Revised Version

Monopoles near the Planck Scale and Unification

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Abstract

Considering our (3 + 1)-dimensional space-time as, in some way, discrete or lattice with a parameter $a = \lambda_P$, where λ_P is the Planck length, we have investigated the additional contributions of lattice artifact monopoles to beta-functions of the renormalisation group equations for the running fine structure constants $\alpha_i(\mu)$ (i=1,2,3 correspond to the U(1), SU(2) and SU(3) gauge groups of the Standard Model) in the Family Replicated Gauge Group Model (FRGGM) which is an extension of the Standard Model at high energies. It was shown that monopoles have N_{fam} times smaller magnetic charge in FRGGM than in SM (N_{fam} is the number of families in FRGGM). We have estimated also the enlargement of a number of fermions in FRGGM leading to the suppression of the asymptotic freedom in the non–Abelian theory. We have shown that, in contrast to the case of AntiGUT when the FRGGM undergoes the breakdown at $\mu = \mu_G \sim 10^{18}$ GeV, we have the possibility of unification if the FRGGM–breakdown occurs at $\mu_G \sim 10^{14}$ GeV. By numerical calculations we obtained an example of the unification of all gauge interactions (including gravity) at the scale $\mu_{GUT} \approx 10^{18.4}$ GeV. We discussed the possibility of $[SU(5)]^3$ or $[SO(10)]^3$ (SUSY or not SUSY) unifications.

1. Introduction

Trying to look insight the Nature and considering the physical processes at very small distances, physicists have made attempts to explain the well–known laws of low–energy physics as a consequence of the more fundamental laws of Nature. The contemporary physics of the electroweak and strong interactions is described by the Standard Model (SM) which combines the Glashow–Salam–Weinberg electroweak theory with QCD — theory of strong interactions.

The gauge group of symmetry in SM is:.

$$SMG = SU(3)_c \times SU(2)_L \times U(1)_Y, \tag{1}$$

which describes the present elementary particle physics up to the scale ≈ 100 GeV.

Having an interest in the fundamental laws of physics, we can consider the two possibilities:

1. At the very small (Planck length) distances our space–time is continuous and there exists the fundamental theory (maybe with a very high symmetry) which we do not know at present time.

2. At the very small distances our space–time is discrete, and this discreteness influences on the Planck scale physics.

The item 2 is a base of Random Dynamics (RD) which was suggested and developed in Refs. [1]- [11] as a theory of physical processes proceeding at small distances of order of the Planck scale $\lambda_P = M_{Pl}^{-1}$:

$$M_{Pl} = 1.22 \cdot 10^{19}\,\mathrm{GeV}. \tag{2}$$

The theory of Scale Relativity (SR) [12]- [14] is also related with the item 2 and has a lot in common with RD. SR assumes that the resolution of experimental measurements plays in quantum physics a completely new role with respect of the classical theory, and the space–time is described by a metric based on generalized, explicitly scale–dependent, metric potentials:

$$g_{\mu\nu} = g_{\mu\nu}(t, x, y, z;\, \Delta t,\, \Delta x,\, \Delta y,\, \Delta z). \tag{3}$$

SR theory predicts that there exists a minimal scale of the space–time resolution equal to the Planck length λ_P, which can be considered as a fundamental scale of our Nature. This gives us a reason to make an assumption that *our (3+1)–dimensional space is discrete on the fundamental level.*

This is an initial (basic) point of view of the present theory, i.e. we take a discreteness as existing, not as the lattice computation trick in QCD, say. In the simplest case we can imagine our (3+1) space–time as a regular hypercubic lattice with a parameter $a = \lambda_P$.

Then the lattice artifact monopoles can play an essential role near the Planck scale. But, of course, it is necessary to comment that we do not know (at least, on the level of our today knowledge), what lattice–like structure (random lattice, or foam, or string lattice, etc.) is realized in the description of physical processes at very small distances [15], even if there should be a lattice.

The main aim of the present paper is to show that monopoles cannot be seen in the usual SM up to the Planck scale, because they have a huge magnetic charge and are completely confined. Supersymmetric extensions of SM, known in literature, does not help to see monopoles. We suggest to consider a possibility of the existence of monopoles in our World by extending the Standard Model Group (SMG) to the Family Replicated Gauge Group $(SMG)^{N_{fam}}$, where N_{fam} is the number of families of quarks and leptons. This Family Replicated Gauge Group (FRGG) was very successful in describing the SM parameters: the authors of Refs. [16]– [18], using the breakdown of FRGG to SM, have found a very good fit to conventional experimental data for all fermion masses and mixing angles in SM, also explained the modern experiments for neutrino oscillations.

In the present paper we have written the renormalisation group equations (RGEs) for electric and magnetic fine structure constants (Section 2) and investigated the evolution of gauge and gravitational fine structure constants in Section 3.

The dropping (diminishing) of the monopole charges in FRGG was considered in Section 4. It was shown that if one wants to see monopoles in Nature, it is necessary to drive in the direction of FRGGs.

Multiple Point Principle (MPP) — the existence of the Multiple Critical Point (MCP) at the Planck scale — is reviewed in Section 5.

Section 6 is devoted to the lattice results for the phase transition coupling constants.

We have described a specific evolution of the U(1) fine structure constant in SM and FRGGM in the presence of the lattice artifact monopoles, or fundamental Higgs magnetic fields (Section 7), confirming the idea of MPP.

In Section 8 we have presented the predictions of FRGGM for the Planck scale values of gauge fine structure constants. We show that AntiGUT exists in the case of the breakdown of FRGG at $\mu_G \sim 10^{18}$ GeV (where μ is the energy scale).

In Section 9 we have considered a new possibility of unification of all interactions, including gravity, at the point $\mu_{GUT} = 10^{18.4}$ GeV and $\alpha_{GUT}^{-1} = 27$, if the breakdown of FRGG occurs at $\mu_G \sim 10^{14}$ GeV.

In Section 10 we have presented the discussion of two various scenarios for the existence of Multiple Citical Point (MCP).

2. Renormalisation Group Equations for Electric and Magnetic Fine Structure Constants

J.Schwinger was first [19] who investigated the problem of renormalisation of the magnetic charge in Quantum ElectroMagnetoDynamics (QEMD), i.e. in the Abelian quantum field theory of electrically and magnetically charged particles (with charges e and g, respectively).

Considering the "bare" charges e_0 and g_0 and renormalised (effective) charges e and g, Schwinger (and later the authors of Refs. [20] and [21]) obtained:

$$e/g = e_0/g_0, \tag{4}$$

what means the absence of the Dirac relation [22] for the renormalised electric and magnetic charges.

But there exists another solution of this problem (see Refs. [23]- [26] and review [27]), which gives:

$$eg = e_0 g_0 = 2\pi n, \quad n \in Z, \tag{5}$$

i.e. the existence of the Dirac relation (charge quantization condition) for both, bare and renormalised electric and magnetic charges. Here we have $n = 1$ for the minimal (elementary) charges.

These two cases lead to the two possibilities for the renormalisation group equations (RGEs) describing the evolution of electric and magnetic fine structure constants:

$$\alpha = \frac{e^2}{4\pi} \quad \text{and} \quad \tilde{\alpha} = \frac{g^2}{4\pi}, \tag{6}$$

which obey the following RGEs containing the electric and magnetic beta–functions:

$$\frac{d(\log \alpha(\mu))}{dt} = \pm \frac{d(\log \tilde{\alpha}(\mu))}{dt} = \beta^{(e)}(\alpha) \pm \beta^{(m)}(\tilde{\alpha}). \tag{7}$$

In Eq.(7) we have:

$$t = \log(\frac{\mu^2}{\mu_R^2}), \tag{8}$$

where μ is the energy scale and μ_R is the renormalisation point.

The second possibility (with minuses) in Eq.(7) corresponds to the validity of the Dirac relation (5) for the renormalised charges. We believe only in this case considered by authors in Ref. [26] where we have used the Zwanziger formalism of QEMD [28], [29]. In the present paper, excluding the Schwinger's renormalisation condition (4), we assume only the Dirac relation for running α and $\tilde{\alpha}$:

$$\alpha\tilde{\alpha} = \frac{1}{4}. \tag{9}$$

It is necessary to comment that RGEs (7) are valid only for $\mu > \mu_{threshold} = m_{mon}$, where m_{mon} is the monopole mass.

In contrast to the method given in Ref. [26], there exists a simple way to obtain Eq.(7) for single electric and magnetic charges of the same type (scalar or fermionic). The general expressions for RGEs are:

$$\frac{d(\log \alpha(\mu))}{dt} = \beta_1(\alpha) + \beta_2(\tilde{\alpha}) + C, \tag{10}$$

$$\frac{d(\log \tilde{\alpha}(\mu))}{dt} = \tilde{\beta}_1(\alpha) + \tilde{\beta}_2(\tilde{\alpha}) + \tilde{C}, \tag{11}$$

The Dirac relation (9) gives:

$$\frac{d(\log \alpha(\mu))}{dt} = -\frac{d(\log \tilde{\alpha}(\mu))}{dt}. \tag{12}$$

Using Eq. (12) and the duality symmetry of QEMD, i.e. the symmetry under the interchange:

$$\alpha \longleftrightarrow \tilde{\alpha}, \tag{13}$$

it is not difficult to obtain:

$$C = \tilde{C} = 0, \quad \beta_1(\alpha) = -\beta_2(\alpha) = \tilde{\beta}_1(\alpha) = -\tilde{\beta}_2(\alpha) = \beta(\alpha), \tag{14}$$

and we have the following RGE:

$$\frac{d(\log \alpha(\mu))}{dt} = -\frac{d(\log \tilde{\alpha}(\mu))}{dt} = \beta(\alpha) - \beta(\tilde{\alpha}). \tag{15}$$

If monopole charges, together with electric ones, are sufficiently small, then β–functions can be considered perturbatively:

$$\beta(\alpha) = \beta_2(\alpha/4\pi) + \beta_4(\alpha/4\pi)^2 + ... \tag{16}$$

and

$$\beta(\tilde{\alpha}) = \beta_2(\tilde{\alpha}/4\pi) + \beta_4(\tilde{\alpha}/4\pi)^2 + ... \tag{17}$$

with (see paper [26] and references there):

$$\beta_2 = \frac{1}{3} \quad \text{and} \quad \beta_4 = 1 \quad - \quad \text{for scalar particles,} \tag{18}$$

and

$$\beta_2 = \frac{4}{3} \quad \text{and} \quad \beta_4 \approx 4 \quad - \quad \text{for fermions.} \tag{19}$$

These cases were investigated in Ref. [26]. For scalar electric and magnetic charges we have [26]:

$$\frac{d(\log \alpha(\mu))}{dt} = -\frac{d(\log \tilde{\alpha}(\mu))}{dt} = \beta_2 \frac{\alpha - \tilde{\alpha}}{4\pi}(1 + 3\frac{\alpha + \tilde{\alpha}}{4\pi} + ...) \tag{20}$$

with $\beta_2 = 1/3$, and approximately the same result is valid for fermionic particles [30] with $\beta_2 = 4/3$. Eq.(20) shows that there exists a region when both fine structure constants are perturbative. Approximately this region is given by the following inequalities:

$$0.2 \lesssim (\alpha, \tilde{\alpha}) \lesssim 1. \tag{21}$$

Using the Dirac relation (9), we see from Eq.(20) that in the region (21) the two–loop contribution is not larger than 30% of the one–loop contribution, and the perturbation theory can be realized in this case (see Refs. [31]- [35]).
It is necessary to comment that the region (21) almost coincides with the region of phase transition couplings obtained in the lattice U(1)–gauge theory (see Subsection 6.3).

3. Evolution of Running Fine Structure Constants

The usual definition of the SM coupling constants is given in *the Modified minimal subtraction scheme*($\overline{MS}$):

$$\alpha_1 = \frac{5}{3}\alpha_Y, \quad \alpha_Y = \frac{\alpha}{\cos^2\theta_{\overline{MS}}}, \quad \alpha_2 = \frac{\alpha}{\sin^2\theta_{\overline{MS}}}, \quad \alpha_3 \equiv \alpha_s = \frac{g_s^2}{4\pi}, \tag{22}$$

where α and α_s are the electromagnetic and SU(3) fine structure constants respectively, Y is the hypercharge, and $\theta_{\overline{MS}}$ is the Weinberg weak angle in $\overline{MS}$ scheme. Using RGEs with experimentally established parameters, it is possible to extrapolate the experimental values of three inverse running constants $\alpha_i^{-1}(\mu)$ (here i=1,2,3 correspond to U(1), SU(2) and SU(3) groups of SM) from the Electroweak scale to the Planck scale.
It is well known (see for example [36]) that (in the absence of monopoles) the one–loop approximation RGEs can be described by the following expressions:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_R) + \frac{b_i}{4\pi}t \tag{23}$$

with slopes b_i given by the following values:

$$b_i = (b_1, b_2, b_3) =$$

$$(-\frac{4N_{gen}}{3} - \frac{1}{10}N_S, \quad \frac{22}{3}N_V - \frac{4N_{gen}}{3} - \frac{1}{6}N_S, \quad 11N_V - \frac{4N_{gen}}{3}). \tag{24}$$

The integers N_{gen}, N_S, N_V are respectively the numbers of generations, Higgs bosons and different vector gauge fields.
In SM we have:

$$N_{gen} = 3, \quad N_S = N_V = 1, \tag{25}$$

and the corresponding slopes (24) describe the evolutions of $\alpha_i^{-1}(\mu)$.

The precision of the LEP data allows to make the extrapolation of RGEs with small errors up to the Planck scale (see [37]) unless the new physics pops, of course. Assuming that these RGEs for $\alpha_i^{-1}(\mu)$ contain only the contributions of the SM particles up to $\mu = \mu_{Pl}$ and doing the extrapolation with one Higgs doublet under the assumption of a "desert" and absence of monopoles, we have the following result obtained in Ref. [37]:

$$\alpha_1^{-1}(\mu_{Pl}) \approx 33.3; \quad \alpha_2^{-1}(\mu_{Pl}) \approx 49.5; \quad \alpha_3^{-1}(\mu_{Pl}) \approx 54.0. \tag{26}$$

The extrapolation of $\alpha_{1,2,3}^{-1}(\mu)$ up to the point $\mu = \mu_{Pl}$ is shown in Fig.1 as function of the variable $x = log_{10}\mu$ (GeV).
In this connection, it is very attractive to consider also the gravitational interaction.

3.1. "Gravitational fine structure constant" evolution

The gravitational interaction between two particles of equal masses M is given by the usual classical Newtonian potential:

$$V_g = -G\frac{M^2}{r} = -\left(\frac{M}{M_{Pl}}\right)^2 \frac{1}{r} = -\frac{\alpha_g(M)}{r}, \tag{27}$$

which always can be imagined as a tree–level approximation of quantum gravity.
Then the quantity:

$$\alpha_g = \left(\frac{\mu}{\mu_{Pl}}\right)^2 \tag{28}$$

plays a role of the running "gravitational fine structure constant" and the evolution of its inverse quantity also is presented in Fig.1 together with the evolutions of $\alpha_i^{-1}(\mu)$.
Then we see the intersection of $\alpha_g^{-1}(\mu)$ with $\alpha_1^{-1}(\mu)$ at the point:

$$(x_0, \alpha_0^{-1}),$$

where ($x_0 = \log_{10}\mu_{int.}$):

$$x_0 \approx 18.3, \quad \alpha_0^{-1} \approx 34.4. \tag{29}$$

4. Dropping of the Monopole Charge in the Family Replicated Gauge Group Model (FRGGM)

In the simplest case, the scalar monopole beta–function in QEMD is is the same one as for the scalar QED (see [38], [59] and [30]):

$$\beta(\tilde{\alpha}) = \frac{\tilde{\alpha}}{12\pi} + (\frac{\tilde{\alpha}}{4\pi})^2 + ... = \frac{\tilde{\alpha}}{12\pi}(1 + 3\frac{\tilde{\alpha}}{4\pi} + ...). \tag{30}$$

It follows from Eq.(30) that the theory of monopoles cannot be considered perturbatively at least for

$$\tilde{\alpha} > \frac{4\pi}{3} \approx 4. \tag{31}$$

This limit is smaller for non–Abelian monopoles.

Using the Dirac relation (9), it is easy to estimate in the simple SM the Planck scale value $\tilde{\alpha}(\mu_{Pl})$ (minimal for $U(1)_Y$ gauge group):

$$\tilde{\alpha}(\mu_{Pl}) = \frac{5}{3}\alpha_1^{-1}(\mu_{Pl})/4 \approx 55.5/4 \approx 14. \tag{32}$$

This value is really very big compared with the estimation which follows from (31) and, of course, with the critical coupling $\tilde{\alpha}_{crit} \approx 1$, corresponding to the confinement—deconfinement phase transition in the $U(1)_Y$ lattice gauge theory (see below Section 6). Clearly we cannot do the perturbation approximation with such a strong coupling $\tilde{\alpha}$. It is hard for such monopoles not to be confined. If they are confined — most easily by having an electrically charged scalar field condensate — they are effectively out of the game for scales much smaller in energy μ than the mass $\mu_h = \mu_{"hadron"}$ of their "hadron particles" — bound states of them. With the huge $\tilde{\alpha}$ practically no running brings them out of the range of being huge and so really the μ_h scale should go (at least) to the cut off scale. This really simply means that such monopoles do not exist at all. So if we make the formal assumption that monopoles exist and combine it with the above suggestion, we get as conclusion that so strong $\tilde{\alpha}$ is impossible.

There is an interesting way out of this problem if one wants to have the existence of monopoles, namely to extend the SM gauge group and let the monopoles correspond to charges which are really linear combinations of the SM and extended ones. Then we might attempt to choose a model so cleverly that certain selected linear combinations get bigger electric couplings than the corresponding SM couplings. That could make the monopoles which for these certain charge linear combinations couple more weakly and thus have a better chance of being allowed "to exist", in agreement with the arguments for the confinement of too strongly coupled particles.

An example of such an extension of SM that can impose the possibility of the allowance of monopoles is just Family Replicated Gauge Group Model (FRGGM).

4.1. Family Replicated Gauge Group Model

Most efforts to explain the Standard Model (SM) describing well the experimental results known today are devoted to Grand Unification Theories (GUTs). The supersymmetric extension of the SM consists of taking the SM and adding the corresponding supersymmetric partners. The Minimal Supersymmetric Standard Model (MSSM) shows [37] the possibility of the existence of the grand unification point at

$$\mu_{GUT} \sim 10^{16} \text{ GeV}.$$

Unfortunately, at present time experiment does not indicate any manifestation of supersymmetry. In this connection, the Anti–Grand Unification Theory (AntiGUT) was developed in Refs. [1], [5]- [9], [16]- [18], [31]- [35], [39]- [43] as a realistic alternative to SUSY GUTs. According to this theory, supersymmetry does not come into the existence up to the Planck energy scale (2), where the multiple critical point [39] exists and governs the behaviour of the running coupling constants.

The Standard Model (SM) is based on the group SMG described by Eq.(1). AntiGUT suggests that at the scale $\mu_G \sim \mu_{Pl} = M_{Pl}$ there exists the more fundamental symmetry described by the family replicated gauge group G containing N_{fam} copies of the Standard Model Group SMG:

$$G = SMG_1 \times SMG_2 \times ... \times SMG_{N_{fam}} \equiv (SMG)^{N_{fam}}, \tag{33}$$

where N_{fam} designates the number of families.

If $N = 3$ (as AntiGUT predicts [9] and experiment confirms), then the fundamental gauge group G is:

$$G = (SMG)^3 = SMG_{1st\ fam.} \times SMG_{2nd\ fam.} \times SMG_{3rd\ fam.}, \tag{34}$$

or the generalized one:

$$G_f = (SMG)^3 \times U(1)_f, \tag{35}$$

which was suggested by the fitting of fermion masses of the SM (see Refs. [16]).

Recently a new generalization of AntiGUT was suggested in Refs. [17], [18]:

$$G_{\text{ext}} = (SMG \times U(1)_{(B-L)})^3 \equiv [SU(3)_c]^3 \times [SU(2)_L]^3 \times [U(1)_Y]^3 \times [U(1)_{(B-L)}]^3, \tag{36}$$

which takes into account the see–saw mechanism with right-handed neutrinos, describes all modern neutrino experiments, and gives the reasonable fitting of the SM fermion masses and mixing angles.

Sometimes, for simplicity, we call the theory with the family replicated gauge group of symmetry G, or G_f, or G_{ext}, given by Eqs.(33)-(36), "G–theory", especially prefering to use this terminology for figures (see for example Fig.2).

The group $G = G_{ext}$ contains: $3 \times 8 = 24$ gluons, $3 \times 3 = 9$ W-bosons and $3 \times 1 + 3 \times 1 = 6$ Abelian gauge bosons.

At first sight, this $(SMG \times U(1)_{(B-L)})^3$ group with its 39 generators seems to be just one among many possible SM gauge group extensions. However, it is not such an arbitrary choice. There are at least reasonable requirements (postulates) on the gauge group G

which have uniquely to specify this group. It should obey the following postulates (the first two are also valid for SU(5) GUT):

1. $G = G_{ext}$ should only contain transformations, transforming the well–known 45 Weyl fermions of the SM (= 3 generations of 15 Weyl particles each — counted as left handed), and the additional three heavy see–saw (right handed) neutrinos — into each other unitarily, so that $G = G_{ext}$ must be a subgroup of U(48): $G \subseteq U(48)$.

2. No anomalies, neither gauge nor mixed. Otherwise the model becomes non–renormalisable. G–theory assumes that only straightforward anomaly cancellation takes place and forbids the Green-Schwarz type anomaly cancellation [44].

3. G–theory should NOT UNIFY the irreducible representations under the SM gauge group, called here SMG (see Eq.(1)).

4. G is the maximal group satisfying the above-mentioned postulates.

In Refs. [17], [18] the gauge group $G = (SMG \times U(1)_{(B-L)})^3$ is spontaneously broken down (at one or two orders of magnitude below the Planck scale) by 7 different Higgs fields to the gauge group $SMG \times U(1)_{(B-L)}$ which is the diagonal subgroup of G. Therefore, these Higgs fields break AGUT to the SM. The field ϕ_{WS} corresponds to the Weinberg–Salam theory (its VEV is known: $< \phi_{WS} >= 246$ GeV), so that we have only six free parameters – six VEVs – to fit the experiment in the framework of this model. The authors of Refs. [18] used them with aim to find the best fit to conventional experimental data for all fermion masses and mixing angles in the SM, also with aim to explain the experiments in neutrino oscillations. The typical fit to the masses and mixing angles for the SM leptons and quarks is encouraging in the crude approximation. It was shown that the family replicated gauge group G_{ext} generates the large mixing angle solution to the solar neutrino oscillation problem. Also the neutrino masses are predicted in Ref. [18]. Finally, we conclude that, in general, the theory with $FRGG$–symmetry is very successful in describing of the SM experiment.

4.2. FRGG-model prediction for the values of electric and magnetic charges

According to the FRGG–model, at some point $\mu = \mu_G < \mu_{Pl}$ (or really in a couple of steps) the fundamental group G undergoes spontaneous breakdown to its diagonal subgroup:

$$G \longrightarrow G_{diag.subgr.} = \{g, g, g || g \in SMG\}, \tag{37}$$

which is identified with the usual (low–energy) group SMG. The point $\mu_G \sim 10^{18}$ GeV is shown in Fig.2 demonstrating the evolution of the inverses $\alpha^{-1}_{Y,2,3}$ up to the Planck scale.

But it seems that this extrapolation is valid only up to the $\mu = \mu_G$, because FRGGM (here named G-theory) works in the region $\mu_G \leq \mu \leq \mu_{Pl}$, giving the quite different behaviour of $\alpha_i^{-1}(\mu)$ in this region, which is not shown in Fig.2, but will be investigated in Section 9.

It should be said that in the FRGG–model each family has its own gluons, own W's, etc. The breaking just makes only a certain linear combination of gluons which exists in the SM: below $\mu = \mu_G$ and down to the low energies. We can say that the phenomenological gluon is a linear combination (with amplitude $1/\sqrt{3}$ for $N_{fam} = 3$) for each of the FRGG gluons of the same colour combination. Then we have the following formula connecting the fine structure constants of non–Abelian FRGG–model to the low energy surviving diagonal subgroup $G_{\text{diag.subg.}} \subseteq (SMG)^3$:

$$\alpha^{-1}_{i,\text{diag}} = \alpha^{-1}_{i,\text{1st fam.}} + \alpha^{-1}_{i,\text{2nd fam.}} + \alpha^{-1}_{i,\text{3rd fam.}}. \tag{38}$$

Here i = SU(2), SU(3), and i=3 means that we talk about the gluon couplings.

Assuming that three FRGG couplings are equal to each other (as comes indeed out of our MPP, see later the Section 5), we obtain:

$$\alpha^{-1}_{i,\text{diag}} \approx 3\alpha^{-1}_{i,\text{one fam.}} \equiv 3\alpha^{-1}_{i,G}. \tag{39}$$

In contrast to non-Abelian theories, in which the gauge invariance forbids the mixed (in families) terms in the Lagrangian of FRGG–theory, the U(1)–sector of FRGG contains such mixed terms (see Ref. [39]):

$$\frac{1}{g^2}\sum_{p,q} F_{\mu\nu,\,p}F^{\mu\nu}_{q} = \frac{1}{g^2_{11}}F_{\mu\nu,\,1}F^{\mu\nu}_{1} + \frac{1}{g^2_{12}}F_{\mu\nu,\,1}F^{\mu\nu}_{2} + ... + \frac{1}{g^2_{23}}F_{\mu\nu,\,2}F^{\mu\nu}_{3} + \frac{1}{g^2_{33}}F_{\mu\nu,\,3}F^{\mu\nu}_{3}, \tag{40}$$

where $p, q = 1, 2, 3$ are the indices of three families of the group $(SMG)^3$. Now it is easily seen that if the different families had specific equal electric charges, i.e. equal α_{pq}, then taking the diagonal subgroup we obtain [39]:

$$\alpha^{-1}_{\text{diag}} \approx 6\alpha^{-1}_{\text{G}}, \tag{41}$$

which shows that we can increase electric α by a factor 6 replacing it by the electric $\alpha_{\text{one fam.}} \equiv \alpha_G$.

In the article [39] it was estimated that a large number (presumably largest possible) phases might be brought to meet by having a certain hexagonal symmetry which actually would mean that various "cross couplings" g_{pq} in (40) are equal to each other.

Taking (41), we can get the new $\tilde{\alpha}_{one\,fam.}$ for the U(1)–sector of FRGG which is smaller by factor 6 in comparison with $\tilde{\alpha}$ in SM:

$$\tilde{\alpha}_G = \frac{1}{6}\tilde{\alpha}_{diag}. \tag{42}$$

Using the result (32), we can estimate $\tilde{\alpha}$ for one family at the Planck scale:

$$\tilde{\alpha}_G(\mu_{Pl}) \approx 14/6 \approx 2.3. \tag{43}$$

But it seems that $\tilde{\alpha}_G(\mu_{Pl}) \approx 1$ in FRGGM (see Section 8), and the perturbation theory works for β–function of scalar monopoles near the Planck scale.
In general, considering the non–Abelian FRG–group $SU(N)^{N_{fam}}$ we have the following value for the one family monopole charge:

$$\tilde{\alpha}_G = \frac{\tilde{\alpha}}{N_{fam}}, \tag{44}$$

where $\tilde{\alpha}$ is the total fine structure constant.
But for $U(1)^{N_{fam}}$ gauge group, taking into account the considerations of Ref. [39], it is easy to obtain the following relation:

$$\tilde{\alpha}_{1,G} = \frac{\tilde{\alpha}_1}{N^*} \quad \text{, where} \quad N^* = \frac{1}{2} N_{fam}(N_{fam}+1). \tag{45}$$

The conclusion: if one wants monopoles "to exist", it is necessary to drive in the direction of a model like FRGG.

5. Multiple Point Principle

AntiGUT approach is often used in conjunction with the Multiple Point Principle (MPP) proposed by D.L.Bennett and H.B.Nielsen [39]. According to this principle, Nature seeks a special point – the Multiple Critical Point (MCP) – which is a point on the phase diagram of the fundamental regularized gauge theory G, where the vacua of all fields existing in Nature are degenerate having the same vacuum energy density. Such a phase diagram has axes given by all coupling constants considered in theory. Then all (or just many) numbers of the phases meet at the MCP.
MPM assumes the existence of MCP at the Planck scale, insofar as gravity may be "critical" at the Planck scale. [1]
The philosophy of MPM leads to the need for investigating the phase transition in various gauge theories. A lattice model of gauge theories is the most convenient formalism for the realization of the MPM ideas. As it was mentioned above, in the simplest case we

[1] We say "at the Planck scale" because we have in mind the definition of the concept of phase transition representing the borderline in space of couplings cross producted with the axis of scales so that we think of phase transitions being at different coupling values depending on the scales considered. There is another, we think more usual, way of thinking of it: a phase transition is along a hypersurface in the space of only parameters of the theory, such as couplings, the temperature and the mass parameters, where the various output properties have jumps.

can imagine our space–time as a regular hypercubic (3+1)–lattice with the parameter a equal to the fundamental (Planck) scale:

$$a = \lambda_P = 1/M_{Pl} \sim 10^{-33}\,\mathrm{cm}. \tag{46}$$

In general, the lattice results are very encouraging for MPM. Taking seriously the ideas of Ref. [39], we were able to predict gauge coupling constants in terms of lattice phase transition couplings, using very strongly FRGG–model.

It is necessary to emphasize, as it was mentioned in the introduction, that we do not know, what lattice–like structure plays role in the description of physical processes at very small distances. We assume and have tried to show (see the review [15]) that one can crudely calculate the phase transition couplings without using any specific lattice, rather only approximating the lattice artifact monopoles as fundamental (pointlike) particles condensing. The details of the lattice — hypercubic or random, with multiplaquette terms or without them, etc. — also the details of the regularization — lattice or Wilson loops, lattice or Higgs monopole model — do not matter too much for the value of the phase transition couplings. Critical couplings depend only on the group, not so much on the regularization. Such an approximate universality of critical couplings is, of course, absolutely needed if there should be any sense in relating lattice phase transition couplings to the experimental couplings found in Nature. Otherwise, such a comparison would only make sense if we could guess the true lattice in the right model, what sounds too ambitious.

6. Phase Transitions in Lattice Gauge Theories

Following the MPM ideas, we consider our space–time as a simple regular hypercubic (3+1)–lattice with the parameter a equal to the fundamental Planck scale (46). Our aim is to investigate the phase transitions in the lattice gauge theories and relate the obtained phase transition couplings with MCP at the Planck scale. The lattice investigations were performed by a lot of authors [45]- [54] using Monte Carlo method of simulations of the lattice gauge theories, and we have considered the Higgs monopole model [31]- [35] to explain their results.

6.1. Lattice actions

A lattice contains sites, links and plaquettes. Link variables are fundamental variables of the lattice theory. These variables are also the elements of gauge group G, describing a symmetry of the corresponding lattice gauge theory:

$$\mathcal{U}(x \bullet\!\!\longrightarrow\!\!\bullet\, y) \in G. \tag{47}$$

The link variable connects the point n and the point $n + a_\mu$, where the index μ indicates the direction of a link in the hypercubic lattice with parameter a:

$$\mathcal{U}(x \bullet\!\!\longrightarrow\!\!\bullet\ y) = e^{i\Theta_\mu(n)} \equiv \mathcal{U}_\mu(n), \tag{48}$$

where

$$\Theta_\mu(n) = a\hat{A}_\mu(x),$$

$$\hat{A}_\mu(x) = gA_\mu(x) \quad \text{— for U(1)-group,}$$

$$\hat{A}_\mu(x) = gA^j_\mu(x)t^j \quad \text{— for SU(N)-group.} \tag{49}$$

Here t^j is the generator of the SU(N)-group, and $t^j = \lambda^j/2$ for SU(3)-group, where λ^j are the well–known Gell–Mann matrices.

Plaquette variables are products of link variables:

$$\mathcal{U}_p \equiv \mathcal{U}(\square) \stackrel{def}{=} \mathcal{U}\left(\square\right)\mathcal{U}\left(\square\right)\mathcal{U}\left(\square\right)\mathcal{U}\left(\square\right). \tag{50}$$

The simplest action of the U(1) lattice gauge theory is given by the following expression:

$$S[\mathcal{U}_p] = \beta \sum_p \cos\Theta_p, \quad \text{where} \quad \mathcal{U}_p = e^{i\Theta_p}. \tag{51}$$

For the compact lattice U(1) theory: $\beta = 1/e_0^2$, where e_0 is the bare charge.

The Villain form of the U(1) lattice gauge theory [45] is:

$$S_V = (\beta/2)\sum_p (\Theta_p - 2\pi k_p)^2, \quad k_p \in Z, \tag{52}$$

where k_p is defined to be that integer giving the term of the smallest value.

The lattice $SU(N)$ gauge theories were first introduced by K.Wilson [46] for studying the problem of confinement. He suggested the following simplest action:

$$S = -\frac{\beta}{N}\sum_p Re(Tr\,\mathcal{U}_p), \tag{53}$$

where the sum runs over all plaquettes of a hypercubic lattice.

Monte Carlo simulations of these simple Wilson lattice theories in four dimensions showed a (or an almost) second–order deconfining phase transition for U(1) [47], a crossover behaviour for SU(2) and SU(3) [48], and a first–order phase transition for SU(N) with $N \geq 4$ [49].

Bhanot and Creutz [50] have generalized the simple Wilson theory, introducing two parameters in the $SU(N)$ action:

$$S = \sum_p [-\frac{\beta_f}{N} Re(Tr\,\mathcal{U}_p) - \frac{\beta_A}{N^2-1} Re(Tr_A \mathcal{U}_p)], \tag{54}$$

where β_f, Tr and β_A, Tr_A are respectively the lattice constants and traces in the fundamental and adjoint representations of $SU(N)$.

Monte Carlo simulations of the U(1) gauge theory described by the two-parameter lattice action [51], [52]:

$$S = \sum_p [\beta^{lat} \cos \Theta_p + \gamma^{lat} \cos 2\Theta_p] \tag{55}$$

show the existence of a triple point on the corresponding phase diagram: "Coulomb–like", totally confining and Z_2 confining phases come together at this triple point.

The phase diagrams were obtained for the generalized lattice SU(2) and SU(3) theories (54) by Monte Carlo methods in Refs. [50] (see also [53]). They indicated the existence of a triple point which is a boundary point of three first–order phase transitions: the "Coulomb–like" and the confining phase and that phase in which Z_N subgroup confines, while the factor group $SU(N)/Z_N$ does not confine, make up the three phases that meet at this point. From the triple point emanate three phase border lines which separate the corresponding phases. All confinement phases exist due to monopole condensations of slightly different sorts.

6.2. The behaviour of the lattice electric fine structure constant α near the phase transition point

The lattice investigations did not give the lattice triple point values of $\alpha_{i,crit}$ by Monte Carlo simulation method. Only in Ref. [52] the critical value of the effective electric fine structure constant α was obtained in the compact U(1) lattice gauge theory described by the Wilson and Villain actions (51) and (52), respectively:

$$\alpha^{lat}_{crit} \approx 0.20 \pm 0.015 \quad \text{and} \quad \tilde{\alpha}^{lat}_{crit} \approx 1.25 \pm 0.10 \quad \text{at} \quad \beta_T \equiv \beta_{crit} \approx 1.011. \tag{56}$$

The result of Ref. [52] for the behaviour of $\alpha(\beta)$ in the vicinity of the phase transition point β_T is shown in Fig.3 for the Wilson and Villain lattice actions. Fig.4 demonstrates the comparison of the function $\alpha^{-1}(\beta)$ obtained by Monte Carlo method for the Wilson lattice action and by our theoretical calculation of the same quantity. The theoretical (dashed) curve was calculated by so-called "Parisi improvement formula" [54]:

$$\alpha(\beta) = [4\pi\beta W_p]^{-1}. \tag{57}$$

Here $W_p = < \cos \Theta_p >$ is a mean value of the plaquette energy. The corresponding values of W_p were taken from Ref. [51].

The theoretical value of α_{crit} is less than the "experimental" (Monte Carlo) value (56):

$$\alpha_{crit.,theor.}(\text{in lattice theory}) \approx 0.12. \tag{58}$$

According to Fig.4:

$$\alpha^{-1}_{crit.,theor.} \approx 8, \quad \text{and} \quad \alpha^{-1}_{crit.,lat.} \approx 5. \tag{59}$$

This discrepancy between the theoretical and "experimental" results, shown in Fig.4, has the following explanation: "Parisi improvement formula" (57) does not take into account the contribution of monopoles running due to the renormalisation group near the phase transition point.

6.3. Lattice artifact monopoles and Higgs Monopole Model

Lattice monopoles are responsible for the confinement in lattice gauge theories what is confirmed by many numerical and theoretical investigations (see reviews [55] and papers [56]). In the compact lattice gauge theory the monopoles are not physical objects: they are lattice artifacts driven to infinite mass in the continuum limit.

In the papers [39]- [41] the calculations of the U(1) phase transition (critical) coupling constant were connected with the existence of artifact monopoles in the lattice gauge theory and also in the Wilson loop action model [41].

In Ref. [41] we have put forward the speculations of Refs. [39] and [40] suggesting that the modifications of the form of the lattice action might not change too much the phase transition value of the effective continuum coupling constant. The purpose was to investigate this approximate stability of the critical coupling with respect to a somewhat new regularization being used instead of the lattice, rather than just modifying the lattice in various ways. In [41] the Wilson loop action was considered in the approximation of circular loops of radii $R \geq a$. It was shown that the phase transition coupling constant is indeed approximately independent of the regularization method: $\alpha_{crit} \approx 0.204$, in correspondence with the Monte Carlo simulation result on lattice [52] given by Eq.(56). It was the first step to confirm the "approximate universality" of the critical couplings which we have considered in other our papers (see review [15]).

In Refs. [31]- [35], instead of using the lattice or Wilson loop cut–off, we have developed the Higgs Monopole Model (HMM) approximating the lattice artifact monopoles as fundamental pointlike particles described by the Higgs scalar field. Indeed, the simplest effective dynamics describing the confinement mechanism in the pure gauge lattice U(1) theory is the dual Abelian Higgs model of scalar monopoles [55], [56]. This model has the following Lagrangian:

$$L = -\frac{1}{4g^2} F^2_{\mu\nu}(B) + \frac{1}{2}|(\partial_\mu - iB_\mu)\Phi|^2 - U(\Phi), \tag{60}$$

where

$$U(\Phi) = \frac{1}{2}\mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 \tag{61}$$

is the Higgs potential of scalar monopoles with magnetic charge g, and B_μ is the dual gauge (photon) field interacting with the scalar monopole field Φ. In this model the positive λ is the self–interaction constant of scalar fields, whereas the mass parameter μ^2 is negative.

Considering the renormalisation group improvement of the effective Coleman–Weinberg potential [38] (see also [57]), written in Refs. [33]- [35] for the dual sector of scalar electrodynamics in the two–loop approximation for β–functions, we have calculated the U(1) critical values of the magnetic fine structure constant:

$$\tilde{\alpha}_{crit} = g^2_{crit}/4\pi \approx 1.20 \tag{62}$$

and (by the Dirac relation) electric fine structure constant:

$$\alpha_{crit} = \pi/g^2_{crit} \approx 0.208. \tag{63}$$

These values coincide with the lattice result (56).

Here it is quite necessary to comment that the lattice result $\alpha_{crit} \approx 0.2$ is valid for U(1) lattice gauge theory with or without electrically charged fermions (at least approximately), what is easy to confirm calculating the partition function with fermions on lattice and integrating over fermion fields [58]. By this reason, considering the U(1) theory with fermions in the vicinity of the phase transition point, we have used the lattice results for pure U(1), what is very important for the predictions of MPM (see Section 8).

7. The Evolution of the Running U(1) fine structure Constant in the Theory with/without the Higgs Monopole Field

Considering the evolution of the running U(1) fine structure constant $\alpha_Y(\mu) = \alpha(\mu)/cos^2\theta_{\overline{MS}}$, we have in SM the following one–loop approximation:

$$\alpha_Y^{-1}(\mu) = \alpha_Y^{-1}(\mu_R) + \frac{b_Y}{4\pi}t, \tag{64}$$

where t is the evolution variable (8), and b_Y is given for N_{gen} generations and N_S Higgs bosons by the following expression:

$$b_Y = -\frac{20}{9}N_{gen} - \frac{1}{6}N_S. \tag{65}$$

The evolution of $\alpha_Y^{-1}(\mu)$ is represented in Fig.2 for $N_{gen} = 3$ and $N_S = 1$ by the straight line going up to $\mu = \mu_{Pl}$.

Let us consider now the exotic (not existing in reality) case when we have, for example, the cut off energy $\mu_{cutoff} \sim 10^{42}$ GeV. In this case the evolution of ${\alpha_Y}^{-1}(\mu)$ is given by Fig.5a, where the straight line 1 (one–loop approximation) goes to the Landau pole at $\alpha_Y^{-1} = 0$. But it is obvious that in the vicinity of the Landau pole, when $\alpha_Y^{-1}(\mu) \to 0$, the charge of U(1) group becomes larger and larger with increasing of μ. This means that the one–loop approximation for β–function is not valid for large μ, and the straight line 1 may change its behaviour. In general, the two–loop approximation for β–function of QED (see Refs. [30], [38] and [59]) shows that this straight line turns and goes down.
Exotically we can consider our space-time as a lattice with parameter a smaller than the Planck scale value λ_P. For example, we can imagine a lattice with $a \sim (10^{42}\,\mathrm{GeV})^{-1}$, or the existence of the fundamental magnetically charged Higgs scalar field in the vicinity of large $\mu_{crit} \sim 10^{38}$ GeV, when we have the phase transition point with $\alpha_{Y,crit}^{-1} \approx 4$ (see below (75) and Fig.5a).
The artifact monopoles, responsible for the confinement of electric charges at the very small distances, can be approximated by the magnetically charged Higgs scalar field, which leads to the confinement–deconfinement phase transition, as it was shown in Refs. [31]- [35]. If we have this phase transition, then there exists a rapid fall in the evolution of $\alpha^{-1}(\mu)$ (α_Y^{-1} or $\alpha_{Y,G}^{-1}$, etc.) near the phase transition (critical) point. This "fall" always accompanies the phase transition from the Coulomb–like phase to the confinement one. Indeed, we can present the effective Lagrangian of our field system as a function of the variable

$$F^2 \equiv F_{\mu\nu}^2, \tag{66}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{67}$$

is the field strength tensor:

$$L_{eff} = -\frac{\alpha_{eff}^{-1}(F^2)}{16\pi} F^2. \tag{68}$$

When $F^2 = \vec{B}^2$ ($\vec{B}$ is the magnetic field) and $\vec{B}^2$ is independent of space–time coordinates (see Refs. [60], [61]), we can write the effective potential:

$$V_{eff} = \frac{\alpha_{eff}^{-1}(\vec{B}^2)}{16\pi} \vec{B}^2 = A \alpha_{eff}^{-1}(t) e^{2t}. \tag{69}$$

Here, choosing $\vec{B}^2 = \mu^4$ and $\mu_R = \mu_{cutoff}$, we have:

$$A = \mu_{cutoff}^4 / 16\pi,$$

and

$$t = \frac{1}{2} \log \frac{F^2}{\mu_{cutoff}^4}. \tag{70}$$

It is well–known [60], [61] that $\alpha_{eff}^{-1}(t)$ has the same evolution over $t = \log(\mu^2/\mu^2_{cutoff})$ whether we consider $\mu^2 = p^2$ (where p is the 4-momentum), or $\mu^4 = F^2 = \vec{B}^2$ (at least, up to the second order perturbation).

In the confinement region ($t > t_{crit}$) the effective potential (69) has a minimum, given by the requirement [61]:

$$[\frac{d\alpha_{eff}^{-1}}{dt} + 2\alpha_{eff}^{-1}(t)]|_{t=t_{min}} = 0. \tag{71}$$

Of course, we need this minimum for t–values above t_{crit} in order to have confinement which namely means that we have a nonzero $F^2 = \vec{B}_0^2 = const$ in the vacuum [62]- [65]. This minimum of the effective potential can exist only if $\alpha_{eff}^{-1}(\mu)$ has a rapid fall near the phase transition point which is illustrated in Fig.5a,b by curve 2. Moreover, the width of this fall must be narrow enough to give a minimum of V_{eff}. Indeed, the condition $V'_{eff} < 0$ gives:

$$[\frac{d\alpha_{eff}^{-1}}{dt} + 2\alpha_{eff}^{-1}(t)]|_{t=t_{crit}} < 0, \tag{72}$$

or

$$\Delta t < -\frac{\Delta(\alpha_{eff}^{-1})}{2\alpha_{eff}^{-1}}|_{t=t_{crit}}. \tag{73}$$

If the value $\Delta\alpha_{eff}^{-1}$ is not large in the critical point, then Δt is small, and we have a sharp jump of V_{eff} (what means the almost first order phase transition) and a sharp jump of α_{eff}^{-1} from the one-loop approximation end point value (we call it $\alpha_{crit.Coul.}$) to the critical one ($\alpha_{crit.conf.}$). This case is realized in AntiGUT and will be discussed later in Section 8.

The existence of minimum of the effective potential (69) explains why the straight line 1 changes its behaviour and rapidly falls: $\alpha_{eff}^{-1}(t)$ is multiplied by $\exp(2t)$ in Eq.(69).

After this "fall" $\alpha_Y^{-1}(\mu)$ has a crook and goes to the constant value, as it is shown in Fig.5a by solid curve 2, demonstrating the phase transition from Coulomb–like phase to the confinement one.

The next step is to give the explanation why $\alpha_{eff}^{-1}(t)$ is arrested when $t \to t_{cutoff}$.

The process of formation of strings in the confinement phase (considered in Ref. [32]) leads to the "freezing" of α: in the confinement phase the effective electric fine structure constant is almost unchanged and approaches its maximal value $\alpha = \alpha_{max}$ when ($\mu \to \infty$). The authors of Ref. [62] predicted that the fine structure constant α cannot be infinitely large, but has the following maximal value:

$$\alpha_{max} \approx \frac{\pi}{12} \approx 0.26, \tag{74}$$

due to the Casimir effect for strings. This viewpoint was developed in spinor QED by authors of Ref. [63]: the vacuum polarization induced by thin "strings"—vortices of

magnetic flux—leads to the suggestion of an analogue of the "spaghetti vacuum" [64] as a possible mechanism for avoiding the divergences of perturbative QED. According to Ref. [63], the non–perturbative sector of QED arrests the growth of the effective α to infinity for small distances and confirms the existence of α_{max}. This phenomenon was called "the freezing of α".

Fig.3 demonstrates the tendency to freezing of α in the compact QED for $\beta < \beta_T$ (i.e. for "bare" constant $e_0 > 1$, what means $\alpha_0^{-1} < 4\pi \approx 12.56$).

Choosing the lattice result (56), which almost coincides with the HMM result (63), we have:

$$\alpha_Y^{-1}(\mu_{crit}) \equiv \alpha_{U(1),crit}^{-1} \approx 5. \tag{75}$$

Analogously, using (74), we have the maximal value for α_Y^{-1}:

$$\alpha_{Y,max}^{-1} \approx 1/0.26 \approx 3.8. \tag{76}$$

An interesting situation arises in the theory with FRGG–symmetry, when it begins to work at $\mu = \mu_G (< \mu_{Pl})$. As it was shown in Section 4, in the vicinity of the phase transition point the U(1)–sector of FRGG has $\alpha_{Y,G} \equiv \alpha_{Y,one\,fam.}$, which is 6 times larger than α_Y. Now the phase transition "deconfinement–confinement" occurs at $\mu_{crit} = \mu_{Pl}$ (but not at $\mu_{crit} \sim 10^{38}$ GeV as it was in the SM prolonged up to the scale $\mu_{cutoff} \sim 10^{42}$ GeV). This case is not "exotic" more, and confirms the MPP idea [39].

The evolution of the inverse one family fine structure constant $\alpha_{Y,G}(\mu)^{-1}(x)$ is given in Fig.5b, which demonstrates the existence of critical point at the Planck scale.

Here it is necessary to comment that we have given a qualitative behaviour of the fall in Fig.5a,b believing in the confinement existing in the $U(1)_Y$ theory. In general, the existence of the confinement (see [63]), as well as the shape of the fall (wide or narrow), depends on the type of theory considered. If the cutoff energy is not very high ($\mu_{cutoff} < \mu_{crit}$), or the lattice spacing a is not too small, then there is no confinement region in such a theory (for example, in the U(1) sector of the SM we have $\mu_{cutoff} = \mu_{Pl}$ which is smaller than μ_{crit}, given by Fig.5a, and the confinement phase is not available). According to MPP [2], Nature has to have the phase transition point at the Planck scale not only for the Abelian U(1) theory, but also for non–Abelian theories. This means that the effective potential of SU(3) gauge theory has the second minimum at the Planck scale (the first one corresponds to the low–energy hadron physics). String states of this second confinement phase are not observed in Nature, because the FRGG–theory approaches the confinement phase at the Planck scale, but does not reach it.

[2] We call it MPP-II to distinguish this definition from the version in which MPP-I is defined only as a phase transition depending on bare couplings while no scale involved

8. FRGGM Prediction of the Planck Scale Values of the U(1), SU(2) and SU(3) Fine Structure Constants

The FRGGM prediction of the values of $\alpha_i(\mu)$ at $\mu = \mu_{Pl}$ is based on the MPM assumption about the existence of phase transition boundary point MCP at the Planck scale, and gives these values in terms of the corresponding critical couplings $\alpha_{i,crit}$ [3]- [9], [39]:

$$\alpha_i(\mu_{Pl}) = \frac{\alpha_{i,crit}}{N_{fam}} = \frac{\alpha_{i,crit}}{3} \quad \text{for} \quad i = 2,3 \quad \text{(also for } i > 3\text{)}, \tag{77}$$

and

$$\alpha_1(\mu_{Pl}) = \frac{\alpha_{1,crit}}{\frac{1}{2}N_{fam}(N_{fam}+1)} = \frac{\alpha_{1,crit}}{6} \quad \text{for} \quad U(1). \tag{78}$$

A simple explanation of the relations (77) and (78) was given in Section 4 by Eqs.(38)-(40).

8.1. Phase transition couplings in the regularized SU(N) gauge theories

It was shown in a number of investigations (see for example [55], [56] and references there), that the confinement in the SU(N) lattice gauge theory effectively comes to the same U(1) formalism. The reason is the Abelian dominance in their monopole vacuum: monopoles of the Yang–Mills theory are the solutions of the U(1)–subgroups, arbitrary embedded into the SU(N) group. After a partial gauge fixing — Abelian projection by 't Hooft [65] — SU(N) gauge theory is reduced to an Abelian $U(1)^{N-1}$ theory with $N-1$ different types of Abelian monopoles. Choosing the Abelian gauge for dual gluons, it is possible to describe the confinement in the lattice SU(N) gauge theories by the analogous dual Abelian Higgs model of scalar monopoles.

Using the Abelian gauge by 't Hooft and taking into account that the direction in the Lie algebra of monopole fields are gauge dependent, we have found in Ref. [33] an average over these directions and obtained the group dependence relation between the phase transition fine structure constants for the groups $U(1)$ and $SU(N)/Z_N$:

$$\alpha^{-1}_{N,crit} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}}\alpha^{-1}_{U(1),crit}. \tag{79}$$

We have calculated this relation using only the one–loop approximation diagrams of non-Abelian theories.

According to Eq.(79), we have the following relations:

$$\alpha^{-1}_{U(1),crit} : \alpha^{-1}_{2,crit} : \alpha^{-1}_{3,crit} = 1 : \sqrt{3} : 3/\sqrt{2}, \tag{80}$$

which together with relations (77) and (78) give:

$$\begin{aligned}
\alpha_1^{-1}(\mu_{Pl}) &\approx 6 \cdot \tfrac{3}{5}\alpha_{U(1),crit}^{-1} \approx 18, \\
\alpha_2^{-1}(\mu_{Pl}) &\approx 3 \cdot \sqrt{3}\alpha_{U(1),crit}^{-1} \approx 26, \\
\alpha_3^{-1}(\mu_{Pl}) &\approx 3 \cdot \tfrac{3}{\sqrt{2}}\alpha_{U(1),crit}^{-1} \approx 32,
\end{aligned} \tag{81}$$

where we have used the value (75).

Here we see that in FRGGM the magnetic fine structure constant of Abelian monopoles approximately is equal to unity at the Planck scale:

$$\tilde{\alpha}_{U(1)}(\mu_{Pl}) = \tilde{\alpha}_{U(1),crit} = \frac{1}{4}\alpha_{U(1),crit}^{-1} \approx \frac{5}{4} \approx 1.25, \tag{82}$$

and the corresponding beta-function can be considered perturbatively (see Section 2, inequality (21)).

One should tell here that monopoles only escape confinement above μ_G scale because there is the Higgsing of the gauge group G down to the diagonal subgroup. These Higgs fields must somehow at least confine the family monopoles.

8.2. Olive's monopoles

The fact that we have one special monopole for each of the three groups SU(3), SU(2) and U(1), which we have considered when calculated the phase transition (critical) couplings (for the confinement due to monopole condensation, either in the SM, or in FRGGM) is indeed not consistent with quarks and leptons as phenomenologically found objects. The point is that the SU(3) monopole we used, for instance, radiates a flux corresponding to a path in the gauge group SU(3) from the unit element to one of the non–trivial center elements. Such a monopole gives rise to a phase factor $exp(2\pi i/3)$ when a quark encircles its Dirac string. Therefore, it does not allow quarks.

What is allowed consistently with the SM representations is what D.Olive [66] has proposed and we call "the Olive–monopole". These monopoles have the magnetic charge *under all three subgroups: SU(3), SU(2) and U(1)* of SMG. Their total magnetic charge corresponds to the center element $(Ie^{i2\pi/3}, -I, 2\pi) \in SU(3) \times SU(2) \times \mathbf{R}$ contained in the covering group $SU(3) \times SU(2) \times \mathbf{R}$ of SMG, which according to the interpretation of Ref. [67], has a meaning of the Lie group, rather than just Lie algebra, fitting the SM representations. That is, Olive–monopole in the SM has at the same time three different monopolic charges with the following sizes:

1) An SU(3) magnetic charge identical to the one that if alone would allow only the representations of the group $SU(3)/Z_3$, i.e. only to the representations with triality t=0.

2) An SU(2) magnetic charge identical to the one that would allow only representations of the group $SU(2)/Z_2 = SO(3)$, i.e. only representations with integer weak isospin.

3) And finally, a U(1) weak hypercharge monopolic charge of a size that if alone would allow only integer values of the weak hypercharge half, i.e. of $y/2$ = integer.

These three magnetic charge contributions would, if alone, not allow the existence neither fermions, nor the Higgs bosons in the SM. However, considering the phase of a quark or lepton field along a little circle encircling the Dirac string for an Olive's SM–monopole, one gets typically a phase rotation from each of the three contributions to the magnetic charge. The consistency condition to have the Dirac string without visible effect is that these phase contributions *together* make up a multiplum of 2π. It can be checked that the quark and lepton representations, as well as the Weinberg–Salam Higgs boson representation, lead to the full phase rotations which are indeed multipla of 2π.

Thus, as it was already mentioned above: if we imagine monopoles with each of these contributions alone they would not allow the phenomenologically observed quarks and leptons, nor the Higgs bosons.

In going to the FRGG–model we can, without problem, postulate one Olive–monopole for each proto–family since the proto–family representations are just analogous to the ones in the SM.

Considering the Olive–monopoles condensation — thereby causing a confinement–deconfinement phase transition — for the different families, we assume that as long as we consider only, say, the SU(3)–coupling to cause the phase transition, the Olive–monopole functions as if it is the SU(3)–monopole consistent only with the representations of $SU(3)/Z_3$. But this is just what gives the phase transition couplings derived with help of Eq.(75) together with Eq.(64). Similarly, it is relatively easy to see that the use of the Olive–monopole for all the (family) gauge groups SU(3), SU(2), U(1) leads to the phase transition couplings obtained by combining Eqs.(64) and (75).

8.3. AntiGUT prediction of the fine structure constants near the Planck scale

According to AntiGUT [3]- [9], the point where FRGG undergoes spontaneous breakdown to its diagonal subgroup (isomorphic to SMG) takes place at $\mu_G \sim 10^{18}$ GeV. This case is shown in Fig.2 demonstrating the evolution of the inverses $\alpha^{-1}_{Y,2,3}(\mu)$. Here $\alpha_Y \equiv \frac{3}{5}\alpha_1$ and:

$$\alpha_Y^{-1}(\mu_{Pl}) \approx 55.5. \tag{83}$$

A region of G–theory (where FRGGM works) also is shown in Fig.2.

If the point $\mu = \mu_G$ is very close to the Planck scale $\mu = \mu_{Pl}$, then according to Eqs.(26) and (78), we have:

$$\alpha^{-1}_{1st\,fam.} \approx \alpha^{-1}_{2nd\,fam.} \approx \alpha^{-1}_{3rd\,fam.} \approx \alpha_{Y,G}(\mu_G) \equiv \frac{\alpha_Y^{-1}(\mu_G)}{6} \approx 9.2, \tag{84}$$

which is almost equal to the value (59): $\alpha^{-1}_{crit,theor} \approx 8$ obtained by the "Parisi improvement method" and illustrated by Fig.4. This means that in the U(1)–sector of AntiGUT we have $\alpha_{i,G}(\mu_G)$ near the (multiple)critical point (MCP), and it is natural to assume that they obey the relations (80) at least approximately:

$$\alpha^{-1}_{Y,G}(\mu_G) : \alpha^{-1}_{2,G}(\mu_G) : \alpha^{-1}_{3,G}(\mu_G) \approx 1 : \sqrt{3} : 3/\sqrt{2} = 1 : 1.73 : 2.12, \tag{85}$$

For $\alpha^{-1}_{Y,G}(\mu_G) \approx 9.2$ (see Eq.(84)) the last equation gives the following result:

$$\alpha^{-1}_{Y,G}(\mu_G) : \alpha^{-1}_{2,G}(\mu_G) : \alpha^{-1}_{3,G}(\mu_G) = 9.2 : 15.9 : 19.5, \tag{86}$$

which confirms the AntiGUT–MPM prediction [39]:

$$\alpha^{-1}_{Y,G}(\mu_G) \approx 9.2 \pm 1, \quad \alpha^{-1}_{2,G}(\mu_G) \approx 16.5 \pm 1, \quad \alpha^{-1}_{3,G}(\mu_G) \approx 18.9 \pm 1. \tag{87}$$

Using the relations (77), (78) and values (87), we see that AntiGUT predicts:

$$\alpha^{-1}_{Y}(\mu_G) \approx 55.5 \pm 6, \quad \alpha^{-1}_{2}(\mu_G) \approx 49.5 \pm 3, \quad \alpha^{-1}_{3}(\mu_G) \approx 57.0 \pm 3, \tag{88}$$

confirming the "experimental" evolutions of $\alpha_{Y,2,3}(\mu)$ shown in Fig.2.
If we believe in FRGGM existing near the Planck scale, then the fall shown in Fig.5 also occurs near the Planck scale. Now the value of $\alpha^{-1}_{U(1)}(\mu_{Pl}) = \alpha^{-1}_{Y,G}(\mu_{Pl})$ is 6 times smaller than $\alpha^{-1}_{Y}(\mu_{Pl})$, and equal to the phase transition (critical) point $\alpha^{-1}_{U(1),crit}$.
We can estimate the width of the effective potential jump Δt at the Planck scale from Eq.(71) (see Section 7):

$$\Delta t \approx -\frac{\Delta \alpha^{-1}_{Y}}{2\alpha^{-1}_{Y}}|_{t=0}. \tag{89}$$

Using the values (75) and (84), we obtain:

$$\Delta t \approx \frac{\alpha_{Y,crit.Coul.} - \alpha_{Y,crit.conf.}}{2\alpha_{y,crot.conf}} \approx (3.7 - 9.2)/8 \approx -0.7, \tag{90}$$

Here we call $\alpha_{Y,crit.Coul.} \equiv \alpha_{Y,G}(\mu_G) \approx 9.2$ — the end point of the one–loop approximation straight line in Fig.5b, whereas $\alpha_{Y,crit.conf.} \approx 3.7$ is the beginning of the confinement phase. Eq.(90) means that the one–loop approximation, shown in Fig.2 for $\alpha^{-1}_{Y}(\mu)$, works up to the value

$$\mu_G \approx \mu_{Pl} e^{-0.35} \approx 0.7\mu_{Pl} \approx 8.5 \cdot 10^{18}\ \text{GeV}. \tag{91}$$

As a consequence, we see that in the AntiGUT approach there is a sharp jump of $\alpha^{-1}_{Y}(\mu)$ from its one–loop value at $\mu = \mu_G$ to its phase transition (critical) value at $\mu = \mu_{Pl}$.

9. The possibility of the Grand Unification Near the Planck Scale

In AntiGUT the FRGG breakdown was considered at $\mu_G \sim 10^{18}$ GeV. It was a significant point for MPM. In this case the evolution of the fine structure constants $\alpha_i(\mu)$, also in the region $\mu > \mu_G$, excludes the existence of the unification point (see Refs. [1]- [9], [16]- [18], [39]- [43]).

But the aim of this Section is to show that we can see quite different consequences of the extension of SM to FRGGM if G–group undergoes the breakdown to its diagonal subgroup (i.e. SM) not at $\mu_G \sim 10^{18}$ GeV, but at $\mu_G \sim 10^{14}$ or 10^{15} GeV, i.e. before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$ at $\mu \approx 10^{16}$ GeV.

In fact, we here want to illustrate the idea that with monopoles we can modify the running of the fine structure constants so much that unification can be arranged without needing SUSY. To avoid confinement of the monopoles at the cutoff scale we need FRGG or some replacement for it (see Section 4).

If we want to realize the behavior shown in Fig.5b for the function $\alpha_{Y,FRGG}^{-1}(\mu) = \alpha_{Y,G}^{-1}$ near the Planck scale we need the evolution of all $\alpha_i^{-1}(\mu)$ to turn away from asymptotic freedom and that could be achieved by having more fermions, i.e. generations, appearing above the μ_G–scale.

Now we shall suggest that such appearance of e.g. more fermions above μ_G is not at all so unlikely.

9.1. Guessing more particles in FRGGM

Once we have learned about the after all not so extremely simple SM, it is not looking likely that the fundamental theory — the true model of everything — should be so simple as to have only one single type of particle — the "urparticle" — unless this "urparticle" should be a particle that can be in many states internally such as say the superstring. We should therefore not necessarily assume that the number of species of particles is minimal anymore — as could have been reasonable in a period of science where one had only electron, proton and perhaps neutron, ignoring the photon exchanges which bind the atom together, so that only three particles were really there. Since now there are too many particles.

Looking at the problem of guessing the physical laws behind the Standard Model in this light, we should rather attempt to guess a set of species of particles to exist and the order of magnitudes of the numbers of such species which one should find at the various scales of energy. Indeed, the historical learning about the species of particles in Nature has

rather been that physicists have learned about many types of particles not much called for at first: It is only rather few of particle types, which physicists know today that have so great significance in the building of matter or other obviously important applications that one could not almost equally well imagine a world without these particles. They have just been found flavour after flavour experimentally studies often as a surprise, and if for instance the charmed quark was needed for making left handed doublets, that could be considered as a very little detail in the weak interactions which maybe was not needed itself.

These remarks are meant to suggest that if we should make our expectations to be more "realistic" in the sense that we should get less surprised next time when the Nature provides us with new and simingly not needed particles, we should rather than guessing on the minimal system of particles seek to make some more statistical considerations as to how many particle types we should really expect to find in different ranges of energy or mass.

To even crudely attack this problem of guessing it is important to have in mind the reasons for particles having the mass order of magnitudes. In this connection, a quite crucial feature of the SM "zoo" is that except for the Higgs particle itself, are mass protected particles, which have no mass in the limit when the Higgs field has no vacuum expectation value (VEV). In principle, you would therefore expect that SM particles have masses of order of the Higgs VEV. Actually they are mostly lighter than that by up to five orders of magnitude.

In the light of this mass protection phenomenon it would be really very strange to assume that there should be no other particles than in SM if one went up the energy scale and looked for heavier particles, because then what one could ask: Why should there be only mass protected particles? After all, there are lots of possibilities for making vector coupled Dirac particles, say. It would be a strange accident if Nature should only have mass protected particles and not a single true Dirac particle being vector coupled to even the weak gauge particles. It is much more natural to think that there are at higher masses lots of different particles, mass protected as well as not mass protected. But because we until now only could "see" the lightest ones among them, we only "saw" the mass protected ones.

In this light the estimate of how many particles are to be found with higher masses should now be a question of estimating how many particles turn out mass protected and getting masses which we can afford to "see" today.

We are already in the present article having the picture that as one goes up in the energy scale μ there will be bigger and bigger gauge group, which will thus be able to mass protect more and more particle types. Each time one passes a breaking scale at which a

part of the gauge group breaks down — or thought the way from infrared towards the ultraviolet: each time we get into having a new set of gauge particles — there will be a bunch of fermions which are mass protected to get — modulo small Yukawa couplings — masses just of the order of magnitude of the Higgs scale corresponding to that scale of diminishing of the gauge group.
If we for example think of the scale of breaking of our FRGG down to its diagonal subgroup, then we must expect that there should be some fermions just mass protected to that scale. However, these particles should be vector coupled w.r.t. the SM gauge fields, and only mass protected by the gauge fields of FRGG, which are not diagonal. We would really like to say that it would be rather strange if indeed there were no such particles just mass protected to that scale.

9.2. Quantitative estimate of number of particles in FRGGM

We might even make an attempt to perform a quantitative estimate of how many particle species we should expect to appear when we pass the scale $\mu = \mu_G$ going above μ_G. Of course, such an estimate can be expected to be very crude and statistical, but we hope that anyway it would be better than the unjustified guess that there should be nothing, although this guess could be in some sense the simplest one.
Since it is going to be dependent on the detailed way of arguing, we should like to make a couple of such estimates:
1) The first estimate is the guessing that, in analogy with the type of particle combinations which we have had in FRGGM already, we find the particles grouped into families which are just copies of the SM families. If only one of the gauge group families is considered, then two others are represented trivially on that family. We shall, however, allow that these families can easily be mirror families, in the sense that they have the weak doublets being the right handed particles and actually every gauge quantum number parity are reflected. But, by some principle, only small representations are realized in Nature and we could assume away the higher representations. Now let us call the number of mirror plus ordinary families which are present above the scale μ_G of the breakdown of FRGG to the diagonal subgroup as $N_{fam,tot}$, and assume that we have a statistical distribution as if these families or mirror families had been made one by one independently of each other with a probability of 50% for it being a mirror family and 50% for it being an ordinary left handed one. The order of the number of families survived under the scale μ_G should then be equal to the order of the difference between two samples of the Poisson distributed numbers with average $N_{fam,tot}/2$. In fact, we might consider respectively the number of mirror families and the number of genuine (left) families as such Poisson distributed numbers. The excess of the one type over the other one is then the number of low energy

scales surviving families, and the physicists living today can afford to see them. It is well known that crudely this difference is of the order of $\sqrt{N_{fam,tot}}$. But we know that this number of surviving families has already been measured to be 3, and so we expect that $\sqrt{N_{fam,tot}} = 3$, what gives $N_{fam,tot} \approx 9$. This would mean that there are 6 more families to be found above the diagonal subgroup breaking scale μ_G.

2) As an alternative way of estimating, we can say very crudely that the fermions above the scale μ_G could be mass protected by 3 times as many possible gauge quantum numbers, as far as there are three families of gauge boson systems in our FRGGM. If we use the "small representations assumption", then going from $\mu > \mu_G$ to $\mu < \mu_G$, two of three fermions loose their mass protection, and these two fermions obtain masses of order of the scale μ_G. But this means that 1/3 of all fermions survive to get masses below the scale μ_G and become "observable in practice". Again we got that there should be two times as many particles with masses at the diagonal subgroup breaking scale μ_G than at the EW scale. We got the same result by two different ways. Notice though that in both case we have used a phenomenologically supported assumption that Nature prefers very small representations. In fact, it seems to be true that the SM representations are typically the smallest ones allowed by the charge quantization rule:

$$\frac{Y}{2} + \frac{d}{2} + \frac{t}{3} = 0 \pmod 1, \tag{92}$$

where d and t are duality and triality, respectively.

9.3. The FRGGM prediction of RGEs. The evolution of fine structure constants near the Planck scale.

Let us consider now, in contrast to AntiGUT having the breakdown of G–group at $\mu_G \sim 10^{18}$ GeV, a new possibility of the FRGG breakdown at $\mu_G \sim 10^{14}$ or 10^{15} GeV (that is, before the intersection of $\alpha_2^{-1}(\mu)$ with $\alpha_3^{-1}(\mu)$, taking place at $\mu \sim 10^{16}$ GeV in SM). Then in the region $\mu_G < \mu < \mu_{Pl}$ we have three $SMG \times U(1)_{(B-L)}$ groups for three FRGG families, as in Refs. [1]- [9], [15]- [18], [31]- [35], [39]- [43]. In this region we have, according to the statistical estimates in Subsection 9.2, a lot of fermions (we designate their total number N_F, maybe different with $N_{fam.tot}$), mass protected or not mass protected, belonging to usual families or to mirror ones.

Also monopoles can be important in the vicinity of the Planck scale: they can give essential contributions to RGEs for $\alpha_i(\mu)$ and change the previously considered evolution of the fine structure constants.

Analogously to Eq.(15), obtained in Ref. [26], we can write the following RGEs for $\alpha_i(\mu)$

containing beta–functions for monopoles:

$$\frac{d(\log \alpha_i(\mu))}{dt} = \beta(\alpha_i) - \beta^{(m)}(\tilde{\alpha}_i), \qquad i = 1, 2, 3. \tag{93}$$

We can use the one–loop approximation for $\beta(\alpha_i)$ because α_i are small, and the two–loop approximation for dual beta–function $\beta^{(m)}(\tilde{\alpha}_i)$ by reason that $\tilde{\alpha}_i$ are not very small. Finally, taking into account that in the non–Abelian sectors of FRGG we have the Abelian artifact monopoles (see Section 8), we obtain the following RGEs:

$$\frac{d(\alpha_i^{-1}(\mu))}{dt} = \frac{b_i}{4\pi} + \frac{N_M^{(i)}}{\alpha_i}\beta^{(m)}(\tilde{\alpha}_{U(1)}), \tag{94}$$

where b_i are given by the following values:

$$b_i = (b_1, b_2, b_3) =$$

$$(-\frac{4N_F}{3} - \frac{1}{10}N_S, \quad \frac{22}{3}N_V - \frac{4N_F}{3} - \frac{1}{6}N_S, \quad 11N_V - \frac{4N_F}{3}). \tag{95}$$

The integers N_F, N_S, N_V are respectively the total numbers of fermions, Higgs bosons and vector gauge fields in FRGGM considered in our theory, while the integers $N_M^{(i)}$ describe the amount of contributions of scalar monopoles.

The Abelian monopole beta–function in the two–loop approximation is:

$$\beta^{(m)}(\tilde{\alpha}_{U(1)}) = \frac{\tilde{\alpha}_{U(1)}}{12\pi}(1 + 3\frac{\tilde{\alpha}_{U(1)}}{4\pi}). \tag{96}$$

Using the Dirac relation (9) we have:

$$\beta^{(m)} = \frac{\alpha_{U(1)}^{-1}}{48\pi}(1 + 3\frac{\alpha_{U(1)}^{-1}}{16\pi}), \tag{97}$$

and the group dependence relation (79) gives:

$$\beta^{(m)} = \frac{C_i {\alpha_i}^{-1}}{48\pi}(1 + 3\frac{C_i {\alpha_i}^{-1}}{16\pi}), \tag{98}$$

where

$$C_i = (C_1, C_2, C_3) = (\frac{5}{3}, \frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{3}). \tag{99}$$

Finally we have the following RGEs:

$$\frac{d(\alpha_i^{-1}(\mu))}{dt} = \frac{b_i}{4\pi} + N_M^{(i)}\frac{C_i {\alpha_i}^{-2}}{48\pi}(1 + 3\frac{C_i {\alpha_i}^{-1}}{16\pi}), \tag{100}$$

where b_i and C_i are given by Eqs.(95) and (99), respectively.

In our FRGG model:

$$N_V = 3, \tag{101}$$

because we have 3 times more gauge fields in comparison with the SM ($N_{fam} = 3$).
As an illustration of the *possibility* of unification in an FRGG–scheme with monopoles we propose some — strictly speaking adjusted — parameter choices within the likely ranges already suggested.
In fact, we take the total number of fermions $N_F = 2N_{fam.tot}$ (usual and mirror families), $N_{fam.tot} = N_{fam}N_{gen} = 3 \times 3 = 9$ (three SMG groups with three generations in each group), we have obtained (see Fig.6) the evolutions of $\alpha_i^{-1}(\mu)$ near the Planck scale by numerical calculations for $N_F = 18$, $N_S = 6$, $N_M^{(1)} = 6$, $N_M^{(2,3)} = 3$ and the following $\alpha_i^{-1}(\mu_{Pl})$:

$$\alpha_1^{-1}(\mu_{Pl}) \approx 13, \qquad \alpha_2^{-1}(\mu_{Pl}) \approx 19, \qquad \alpha_3^{-1}(\mu_{Pl}) \approx 24, \tag{102}$$

which were considered instead of Eqs.(81).
We think that the values $N_M^{(i)}$, which we have used here, are in agreement with Eqs.(38)–(42), and $N_S = 6$ shows the existence of the six scalar Higgs bosons breaking FRGG to SMG (compare with the similar descriptions in Refs. [16]- [18]).
Fig.6 shows the existence of the unification point.
We see that a lot of new fermions in the region $\mu > \mu_G$ and monopoles near the Planck scale change the one–loop approximation behaviour of $\alpha_i^{-1}(\mu)$ in SM. In the vicinity of the Planck scale these evolutions begin to decrease, approaching the phase transition (multiple critical) point at $\mu = \mu_{Pl}$ what means the suppression of the asymptotic freedom in the non–Abelian theories.
Here it is necessary to emphasize that these results do not depend on the fact, whether we have in Nature lattice artifact monopoles or the fundamental Higgs scalar particles having a magnetic charge (scalar monopoles).
Fig.7 demonstrates the unification of all gauge interactions, including gravity (the intersection of α_g^{-1} with α_i^{-1}), at

$$\alpha_{GUT}^{-1} \approx 27 \quad \text{and} \quad x_{GUT} \approx 18.4. \tag{103}$$

Here we can expect the existence of $[SU(5)]^3$ or $[SO(10)]^3$ unification (with/without SUSY). Of course, the results obtained in this article are preliminary and in future we are going to perform the spacious investigation of the unification possibility.
But calculating the GUT–values for one family fine structure constants in the case, considered in this paper, we have for i=5:

$$\alpha_{GUT,one\ fam}^{-1} = \frac{\alpha_{GUT}}{3} \approx 9, \tag{104}$$

what corresponds to the Abelian monopole (for the $SU(5)/Z_5$ lattice artifact) coupling with the total average monopolic fine structure constant $\tilde{\alpha}_{eff}$ and the "genuine" monopole

fine structure constant $\tilde{\alpha}_{genuine}$ as defined in Ref. [33] (see Section 5, Eq.(81) of this reference), determined from Eq.(79) and the Dirac relation by

$$\alpha_N^{-1} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}} \cdot 4\tilde{\alpha}_{eff} = \frac{2N}{N-1}\tilde{\alpha}_{genuine} \tag{105}$$

leading to

$$\tilde{\alpha}_{eff} = 9 \cdot \frac{2}{5 \cdot 4}\sqrt{\frac{4}{6}} = 0.7, \tag{106}$$

and

$$\tilde{\alpha}_{genuine} \approx \frac{2 \cdot 4}{4 \cdot 5} \cdot 9 \approx 3.6. \tag{107}$$

This value $\tilde{\alpha}_{eff}$ suggests that we may apply crude perturbation theory both for monopoles and charges ($\alpha_{eff} \approx 0.35$) if accepting that in the region (21).
Critical coupling corresponds (see (62)) to $\tilde{\alpha}_{eff,crit} = 1.20$ giving for $SU(5)/Z_5$:

$$\alpha_{5,crit}^{-1} \approx \frac{5}{2}\sqrt{\frac{6}{4}} \cdot 4 \cdot 1.20 \approx 14.5, \tag{108}$$

meaning that the unified couplings for $[SU(5)]^3$ are already of confinement strength.
Within the uncertainties we might, however, consider from (104) the coupling strength $\alpha_{GUT} \approx 1/9$ as being equal to the critical value $\alpha_{5,crit} \approx 1/14$ from Eq.(108).
If indeed the α_{GUT} were so strong as to suggest confinement at the unification point, it will cause the problem that the fundamental fermions in SU(5) representations would be confined and never show up at lower energy scales.
Assuming the appearance of SUSY, we can expect to have sparticles at the GUT-scale with masses:

$$M \approx 10^{18.4} \quad \text{GeV}. \tag{109}$$

Then the scale $\mu_{GUT} = M$, given by Eq.(109), can be considered as a SUSY breaking scale.
The unification theory with $[SU(5)]^3$–symmetry was suggested first by S.Rajpoot [68] (see also [69]).
Note that since our possibility of unification has the very high scale (109), it allows for much longer proton lifetime than corresponding models with more usual unification scales, arround 10^{16} GeV. This is true not only for proton decay caused by the gauge boson exchange, but also by the triplet Higgs exchange, since then the mass of the latter also may be put up in scale.
Considering the predictions of such a theory for the low–energy physics and cosmology, maybe in future we shall be able to answer the question: "Does the unification of $[SU(5)]^3$ or $[SO(10)]^3$ type (SUSY or not SUSY) really exist near the Planck scale?"

10. Discussion of Some Various Scenarios of Working MPP

In the present article we have — mostly, i.e.except for Subsection 8.2 — taken the picture that the inverse fine structure constants α_i^{-1} run very fast being smaller as μ gets bigger in the interval $[\mu_G, \mu_{Pl}]$. The old literature [39] and Subsection 8.2, however, rather take it that the running of the fine structure constants between μ_G and μ_{Pl} is minute. It could be taken, for instance by the philosophy of Ref. [39], as lowest order perturbation theory gives it because the scale ratio logarithm $\log(\mu_G/\mu_{Pl})$ is supposed so small that the details of β–functions are hardly of any importance. This point of view is in disagreement with the expectation of quick strong jump put forward in Subsection 8.2. The argument for there having to be such a jump in α_i^{-1} just before $t = 0$ (when $\mu = \mu_{Pl}$) is based on *the assumption that there is a phase transition at the Planck scale as a function of* t. In principle, however, the occurrence of the jump depends on hard computations *and on (precisely) how big the running* $\alpha_i^{-1}(t)$ *are when approaching the Planck scale*. Also that depends again on what (matter) particles exist and influence the β–function (monopoles, extra fermions above μ_G, etc.).

We would like here to list some options for obtaining MCP–agreement in one version or the other one combined with pictures of associated matter:

1) The option of Ref. [39] is to use

1a) the MCP-I definition;

1b) the approximation of very little running between μ_G and μ_{Pl}.

In this interpretation MCP-I, we do not really think of phases as a function of the scale, but only as a function of the bare parameters, e.g. the bare fine structure constants. Rather we have to estimate what are the bare (or the Planck scale) couplings at the phase transition point conceived of as a transition point in the coupling constant space, but not as a function of scale. Here we may simply claim that the Parisi improvement approximation (58) is close to the calculation of the "bare" coupling. The Parisi improvement namely calculates an effective coupling as it would be measured by making small tests of the effective action of the theory on a very local basis just around one plaquette. Indeed, we expect that if in the lattice model one seeks to measure the running coupling at a scale only tinily under the lattice scale, one should really get the result to be the Parisi improved value corrected by a tiny running only.

Since even in this option 1), corresponding to Ref. [39], we want a phase transition which we here like to interpret as due to monopole condensation — in an other phase though — we need to have monopoles. Thus, we get the problem of avoiding these monopoles in the β–function except for an extremely small amount of scales. In the phase in which we

live it is suggested that the monopoles are made unimportant in the β–function below μ_G by being *confined* by the Higgs field VEVs which break the FRGG down to the SMG (possibly extended with $U(1)_{(B-L)}$). Since the monopoles, we need, are monopoles for the separate family-gauge-groups and since most of the latter are Higgsed at the μ_G–scale, we expect the monopoles to be confined into hadron–like combinations/bound states by the Higgs fields at the scale μ_G. Thus if μ_G is close to μ_{Pl} — in logarithm — there will be very little contribution of the β–function running due to monopoles.
In such a picture even extra fermions, as suggested in Subsection 9.1, and the SM particles will not give much running between μ_G and μ_{Pl}.
Now there seems to be a discrepancy with calculations in the MCP-II approach (Refs. [32]-[35] and [52]) which gave the phase transition point what in (59) is called $\alpha^{-1}_{crit.lat} \approx 5$. But now we must remember that this value was calculated as a long distance value, i.e. not a "bare" value of the fine structure constant in the lattice calculations of Jersak et al. [52]. Also our Coleman–Weinberg type calculation of it [32]- [35] was rather a calculation of the renormalised (or dressed) coupling than of a bare coupling.
The disagreement between $\alpha^{-1}_{crit.lat} \approx 5$ (= the critical coupling) and the bare coupling in the picture 1 sketched here is suggested to be due to renormgroup running in other phases caused by monopoles there.
In fact, we have in this picture 1 other phases — existing somewhere else or at some other time — in which there is no breaking down to the diagonal subgroup, as in our phase. In such phases the monopoles for the family groups can be active in the renormalisation group over a longer range of scales provided they are sufficiently light. Assuming that the Schwinger's renormalisation scheme is wrong and that the running due to monopoles make the coupling weaker at the higher scales than at the lower energies, it could be *in the other phase* a value corresponding to $\alpha^{-1}_{crit.lat} \approx 5$ in Eq.(59) for some lower μ, say at the mass of monopoles, while it is still the Parisi improved value at the bare or fundamental (Planck) scale.
Preliminary calculations indicate that requiring a large positive value of the running $\lambda(\mu)$ at the cut off scale, i.e. a large positive bare λ_0 in a coupling constant combination at the phase transition makes the value $\alpha_{crit.lat} \approx 0.2$ (or $\alpha_{crit} \approx 0.208$ given by Refs. [33]- [35]) run to a bare α rather close to the Parisi improvement phase transition coupling value (58).
In order to explain this picture we need to talk about at least three phases, namely, two phases without the Higgs fields performing the breaking the group G to the diagonal subgroup and our own phase. In one of these phases the monopoles condense and provide the "electric" confinement, while the other one has essentially massless gauge particles in the family gauge group discussed. In reality, we need a lot of phases in addition to

our own because we need the different combinations of the monopoles being condensed or not for the different family groups.
2) The second picture uses MCP-II, which interprets the phase transition required by any MCP–version as being a phase transition as a function of the scale parameter, μ or t, and the requirement of MCP-II is that it occurs just at the fundamental scale, identified with the Planck scale.
Since the gauge couplings, if their running is provided (perturbatively) only by the SM particles, would need $\alpha_{crit}^{-1} \approx 9$ rather than $\alpha_{crit}^{-1} \approx 5$, further β–function effects are needed.
Once the couplings get — as function of t — sufficiently strong, then, of course, perturbation theory gets unjustified, however, and higher orders or nonperturbative effects are important. Indeed, higher order seems to help strengthening of the electric couplings approaching the Planck scale. But most crucially it is argued that the very fact of finding a phase transition at μ_{Pl}, as postulated by MCP-II in itself — see Sections 7 and 9 — suggests that there must be a rather quick running just below μ_{Pl}. This picture 2 has such a property because of the extra fermions — or whatever — which bring the strength of the fine structure constants up for $\mu > \mu_G$, so that nonperturbative and higher order effects can take over and manage to realize MCP-II.

11. Conclusions

In the present paper we have shown that the existence of monopoles in Nature leads to the consideration of the Family Replicated Gauge Groups (FRGG) of symmetry as an extension of the Standard Model in the sense that the use of monopoles corresponding to the family replicated gauge fields can bring the monopole charge down from the unbelievably large charge which it gets in the simple SM, according to the Dirac relation. We have considered the case when our (3+1)–dimensional space–time is discrete and has a lattice–like structure. As a consequence of such an assumption, we have seen that the lattice artifact monopoles play an essential role in FRGGM near the Planck scale: then these FRGG–monopoles give perturbative contributions to β–functions for the renormalization group equations written for both, electric and magnetic fine structure constants, and change the evolution of $\alpha_i(\mu)$ in the vicinity of the Planck scale.
We have shown that in FRGG–model monopoles can produce the phase transition (multiple critical) point at the Planck scale, indicating the realization in Nature of the Multiple Point Principle (MPP) by D.L.Bennett and H.B.Nielsen [39].
We have calculated the critical couplings $\alpha_{i,crit} = \alpha_i(\mu_{Pl})$ using the results obtained previously in the Higgs Monopole Model [33]- [35] where the lattice artifact monopoles

are approximated by the Higgs scalar fields.

Here it is necessary to point out the coincidence of the results, obtained in this paper, if we have in Nature lattice artifact monopoles, or the Higgs scalar magnetically charged fields.

We discussed two scenarios implementing Multiple Critical Point (MCP) existence.

It was also considered that the breakdown of FRGG at $\mu_G \sim 10^{18}$ GeV leads to the AntiGUT by C.D.Froggatt and H.B.Nielsen [9] with the absence of any unification up to the scale $\mu \sim 10^{18}$ GeV.

Finally we have investigated the case when the breakdown of FRGG undergoes at $\mu_G \sim 10^{14}$ GeV and is accompanied by a lot of extra fermions in the region $\mu_G < \mu < \mu_{Pl}$. These extra fermions suppressing the asymptotic freedom of non-Abelian theories lead, together with monopoles, to the possible existence of unification of all interactions including gravity at $\mu_{GUT} = 10^{18.4}$ GeV and $\alpha_{GUT}^{-1} = 27$.

We discussed the possibility of the existence of the family replicated unifications $[SU(5)]^3$ or $[SO(10)]^3$ (SUSY or not SUSY).

We have given a special case of new type of unification. The realistic family replicated unification needs a serious program of investigations showing the consequences for the low–energy physics and cosmology what we plan to do in future.

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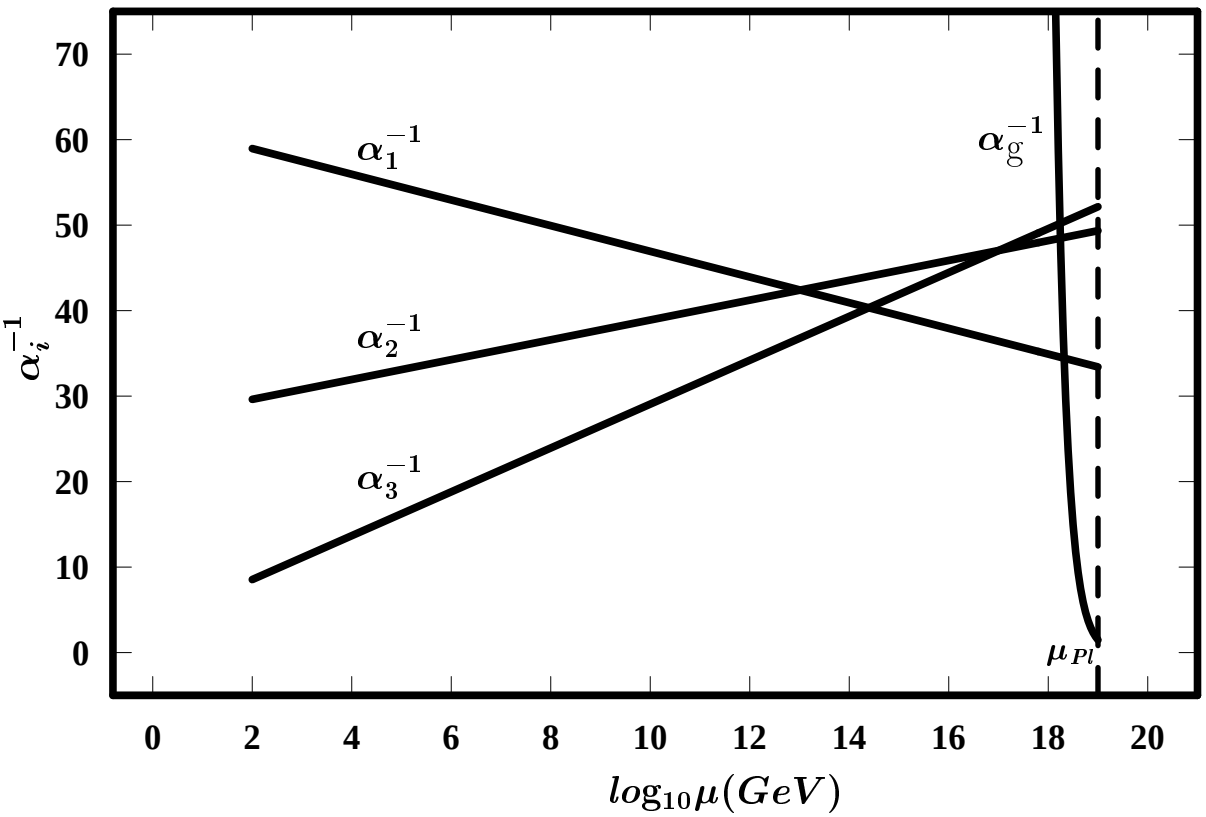

Fig.1: The evolution of three inverse running constants $\alpha_i^{-1}(\mu)$, where i=1,2,3 correspond to U(1), SU(2) and SU(3) groups of the SM. The extrapolation of their experimental values from the Electroweak scale to the Planck scale was obtained by using the renormlization group equations with one Higgs doublet under the assumption of a "desert". The precision of the LEP data allows to make this extrapolation with small errors. The intersection of the inverse "gravitational fine structure constant" $\alpha_g^{-1}(\mu)$ with $\alpha_1^{-1}(\mu)$ occurs at the point $(x_0,\ \alpha_0^{-1})$:

$\alpha_0^{-1} \approx 34.4$, and $x_0 \approx 18.3$, where $x = \log_{10}\mu(GeV)$.

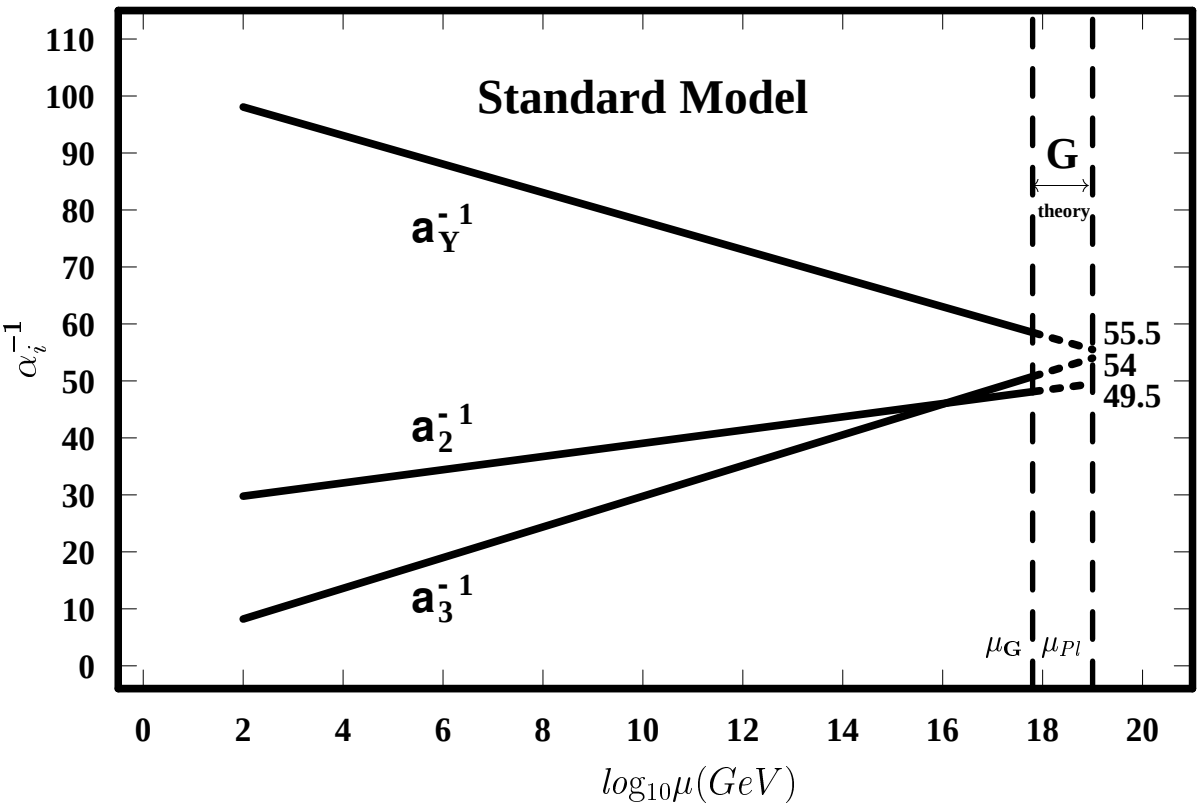

Fig.2: The evolution of three inverse running constants $\alpha_i^{-1}(\mu)$, where i=1,2,3 correspond to U(1), SU(2) and SU(3) groups of the SM. The extrapolation of their experimental values from the Electroweak scale to the Planck scale was obtained by using the renormlization group equations with one Higgs doublet under the assumption of a "desert". The precision of the LEP data allows to make this extrapolation with small errors. AGUT works in the region $\mu_G \leq \mu \leq \mu_{Pl}$.

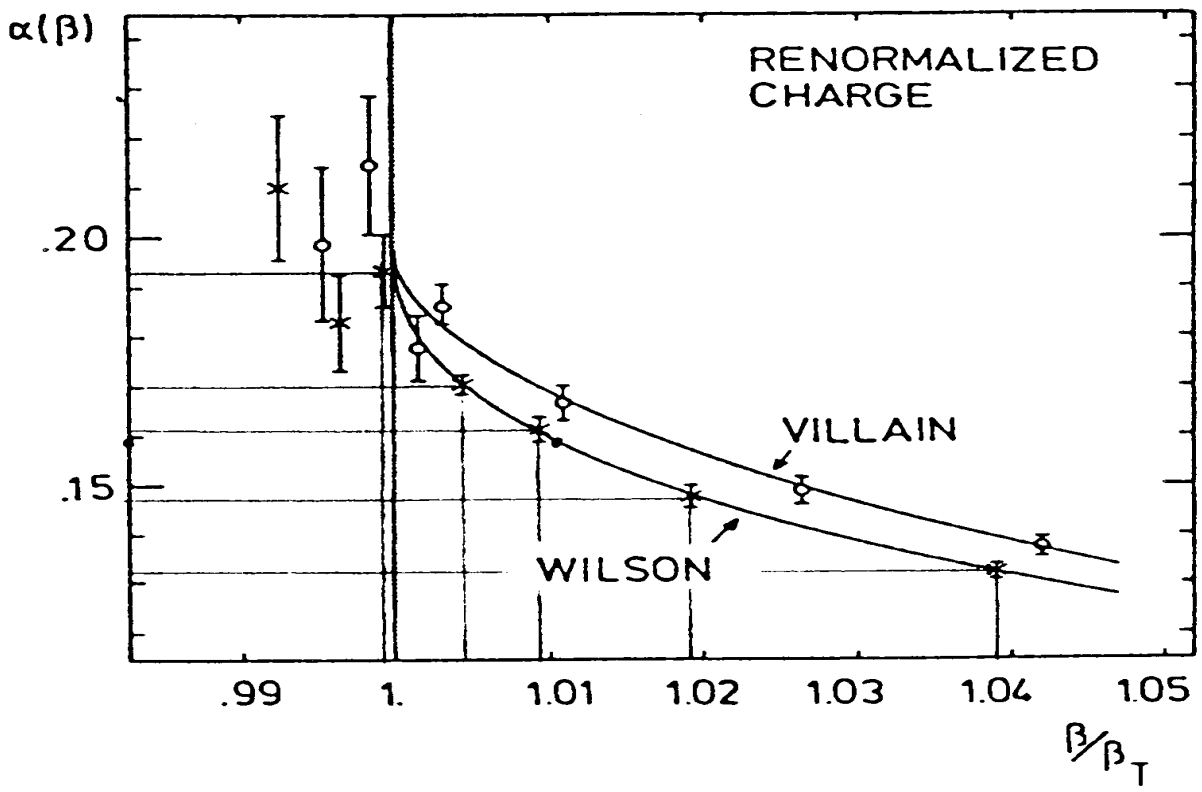

Fig.3: The renormalized electric fine structure constant plotted versus β/β_T for the Villain action (circles) and the Wilson action (crosses). The points are obtained by the Monte–Carlo simulations method for the compact QED.

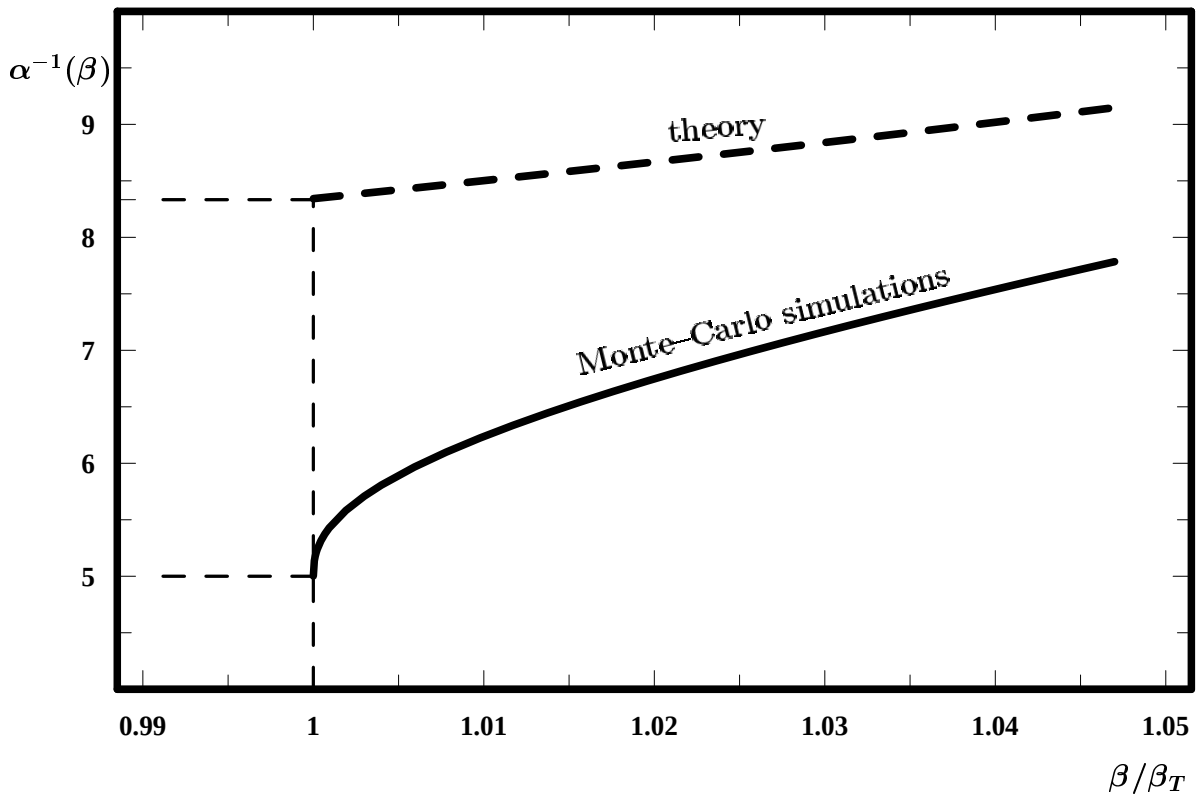

Fig.4: The behavior of the inverse effective electric fine structure constant in the vicinity of the phase transition point plotted versus β/β_T for the simple Wilson lattice action. The dashed curve corresponds to the theoretical calculations by the "Parisi improvement method".

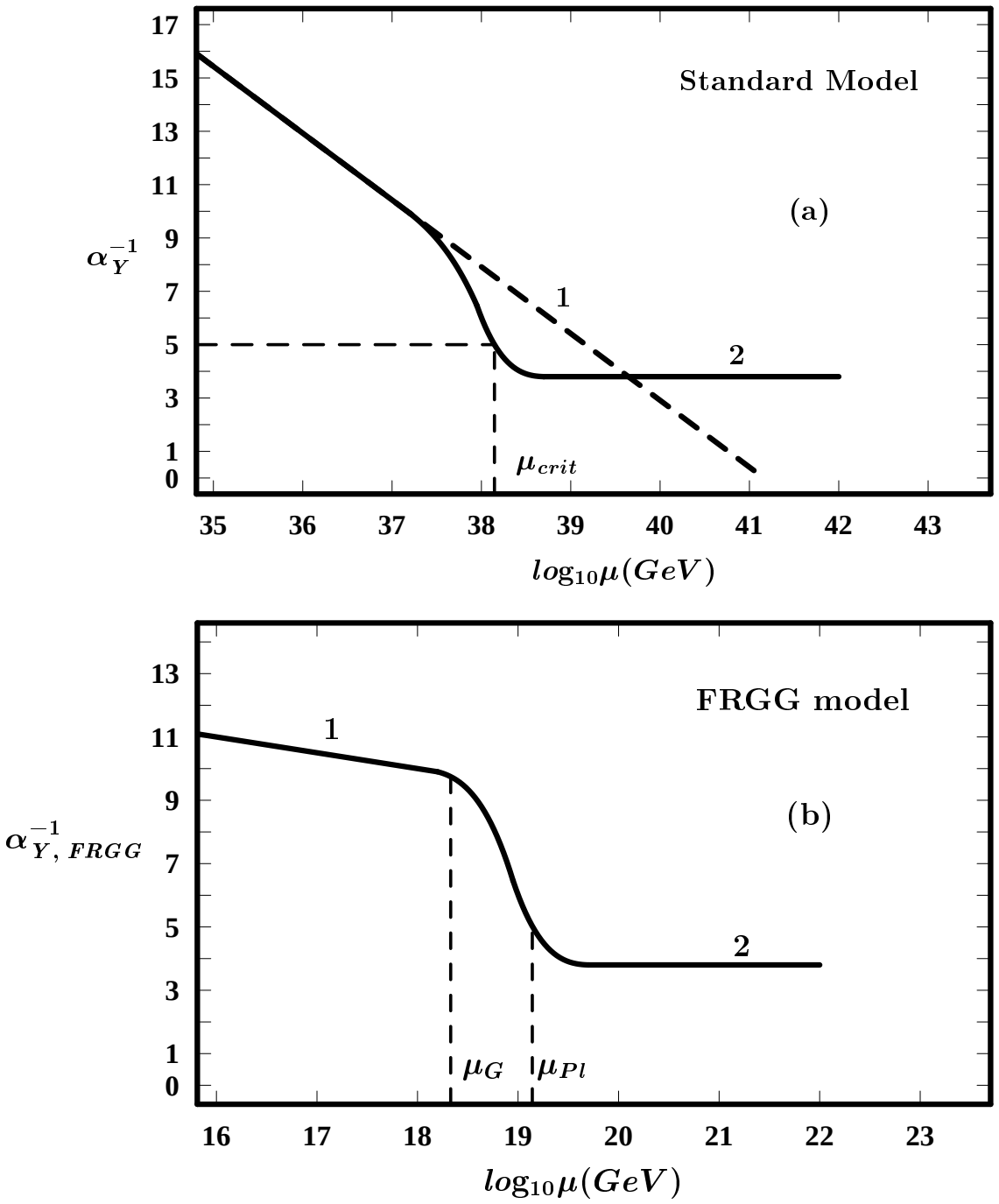

Fig.5a,b: (a) The evolution of the $U(1)_Y$ fine structure constant in the Standard Model with influence of monopoles at very high energies; $\mu = \mu_{crit}$ is a critical point corresponding to the phase transition "confinement – deconfinement". (b) The evolution of the one family fine structure constant in the Family replicated gauge group model with the phase transition point at $\mu_{crit} = \mu_{Pl}$, and the FRGG symmetry breaking point at $\mu = \mu_G$.

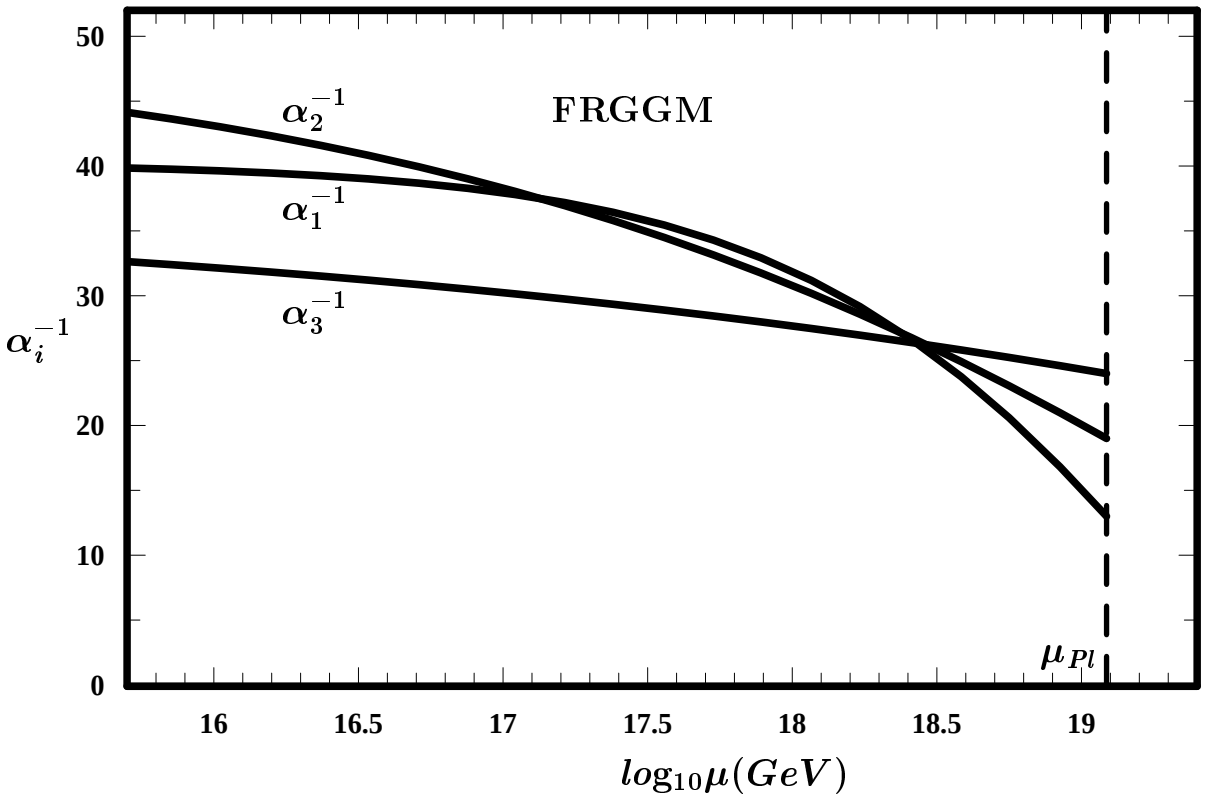

Fig.6: The evolution of fine structure constants $\alpha_{1,2,3}^{-1}(\mu)$ beyond the Standard model in the Family replicated gauge group model (FRGGM) with influence of monopoles near the Planck scale.

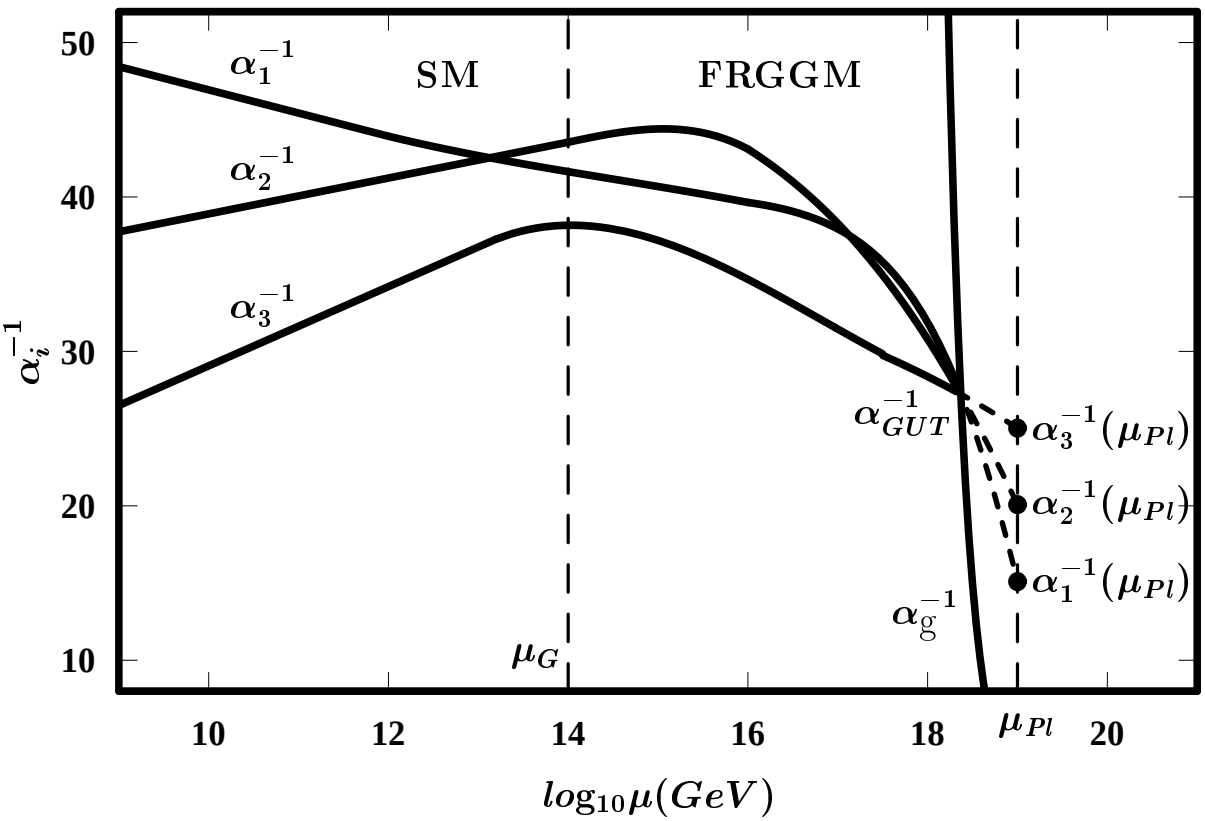

Fig.7: The evolution of $\alpha_{1,2,3}^{-1}(\mu)$ in the Standard Model (SM) and beyond it. The breakdown of FRGG occurs at $\mu_G \sim 10^{14}$GeV. It is shown the possibility of the $[SU(5)]^3$ SUSY unification of all gauge interactions, including gravity, at $\alpha_{GUT}^{-1} \approx 27$ and $x_{GUT} \approx 18.4$, where $x = \log_{10}\mu(\mathrm{GeV})$.

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