Revised Version

Universality and RG-improved gauge actions

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The reference energy scale r_0 is evaluated for RG-improved Iwasaki and DBW2 gauge actions ($N_f = 0$), at values of the deconfinement coupling β_c corresponding to $N_t = 3, 4, 6, 8$. The universality of $r_0 T_c$ between Iwasaki and Wilson action is confirmed; the scaling behaviour and the influence of the violation of positivity in the extraction of effective masses are investigated.

1. INTRODUCTION

In the last years the lattice comunity made a big effort in the improvement of both fermionic and gauge actions. Concerning the latter, the tree-level and one-loop improved action were introduced in the Symanzik framework [1]. Iwasaki [2] and the QCD-TAR0 collaboration [3] followed another approach based on the renormalization group (RG); the improvement is obtained estimating the renormalized trajectory, in [2] through perturbation theory, in [3] using a Schwinger-Dyson method.
The Iwasaki action has been used by the CP-PACS collaboration [4] for the most advanced computation of the light hadron spectrum; moreover, the RG-improved actions have been considered as candidates to be used in the next simulations on Ginsparg-Wilson/domain wall fermions, due to the suppression of small instantons and dislocations and a possible remedy of the problem of residual chiral symmetry breaking for domain wall fermions [5,6].
This motivates more investigations about their properties, in particular the universality between the Iwasaki and the standard plaquette action, the scaling properties and the potential problem in the evaluation of the physical observables due to the violation of physical positivity.

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2. IMPROVED GAUGE ACTIONS AND LOSS OF PHYSICAL POSITIVITY

We will consider improved gauge actions of the form

$$S_{\rm G} = \frac{\beta}{3}\Big(\sum_{x;\mu<\nu} (1-8c_1) P_{\mu\nu}^{1\times 1} + c_1 \sum_{x;\mu\neq\nu} P_{\mu\nu}^{1\times 2}\Big), \quad (1)$$

where $P_{\mu\nu}^{1\times 1}$ is the usual plaquette term and $P_{\mu\nu}^{1\times 2}$ contains all loops enclosing two coplanar plaquettes. The coefficient c_1 takes different values for various choices of the improved actions

$$c_1 = \begin{cases} -1/12 & \text{Symanzik, tree level impr.} \\ -0.331 & \text{Iwasaki, RG} \\ -1.4088 & \text{DBW2 (QCD-TAR0), RG} \end{cases}$$

In general, adding operators of dimension 6 to the standard Wilson action violates the physical positivity at finite lattice spacing.
A transfer matrix, $\mathbb{T}$, can be nevertheless defined, but complex eigenvalues may occur and there are contributions in the spectral decomposition of two-point functions with negative weight [7]. Near the continuum limit, positivity is only lost at energies of the order of the cutoff: if we restrict ourselves to the states with small energy relative to the cutoff the physical positivity is completely restored. Perturbative and free field calculations suggest there exists a $0 < \epsilon < 1$ such that independently of the cutoff all spectral values λ of $\mathbb{T}$ with $|\lambda| \geq \epsilon\Lambda$ are real and positive, where Λ is the largest eigenvalue of $\mathbb{T}$. These eigenvalues λ can be interpreted as energy values through

$$E = -\tfrac{1}{a}\ln(\lambda/\Lambda). \quad (2)$$

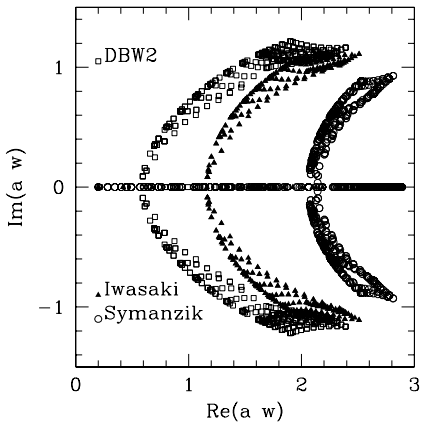

Figure 1. The poles of the propagator for different actions

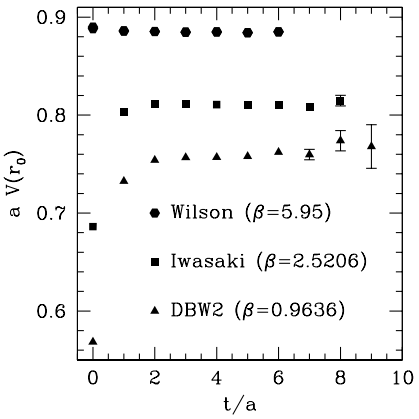

Figure 2. The effective potential at $r \sim r_0$ for different actions, from smeared Wilson loops at smearing level $\sim (r_0/a)^2$.

An estimate of ϵ can be obtained in weak coupling perturbation theory by determining the location of unphysical (but gauge independent) poles in the propagator. The general solution will be $w(k_1,k_2,k_3) = \mathfrak{Re}(w) + i\mathfrak{Im}(w)$, with $(k_1,k_2,k_3) \in$ Brillouin zone. In Fig. 1 the solutions for different improved actions are shown; for $\mathfrak{Re}(w) < E_{max}$ one has $\mathfrak{Im}(w) = 0$ for all solutions at a given momentum. The value $aE_{max} = -\ln(\epsilon)$ is related to the parameter c_1 by the relation:

$$aE_{max} = 2\mathrm{arcsinh}\left(\frac{1}{2\sqrt{-2c_1}}\right). \qquad (3)$$

One estimates that the contribution of the unphysical states in asymptotic decay of euclidean 2-point functions can then be neglected if

$$t/a \gg t_{min}/a = \frac{1}{aE_{max}} = \begin{cases} 0.485 & \text{Symanzik} \\ 0.860 & \text{Iwasaki} \\ 1.703 & \text{DBW2} \end{cases} \qquad (4)$$

This turns out to play a rôle also in MC simulations; Fig. 2 shows the effective potential at a fixed distance $r \sim r_0$ for the different actions. The APE smearing procedure [8] was used, and the smearing level considered in the plot is the one estimated to be optimal for the Wilson plaquette action [9]. Negative contributions at small t/a are strong and they can be a problem for the applicability of the variational method [10] to extract the mass spectrum. That procedure is well defined only for positive definite correlation functions, but this is verified only if $t \gg t_{min}$, where the statistical fluctuations can be large. Moreover, although one can observe a plateau in Fig. 2, it is clear that for the improved actions there is no unambiguous criterion for the choice of the smearing operators.

3. SCALING OF $T_c r_0$

The deconfinement transition occurs at the critical temperature

$$T_c = \frac{1}{N_t a(\beta_c)}. \qquad (5)$$

For known values of β_c at given N_t for different actions one can refer to [11–14]. Expressing the critical temperature in units of the string tension, the following results were obtained

$$T_c/\sqrt{\sigma} = \begin{cases} 0.630(5) & (N_t=\infty) & \text{Wilson[12]} \\ 0.650(5) & (N_t=\infty) & \text{Iwasaki[13]} \\ 0.6301(65) & (N_t=6) & \text{DBW2[14]} \end{cases} \qquad (6)$$

For the Iwasaki and the Wilson action a difference of the order 4σ in the continuum results is observed. It is unrealistic that the origin of the discrepancy is a violation of the universality; the most natural explanation is that the string tension is difficult to determine, above all at small lattice spacings. We decided to evaluate instead the Sommer scale r_0 [15] defined through the force

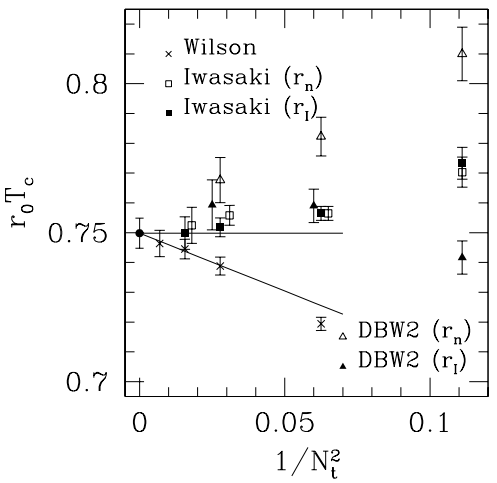

Figure 3. $T_c r_0$ for different actions. The x coordinates were slighly shifted for clarity reasons.

between a pair of static quarks, $F(r_0)r_0^2 = 1.65$, at β_c. For the Wilson action a parametrization formula for $5.7 \leq \beta \leq 6.57$ [9] which relates r_0/a to β can be used; for the Iwasaki and DBW2 action there is no available evaluation of r_0, so it was measured from Wilson loop correlation matrices, using

$$\frac{V(r)-V(r-a)}{a} = \begin{cases} F\left(r-\frac{a}{2}\right) & \text{naive definition} \\ F(r_I) & \text{tree-level improved,} \end{cases} \quad (7)$$

where r_I is defined through the scalar lattice propagator in 3 dimensions,

$$(4\pi r_I^2)^{-1} = [G(r,0,0) - G(r-a,0,0)]/a.$$

The operator corresponding to the best smearing for the Wilson action was considered; due to the negative contributions in the correlation functions and the previous discussion, we decided to avoid the variational method.

The results for $T_c r_0$, together with the continuum extrapolation, are shown in Fig. 3. For the Iwasaki action there is no appreciable difference between the results obtained with r_{naive} and r_I; the data show better scaling properties in comparison to the Wilson action and moreover the universality is confirmed. Also for the $DBW2$ action the scaling properties are improved, although only using r_I instead of r_{naive}. A constrained fit

$$T_c r_0 = T_c r_0|_{a=0} + s \cdot \left(\frac{a}{T_c}\right)^2, \text{with} \quad N_t \geq 6 \quad (8)$$

for Iwasaki and Wilson action yields the continuum result

$$T_c r_0 = 0.7498(50). \quad (9)$$

At $N_t = 6$ the Wilson action shows scaling violations for $r_0 T_c$ of about 1.5%, while they are 0.3% for the Iwasaki action.

4. CONCLUSIONS

The loss of physical positivity can be a serious problem for the extraction of the physical observables, in particular the variational technique is not easily applicable. For the Iwasaki and the Wilson action the universality of $T_c r_0$ is demonstrated. In general the lattice artefacts seem to be reduced for the Iwasaki action; it would be anyhow useful to have more investigations on the scaling behavior, for example evaluating glueball masses.

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