Bicocca-FT-03-7
DCPT/03/32
DESY 03-027
IPPP/03/16

arXiv:hep-ph/0304272 v1 29 Apr 2003

On a possible measurement of α_S from B-$\overline{B}$ correlations in Z^0 decay

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Abstract

Motivated by recent preliminary results from the SLD Collaboration on the measurement of angle-dependent B-$\overline{B}$ energy correlations in $Z^0 \to b\bar{b}$ events, we propose a class of observables that can be computed as a power expansion in the strong coupling constant α_S, of order α_S at the Born level and that can be used for a precision measurement of $\alpha_S(M_Z)$. We compute their next-to-leading order $\mathcal{O}(\alpha_S^2)$ corrections in the strong coupling constant, including exactly quark-mass effects. We show that, in the theoretical evaluation of these quantities, large logarithms of the ratio of the mass of the final quark over the centre-of-mass energy cancel out. Thus, these variables have a well-behaved perturbative expansion in $\alpha_S(M_Z)$. We study the theoretical uncertainties due to the renormalization-scale dependence and the quark-mass scheme and we address the question of which mass scheme is more appropriate for these variables.

1 Introduction

Hadronic final states in e^+e^- annihilation have been intensively studied in recent years, both at LEP and SLC, in order to perform QCD tests and to determine α_s [1]. Interesting studies on bottom flavoured hadronic events have also been performed, in order to test the flavour independence of the strong coupling constant and to search for "running mass" effects [2, 3, 4, 5, 6, 7]. However, up to now, heavy flavoured jets have been studied with the same tools used to study light flavoured jets. For example, the same shape variables used for light-quark jets (like thrust, cluster multiplicities, etc.) have been applied to heavy flavoured events. On the other hand, heavy flavoured events may have some advantages, because the kinematic distributions of the produced heavy quark can be computed theoretically with much better accuracy than the corresponding light-quark quantities. Furthermore, the kinematic of the final-state heavy flavoured hadron is more closely related to that of the final-state quark, since hadronization effects involve energy scales that are much smaller than the quark mass.

Recently, the SLD Collaboration [8] has published preliminary data on the double inclusive B-$\overline{B}$ production, and studied quantities (proposed in Ref. [9]) that carry information about the angular dependence of the B-$\overline{B}$ energy correlations.

The main purpose of the present work is to demonstrate that a slight variant of the quantities introduced in Ref. [9] has suitable properties to perform a novel, heavy-flavour-specific determination of the strong coupling constant. We will demonstrate the following facts:

- The observables we are considering have a well-defined, finite expansion in the strong coupling constant.
- Non-perturbative corrections of order $\Lambda_{\rm QCD}/m$, where m is the heavy-quark mass, cancel in these variables. The only remaining non-perturbative corrections are of order $\Lambda_{\rm QCD}/E_{\rm CM}$, where $E_{\rm CM}$ is the centre-of-mass energy.
- Potentially large terms, enhanced by powers of $\log(E_{\rm CM}/m)$, cancel at all orders in perturbation theory, provided α_s is evaluated at a scale near $E_{\rm CM}$. This cancellation holds as long as one can neglect multiple heavy-flavour pair production in our process, which is certainly the case for b production in Z^0 decays.

The quantities we are considering have a well defined massless limit, provided one can neglect multiple heavy-flavour pair production. This limitation is what makes these quantities heavy-flavour specific. In fact, only for heavy flavours multiple pair production is truly negligible. Thus, although these quantities have formally a well defined massless limit, they cannot be used in a straightforward way to study light-flavour production.

Tools to compute heavy-flavour production at the next-to-leading-order (NLO) level have been available in the literature for the past few years [10, 11, 12, 13, 14]. We have used our calculations [12, 13, 14] in order to compute the NLO corrections to the observables we propose.

In Sec. 2, we define the variables we are considering and show that their perturbative expansion in terms of the strong coupling constant $\alpha_{\rm S}\,(M_Z)$ is well behaved, and free from large logarithms of the ratio of the quark mass over the centre-of-mass energy. The SLD Collaboration has provided us with (preliminary) data for these new variables [15], and thus we have been able to check our NLO predictions against the data.

In Sec. 3 we investigate the massless limit of these variables, and show explicitly that potentially large logarithms cancel out.

In Sec. 4 we study theoretical uncertainties of our results. We analyze the dependence of the NLO results on the renormalization scale and compare the results obtained with the pole-mass definition to those obtained using the $\overline{\rm MS}$ mass, addressing the question of which of the two mass definitions is more suitable for this calculation. Finally, in Sec. 5 we give our conclusions.

In Appendix A, we collect some leading order (LO) formulae used in our analysis, and in Appendix B we comment on the renormalization scheme used in this paper.

2 Double-inclusive heavy-quark cross section

Heavy flavoured hadron production proceeds through the process

$$e^+e^- \to Z^0/\gamma \to Q + \overline{Q} + X\,,$$

(where Q is a heavy quark of mass m) followed by the hadronization of the final heavy quarks. In the following we will focus upon B meson production.

Defining the B hadron scaled energy $x_{_B} = 2E_{_B}/E_{\rm CM}$, we consider the following quantities [1]

$$D_i = \frac{1}{\sigma_{_B}} \int x_{_B}^{i-1} \frac{d\sigma_{_B}}{dx_{_B}}\, dx_{_B}\,, \tag{2.1}$$

$$D_{ij}(\phi) = \frac{1}{\sigma_{_B}} \int x_{_{B_1}}^{i-1} x_{_{B_2}}^{j-1} \frac{d^3\sigma_{_B}}{dx_{_{B_1}} dx_{_{B_2}} d\cos\phi}\, dx_{_{B_1}} dx_{_{B_2}}\,, \tag{2.2}$$

$$G_{ij}(\phi) = \frac{D_{ij}(\phi)}{D_i D_j}\,, \tag{2.3}$$

where $d\sigma_{_B}/dx_{_B}$ is the differential cross section for the final-state hadron production, ϕ is the angle between the two hadrons, $\sigma_{_B}$ the total cross section and $x_{_{B_{1/2}}}$ are the scaled energies of the two final B hadrons.

The quantities studied by the authors of [9] are related to the ratios G_{ij}/G_{11} by a calculable perturbative factor (P_iP_j/P_1^2 in the notation of [9]). In Ref. [8] these ratios were compared to experimental results. However, these quantities have a few drawbacks. First of all, they depend upon a "theoretical" factor. It would be preferable to deal with quantities that have an experimental definition, free of theoretical assumptions. Second and most important, they have a perturbative expansion that starts at leading order with

[1] The quantities D_i and D_{ij} were also considered in [9] and the ratios G_{ij} in [16].

a constant. Therefore, even when corrected at the NLO level, they would allow only for a LO determination of $\alpha_{\rm S}$. Conversely, they can be used to test the production mechanism, since in the ratios $\alpha_{\rm S}$ cancels at first approximation.

In this work, we claim that the quantities G_{ij}, defined in Eq. (2.3), should instead be studied in order to perform precision QCD tests. We will in fact show in the following that these quantities have a perturbative expansion in $\alpha_{\rm S}(E_{\rm CM})$ with coefficients that remain finite even in the limit of large $E_{\rm CM}/m$ ratios.

We write the hadronic differential cross sections as

$$\frac{d\sigma_B}{dx_B} = \int \frac{d\sigma_b}{dx_b} \mathcal{D}^B_{\rm NP}(x)\, \delta(x_B - x x_b)\, dx_b dx \,, \tag{2.4}$$

$$\frac{d^3\sigma_B}{dx_{B_1} dx_{B_2} d\cos\phi} = \int \frac{d^3\sigma_b}{dx_{b_1} dx_{b_2} d\cos\phi} \mathcal{D}^B_{\rm NP}(x_1) \mathcal{D}^B_{\rm NP}(x_2) \times \delta(x_{B_1} - x_1 x_{b_1})\, \delta(x_{B_2} - x_2 x_{b_2})\, dx_{b_1} dx_{b_2} dx_1 dx_2 \,, \tag{2.5}$$

where $d\sigma_b$, x_b, $x_{b_{1/2}}$ are the quark differential cross section and the quark scaled energies. The quark differential cross sections $d\sigma_b$ are calculable order-by-order in perturbative QCD, since the heavy-quark mass m acts as a cut-off for final-state collinear singularities. However, in order to go from the QCD partonic cross section $d\sigma_b/dx_b$ to the hadronic one $d\sigma_B/dx_B$, we have to take into consideration non-perturbative effects, of order $\Lambda_{\rm QCD}/m$, that are present in the hadronic cross section. We assume that all these effects are described by a non-perturbative fragmentation function $\mathcal{D}^B_{\rm NP}$.

The quark differential cross sections, although calculable, contain logarithmically enhanced terms of the form $\alpha_{\rm S}^k \log^l(m/E_{\rm CM})$, so that their perturbative expansion is not well behaved for $E_{\rm CM} \gg m$.

In the quantities $G_{ij}(\phi)$ these large logarithms cancel, together with the dependence upon the non-perturbative fragmentation function. This is a consequence of the factorization theorem, if one makes the additional assumption that secondary production of heavy-flavour pairs is negligible, which is in fact the case at the Z^0 peak.

The factorization theorem states that mass singularities in inclusive cross sections can be absorbed into process independent, universal factors. In our case, mass singularities are precisely the $\log(E_{\rm CM}/m)$ terms. Thus, for $E_{\rm CM} \gg m$ we have

$$\frac{d\sigma_b}{dx_b} = \int \frac{d\hat\sigma}{d\hat x_b} \mathcal{D}_b(z)\, \delta(x_b - z\hat x_b)\, d\hat x_b dz \,, \tag{2.6}$$

$$\frac{d^3\sigma_b}{dx_{b_1} dx_{b_2} d\cos\phi} = \int \frac{d^3\hat\sigma}{d\hat x_{b_1} d\hat x_{b_2} d\cos\phi} \mathcal{D}_b(z_1) \mathcal{D}_b(z_2) \times \delta(x_{b_1} - z_1 \hat x_{b_1})\, \delta(x_{b_2} - z_2 \hat x_{b_2})\, d\hat x_{b_1} d\hat x_{b_2} dz_1 dz_2 \,, \tag{2.7}$$

where the hatted cross sections do not depend upon m, and all terms that are large in the $m \to 0$ limit (i.e. terms proportional to powers of $\log(E_{\rm CM}/m)$) are absorbed in the fragmentation functions $\mathcal{D}_b(z)$ [2]. Inserting Eqs. (2.6) and (2.7) into Eqs. (2.4) and (2.5)

[2] Notice that if we had not assumed a negligible secondary heavy-flavour pair production, we should have introduced also the gluon and light-quark component of the heavy-quark fragmentation function.

and considering the definitions in Eqs. (2.1) and (2.2), we get

$$D_i = \left(\int x^{i-1}\mathcal{D}^B_{\rm NP}(x)\,dx\right)\left(\int z^{i-1}\mathcal{D}_b(z)\,dz\right)\,\frac{1}{\sigma_B}\int \hat{x}_b^{i-1}\frac{d\hat{\sigma}}{d\hat{x}_b}\,d\hat{x}_b, \tag{2.8}$$

$$\begin{aligned} D_{ij}(\phi) &= \left(\int x_1^{i-1}\mathcal{D}^B_{\rm NP}(x_1)\,dx_1\right)\left(\int x_2^{j-1}\mathcal{D}^B_{\rm NP}(x_2)\,dx_2\right) \\ &\times\left(\int z_1^{i-1}\mathcal{D}_b(z_1)\,dz_1\right)\left(\int z_2^{j-1}\mathcal{D}_b(z_2)\,dz_2\right) \\ &\times\frac{1}{\sigma_B}\int \hat{x}_{b_1}^{i-1}\hat{x}_{b_2}^{j-1}\frac{d^3\hat{\sigma}}{d\hat{x}_{b_1}d\hat{x}_{b_2}d\cos\phi}\,d\hat{x}_{b_1}d\hat{x}_{b_2}\,, \end{aligned} \tag{2.9}$$

and we see that, in the ratio defining $G_{ij}(\phi)$ (see Eq. (2.3)), the $\mathcal{D}_b$ factors (that contain mass singularities) cancel out, leaving a finite expression [3].

Since non-perturbative effects cancel in the definition of G_{ij}, from now on we will make no distinction between the quantities D_i, D_{ij} defined at the quark and at the hadron level. With the available codes for the calculation of NLO corrections in the production of heavy quarks [13, 14] at e^+e^- colliders [4], we can compute the contributions up to order $\alpha_{\rm S}^2$ to the quantities D_i, $D_{ij}(\phi)$ and $G_{ij}(\phi)$. We can write the perturbative expansions for D_i and $D_{ij}(\phi)$ for massive final-state quarks, renormalized in the pole-mass scheme, in the region away from the two-jet final state (the two parton final state gives a contribution proportional to $\delta(\phi-\pi)$) as

$$D_i = 1 - a_i\frac{\alpha_{\rm S}(\mu)}{2\pi} + \mathcal{O}\left(\alpha_{\rm S}^2\right)\,, \tag{2.10}$$

$$\begin{aligned} D_{ij}(\phi) &= B_{ij}(\phi)\frac{\alpha_{\rm S}(\mu)}{2\pi} + \left[C_{ij}(\phi) + 2\pi\,b_0\,B_{ij}(\phi)\log\left(\frac{\mu^2}{E^2_{\rm CM}}\right)\right]\left(\frac{\alpha_{\rm S}(\mu)}{2\pi}\right)^2 \\ &+ \mathcal{O}(\alpha_{\rm S}^3), \qquad\qquad \text{for } \phi \neq \pi\,, \end{aligned} \tag{2.11}$$

so that the expression for G_{ij} becomes

$$\begin{aligned} G_{ij}(\phi) &= B_{ij}(\phi)\frac{\alpha_{\rm S}(\mu)}{2\pi} + \Bigg\{C_{ij}(\phi) \\ &+ \left[2\pi\,b_0\log\left(\frac{\mu^2}{E^2_{\rm CM}}\right) + a_i + a_j\right]B_{ij}(\phi)\Bigg\}\left(\frac{\alpha_{\rm S}(\mu)}{2\pi}\right)^2 + \mathcal{O}(\alpha_{\rm S}^3)\,, \end{aligned} \tag{2.12}$$

where

$$b_0 = \frac{11C_A - 4n_f T_F}{12\pi}, \tag{2.13}$$

μ is the renormalization scale, $C_A = N_c = 3$ for an SU(3) gauge theory, $T_F = 1/2$ and n_f is the number of active flavours.

[3] Of course, this holds if we neglect terms that are suppressed by powers of m. Terms suppressed by powers of m and enhanced by powers of $\log(E_{\rm CM}/m)$ may still (and in fact are) present in G_{ij}. Furthermore, uncalculable effects of order $\Lambda_{\rm QCD}/E_{\rm CM}$ may still be present.

[4] A version of the program described in [13] was used which implements the dipole subtraction method for massive partons [17] to combine real and virtual corrections.

We observe that expanding the ratio in powers of α_s is essential to implement the cancellation of large logarithms in G_{ij} order-by-order in the perturbative series. If we had not done so, the numerator and denominator of G_{ij} would contain large logarithms at some fixed order, and their ratio would be misleading (i.e. it would be sensitive to higher order terms with higher powers of logarithms).

In Fig. 1 we have plotted G_{ij} at leading and next-to-leading order in α_s, for $i, j = 1, 2, 3$, at a renormalization scale $\mu = E_{CM} = 91.2$ GeV, using $\alpha_s(E_{CM}) = 0.12$, with $n_f = 5$ active flavours. The NLO curves are in good agreement with the SLD preliminary data.

Conversely, the SLD data can be used to extract the value of α_s. In fact, from the knowledge of the numerical values of the coefficients of the α_s and α_s^2 contributions in Eq. (2.12), we can fit $G_{ij}(\phi)$ to the data, keeping α_s as a free parameter.

We have computed tables for the α_s and α_s^2 coefficients of Eq. (2.12), that can be used in a more detailed analysis. These tables (and the programs to generate them) can be obtained from the authors.

2.1 Secondary *b*-quark and the running coupling

We should mention here that, in the calculations of Ref. [14], the b quark is treated as a heavy particle, and thus does not contribute to the running of the coupling constant. Thus, in principle, one should use this calculation in conjunction with the 4-flavour coupling constant $\alpha_s^{(4)}$. Since we are dealing with Z^0 decays, however, secondary b-quark pairs may in fact be produced (from gluon splitting), and when this happens, they behave, in some sense, as light flavours, contributing terms that effectively modify the running of α_s.

In Fig. 2, we show the Feynman graphs for secondary heavy-flavour production, giving rise to four massive-quark final state. The graphs in Fig. 2 also appear with the two identical quarks and/or the two identical antiquarks exchanged. When taking the amplitude squared, this gives rise to two interference terms. Their contribution is known to be extremely small, and will be neglected in the following.

In addition, virtual b-quark loops arise only in the self-energy insertion of the emitted (on-shell) gluon. Their contribution is zero in the renormalization scheme we use, since, in this scheme, the heavy flavour decouples for small momenta (see Appendix B for more details). Thus, the only effect of the heavy flavour is given by the graphs of Fig. 2.

In the quantities we are considering, the possibility of detecting secondary (instead of primary) B mesons has little impact, since secondary B's are very soft. If we make the approximation of neglecting effects that are suppressed by powers of the mass in the secondary b line, the contribution of the graphs of Fig. 2 (without interference) is easily included using the following prescription:

- compute the cross section with the number of light flavours n_{lf} equal to 5;
- use the 5 flavour coupling $\alpha_s^{(5)}$;
- do not include any other graph with secondary heavy flavours.

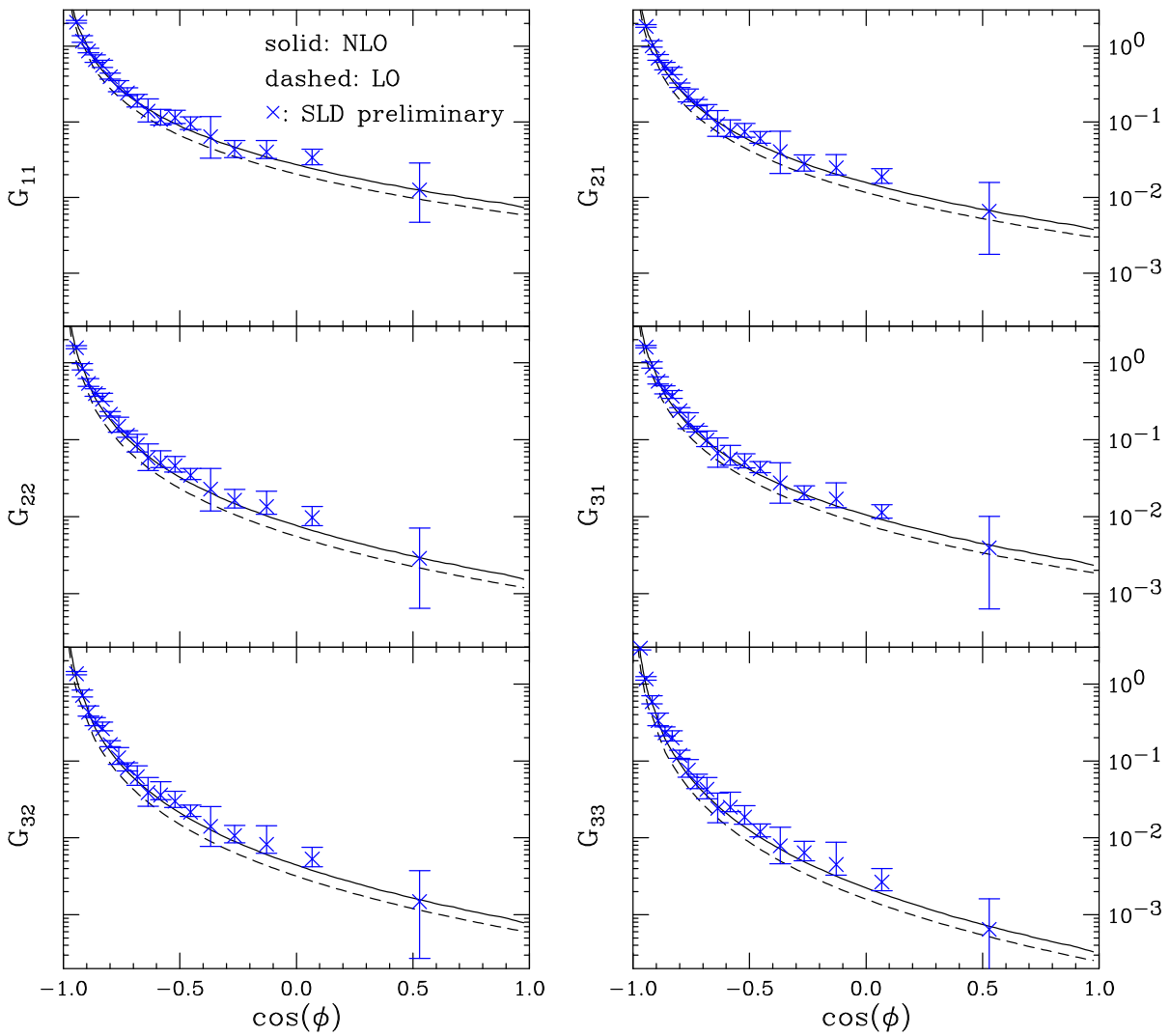

Figure 1: Leading order (dashed) and next-to-leading order (solid) curves for G_{ij}, $i,j = 1,2,3$, at a renormalization scale $\mu = E_{\rm CM} = 91.2$ GeV, using $\alpha_{\rm S}(E_{\rm CM}) = 0.12$. The error bars in the preliminary data from the SLD Collaboration [15] are the quadratic sum of systematic and statistical errors.

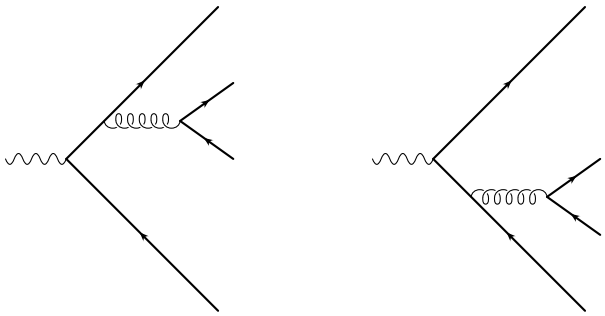

Figure 2: Secondary b-quark production.

In essence, the prescription amounts to including the secondary heavy flavour by simply increasing n_{lf} by one. When doing this, one in fact is also changing the renormalization scheme for the heavy flavour, by treating it as if it were light. Thus, the correct coupling constant to use in this framework is $\alpha_{\rm S}^{(5)}$. In Appendix B we discuss this change of scheme in more detail and from a slightly different point of view.

Secondary b-quark production may also arise from primary light quarks, connected with the Z^0/γ vertex. We neglect altogether this mechanism, since it generates only soft b quarks.

3 The $m \to 0$ limit

As explained in Sec. 2, $G_{ij}(\phi)$ are free from potentially dangerous logarithms of the ratio $E_{\rm CM}/m$. In order to investigate the stability of our expressions in the $m \to 0$ limit, we introduce the integral of G_{ij} over ϕ and we define (we drop the B/b subscript for ease of notation)

$$r_{ij} = \int_{-1}^{1} G_{ij}(\phi)\, d\cos\phi = \frac{\langle x_1^{i-1} x_2^{j-1}\rangle}{\langle x_1^{i-1}\rangle\langle x_2^{j-1}\rangle}\,, \tag{3.1}$$

where

$$\langle x_1^{i-1}\rangle = \langle x_2^{i-1}\rangle \equiv D_i\,, \tag{3.2}$$

$$\langle x_1^{i-1} x_2^{j-1}\rangle \equiv \int_{-1}^{1} D_{ij}(\phi)\, d\cos\phi\,. \tag{3.3}$$

To evaluate Eq. (3.1) up to order $\alpha_{\rm S}^2$ and for $i,j = 2,3$, we rewrite r_{ij} as follows

$$\begin{aligned} r_{ij} &= 1 + \frac{\langle x_1^{i-1} x_2^{j-1}\rangle - \langle x_1^{i-1}\rangle\langle x_2^{j-1}\rangle}{\langle x_1^{i-1}\rangle\langle x_2^{j-1}\rangle} \\ &= 1 + \frac{\langle (1-x_1^{i-1})(1-x_2^{j-1})\rangle - \langle 1-x_1^{i-1}\rangle\langle 1-x_2^{j-1}\rangle}{\langle x_1^{i-1}\rangle\langle x_2^{j-1}\rangle}\,, \end{aligned} \tag{3.4}$$

and we expand in $\alpha_{\rm S}$

$$\langle 1 - x_1^{i-1} \rangle = \langle 1 - x_2^{i-1} \rangle = a_i \, \frac{\alpha_{\rm S}(\mu)}{2\pi} + \mathcal{O}\left(\alpha_{\rm S}^2\right), \tag{3.5}$$

$$\langle (1 - x_1^{i-1})(1 - x_2^{j-1}) \rangle = b_{ij} \, \frac{\alpha_{\rm S}(\mu)}{2\pi} + \left[c_{ij} + 2\pi \, b_0 \, b_{ij} \log\left(\frac{\mu^2}{E_{\rm CM}^2}\right)\right] \left(\frac{\alpha_{\rm S}(\mu)}{2\pi}\right)^2 + \mathcal{O}\left(\alpha_{\rm S}^3\right), \tag{3.6}$$

to obtain

$$r_{ij} = 1 + b_{ij} \, \frac{\alpha_{\rm S}(\mu)}{2\pi} + \left[c_{ij} + (a_i + a_j) \, b_{ij} - a_i a_j + 2\pi \, b_0 \, b_{ij} \log\left(\frac{\mu^2}{E_{\rm CM}^2}\right)\right] \left(\frac{\alpha_{\rm S}(\mu)}{2\pi}\right)^2 + \mathcal{O}\left(\alpha_{\rm S}^3\right). \tag{3.7}$$

Note that, in Eq. (3.4), the expectation value of x_1^{i-1} and x_2^{j-1} should be known only up to order $\alpha_{\rm S}$, and that, in the correlation term $\langle (1 - x_1^{i-1})(1 - x_2^{j-1}) \rangle$, the two-jet final state gives zero contribution, since it is proportional to $\delta(1 - x_1)\delta(1 - x_2)$, at all orders in $\alpha_{\rm S}$. This is the reason why we do not need to know the $\alpha_{\rm S}^2$ two-loop virtual term for $e^+e^- \to Q\overline{Q}$.

The coefficients a_i and b_{ij} can be computed analytically. We have collected their expressions in Appendix A. In Table 1, we give the numerical values for the coefficients

Table 1: Results for the coefficients a_i, b_{ij} and c_{ij} appearing in the definition of r_{ij} (see Eqs. (3.5)–(3.7)). The mass m is in GeV and $\mu = E_{\rm CM} = 91.2$ GeV.

	$m = 0.01$	$m = 0.1$	$m = 1$	$m = 3$	$m = 5$	$m = 10$
a_2	29.1611	20.9741	12.7866	8.87675	7.05412	4.56726
a_3	46.6475	33.8553	21.0574	14.9209	12.0317	8.01007
b_{22}	0.666666	0.666589	0.661895	0.636326	0.598452	0.478637
b_{32}	1.13333	1.13321	1.12569	1.08448	1.02307	0.826778
b_{33}	1.91111	1.91093	1.89945	1.83563	1.73915	1.42415
c_{22}	820.1(3)	420.51(4)	155.14(1)	75.787(7)	49.176(3)	22.943(1)
c_{32}	1290.5(4)	663.99(7)	247.19(2)	122.15(1)	80.053(6)	38.268(3)
c_{33}	2027.5(5)	1046.5(1)	393.09(3)	196.60(2)	130.23(1)	63.871(5)

a_i, b_{ij} and c_{ij}, for a down-type quark with mass $m = 0.01$, 0.1, 1, 3, 5 and 10 GeV, at a center-of-mass energy $E_{\rm CM} = \mu = 91.2$ GeV. Observe that, while the individual coefficients a_i and c_{ij} can be quite large, due to the presence of logarithmic terms, the ratio r_{ij} is well behaved, as illustrated in Table 2. This is due to the fact that the large logarithms present in the single coefficients cancel out in the ratio r_{ij}.

Table 2: Results for r_{ij}. The mass m is in GeV and $\mu = E_{\rm CM} = 91.2$ GeV.

m	$r_{22}-1$	$r_{32}-1$	$r_{33}-1$
0.01	$0.106103\,\alpha_{\rm S}+0.215(4)\,\alpha_{\rm S}^2$	$0.180375\,\alpha_{\rm S}+0.403(6)\,\alpha_{\rm S}^2$	$0.304162\,\alpha_{\rm S}+0.750(9)\,\alpha_{\rm S}^2$
0.1	$0.106091\,\alpha_{\rm S}+0.217(1)\,\alpha_{\rm S}^2$	$0.180356\,\alpha_{\rm S}+0.407(2)\,\alpha_{\rm S}^2$	$0.304133\,\alpha_{\rm S}+0.755(3)\,\alpha_{\rm S}^2$
1	$0.105344\,\alpha_{\rm S}+0.2175(4)\,\alpha_{\rm S}^2$	$0.179159\,\alpha_{\rm S}+0.4066(7)\,\alpha_{\rm S}^2$	$0.302307\,\alpha_{\rm S}+0.7524(1)\,\alpha_{\rm S}^2$
3	$0.101274\,\alpha_{\rm S}+0.2100(1)\,\alpha_{\rm S}^2$	$0.172600\,\alpha_{\rm S}+0.3930(3)\,\alpha_{\rm S}^2$	$0.292149\,\alpha_{\rm S}+0.7287(5)\,\alpha_{\rm S}^2$
5	$0.0952466\,\alpha_{\rm S}+0.19905(9)\,\alpha_{\rm S}^2$	$0.162827\,\alpha_{\rm S}+0.3725(2)\,\alpha_{\rm S}^2$	$0.276794\,\alpha_{\rm S}+0.6919(3)\,\alpha_{\rm S}^2$
10	$0.0761774\,\alpha_{\rm S}+0.16351(4)\,\alpha_{\rm S}^2$	$0.131586\,\alpha_{\rm S}+0.30605(7)\,\alpha_{\rm S}^2$	$0.226661\,\alpha_{\rm S}+0.5706(1)\,\alpha_{\rm S}^2$

4 Theoretical uncertainties

In this section, we investigate the theoretical uncertainties related to the renormalization scale and mass scheme used in the calculation.

The renormalization-scale dependence of r_{ij} is illustrated in Table 3 for the case $m = 5$ GeV, using the two-loop evolution of $\alpha_{\rm S}$ with $\alpha_{\rm S} = 0.12$ for $\mu = E_{\rm CM} = 91.2$ GeV. For $E_{\rm CM}/2 < \mu < 2E_{\rm CM}$ the variation of $(r_{ij}-1)$ around the central value $\mu = E_{\rm CM}$ is roughly $\pm 10\%$ at leading order. It is reduced to $\pm 5\%$ at NLO.

Table 3: Results for r_{ij} for different renormalization scales with $\alpha_{\rm S}(E_{\rm CM}) = 0.12$, $E_{\rm CM} = 91.2$ GeV and $m = 5$ GeV.

	$\mu = E_{\rm CM}/2$		$\mu = E_{\rm CM}$		$\mu = 2E_{\rm CM}$	
	LO	NLO	LO	NLO	LO	NLO
$r_{22}-1$	0.0127974	0.014937(2)	0.0114296	0.014296(1)	0.0103320	0.013622(1)
$r_{32}-1$	0.0218776	0.026116(3)	0.0195393	0.024903(2)	0.0176629	0.023667(2)
$r_{33}-1$	0.0371903	0.045455(5)	0.0332153	0.043179(4)	0.0300257	0.040922(3)

In order to investigate the mass-scheme dependence, we now explicitly distinguish between the pole mass m and the $\overline{\rm MS}$ mass $\overline{m}(\mu)$ (also called running mass). Using the relation between m and $\overline{m}(\mu)$

$$m = \overline{m}(\mu)\left\{1 + \frac{\alpha_{\rm S}(\mu)}{2\pi}C_F\left[2 - \frac{3}{2}\log\left(\frac{\overline{m}(\mu)^2}{\mu^2}\right)\right] + \mathcal{O}\left(\alpha_{\rm S}^2\right)\right\}, \tag{4.1}$$

and defining

$$z = \left(\frac{m}{E_{\rm CM}}\right)^2, \qquad \bar{z}(\mu) = \left(\frac{\overline{m}(\mu)}{E_{\rm CM}}\right)^2, \tag{4.2}$$

we have

$$\bar{r}_{ij}\left(\bar{z}\right) \equiv r_{ij}\left(\bar{z}\left(\mu\right) + \Delta\bar{z}\left(\mu\right)\right) = r_{ij}(\bar{z}(\mu)) + \left(\frac{\alpha_{\rm S}(\mu)}{2\pi}\right)^2 \Delta b_{ij}\left(\bar{z}\left(\mu\right)\right) + \mathcal{O}\left(\alpha_{\rm S}^3\right) , \qquad (4.3)$$

with

$$\Delta b_{ij}(\bar{z}(\mu)) = 4C_F\bar{z}(\mu)\left[1 - \frac{3}{4}\ln\left(\bar{z}(\mu)\frac{E_{\rm CM}^2}{\mu^2}\right)\right]\frac{db_{ij}}{dz}(\bar{z}(\mu))\,. \qquad (4.4)$$

We have evaluated Eq. (4.3) for $\overline{m}(E_{\rm CM}) = 3$ GeV using the analytic expressions for the coefficients b_{ij} collected in Appendix A. The value $\overline{m}(E_{\rm CM}) = 3$ GeV corresponds roughly to what one gets by converting m at the b-mass scale to $\overline{m}$, and then evolving $\overline{m}$ up to $\mu = E_{\rm CM}$. We have further studied the dependence of Eq. (4.3) on the renormalization scale μ. We have used the 2-loop evolution of $\overline{m}(\mu)$, which gives $\overline{m}(E_{\rm CM}/2) = 3.20$ GeV and $\overline{m}(2E_{\rm CM}) = 2.83$ GeV. The results for $(\bar{r}_{ij} - 1)$ are listed in Table 4. Using the $\overline{\rm MS}$ definition for the mass, the theoretical uncertainties due to the scale ambiguity are practically of the same size as in the pole-mass scheme.

Table 4: Results for $\bar{r}_{ij}$ for different renormalization scales with $\alpha_{\rm S}(E_{\rm CM}) = 0.12$, $E_{\rm CM} = 91.2$ GeV and $\overline{m}(E_{\rm CM}) = 3$ GeV.

	$\mu = E_{\rm CM}/2$		$\mu = E_{\rm CM}$		$\mu = 2E_{\rm CM}$	
	LO	NLO	LO	NLO	LO	NLO
$\bar{r}_{22} - 1$	0.0135360	0.015444(2)	0.0121529	0.014884(2)	0.0110323	0.014265(2)
$\bar{r}_{32} - 1$	0.0230755	0.026986(4)	0.0207120	0.025897(4)	0.0187981	0.024743(3)
$\bar{r}_{33} - 1$	0.0390735	0.046904(8)	0.0350579	0.044811(7)	0.0318083	0.042675(5)

Comparing the results for r_{ij} obtained with an on-shell mass $m = 5$ GeV with those obtained with an $\overline{\rm MS}$ mass $\overline{m}(E_{\rm CM}) = 3$ GeV, we found that, at leading order, the differences between $(r_{ij} - 1)$ using the two different mass definitions amount to $5 \div 6\%$. This theoretical uncertainty is reduced to less than 4% at NLO.

An interesting question can be raised about the use of the pole mass versus the $\overline{\rm MS}$ mass, that is to say, which one is more natural in this context. In practice, this is a very difficult question to answer. In inclusive processes, in general, the $\overline{\rm MS}$ mass is better, since it accounts for large powers of logarithms of $m/E_{\rm CM}$ multiplying powers of the mass at all orders in perturbation theory. The case we are considering, however, is not an inclusive process, since we are weighting the cross section with final-state kinematic variables. We are thus unable to give a full answer to this question. However, we can certainly ask whether, at the order we are considering, there is some numerical indication that using the $\overline{\rm MS}$ mass accounts at least for a large part of the logarithmic terms in our expression. The mass dependence of the coefficients of r_{ij} is reported in Table 2. Calling $r_{ij}^{(2)}$ the

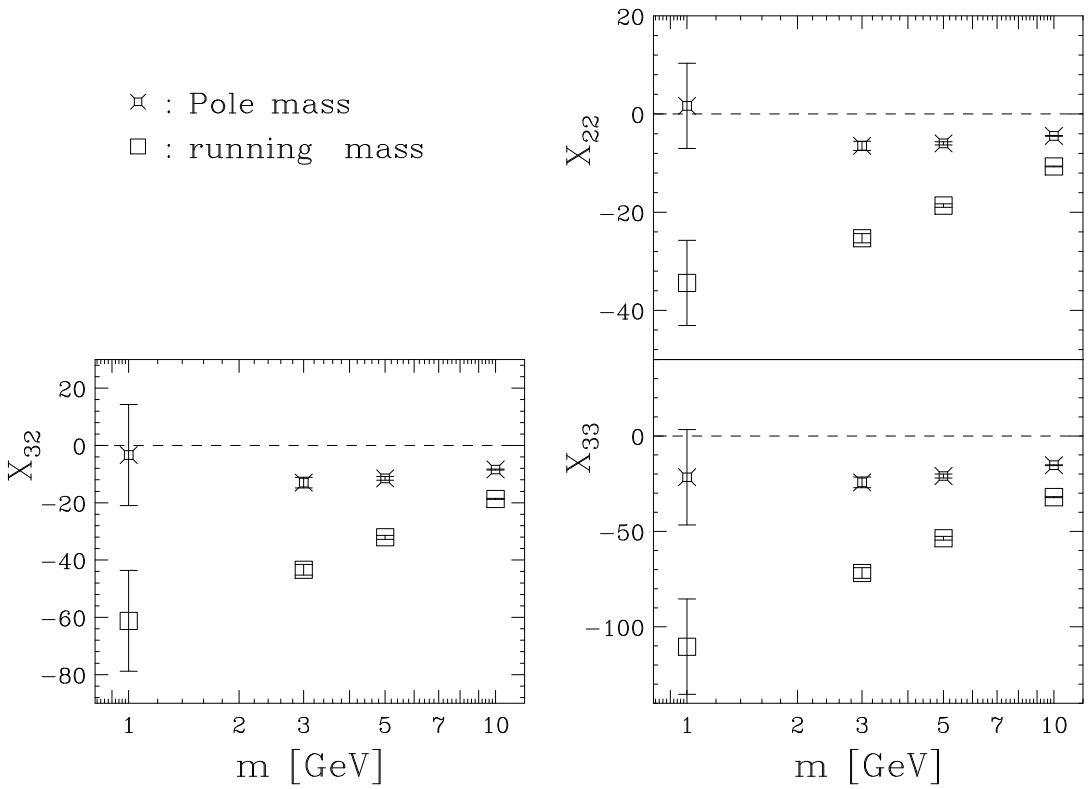

Figure 3: The quantities X_{ij}, defined in Eq. (4.5), plotted as a function of m, both in the pole-mass scheme and in the $\overline{\rm MS}$-mass scheme, with $E_{\rm CM} = \mu = 91.2$ GeV.

coefficient of $\alpha_{\rm S}^2$ in r_{ij}, we consider the expression

$$X_{ij}(z) = \frac{r_{ij}^{(2)}(z) - r_{ij}^{(2)}(0)}{z} \tag{4.5}$$

as a function of $\log z$. The same expression in the $\overline{\rm MS}$ scheme is given by

$$\overline{X}_{ij}(\bar{z}) = \frac{r_{ij}^{(2)}(\bar{z}) - r_{ij}^{(2)}(0)}{\bar{z}} + \frac{4}{(2\pi)^2} C_F \left[1 - \frac{3}{4}\log\bar{z}\right] \frac{db_{ij}}{dz}(\bar{z}) \,, \tag{4.6}$$

where we use $\mu = E_{\rm CM}$, and $\bar{z} = \bar{z}(\mu)$. If the slope of the second expression versus $\log m$ is smaller than the slope of the first one, we can take this as an indication of the fact that it is more appropriate to use the $\overline{\rm MS}$ mass instead of the pole mass. The results are displayed in Fig. 3. The value $r_{ij}^{(2)}(0)$ has been taken to correspond to $m = 0.1$ GeV in Table 2 (the point $m = 0.01$ GeV has been excluded, because of the large errors). The points at $m = 1$, 3, 5 and 10 GeV are then plotted. It is quite clear from the figure that the logarithmic slope is smaller for the pole-mass scheme. Furthermore, the radiative correction to the mass term is always smaller for the pole-mass scheme than for the $\overline{\rm MS}$ scheme. We thus conclude that the quantities r_{ij} seem to be more naturally described in the pole-mass scheme rather than in the $\overline{\rm MS}$-mass scheme.

5 Conclusions

In this paper we have investigated new measurable quantities (related to the quantities introduced in Ref. [9]) defined in e^+e^- annihilation, that carry information about the angle-dependent B-$\overline{B}$ energy correlation. We have shown that these variables have a well-behaved expansion in terms of the strong coupling constant $\alpha_{\rm S}$, since large logarithms in the ratio of the mass of the final quarks over the centre-of-mass-energy cancel out order by order, together with non-perturbative effects of order $\Lambda_{\rm QCD}/m$.

We have compared the computed NLO results with the available preliminary data from the SLD Collaboration [15], and found good agreement. In addition, we have generated tables [5] that can be used in a fit to the data, in order to extract the strong coupling constant $\alpha_{\rm S}$ at next-to-leading order.

We have investigated the renormalization-scale and mass-scheme dependence of our results, and found that, varying the renormalization scale μ in the range $E_{\rm CM}/2 < \mu < 2E_{\rm CM}$, the scale dependence of $(r_{ij}-1)$ is reduced from roughly $\pm 10\%$ at leading order to $\pm 5\%$ at next-to-leading order. The theoretical uncertainties related to the mass definition are less than 4% at NLO. We have shown that these quantities seem to be more naturally described in the pole-mass rather than in the $\overline{\rm MS}$-mass scheme.

From the discussion given in this paper, one may wonder if similar quantities may be defined for massless quarks, given the fact that mass singularities cancel in the variables we are considering, and thus they remain well defined in the massless limit. This is not quite the case. In fact, it is crucial for our discussion that the secondary heavy-flavour production may be neglected. If this were not the case, evolution of the fragmentation function would become multi-dimensional, involving several flavour components and a singlet contribution. Thus, the advantage of having a heavy flavour in the final state lies in the fact that the evolution becomes essentially non-singlet, so that we are left with a single component of the fragmentation function, that simply cancels in the ratios we propose.

Acknowledgements

We would like to thank P. N. Burrows and G. Nesom for numerous discussions and for making available to us experimental data on the B-$\overline{B}$ correlations. Thanks also to P. Uwer for discussions and comments on the manuscript. The research of A.B. was supported by a Heisenberg grant of the DFG and C.O. thanks the UK Particle Physics and Astronomy Research Council (PPARC) for supporting his research.

A Analytic results for $a_2, a_3, b_{22}, b_{32}, b_{33}$

In this appendix, we collect the analytic results for the coefficients a_i and b_{ij}, $i,j = 2,3$, defined in Eqs. (3.5) and (3.6). Calling the generic coefficient $F(z)$, where $z = m^2/s$ and

[5] The programs and the tables can be obtained upon request from the authors.

$s = E_{\rm CM}^2$, we write

$$F(z) = \frac{2C_F}{\beta\,[c_V(1+2z)+c_A(1-4z)]}\left[c_V F^V(z) + c_A F^A(z)\right], \tag{A.1}$$

where $\beta = \sqrt{1-4z}$ and $\omega = (1-\beta)/(1+\beta)$. The electroweak couplings c_V and c_A are given by

$$\begin{aligned} c_V &= Q_b^2\, f^{\gamma\gamma} + 2\, g_V^b\, Q_b\ {\rm Re}[\chi(s)]\ f^{\gamma Z} + g_V^{b\,2}\,|\chi(s)|^2\, f^{ZZ}, \\ c_A &= g_A^{b\,2}|\chi(s)|^2 f^{ZZ}, \end{aligned} \tag{A.2}$$

with

$$\begin{aligned} f^{\gamma\gamma} &= 1-\lambda_-\lambda_+\,, \\ f^{ZZ} &= (1-\lambda_-\lambda_+)(g_V^{e2}+g_A^{e2}) - 2(\lambda_- - \lambda_+) g_V^e g_A^e\,, \\ f^{\gamma Z} &= -(1-\lambda_-\lambda_+) g_V^e + (\lambda_- - \lambda_+) g_A^e\,. \end{aligned} \tag{A.3}$$

Here, $Q_b = -1/3$ is the electric charge of the bottom quark, and $g_{A/V}^f$ denote the axial and vector couplings of the fermion f. In particular, $g_V^e = -\frac{1}{2} + 2\sin^2\vartheta_W$, $g_A^e = -\frac{1}{2}$ for an electron, and $g_V^b = -\frac{1}{2} + \frac{2}{3}\sin^2\vartheta_W$, $g_A^b = -\frac{1}{2}$ for a bottom quark, where ϑ_W is the weak mixing angle. The function $\chi(s)$ reads

$$\chi(s) = \frac{1}{4\sin^2\vartheta_W\cos^2\vartheta_W}\,\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}, \tag{A.4}$$

where M_Z and Γ_Z stand for the mass and the width of the Z boson. Finally, λ_- (λ_+) denotes the longitudinal polarization of the electron (positron) beam.

A simple calculation using the Born differential cross section for $e^+e^- \to b\bar{b}\,g$ gives

$$a_2^V(z) = \left(-\frac{2}{3}+z-\frac{14}{3}z^3\right)\ln(\omega) + \left(-\frac{11}{3}+\frac{7}{6}z+7z^2\right)\frac{\beta}{3}, \tag{A.5}$$

$$\frac{a_2^A(z)-a_2^V(z)}{z} = \left(\frac{8}{3}-10z+12z^2+\frac{20}{3}z^3\right)\ln(\omega) + \left(\frac{35}{3}-\frac{59}{3}z-10z^2\right)\frac{\beta}{3}, \tag{A.6}$$

$$\begin{aligned} a_3^V(z) &= \left(-\frac{25}{24}+\frac{10}{3}z+\frac{5}{2}z^2-\frac{32}{3}z^3-\frac{11}{12}z^4\right)\ln(\omega) \\ &+ \left(-\frac{433}{12}+\frac{137}{2}z+\frac{779}{6}z^2+11z^3\right)\frac{\beta}{24}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} \frac{a_3^A(z)-a_3^V(z)}{z} &= \left(\frac{55}{12}-20z+21z^2+\frac{40}{3}z^3+\frac{15}{2}z^4\right)\ln(\omega) \\ &+ \left(\frac{823}{36}-\frac{845}{18}z-\frac{175}{6}z^2-15z^3\right)\frac{\beta}{4}, \end{aligned} \tag{A.8}$$

$$b_{22}^V(z) = 2z\left(1+z^3\right)\ln(\omega) + \left(\frac{1}{4}+\frac{19}{6}z-\frac{1}{6}z^2-z^3\right)\beta, \tag{A.9}$$

$$\frac{b_{22}^A(z)-b_{22}^V(z)}{z} = 2z\left(-2+3z-5z^3\right)\ln(\omega) + \left(-\frac{3}{4}-\frac{17}{6}z+\frac{5}{6}z^2+5z^3\right)\beta, \tag{A.10}$$

$$
\begin{aligned}
b_{32}^V(z) &= z\left(\frac{19}{6}+\frac{1}{3}z-2z^2+\frac{17}{3}z^3+\frac{1}{3}z^4\right)\ln(\omega) \\
&+ \left(\frac{17}{20}+\frac{1721}{180}z+\frac{47}{45}z^2-\frac{103}{18}z^3-\frac{1}{3}z^4\right)\frac{\beta}{2}, \qquad \text{(A.11)} \\
\frac{b_{32}^A(z)-b_{32}^V(z)}{z} &= z\left(-7+10z-10z^3-14z^4\right)\ln(\omega) \\
&+ \left(-\frac{27}{20}-\frac{287}{60}z+\frac{16}{15}z^2+\frac{37}{6}z^3+7z^4\right)\beta, \qquad \text{(A.12)} \\
b_{33}^V(z) &= z\left(\frac{14}{3}-\frac{13}{6}z-8z^2+\frac{32}{3}z^3+\frac{16}{3}z^4-z^5\right)\ln(\omega) \\
&+ \left(\frac{43}{30}+\frac{533}{45}z-\frac{221}{36}z^2-\frac{1037}{90}z^3-\frac{31}{6}z^4+z^5\right)\frac{\beta}{2}, \qquad \text{(A.13)} \\
\frac{b_{33}^A(z)-b_{33}^V(z)}{z} &= z\left(-12+17z+\frac{4}{3}z^2-12z^3-28z^4-\frac{70}{3}z^5\right)\ln(\omega) \\
&+ \left(-\frac{109}{15}-\frac{661}{30}z+\frac{53}{20}z^2+\frac{157}{6}z^3+\frac{287}{6}z^4+35z^5\right)\frac{\beta}{3}. \qquad \text{(A.14)}
\end{aligned}
$$

In the $m \to 0$ limit, we have $\beta \to 1$, $\omega \to z = m^2/s \to 0$, so that

$$
\begin{aligned}
a_2 &= 2C_F\left(-\frac{2}{3}\ln(z)-\frac{11}{9}\right), \\
a_3 &= 2C_F\left(-\frac{25}{24}\ln(z)-\frac{433}{288}\right), \\
b_{22} &= 2C_F\frac{1}{4}, \\
b_{32} &= 2C_F\frac{17}{40}, \\
b_{33} &= 2C_F\frac{43}{60}. \qquad \text{(A.15)}
\end{aligned}
$$

This behavior is well illustrated from the collections of values in Table 1: while the b_{ij} are finite in the $m \to 0$ limit, the coefficients a_i diverge as $\log(m/E_{\rm CM})$.

B Renormalization schemes

Our calculation was carried out in the "mixed" renormalization scheme of Ref. [18], in which the light flavours n_{lf} are subtracted in the $\overline{\rm MS}$ scheme, while the heavy-flavour loops are subtracted at zero momentum. In this scheme the heavy flavour decouples at low energy. In fact, convergent heavy-flavour loops are suppressed by powers of the mass of the heavy flavour. The only unsuppressed contributions come from divergent graphs. But those are subtracted at zero external momenta, so their contribution is removed by renormalization for small momenta. In the mixed scheme, charge renormalization is given by the prescription

$$
\alpha_{\rm s}^b = \mu^{2\epsilon}\tilde{\alpha}_{\rm s}(\mu)\left\{1-\tilde{\alpha}_{\rm s}(\mu)\frac{1}{\bar{\epsilon}}\left[b_0^{(n_{lf})}-\left(\frac{\mu^2}{m^2}\right)^{\epsilon}\frac{T_F}{3\pi}\right]+\mathcal{O}\left(\alpha_{\rm s}^2\right)\right\}, \qquad \text{(B.1)}
$$

where $\alpha_{\rm S}^b$ is the bare coupling constant, and $\tilde{\alpha}_{\rm S}(\mu)$ is the mixed-scheme coupling constant at the scale μ. We have defined

$$b_0^{(n_{lf})} = \frac{11C_A - 4T_F n_{lf}}{12\pi} , \tag{B.2}$$

and

$$\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \log(4\pi) - \gamma_E , \tag{B.3}$$

where dimensional regularization is used, with the number of space-time dimensions equal to $4 - 2\epsilon$.

In the $\overline{\rm MS}$ scheme the renormalization prescription is

$$\alpha_{\rm S}^b = \mu^{2\epsilon}\alpha_{\rm S}(\mu)\left\{1 - \alpha_{\rm S}(\mu)\frac{1}{\bar{\epsilon}}b_0^{(n_f)}\right\} , \tag{B.4}$$

where $n_f = n_{lf} + 1$, and $\alpha_{\rm S}(\mu)$ is the $\overline{\rm MS}$ coupling constant at the scale μ. The couplings in the two schemes obey a 4-flavour and 5-flavour renormalization-group equation respectively

$$\frac{d}{d\log\mu^2}\tilde{\alpha}_{\rm S}(\mu) = -b_0^{(n_{lf})}\tilde{\alpha}_{\rm S}^2(\mu) , \qquad \frac{d}{d\log\mu^2}\alpha_{\rm S}(\mu) = -b_0^{(n_f)}\alpha_{\rm S}^2(\mu) , \tag{B.5}$$

that can be easily derived by imposing the constancy of the bare coupling $\alpha_{\rm S}^b$ under a renormalization group transformation. Combining Eqs. (B.1) and (B.4) we have

$$\tilde{\alpha}_{\rm S}(\mu) = \alpha_{\rm S}(\mu) - \frac{T_F}{3\pi}\log\left(\frac{\mu^2}{m^2}\right)\alpha_{\rm S}^2(\mu) + \mathcal{O}\left(\alpha_{\rm S}^3\right) , \tag{B.6}$$

which is the standard $\overline{\rm MS}$ matching condition for flavour crossing.

In the heavy flavour production calculation we are considering, if we express the result (usually given in terms of $\tilde{\alpha}_{\rm S}$) in terms of $\alpha_{\rm S}$, an extra term arises, equal to

$$-\,{\rm Born}\times\alpha_{\rm S}(\mu)\frac{T_F}{3\pi}\log\left(\frac{\mu^2}{m^2}\right) . \tag{B.7}$$

The only other diagrams where secondary heavy flavours appear are the graphs with a gluon splitting into a heavy-flavour pair. It can be shown that, if we neglect powers of $m/E_{\rm CM}$, the term in Eq. (B.7) is exactly the term one needs to turn the calculation of the heavy-flavour splitting, which is regulated in the collinear limit by the heavy-flavour mass, into the corresponding $\overline{\rm MS}$ subtracted calculation, provided that interference terms of the heavy flavours coming from gluon splitting with the primary heavy flavours are fully neglected.

Equation (B.7) follows immediately from the theory of heavy-flavour fragmentation functions. According to Ref. [19], Eq. (3.16), the cross section for the inclusive production

of a heavy quark via gluon splitting, with a fraction x of the energy of the gluon, is (at leading order and neglecting powers of $m/E_{\rm CM}$)

$$\frac{d\sigma}{dx} = \sigma_0 \frac{\alpha_{\rm S} T_F}{2\pi} \left[x^2 + (1-x)^2\right] \log\left(\frac{\mu^2}{m^2}\right) + \frac{d\hat{\sigma}}{dx} + \mathcal{O}(\alpha_{\rm S}^2)\,, \tag{B.8}$$

where σ_0 is the cross section for the production of the gluon and $\hat{\sigma}$ stands for the massless limit, $\overline{\rm MS}$ subtracted cross section for the gluon splitting process. Integrating both sides in x one finds

$$\hat{\sigma} = \sigma - \sigma_0 \frac{\alpha_{\rm S} T_F}{3\pi} \log\left(\frac{\mu^2}{m^2}\right)\,. \tag{B.9}$$

The second term on the right-hand-side is precisely the contribution of Eq. (B.7).

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