofermion fields (PFs).

points

$$\frac{\overline{m}(\mu_k)}{M} \quad = \quad \frac{1}{v_k}\frac{\overline{m}(\mu_{\rm min})}{M} \qquad (18)$$

as a function of $\mu_k = 2^k \mu_{\rm min}$, together with the pertubative curve using the 2–loop $\tau$ function and the 3–loop $\beta$ function. The errors of the points come from the statistical errors in the coefficients $v_k$. The overall uncertainties in the quantities $\Lambda L_{\rm max}$ and $M/\overline{m}(\mu_{\rm min})$ are not shown. The points show a dependence on the lattice spacing but for an extrapolation to the continuum limit we need additional simulations on finer lattices.

## 3. ALGORITHM STUDY

In our simulations we compared the computational cost of the HMC algorithm for the standard action and for an action which uses two pseudofermion fields (PFs) [5]. We denote by $\hat{Q}$ the even–odd preconditioned hermitian Dirac operator for $O(a)$ improved Wilson fermions. The effect of $\det(\hat{Q}^2)$ is taken into account by two PFs $\phi_1$ and $\phi_2$ with actions

$$S_{\rm F_1} \quad = \quad \phi_1^\dagger \left[ (\hat{Q}^2 + \rho^2)^{-1} \right] \phi_1 \,, \qquad (19)$$

$$S_{\rm F_2} \quad = \quad \phi_2^\dagger \left[ \rho^{-2} + \hat{Q}^{-2} \right] \phi_2 \,. \qquad (20)$$

This splitting of the action is valid for any real parameter $\rho$. We take

$$\rho \quad = \quad \left( \lambda_{\rm min}(\hat{Q}^2) \lambda_{\rm max}(\hat{Q}^2) \right)^{1/4} \,, \qquad (21)$$

which minimizes the sum of the condition numbers in eq. (19) and eq. (20). This is expected to reduce the fermionic force allowing for larger step sizes in molecular dynamics.

The implementation of the Hybrid Monte Carlo (HMC) algorithm to simulate the system described by eq. (19) and eq. (20) is straightforward. Simulations with the same physical parameters show that the step size of molecular dynamics with two PFs can be doubled with respect to the standard case with one PF by keeping the same acceptance (optimal around 80%). In Fig. 2 we show the performance of the HMC algorithm on the APEmille machine, 1 crate of the machine consists of 128 nodes and sustains 68 GFlops peak. The computational cost to achieve 1% error in $Z_{\rm P}$ at constant value of $\overline{g}^2 \approx 2.48$ is almost a factor 2 lower for two PFs than for one PF on lattices $L/a = 12$ and $L/a = 16$. This gain in performance is important in view of the next simulations on lattices $L/a = 12$ and $2L/a = 24$ needed to obtain $\sigma_{\rm P}(u)$ in the continuum limit.

**Acknowledgement.** We thank NIC/DESY Zeuthen for allocating computer time on the APEmille machine indispensable to this project and the APE group for their support.

## REFERENCES

1. S. Capitani et al., Nucl. Phys. B544, (1999) 669.
2. J. Garden et al., Nucl. Phys. B571, (2000) 237; J. Rolf and S. Sint, Nucl. Phys. Proc. Suppl. 106, (2002) 239.
3. S. Sint and P. Weisz, Nucl. Phys. B545, (1999) 529.
4. M. Della Morte et al. (ALPHA collaboration), these proceedings [hep-lat/0209023]; A. Bode et al., Phys. Lett. B515, (2001) 49.
5. M. Hasenbusch, Phys. Lett. B519, (2001) 177.