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Twist–2 Heavy Flavor Contributions to the Structure Function $g_2(x, Q^2)$

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Abstract

The twist–2 heavy flavor contributions to the polarized structure function $g_2(x, Q^2)$ are calculated. We show that this part of $g_2(x, Q^2)$ is related to the heavy flavor contribution to $g_1(x, Q^2)$ by the Wandzura–Wilczek relation to all orders in the strong coupling constant. Numerical results are presented.

1 Introduction

Heavy flavor contributions to polarized and unpolarized deep inelastic structure functions due to charm and bottom–quark production cause different scaling violations if compared to those due to massless partons. Moreover it is known that in certain phase space regions of x and Q^2 these contributions can be large [1]. Reliable determinations of the QCD scale $\Lambda_{\rm QCD}$ therefore require a careful account of the heavy flavor contributions. In the case of the unpolarized structure functions $F_{2,L}(x,Q^2)$ and $xF_3(x,Q^2)$ the twist–2 contributions with $Q\overline{Q}$–final states, $Q = c, b$ were calculated to next–to–leading order (NLO) [2, 3]. For polarized deep–inelastic scattering only the leading order corrections are known for the structure function $g_1(x,Q^2)$ at present [4]. All corrections mentioned above have been calculated using mass factorization. As shown in [5–8] this method fails in the case of the polarized structure functions which emerge for transverse polarization already in the massless quark limit. The reason for this lies in a violation of covariance due to the omission of contributions $\propto S.k$, where S denotes the spin vector of the nucleon and k the parton 4–momentum.

In Refs. [6, 7] it was shown that the quarkonic contributions to the transverse structure functions can be correctly obtained using the covariant parton model. The results in this approach are identical to those derived in the local light–cone expansion, cf. [9, 8], for massless quarks on the level of the twist–2 contributions. In particular the twist–2 part of $g_2(x,Q^2)$ is obtained by the Wandzura–Wilczek relation. The effect of quark masses for quarkonic matrix elements was further investigated in Ref. [10] using the method of [11]. Also in this case the Wandzura–Wilczek relation was found to hold, irrespective of the values of the quark masses chosen. In [12, 10] the target mass corrections to the quarkonic contributions to $g_2(x,Q^2)$ and the other four polarized structure functions [10] were studied. Again the Wandzura–Wilczek relation was found to hold relating the twist–2 contributions of $g_1(x,Q^2)$ and $g_2(x,Q^2)$. In a more general approach the amplitudes which contribute to the quarkonic matrix elements for deeply virtual non–forward scattering were investigated in [13]. In the generalized Bjorken limit the non–forward Compton amplitude is expressed by operator matrix elements of vector operators. However, one usually parameterizes it in terms of scalar operator matrix elements to which the (generalized) parton distributions correspond. Therefore relations between the operator matrix elements of the vector and scalar operators are implied, which form the origin of the Wandzura–Wilczek relations and other integral–relations [8, 10]. The absence of the typical integral terms e.g. in the Callan–Gross [14] relation is merely the exception and caused by a cancellation of the former, see Ref. [15]. For the polarized case it was shown that the Wandzura–Wilczek relation holds for the twist–2 contributions to the respective non–forward amplitudes, cf. [15]. This result could be generalized allowing multiple meson production in the final state in [16]. Also semi–inclusive processes such as diffrative scattering have been investigated w.r.t. the emergence of Callan–Gross and Wandzura–Wilczek relations [17]. Although the variables change, the Wandzura–Wilczek relation relates the twist–2 contributions of $g_1^{\rm diffr}(x,x_P,Q^2)$ to $g_2^{\rm diffr}(x,x_P,Q^2)$ independently of the value for x_P. Integral relations of similar kind were also established for other structure functions and matrix elements [18] and for the fermionic twist–3 contributions in [10].

In the present paper we calculate the heavy flavor twist–2 contribution to the structure function $g_2(x,Q^2)$. We show that the Wandzura–Wilczek relation holds also in this case and present numerical predictions.

2 Longitudinal Gluon Polarization

The hadronic tensor of the polarized part of the eN–scattering cross section in case of pure photon exchange[1] reads

$$W_{\mu\nu}^{(A)} = i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{p.q}g_1(x,Q^2) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(p.qS^\sigma - S.qp^\sigma)}{(p.q)^2}g_2(x,Q^2)\ . \tag{1}$$

S_σ denotes the nucleon spin vector, p the nucleon momentum, and q the vector of the 4–momentum transfer, with $Q^2 = -q^2$ and $x = Q^2/(2p.q)$. The polarized part of the scattering cross section for longitudinal nucleon polarization S_L, integrated over the azimuthal angle ϕ, is

$$\frac{d^2\sigma(\lambda,\pm S_L)}{dxdy} = \pm 2\pi \mathsf{S}\frac{\alpha^2}{Q^4}\left[-2\lambda y\left(2 - y - \frac{2xyM^2}{\mathsf{S}}\right)xg_1(x,Q^2) + 8\lambda\frac{yx^2M^2}{\mathsf{S}}g_2(x,Q^2)\right]\ . \tag{2}$$

Correspondingly, for transversely polarized nucleons one obtains

$$\begin{aligned}\frac{d^3\sigma(\lambda,\pm S_T)}{dxdyd\phi} = \pm\mathsf{S}\frac{\alpha^2}{Q^4}2\sqrt{\frac{M^2}{\mathsf{S}}}\sqrt{xy\left[1 - y - \frac{xyM^2}{\mathsf{S}}\right]}\cos(\chi-\phi)\\ \times\left[-2\lambda yxg_1(x,Q^2) - 4\lambda xg_2(x,Q^2)\right]\ .\end{aligned} \tag{3}$$

Here M is the nucleon mass, S the cms energy, α the fine structure constant, $y = 2p.q/\mathsf{S}$, λ is the degree of lepton polarization and S_T the degree of hadronic transverse polarization, χ denotes the azimuthal angle associated with S_T^μ, (10), and $g_1(x,Q^2)$ and $g_2(x,Q^2)$ are the polarized structure functions which contribute in this case.

The heavy flavor contributions to the longitudinal and transverse differential scattering cross sections are obtained in calculating the corresponding contributions to the polarized structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$.

Let us consider the sub–system hadronic tensor for photon–interactions with on–shell initial state partons

$$w_{\mu\nu}^{(A)} = \frac{i}{q.k}\varepsilon_{\mu\nu\rho\sigma}\left\{q^\rho s^\sigma g_1^{\rm parton}(z,Q^2) + \left(q^\rho s^\sigma - \frac{s.q}{q.k}q^\rho k^\sigma\right)g_2^{\rm parton}(z,Q^2)\right\}\ . \tag{4}$$

In assuming that the gluon is longitudinally polarized, i.e. parallel to the proton and parton 4–momentum

$$s_\mu = \xi_1 S_\mu = \xi_2 k_\mu\ , \tag{5}$$

with $k = zp$, the sub–system hadronic tensor of the polarization asymmetry (4) receives purely longitudinal contributions, since the term $\propto g_2^{\rm parton}$ vanishes. Clearly (5) is a special model assumption which does not describe the general case being discussed in section 3. However, it is possible to derive in this approximation the correct expression for the coefficient functions $C_{g_1}^{Q\overline{Q}}$ at twist–2 which contributes to the structure function $g_1^{Q\overline{Q}}(x,Q^2)$. At leading order in α_s the structure function $g_1^{Q\overline{Q}}(x,Q^2)$ receives only gluonic contributions and is obtained as [4]

$$g_1^{Q\overline{Q}}(x,Q^2) = 2e_Q^2\frac{\alpha_s(Q^2)}{2\pi}\int_{ax}^1\frac{dy}{y}C_{g_1}^{Q\overline{Q}}\left(\frac{x}{y},M_Q^2,Q^2\right)\Delta G(y,Q^2)\ , \tag{6}$$

[1]The corresponding expressions in the case of additional weak boson exchange are given in [10].

with

$$C_{g_1}^{Q\overline{Q}}(z,Q^2) = \frac{1}{2}\left[\beta(3-4z) - (1-2z)\ln\left|\frac{1+\beta}{1-\beta}\right|\right] \tag{7}$$

and $\Delta G(x,Q^2)$ the polarized gluon distribution. Here a denotes the threshold $a = 1 + 4M_Q^2/Q^2$, e_Q the charge of the produced heavy quarks, and β is the cms velocity

$$\beta = \sqrt{1 - \frac{4M_Q^2}{Q^2}\frac{z}{1-z}} \tag{8}$$

of the final state quarks, with $\beta \in \left[0, \sqrt{1 - 4M_Q^2 x/[Q^2(1-x)]}\,\right]$. Eq. (6) defines the LO twist–2 contribution to $g_1^{Q\overline{Q}}(x,Q^2)$ [2]. Note that

$$\int_0^{1/a} dz C_{g_1}^{Q\overline{Q}}(z, M_Q^2, Q^2) = 0 \ , \tag{9}$$

cf. [19], which leads to a positive and a negative branch of the structure function $g_1^{Q\overline{Q}}(x,Q^2)$ in leading order for a positive definite polarized gluon density $\Delta G(x,Q^2)$, see Figure 2a. In this order also the first moment of $g_1^{Q\overline{Q}}(x,Q^2)$ vanishes, since the r.h.s. of (6) is a Mellin–convolution of two functions with the support of $C_{g_1}^{Q\overline{Q}}(z, M_Q^2, Q^2)$ being $z \in [0, 1/a]$. Eq. (9) also holds for the gluonic contribution to $C_{g_1}^{Q\overline{Q}}(z, M_Q^2, Q^2)$ in the asymptotic limit $Q^2 \gg M_Q^2$ in NLO [3]. For quarkonic initial states this relation does not hold, see [20].

The choice of the collinear factorization leads to difficulties in deriving the correct twist–2 terms in the case of structure functions which contain also twist–3 contributions in the limit of vanishing mass scales, as $g_2(x,Q^2)$ and $g_3(x,Q^2)$ in the case of electro–weak currents. This was extensively studied in the past [7, 8, 10] using the light–cone expansion and comparing the results to those being obtained in parton–model approaches[3]. In Refs. [5–7] it was shown, that for fermionic contributions to the polarized structure functions the well–known results being obtained in the light–cone expansion, see e.g. [8], can be obtained if one refers to the covariant parton model, cf. [21]. This is due to the fact that the kinematic assumption (5) which neglects all parton momenta in the transverse direction is in conflict with the fact that the nucleon spin vector has both transverse and longitudinal components. This may have impact on predictions of relations between the moments of polarized structure functions, the Burkhardt–Cottingham sum rule [22–24] or integral relations between these functions. In the following we will therefore refer to a general orientation of the parton spin vector and use the covariant parton model instead of the collinear approach.

3 The general Case

We now consider the polarization asymmetry of the hadronic tensor $W_{\mu\nu}^{(A)}$ for a general nucleon spin vector S_μ and general gluon spin vector s_μ, respectively. The nucleon spin vector obeys :

$$\begin{aligned} S_\mu P^\mu &= 0, \quad S^\mu = S_{||}^\mu + S_\perp^\mu \\ S_{||}^\mu &= (0;0,0,M) \\ S_\perp^\mu &= M(0;\cos\chi,\sin\chi,0) \ . \end{aligned} \tag{10}$$

[2]Note, that mass–scale effects may introduce twist–3 contributions to the structure function $g_1(x,Q^2)$ as well, as has been shown for target mass corrections in Ref. [10].

[3]For a review of earlier results see the comparison given in Ref. [8].

The latter two relations hold in the nucleon rest frame.

The polarization asymmetry of the hadronic tensor reads

$$W^{(A)}_{\mu\nu}(q,p,S) \;=\; \int d^4k\,[f_+(p,k,S)-f_-(p,k,S)]\,w^{(A)}_{\mu\nu}(p,q,k,s)\;. \tag{11}$$

The nucleon spin S is assumed to enter (11) linearly as usually the case in single photon–fermion interactions [25]. Here, k denotes the gluon 4–momentum. The gluon distribution functions $f_\pm(p,k,S)$ refer to opposite proton spin directions and are supposed to be twist–2 parton distribution functions in the present paper. The sub–system hadronic tensor asymmetry $w^{(A)}_{\mu\nu}(p,q,k,s)$ depends in addition on the virtual photon momentum exchanged, q, and the gluonic momentum– and spin vectors. s_μ obeys

$$s_\mu = \frac{p.k}{\sqrt{(p.k)^2k^2-M^2k^4}}\left[k_\mu-\frac{k^2}{p.k}p_\mu\right]\;, \tag{12}$$

with $s.k=0,\ k.k=-k^2$. We consider general values of the gluon virtuality k^2, which is assumed to be sufficiently damped by the difference of the distribution functions $f_\pm(p,k,S)$ as $k^2\to\infty,0$ in order to keep (11) a well defined relation. Let us denote

$$\sqrt{(p.k)^2k^2-M^2k^4}=N\;. \tag{13}$$

The following relations are obtained :

$$\begin{aligned} S.s &= \frac{1}{N}\;p.k\;S.k \\ \frac{S.q}{q.k} &= \frac{1}{N}\;p.k\left(1-\frac{k^2\;p.q}{p.k\;q.k}\right)\;. \end{aligned} \tag{14}$$

We define now

$$\Delta f=\frac{M\;p.k}{N}\,(f_+-f_-)\equiv\frac{S.k}{M^2}\tilde{f}(p^2,p.k,k^2) \tag{15}$$

and construct the sub–system hadronic tensor $w^{(A)}_{\mu\nu}(p,q,k,s)$ for single photon exchange and general values of the virtuality k^2. The Lorentz structure is determined by the Levi–Civita symbol contracted with two 4–vectors of the problem. $w^{(A)}_{\mu\nu}(p,q,k,s)$ obeys the representation

$$\begin{aligned} w^{(A)}_{\mu\nu}(p,q,k,s) \;=\; & i\varepsilon_{\mu\nu\alpha\beta}\frac{M\;p.k}{N\;q.k}\Bigg\{q^\alpha\left[k^\beta-\frac{k^2p^\beta}{p.k}\right][\hat{g}_1+\hat{g}_2]-q^\alpha k^\beta\left[1-\frac{k^2p.q}{p.kq.k}\right]\hat{g}_2 \\ & +k^\alpha\left[k^\beta-\frac{k^2}{p.k}p^\beta\right]\hat{v}\Bigg\}\;. \end{aligned} \tag{16}$$

The hadronic tensor is thus given by

$$\begin{aligned} W^{(A)}_{\mu\nu}(q,p,S) \;=\; & \frac{i}{M^2}\varepsilon_{\mu\nu\alpha\beta}\int d^4k\tilde{f}(p^2,p.k,k^2)\frac{S.k}{q.k}\left[q^\alpha k^\beta\left(\hat{g}_1+\frac{k^2p.q}{q.kp.k}\hat{g}_2\right)-\frac{q^\alpha p^\beta k^2}{p.k}\,(\hat{g}_1+\hat{g}_2)\right. \\ & \left.-\frac{k^\alpha p^\beta k^2}{p.k}\;\hat{v}\right]\;. \end{aligned} \tag{17}$$

Here $\widehat{g}_i = \widehat{g}_i(q^2, q.k, k^2)$ and $\widehat{v} = \widehat{v}(q^2, q.k, k^2)$ denote the respective sub–system structure functions. The function $\widehat{v}$ emerges for $k^2 \neq 0$.

For later use we rewrite (17) as

$$\begin{aligned} W^{(A)}_{\mu\nu}(q,p,S) &= \frac{i}{M^2}\varepsilon_{\mu\nu\alpha\beta}\int d^4k\ S.k\ \left\{ q^\alpha k^\beta \left[\frac{\partial a_1}{\partial p.k}\frac{\partial b_1}{\partial q.k} + p.q\ \frac{\partial a_2}{\partial p.k}\frac{\partial b_2}{\partial q.k}\right] \right. \\ &\quad + q^\alpha p^\beta \left[\frac{\partial a_3}{\partial p.k}\frac{\partial b_3}{\partial q.k} + \frac{\partial a_4}{\partial p.k}\frac{\partial b_4}{\partial q.k}\right] \\ &\quad \left. + k^\alpha p^\beta \frac{\partial a_5}{\partial p.k}\frac{\partial b_5}{\partial q.k}\right\}, \end{aligned} \tag{18}$$

where

$$\frac{\partial a_1}{\partial p.k} = \widetilde{f}(p^2, p.k, k^2) \tag{19}$$

$$\frac{\partial a_i}{\partial p.k} = \frac{k^2}{p.k}\ \widetilde{f}(p^2, p.k, k^2), \quad \text{for} \quad i = 2 \ldots 5 \tag{20}$$

$$\frac{\partial b_1}{\partial q.k} = \frac{1}{q.k}\ \widehat{g}_1(q^2, q.k, k^2) \tag{21}$$

$$\frac{\partial b_2}{\partial q.k} = \frac{1}{(q.k)^2}\ \widehat{g}_2(q^2, q.k, k^2) \tag{22}$$

$$\frac{\partial b_3}{\partial q.k} = -\frac{1}{q.k}\ \widehat{g}_1(q^2, q.k, k^2) \tag{23}$$

$$\frac{\partial b_4}{\partial q.k} = -\frac{1}{q.k}\ \widehat{g}_2(q^2, q.k, k^2) \tag{24}$$

$$\frac{\partial b_5}{\partial q.k} = -\frac{1}{q.k}\ \widehat{v}(q^2, q.k, k^2)\ . \tag{25}$$

The functions $a_i = a_i(p^2, p.k, k^2)$ and $b_i = b_i(q^2, q.k, k^2)$ will be used in Eq. (42).

The tensor structure in Eq. (17) is yet different of that in (1). To determine the structure functions $g_{1,2}(x, Q^2)$ a tensor decomposition in terms of the outer variables p, q and S is performed :

$$W^{(A)}_{\mu\nu}(q,p,S) = \frac{i}{M^2}\varepsilon_{\mu\nu\alpha\beta}q^\alpha S_\tau \left[I_1^{\beta\tau} + I_2^{\beta\tau} + p^\beta J_1^\tau + p^\beta J_2^\tau \right] + \frac{i}{M^2}\varepsilon_{\mu\nu\alpha\beta}S_\tau p^\beta K^{\alpha\tau}\ , \tag{26}$$

with

$$I_1^{\beta\tau} = \int d^4k \frac{k^\beta k^\tau}{q.k}\ \widetilde{f}\cdot\widehat{g}_1 = A_1 g^{\beta\tau} + B_1 p^\beta p^\tau + C_1 q^\beta q^\tau + D_1\left(p^\beta q^\tau + p^\tau q^\beta\right) \tag{27}$$

$$I_2^{\beta\tau} = \int d^4k \frac{k^\beta k^\tau\ k^2\ p.q}{(q.k)^2 p.k}\ \widetilde{f}\cdot\widehat{g}_2 = A_2 g^{\beta\tau} + B_2 p^\beta p^\tau + C_2 q^\beta q^\tau + D_2\left(p^\beta q^\tau + p^\tau q^\beta\right) \tag{28}$$

$$J_1^\tau = -\int d^4k \frac{k^2\ k^\tau}{p.k\ q.k}\ \widetilde{f}\cdot\widehat{g}_1 = E_1 p^\tau + H_1 q^\tau \tag{29}$$

$$J_2^\tau = -\int d^4k \frac{k^2\ k^\tau}{p.k\ q.k}\ \widetilde{f}\cdot\widehat{g}_2 = E_2 p^\tau + H_2 q^\tau \tag{30}$$

$$K^{\alpha\tau} = -\int d^4k \frac{k^2\ k^\alpha k^\tau}{p.k\ q.k}\ \widetilde{f}\cdot\widehat{v} = A_v g^{\alpha\tau} + B_v p^\alpha p^\tau + C_v q^\alpha q^\tau + D_v\left(p^\alpha q^\tau + q^\alpha p^\tau\right)\ . \tag{31}$$

The contributions due to $B_{1,2}, C_{1,2}, E_{1,2}, B_v$ and D_v vanish. A_v has to vanish because of current conservation. Therefore $W^{(A)}_{\mu\nu}$ is given by

$$W^{(A)}_{\mu\nu}(q,p,S) = \frac{i}{M^2}\varepsilon_{\mu\nu\alpha\beta}q^\alpha S^\beta\left[A_1+A_2\right]+\frac{i}{M^2}\varepsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta S.q\left[D_1+D_2+H_1+H_2+C_v\right] . \tag{32}$$

The coefficients $A_{1,2}, D_{1,2}, H_{1,2}$ and C_v read :

$$A_1 = \int d^4k\left[\frac{k^2}{2(q.k)}+\frac{q^2(p.k)^2}{2(p.q)^2(q.k)}-\frac{(p.k)}{(p.q)}\right]\widetilde{f}\cdot\widehat{g}_1 \tag{33}$$

$$A_2 = \int d^4k\left[\frac{k^4(p.q)}{2(q.k)^2(p.k)}+\frac{q^2k^2(p.k)}{2(p.q)(q.k)^2}-\frac{k^2}{(q.k)}\right]\widetilde{f}\cdot\widehat{g}_2 \tag{34}$$

$$D_1 = \int d^4k\left[-\frac{k^2}{2(p.q)(q.k)}-\frac{3q^2(p.k)^2}{2(p.q)^3(q.k)}+\frac{2(p.k)}{(p.q)^2}\right]\widetilde{f}\cdot\widehat{g}_1 \tag{35}$$

$$D_2 = \int d^4k\left[-\frac{k^4}{2(p.k)(q.k)^2}-\frac{3q^2k^2(p.k)}{2(p.q)^2(q.k)^2}+\frac{2k^2}{(p.q)(q.k)}\right]\widetilde{f}\cdot\widehat{g}_2 \tag{36}$$

$$H_1 = -\int d^4k\frac{k^2}{(p.q)(q.k)}\widetilde{f}\cdot\widehat{g}_1 \tag{37}$$

$$H_2 = -\int d^4k\frac{k^2}{(p.q)(q.k)}\widetilde{f}\cdot\widehat{g}_2 \tag{38}$$

$$C_v = -\int d^4k\frac{k^2(p.k)}{(p.q)^2(q.k)}\widetilde{f}\cdot\widehat{v} . \tag{39}$$

We finally obtain the following representations for $g_1(x,Q^2)$ and $g_2(x,Q^2)$:

$$\begin{aligned} g_1(x,Q^2)+g_2(x,Q^2) &= \frac{p.q}{M^2}[A_1+A_2] \\ &= \int d^4k\ \frac{p.q}{M^2}\Bigg\{\left[\frac{k^2}{2q.k}+\frac{q^2(p.k)^2}{2q.k\ (p.q)^2}-\frac{p.k}{p.q}\right]\widetilde{f}\cdot\widehat{g}_1 \\ &\qquad +\left[\frac{k^4\ p.q}{2\ (q.k)^2p.k}+\frac{q^2\ k^2\ p.k}{2p.q(q.k)^2}-\frac{k^2}{q.k}\right]\widetilde{f}\cdot\widehat{g}_2\Bigg\} \\ &= \int d^4k\left[\frac{q^2\ (p.k)^2}{2q.k\ p.q}-p.k\right]\frac{\widetilde{f}\cdot\widehat{g}_1}{M^2}+\int d^4k\left(\frac{k^2}{Q^2}\right)\ \Phi_1(k,p,q) , \end{aligned} \tag{40}$$

$$\begin{aligned} g_1(x,Q^2) &= \frac{p.q}{M^2}[A_1+A_2]+\frac{(p.q)^2}{M^2}[D_1+D_2+H_1+H_2+C_v] \\ &= \int d^4k\ \frac{p.q}{M^2}\Bigg\{\left[-\frac{k^2}{q.k}-\frac{q^2(p.k)^2}{q.k\ (p.q)^2}+\frac{p.k}{p.q}\right]\widetilde{f}\cdot\widehat{g}_1 \\ &\qquad -\left[\frac{q^2k^2\ p.k}{(q.k)^2\ p.q}\right]\widetilde{f}\cdot\widehat{g}_2-\left[\frac{p.k\ k^2}{q.k\ p.q}\right]\widetilde{f}\cdot\widehat{v}\Bigg\} \\ &= \int d^4k\left[-\frac{q^2\ (p.k)^2}{q.k\ p.q}+p.k\right]\frac{\widetilde{f}\cdot\widehat{g}_1}{M^2}+\int d^4k\left(\frac{k^2}{Q^2}\right)\ \Phi_2(k,p,q) . \end{aligned} \tag{41}$$

Here the functions $\Phi_{1,2}(k,p,q)$ are finite as $k^2\to 0$.

4 Representation of the structure functions in terms of a generating functional

Equivalently to the representation of the polarized structure functions in the previous section one may represent them referring to a generating functional[4]. The hadronic tensor can be represented by functions of the form

$$F_i(p^2, p.q, q^2) = \int d^4k \; a_i(p^2, p.k.k^2) \; b_i(q^2, q.k, k^2) \; , \tag{42}$$

where the functions a_i represent the parts depending on p and k and b_i have a dependence on q and k only. As shown in the foregoing this separation is possible accounting for other factors, which do not depend on k. Here $\widetilde{f}$ belongs to the former and $\widehat{g}_1$, $\widehat{g}_2$, and $\widehat{v}$ to the latter besides of scalar products.

The partial derivatives of F by q and p can be represented by partial derivatives of the functions a and b as follows :

$$\begin{aligned}
\frac{\partial F}{\partial q^\sigma} &= p_\sigma \frac{\partial F}{\partial p.q} + 2q_\sigma \frac{\partial F}{\partial q^2} \\
&= \int d^4k \;\; a \;\; \cdot \;\; \left[k_\sigma \frac{\partial b}{\partial q.k} + 2q_\sigma \frac{\partial b}{\partial q^2} \right] \qquad (43) \\
\frac{\partial F}{\partial p^\sigma} &= q_\sigma \frac{\partial F}{\partial p.q} + 2p_\sigma \frac{\partial F}{\partial p^2} \\
&= \int d^4k \;\; b \;\; \cdot \;\; \left[k_\sigma \frac{\partial a}{\partial p.k} + 2p_\sigma \frac{\partial a}{\partial p^2} \right] \qquad (44) \\
\frac{\partial^2 F}{\partial p^\lambda \partial q^\sigma} &= g_{\lambda\sigma} \frac{\partial F}{\partial p.q} + 2p_\sigma p_\lambda \frac{\partial^2 F}{\partial p^2 \partial p.q} + 2q_\sigma q_\lambda \frac{\partial^2 F}{\partial q^2 \partial p.q} + p_\sigma q_\lambda \frac{\partial^2 F}{\partial (p.q)^2} + 4q_\sigma p_\lambda \frac{\partial^2 F}{\partial q^2 \partial p^2} \\
&= \int d^4k \left[k_\lambda k_\sigma \frac{\partial a}{\partial p.k} \frac{\partial b}{\partial q.k} + 2k_\lambda q_\sigma \frac{\partial a}{\partial p.k} \frac{\partial b}{\partial q^2} + 2p_\lambda k_\sigma \frac{\partial a}{\partial p^2} \frac{\partial b}{\partial q.k} + 4p_\lambda q_\sigma \frac{\partial a}{\partial p^2} \frac{\partial b}{\partial q^2} \right] . \qquad (45)
\end{aligned}$$

The following structures contribute to the polarized part of the hadronic tensor :

$$\begin{aligned}
\varepsilon_{\mu\nu\alpha\sigma} q^\alpha S_\lambda \int d^4k \; k^\lambda k^\sigma \frac{\partial a}{\partial p.k} \frac{\partial b}{\partial q.k} &= \varepsilon_{\mu\nu\alpha\beta} q^\alpha \left[S^\beta \frac{\partial F}{\partial p.q} + S.q p^\beta \frac{\partial^2 F}{\partial (p.q)^2} \right] \qquad (46) \\
\varepsilon_{\mu\nu\alpha\sigma} p^\alpha S_\lambda \int d^4k \; k^\lambda k^\sigma \frac{\partial a}{\partial p.k} \frac{\partial b}{\partial q.k} &= \varepsilon_{\mu\nu\alpha\sigma} p^\alpha \left[S^\sigma \frac{\partial F}{\partial p.q} + 2S.q q^\sigma \frac{\partial^2 F}{\partial (p.q)^2} \right. \\
&\qquad \left. -2q^\sigma S_\lambda \int d^4k \; k^\lambda \frac{\partial a}{\partial p.k} \frac{\partial b}{\partial q^2} \right] \qquad (47) \\
\varepsilon_{\mu\nu\alpha\sigma} q^\alpha p^\beta S_\lambda \int d^4k k^\lambda \frac{\partial a}{\partial p.k} \;\cdot\; b &= \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta S.q \frac{\partial F}{\partial p.q} \; , \qquad (48)
\end{aligned}$$

which is given by

$$W^{(A)}_{\mu\nu}(q,p,S) = \frac{i}{M^2} \varepsilon_{\mu\nu\alpha\beta} q^\alpha \left\{ S^\beta \left[\frac{\partial F_1}{\partial (p.q)} + p.q \frac{\partial F_2}{\partial (p.q)} \right] + S.q \; p^\beta \left[\frac{\partial^2 F_1}{\partial (p.q)^2} + p.q \frac{\partial^2 F_2}{\partial (p.q)^2} \right. \right.$$

[4] This case was studied for polarized light quarks in the presence of an on–shell condition in Ref. [5] before.

$$+\frac{\partial^2(F_3+F_4)}{\partial(p.q)^2}\Big]$$
$$+2S.q\;p^\beta\Big[\frac{\partial^2 F_5}{\partial(p.q)\;\partial q^2}-\int d^4k\frac{S.k}{S.q}\;\frac{\partial a_5}{\partial(p.q)}\frac{\partial b_5}{\partial q^2}\Big]\Big\}\;. \tag{49}$$

The comparison with (1) yields the following expressions for the structure functions :

$$g_1(x,Q^2)+g_2(x,Q^2) \;=\; \frac{p.q}{M^2}\left[\frac{\partial F_1}{\partial p.q}+p.q\frac{\partial F_2}{\partial p.q}\right]\;, \tag{50}$$

$$g_2(x,Q^2) \;=\; -\frac{(p.q)^2}{M^2}\Big[\frac{\partial^2 F_1}{\partial(p.q)^2}+p.q\frac{\partial^2 F_2}{\partial(p.q)^2}+\frac{\partial^2(F_3+F_4)}{\partial(p.q)^2}+2\frac{\partial^2 F_5}{\partial p.q\partial q^2}$$
$$-2\int d^4k\;\frac{S.k}{S.q}\frac{\partial a_5}{\partial p.q}\frac{\partial b_5}{\partial q^2}\Big]\;. \tag{51}$$

We now separate the finite contributions to the polarized structure functions in the limit $k^2\to 0$ from those which vanish. We consider

$$\frac{d}{dx}\Big\{x\left[g_1(x,Q^2)+g_2(x,Q^2)\right]\Big\} \;=\; -\frac{(p.q)^2}{M^2}\left[\frac{\partial^2 F_1}{\partial(p.q)^2}+p.q\frac{\partial^2 F_2}{\partial(p.q)^2}+\frac{\partial F_2}{\partial(p.q)}\right]\;, \tag{52}$$

where $x=-q^2/(2p.q)$. Likewise one obtains

$$-x\frac{d}{dx}\Big\{g_1(x,Q^2)+g_2(x,Q^2)\Big\} \;=\; g_1(x,Q^2)+g_2(x,Q^2)$$
$$+\frac{(p.q)^2}{M^2}\left[\frac{\partial^2 F_1}{\partial(p.q)^2}+p.q\frac{\partial^2 F_2}{\partial(p.q)^2}+\frac{\partial F_1}{\partial(p.q)}\right]\;. \tag{53}$$

On the r.h.s. of (53) one may express the structure function $g_2(x,Q^2)$ inserting (51) which yields

$$-x\frac{d}{dx}\Big\{g_1(x,Q^2)+g_2(x,Q^2)\Big\} \;=\; g_1(x,Q^2)-\Phi(x,Q^2)\;, \tag{54}$$

with

$$\Phi(x,Q^2) \;=\; \frac{(p.q)^2}{M^2}\left[\frac{\partial(F_3+F_4-F_2)}{\partial(p.q)}+2\frac{\partial^2 F_5}{\partial(p.q)\partial q^2}-2\int d^4k\frac{S.k}{S.q}\frac{\partial a_5}{\partial p.k}\frac{\partial b_5}{\partial q^2}\right]\;. \tag{55}$$

Let us investigate the structure of the function $\Phi(x,Q^2)$ more closely. To do this we refer to (46–48) from which follows

$$\frac{\partial F_i}{\partial p.q} \;=\; \int\; d^4k\frac{S.k}{S.q}\frac{\partial a_i}{\partial p.k}\;\cdot\; b\;. \tag{56}$$

One notices that

$$\frac{\partial a_i}{\partial p.k}=\frac{k^2}{p.k}\cdot\widehat{f},\quad \text{for}\quad k=2\ldots 5\;. \tag{57}$$

The remainder terms contributing to $\Phi(x,Q^2)$ result from (48). Using Eqs. (56,57) one thus concludes that $\Phi(x,Q^2)$ obeys the representation

$$\Phi(x,Q^2)=\int d^4k\left(\frac{k^2}{Q^2}\right)\;\phi(p.k,q.k,p^2,q^2,k^2)\;, \tag{58}$$

where $\phi(p.k,q.k,p^2,q^2,k^2)$ is finite for $k^2\to 0$.

5 The relation between $g_1(x,Q^2)$ and $g_2(x,Q^2)$

In (40, 41, 50, 51) the virtuality of the gluon field is revealed by power corrections in $(k^2/Q^2)^l$. These functions are not yet projected onto the twist–2 contributions.[5] Any kind of partonic approach is only valid if the partonic virtualities k^2 obey

$$|k^2| \ll Q^2 \ . \tag{59}$$

This is an analogous condition to the requirement that the parton lifetime has to be much larger than the interaction time in the deeply inelastic scattering process for all associated infinite momentum frames, [27]. To project out the twist–2 contributions we refer to the collinear basis, cf. [28], and expand these functions into a Taylor series in k^μ at $k^\mu = zp^\mu$. Here $p.p = 0$ and the lowest twist contribution is obtained in setting $k^2 \to 0$ in (40, 41, 50, 51). Note, however, that the corresponding expressions may contribute at higher twist due to the associated derivatives in k^μ. We denote the twist–2 contributions to the structure functions $g_i(x,Q^2)$ by $g_i^{\rm II}(x,Q^2)$.

Whereas the relation between the structure functions $g_1^{\rm II}$ and $g_2^{\rm II}$ is not easily seen from Eqs. (40, 41) [6], it can be directly obtained from (54),

$$-x\frac{d}{dx}\left[g_1^{\rm II}(x,Q^2) + g_2^{\rm II}(x,Q^2)\right] = g_1^{\rm II}(x,Q^2) \ , \tag{60}$$

the differential form of the Wandzura–Wilczek relation, see [9, 10]. The integral form can be obtained from (60) using the condition

$$\lim_{x\to 1} g_i^{\rm II}(x,Q^2) = 0 \tag{61}$$

which yields

$$g_2^{\rm II}(x,Q^2) = -g_1^{\rm II}(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^{\rm II}(y,Q^2) \ . \tag{62}$$

At the level of twist–2 all structure functions depend on the same non–perturbative function $\widetilde{f}$ and sub–system structure function $\widehat{g}_1^{\rm par}$. Since the Wandzura–Wilczek relation holds as well for quarkonic initial states [6, 7] all higher order radiative corrections can be absorbed into $\widehat{g}_1^{\rm par}$, respectively, with par $= g,\ q_i$ denoting the quark species and the gluon, and holds thus in all orders.

The validity of the Wandzura–Wilczek relation also in the case of gluonic operator matrix elements is in accordance with the observation of the general nature of this relation connecting vector operator matrix–elements with the associated scalar operator matrix–elements on the light cone, which was shown for fermionic fields in [13, 15].

To compare the heavy flavor contributions to $g_{1,2}(x,Q^2)$ to the usual parameterizations of these structure functions numerically we show the light flavor contributions to $xg_{1,2}(x,Q^2)$ in Figures 1a,b at leading order. For the parton distributions we refer to the parameterization [29]. Other recent parameterizations [30, 31] are well in accordance with [29] within the current experimental errors. The polarized gluon and sea–quark distributions have still a rather large uncertainty which can only be lowered by more precise data from future experiments. The present parameterizations were derived assuming three light quarks through a fit to the data ignoring

[5] Plain consideration of off–shell contributions in k^2 to hadronic structure functions may introduce unphysical contributions. One example are wrong target mass corrections as noted in [26] a long time ago.

[6] For this the explicit dependence on $(p.q)$ had to be known for Eq. (40).

any heavy flavor contribution. Part of the present gluon–distribution is thus corresponding to the contribution due to heavy flavors and one cannot add the distributions in Figures 1a,2a or Figures 1b,2b in a simple way.

As Figure 2a shows the effect of $xg_1^{c\overline{c}}(x,Q^2)$ is small at low scales Q^2 but rises rapidly with Q^2 and should be taken into account in future refined QCD analyses. Due to the larger quark mass and charge–suppression the b–quark contribution to $xg_1(x,Q^2)$ is smaller than that due to charm–quarks. While the distributions are positive for $x > 5 \cdot 10^{-3}$ they change sign for lower values of x. In the present example the polarized gluon distribution is positive. The Wilson coefficient (7) changes sign, since

$$\lim_{z\to 0} C_{g_1}^{Q\overline{Q}}(z,Q^2) = \frac{1}{2}\left[3 - \ln\left(\frac{Q^2}{zM_Q^2}\right)\right] , \tag{63}$$

$$\lim_{\beta\to 0} C_{g_1}^{Q\overline{Q}}(z,Q^2) = \beta > 0, \tag{64}$$

and vanishes at threshold $z = Q^2/(1+4M_Q^2)$. Furthermore Eq. (9) holds. The oscillating behaviour of the Wilson coefficient implies lower relative heavy flavor contributions than in the unpolarized case.

Figure 1b shows $xg_2(x,Q^2)$ for the light flavors due to the Wandzura–Wilczek relation. $xg_2(x,Q^2)$ is positive for small x values up to $x \sim 10^{-1}$ and changes sign then. The integral over the positive function $xg_1(x,Q^2)$ is such larger than the subtraction term $xg_1(x,Q^2)$ in the former region while the subtraction term dominates in the latter region. The function has to have a positive and a negative branch since the Wandzura–Wilczek relation formally covers the Burkhardt–Cottingham relation[7],

$$\int_0^1 dx g_2(x,Q^2) = 0 \ . \tag{65}$$

Due to the additivity of twist–2 structure functions w.r.t. their parton contents the relation has to hold for each flavor separately.

Similar conclusions can be drawn for $xg_2^{Q\overline{Q}}(x,Q^2)$ shown in Figure 2b. $xg_2^{Q\overline{Q}}(x,Q^2)$ is widely positive for smaller values of $x \lesssim 2 \cdot 10^{-2}$ and negative for larger x. Again we would like to stress that the present numerical results on $xg_{1,2}^{Q\overline{Q}}(x,Q^2)$ are very sensitive on the polarized gluon distribution.

6 Conclusions

We calculated the heavy flavor contribution to $g_2(x,Q^2)$ at leading order in the strong coupling constant using the covariant parton model for finite values of the gluon virtuality k^2. The representation of the polarized structure functions $g_{1,2}^{Q\overline{Q}}(x,Q^2)$ can be obtained applying a tensor decomposition. Furthermore, a generating functional in which the k–dependent parton densities and coefficient functions are connected can be used to obtain a representation of the structure functions from which their possible relation can be derived. The twist–2 contributions are obtained in the limit $k^2 \ll Q^2$. The functions $g_{1,2}^{Q\overline{Q}}(x,Q^2)$ obey the Wandzura–Wilczek relation

[7] Note that the Wandzura–Wilczek relation is the analytic continuation from the positive moments, cf. e.g. [8, 10] in the local light cone expansion, where the 0th moment, which corresponds to the Burkhardt–Cottingham sum rule, does not contribute.

for a gluon induced process similar to earlier findings in fermion induced processes [5–10,12,13,15–18]. The absolute value of the relative numerical effect on both structure functions due to the LO heavy flavor contributions is about the same and may reach values of up to 5–10% in some kinematic ranges. [8] To make future QCD analyses of even more precise experimental data consistent at the level of the twist–2 contributions the heavy flavor distributions have to be taken into account.

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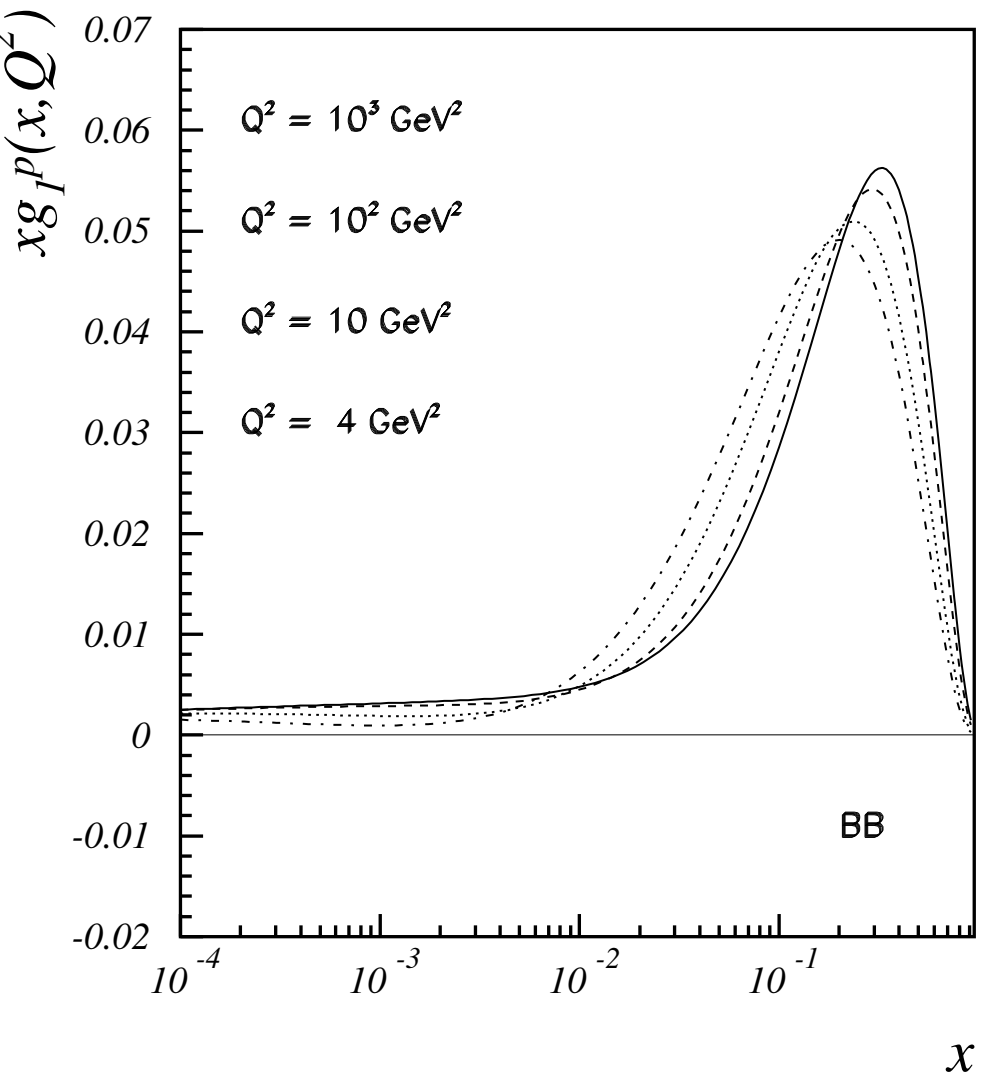

Figure 1a: The light flavor contributions to the polarized structure function $xg_1^p(x,Q^2)$ in leading order (parameterization `ISET=1` of [29], $\Lambda_{\rm QCD} = 203{\rm MeV}$) as a function of x and Q^2. Full line: $Q^2 = 4{\rm GeV}^2$; dashed line: $Q^2 = 10{\rm GeV}^2$; dotted line: $Q^2 = 100{\rm GeV}^2$; dash–dotted line: $Q^2 = 1000{\rm GeV}^2$.

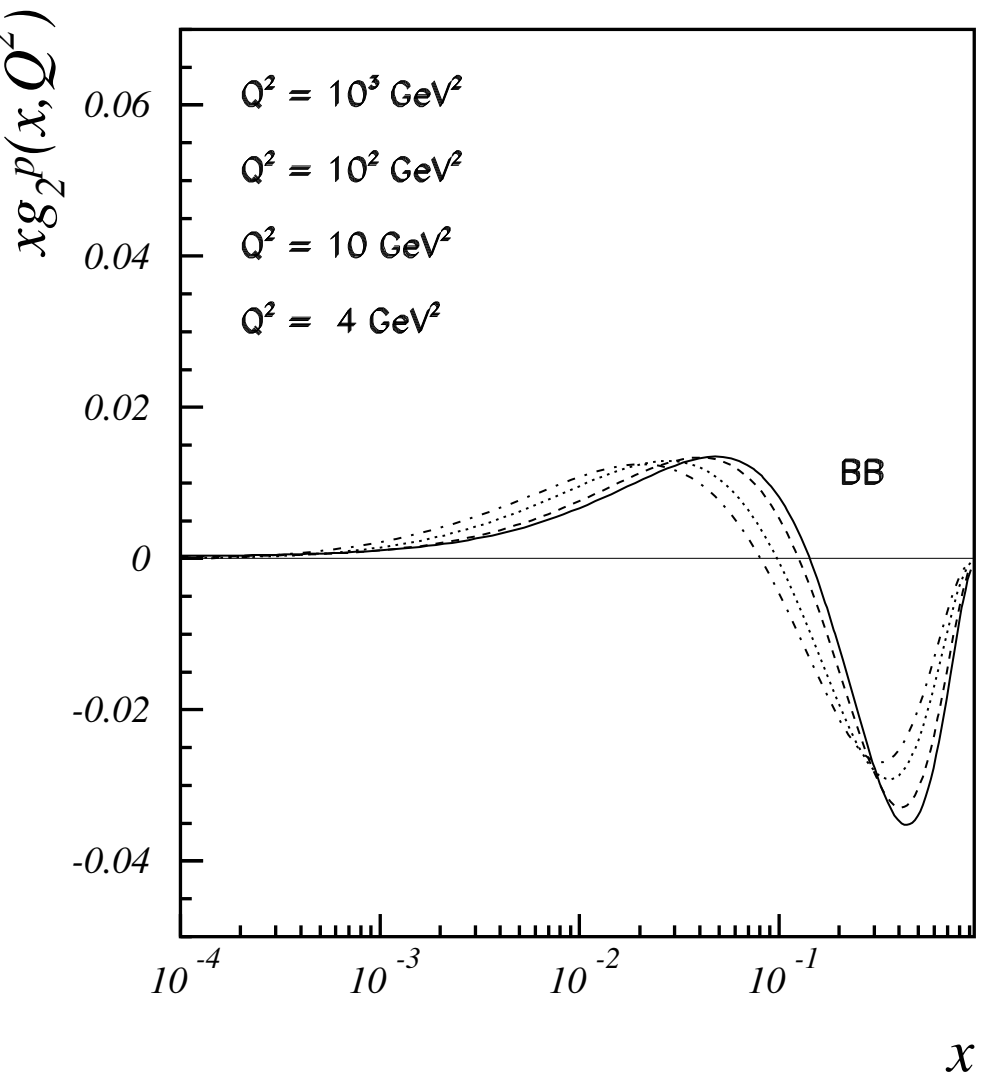

Figure 1b: The light flavor contributions to the polarized structure function $xg_2^p(x,Q^2)$. All conditions are the same as in Figure 1a.

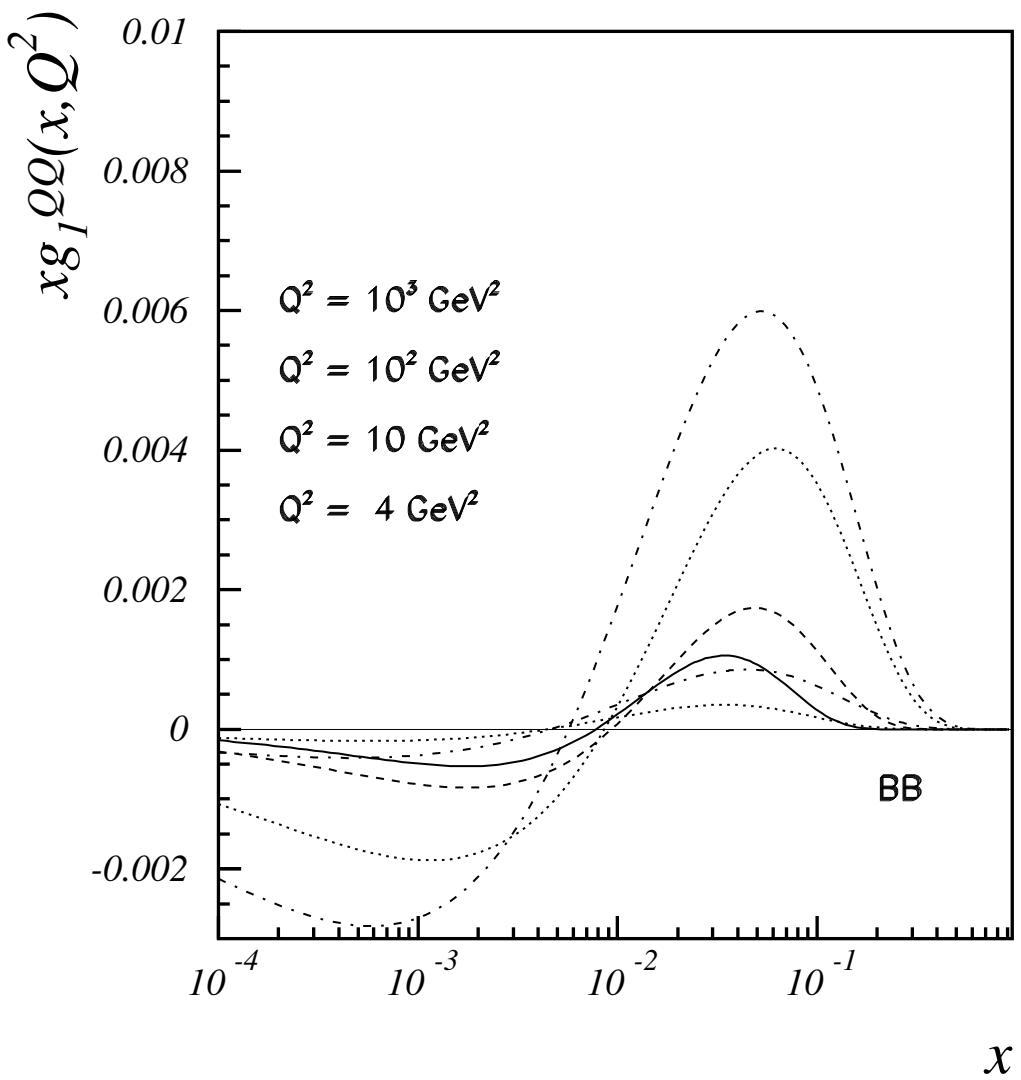

Figure 2a: Heavy flavor contributions to the polarized structure function $xg_1(x,Q^2)$ in leading order (parameterization ISET=1 of [29], $\Lambda_{\rm QCD} = 203{\rm MeV}$) as a function of x and Q^2. Upper lines: $g_1^{c\overline{c}}(x,Q^2)$ for $m_c = 1.5{\rm GeV}$. Full line: $Q^2 = 4{\rm GeV}^2$; dashed line: $Q^2 = 10{\rm GeV}^2$; dotted line: $Q^2 = 100{\rm GeV}^2$; dash–dotted line: $Q^2 = 1000{\rm GeV}^2$. Lower lines: $g_2^{c\overline{c}}(x,Q^2)$ for $m_b = 4.3{\rm GeV}$. Dotted line: $Q^2 = 100{\rm GeV}^2$; dash–dotted line: $Q^2 = 1000{\rm GeV}^2$. Figure 2b: Heavy flavor contributions to the polarized structure function $xg_2^p(x,Q^2)$.

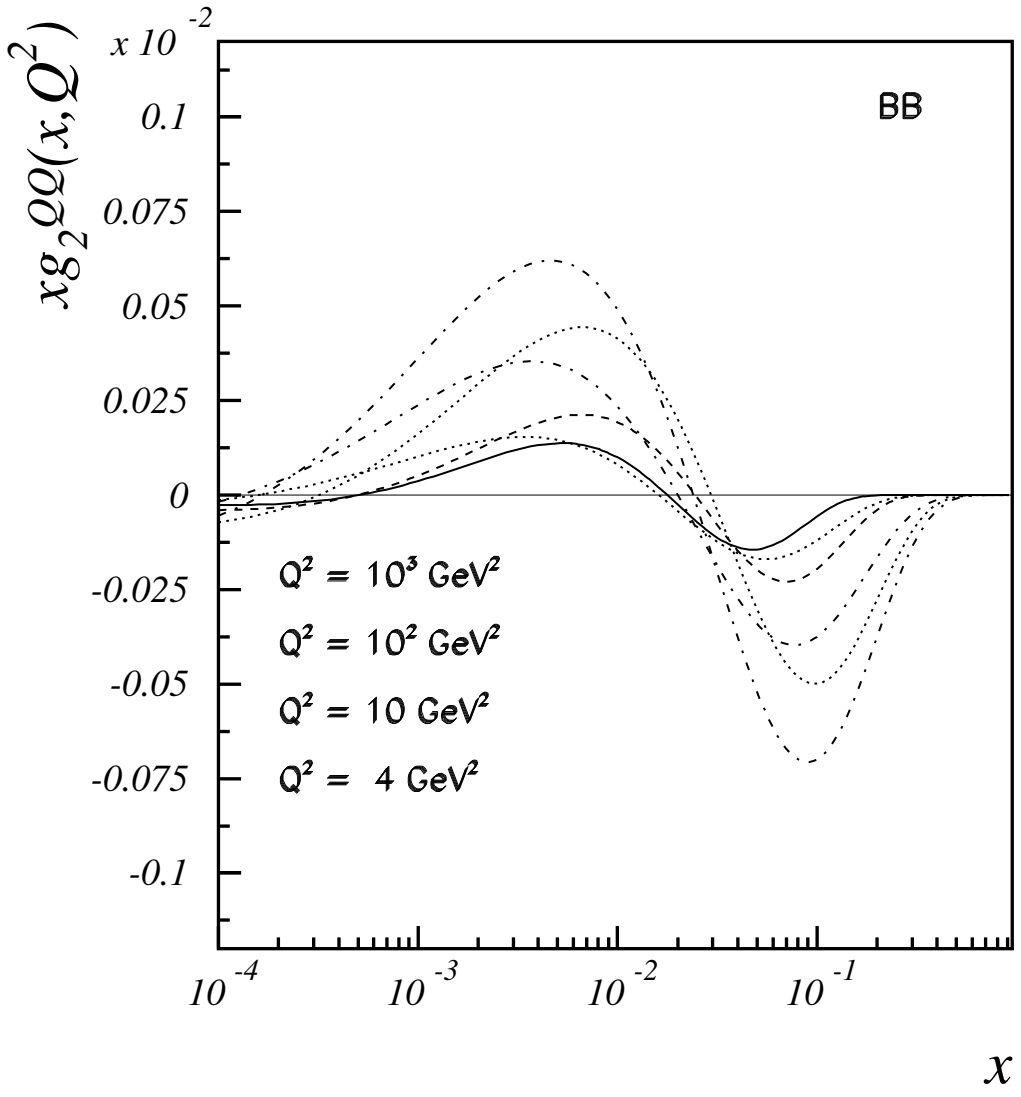

Figure 2b: Heavy flavor contributions to the polarized structure function $xg_2^p(x,Q^2)$. All conditions are the same as in Figure 2a.