arXiv:hep-ph/0306311 v1 30 Jun 2003

DESY-03-077
hep-ph/0306311

Exclusive two-photon annihilation at large energy or large virtuality*

M. Diehl [a]

[a]Deutsches Elektronen-Synchroton DESY, 22603 Hamburg, Germany

I review recent progress in the theory of $\gamma\gamma$ annihilation into meson or baryon pairs at large energy, and of the process $\gamma^*\gamma^* \to \pi^0$ at large photon virtuality.

1. INTRODUCTION

In this contribution I discuss two topics of two-photon physics. In both of them, handbag diagrams play a major role, but the physics issues are rather different.

Several talks in this session have shown the ongoing experimental progress in measuring $\gamma\gamma$ annihilation into meson or baryon pairs at large energy. Recent theoretical work suggests that these processes might help clarify the long-standing problems in understanding the dynamics of exclusive reactions in fixed-angle kinematics (i.e. at large Mandelstam variables s, t, u). In a second part I investigate what one might learn from the annihilation of two virtual photons into a single π^0, beyond what we have already learned from the measurements with one virtual photon and their theoretical analysis.

This presentation is based on work with P. Kroll and C. Vogt [1,2,3], where further detail and references can be found.

2. REAL PHOTONS AND LARGE s, t, u

Exclusive two-photon processes like $\gamma\gamma \to \pi\pi$ or $\gamma\gamma \to p\bar{p}$ are described by the hard-scattering mechanism of Brodsky and Lepage in the limit $s \to \infty$ at fixed t/s and u/s [4]. In this limit the process amplitudes factorize into a subprocess involving quarks and gluons and into the distribution amplitudes for the lowest Fock states of the produced hadrons. Example graphs are shown in Fig. 1.

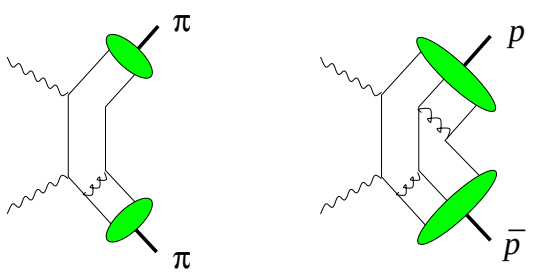

Figure 1. Example graphs for $\gamma\gamma \to \pi\pi$ and for $\gamma\gamma \to p\bar{p}$ in the hard-scattering mechanism.

In evaluating these graphs, one encounters regions of loop momentum where all parton lines except those attached to one or both photons are soft, with momenta of a few 100 MeV in the collision c.m. In these regions the assumptions and approximations of the hard-scattering calculation break down and the result is not trustworthy. In the limit specified above this is not a problem: the contribution from such soft regions is power

*Presented at the International Conference on the Structure and Interactions of the Photon (PHOTON 2003), Frascati, Italy, 7–11 April 2003. To appear in the proceedings.

suppressed in $1/s$ and thus harmless within the accuracy of the calculation. Problems do however occur in practical calculations for s in the range of a few to a few 10 GeV^2. Depending on what one takes for the distribution amplitudes of the produced particles, the result either undershoots data by an order of magnitude or more, or it receives substantial contributions from the soft phase space regions where the result cannot be trusted.

One may take this as an indication that the soft contribution just described plays an important role at experimentally accessible values of s, and there are recent theoretical attempts to calculate this contribution in a consistent way [1,2,5]. To justify the approximations of the calculations we must still require that s, t, u be large compared with a typical hadronic scale; in particular one has to avoid the region where $\sqrt{s}$ is close to resonance masses and the regions of forward or backward scattering angle θ in the c.m. The separation of the dynamics into soft and hard pieces now differs from the one in Fig. 1 and is shown in Fig. 2: a hard subprocess $\gamma\gamma \to q\bar{q}$ is followed by soft hadronization into the final state. In order for the second step to be soft, the initial quark and antiquark must each carry approximately the full four-momentum of one final-state hadron; all other partons involved in the hadronization process must have soft four-momenta in the c.m. This soft subprocess can be described by matrix elements of the form $\langle \pi\pi |\, \bar{q}_\alpha(0)\, q_\beta(z) |0\rangle$ between the vacuum and the final state, with a quark-antiquark operator separated along the light-cone $z^2 = 0$. Such matrix elements are parameterized by generalized distribution amplitudes [6,7], which are related to generalized parton distributions by crossing symmetry.

In the limit $s \to \infty$ the soft annihilation contribution is power suppressed in $1/s$ compared with the hard-scattering contribution, but it is in finite s that we are interested in. At present we have no theory for consistently adding the contributions from the two mechanisms without double counting. Since in the kinematics of interest the hard-scattering results are small (at least when the region of soft loop momenta is discarded) I will in the following neglect this contribution.

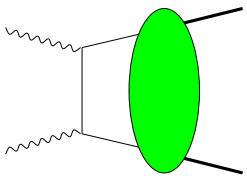

Figure 2. Handbag graph for the soft annihilation contribution to $\gamma\gamma \to \pi\pi$ or $\gamma\gamma \to p\bar{p}$.

I will in fact assume that the soft annihilation mechanism dominates over all other mechanisms (for instance of vector dominance type, where the hadronic components of the photons take part in the reaction). This leads to predictions which can be confronted with data.

2.1. Results: meson pairs

For pseudoscalar mesons P the soft annihilation mechanism gives the $\gamma\gamma \to P\bar{P}$ cross section as [1]

$$\frac{d\sigma(P\bar{P})}{dt} = \frac{8\pi\alpha_{\rm em}^2}{s^2}\,\frac{1}{\sin^4\theta}\,|R_{2P}(s)|^2. \qquad (1)$$

The information on the soft subprocess is contained in $R_{2P}(s)$, which is a form factor of the local operator

$$\sum_q e_q^2 \left(\bar{q}\gamma^\mu i\overleftrightarrow{D}^\nu q + \{\mu \leftrightarrow \nu\}\right) \qquad (2)$$

and hence related with the quark energy-momentum tensor. The mechanism makes a *prediction* for the cross section dependence on the scattering angle θ in the two-photon c.m., keeping in mind that the result (1) is not valid for θ close to 0° or 180°.

A general feature of the handbag graphs (independently of the approximations needed to evaluate them) is that the reaction proceeds through a single $q\bar{q}$ pair in the s-channel. This leads to *predictions* of the soft annihilation mechanism for the production ratios of different meson pairs. The cleanest of them makes only use of isospin symmetry and is

$$\frac{d\sigma(\pi^0\pi^0)}{dt} = \frac{d\sigma(\pi^+\pi^-)}{dt}. \qquad (3)$$

This result is easily derived: the two-photon collision produces the pion pair only in a C-even state, which must have isospin $I = 0$ or $I = 2$. The $I = 2$ state is however forbidden in the handbag mechanism since it is unaccessible for a $q\bar{q}$ pair. The prediction (3) stands out against the hard-scattering mechanism, where one obtains $\sigma(\pi^0\pi^0) \ll \sigma(\pi^+\pi^-)$ for pion distribution amplitudes close to the asymptotic one [4]. For $\rho\rho$ production we correspondingly have

$$\frac{d\sigma(\rho^0\rho^0)}{dt} = \frac{d\sigma(\rho^+\rho^-)}{dt}. \tag{4}$$

Since the isospin argument works at the amplitude level, the relation (4) readily generalizes to the cross sections for definite polarization states of the ρ mesons. To my knowledge, a prediction of the hard-scattering mechanism is unfortunately not available for the $\rho\rho$ channel. Returning to pseudoscalars, one obtains as a further consequence of the handbag mechanism that

$$\frac{d\sigma(K^0\bar{K}^0)}{dt} \simeq \frac{4}{25}\,\frac{d\sigma(K^+K^-)}{dt}, \tag{5}$$

where the $\simeq$ sign signals that to obtain this result one needs SU(3) flavor symmetry, which is only approximately satisfied. A further consequence of SU(3) symmetry is

$$\frac{d\sigma(K^+K^-)}{dt} \simeq \frac{d\sigma(\pi^+\pi^-)}{dt}, \tag{6}$$

which—contrary to the previous relations—is independent of the reaction mechanism. Its violation in the real world may be taken as a measure of how strongly SU(3) flavor symmetry is broken in this type of process. Notice that the often cited ratio $\sigma(K^+K^-)/\sigma(\pi^+\pi^-) = (f_K/f_\pi)^4$ is *not* a prediction of the hard-scattering mechanism, but assumes in addition that the distribution amplitudes of pions and kaons have the same shape—an assumption one may or may not wish to make.

We cannot presently calculate the form factors $R_{2P}(s)$ within QCD, so that we have *no prediction* for the s dependence of the cross section, nor for its absolute size. We found that the preliminary data for $\pi^+\pi^-$ production from ALEPH [8] and DELPHI [9] can be described by a form factor $R_{2\pi}(s)$ which for s between 6 and 30 GeV^2 behaves as s^{-1} and is similar in size to the electromagnetic pion form factor $F_\pi(s)$ at large timelike s. There is no reason for these form factors to be equal, since they belong to different currents, but it seems plausible that they should be of similar size. Notice that both $R_{2\pi}(s)$ and $\sigma(\pi^+\pi^-)$ approximately follow the s dependence obtained in the hard scattering picture (which fails badly for the absolute normalization). That this might happen over a finite interval in s is not in contradiction with the soft annihilation mechanism. I note that this mechanism has an analog in the crossed channel, namely the Feynman mechanism for spacelike form factors and large-angle Compton scattering, see [10] for a recent overview. In these cases, a model study has in fact shown how the soft mechanism can systematically mimic the hard-scattering scaling behavior in the t region of several GeV^2 [11].

2.2. Results: baryon pairs

For a baryon B the soft annihilation mechanism leads to a $\gamma\gamma \to B\bar{B}$ cross section [2]

$$\frac{d\sigma(B\bar{B})}{dt} = \frac{4\pi\alpha_{\rm em}^2}{s^2} \times \frac{1}{\sin^2\theta}\left(|R^B_{\rm eff}(s)|^2 + \cos^2\theta\,|R^B_V(s)|^2\right) \tag{7}$$

with

$$|R^B_{\rm eff}(s)|^2 = |R^B_A + R^B_P|^2 + \frac{s}{4m_B^2}\,|R^B_P|^2. \tag{8}$$

The form factors R^B_A and R^B_P belong to the axial current and R^B_V belongs to the vector current of quarks, with the different flavors weighted by their squared electric charges as in (2).

As in the meson case, we cannot calculate these form factors at present and hence have *no prediction* for the s dependence or the size of the cross section. We found the CLEO and VENUS data [12] for $p\bar{p}$ production at s between 6.5 and 11 GeV^2 rather well described by a form factor $R^p_{\rm eff}(s)$ that falls like s^{-2} and is somewhat larger in size than the measured magnetic proton form factor $G_M(s)$ at similar s. We neglected $R_V(s)$ at this stage since its prefactor in the integrated cross section is rather small in the range $|\cos\theta| \le 0.6$ of the data. In the s range under study we find again the same s behavior one

would obtain in the hard-scattering mechanism, and our discussion of the meson case equally applies here.

Our result (7) *does predict* the form of the θ dependence in terms of one unknown parameter $|R_V^B(s)/R_{\rm eff}^B(s)|$. We obtain more predictions if we assume flavor SU(3) symmetry and make further approximations, spelled out in [2]. The production ratios of all $B\bar{B}$ channels where B is in the ground state baryon octet are then expressed in terms of a single parameter ρ, which describes the relative strength of the transitions $d\bar{d} \to p\bar{p}$ and $u\bar{u} \to p\bar{p}$. Taking ρ between 0.25 and 0.75 we find fair agreement with the data of CLEO and L3 [13] for $\Sigma^0\bar{\Sigma}^0$ and $\Lambda\bar{\Lambda}$ production, and predict in particular that the cross section for the mixed $\Lambda\bar{\Sigma}^0 + \Sigma^0\bar{\Lambda}$ channel should be much lower than $\sigma(p\bar{p})$.

The experimental progress reported at this meeting, both for meson and for baryon pairs, will allow one to refine the phenomenological studies I have reported on. The θ dependence of the cross section and production ratios for various channels can be used to test the soft annihilation mechanism. If it stands up to these tests, the data may be used to extract the various form factors which describe the transition from $q\bar{q}$ to simple hadronic systems at a quantitative level.

3. VIRTUAL PHOTONS AND $s = m_\pi^2$

The annihilation of two spacelike photons into a single π^0 is one of the simplest exclusive processes and has been studied for a long time. In the limit where the sum $Q^2 + Q'^2$ of the photon virtualities becomes infinite, the amplitude factorizes into the short-distance annihilation process $\gamma^*\gamma^* \to q\bar{q}$ and the pion distribution amplitude, as shown in Fig. 3. Compared to the processes discussed in Section 2, the theory of $\gamma^*\gamma^* \to \pi^0$ is in much better shape: the hard-scattering kernel at $O(\alpha_s)$ has been known and used to for a long time, and the calculation of the $O(\alpha_s^2)$ corrections has recently been reported [14]. Power corrections to the leading-twist result have been estimated by various methods and are found to be moderate, even if one of the photon virtualities is zero.

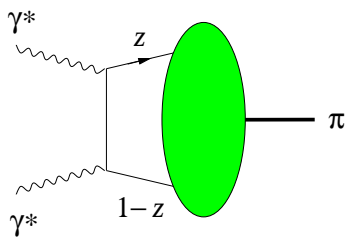

Figure 3. Handbag graph for $\gamma^*\gamma^* \to \pi^0$ at leading order in α_s.

The pion distribution amplitude $\phi(z,\mu^2)$ is a fundamental quantity describing the structure of the pion. It is useful to expand it on Gegenbauer polynomials in $2z-1$,

$$\phi(z,\mu^2) = 6z(1-z) \times \Big[1 + \sum_{n=2,4,\ldots}^{\infty} B_n(\mu^2)\, C_n^{3/2}(2z-1)\Big], \qquad (9)$$

where z denotes the momentum fraction of the quark in the pion and μ^2 the factorization scale. The cleanest experimental constraints on the distribution amplitude come from the CLEO measurement [15] of $\gamma^*\gamma \to \pi^0$, where one of the photons is quasi-real. The amplitude is parameterized in terms of the photon-pion transition form factor, which reads

$$F_{\pi\gamma}(Q^2) = \frac{f_\pi}{3\sqrt{2}Q^2}\int dz\, \frac{\phi(z,\mu^2)}{z(1-z)} = \frac{\sqrt{2}f_\pi}{Q^2}\Big[1 + \sum_{n=2,4,\ldots}^{\infty} B_n(\mu^2)\Big] \qquad (10)$$

with $f_\pi \approx 131$ MeV, up to relative corrections of order α_s or $1/Q^2$. The Q^2 dependence of the CLEO data is well described by this approximation already at a few GeV^2. Analyzing the data in terms of the leading-order formula (10) one finds that the sum $\sum B_n(\mu^2)$ is small already at $\mu \sim 1$ GeV. This conclusion does not change much when taking the $O(\alpha_s)$ corrections into account. Without further theoretical assumptions one can however not conclude that the *individual* Gegenbauer moments $B_n(\mu^2)$ must be small,

although this would be in line with theoretical prejudice and with the analysis of other processes (where however the errors of either theory or experiment are considerably larger). For a discussion I refer to [3]; a conflicting point of view concerning theoretical errors is taken in [16]. The different μ dependence of the $B_n(\mu^2)$ provides a handle to gain separate information about them from the Q^2 dependence of $F_{\pi\gamma}$, but in order to use this one must have sufficient control over power corrections, whose Q^2 dependence is much stronger. Data with higher statistics at larger Q^2 would greatly help in this.

It is natural to ask whether more information can be obtained from data on $\gamma^*\gamma^* \to \pi^0$ with both photons off-shell. To leading-power accuracy in $1/\overline{Q}^2$, the transition form factor for a virtual photon can be written as

$$F_{\pi\gamma^*}(\overline{Q}^2,\omega) \;=\; \frac{f_\pi}{\sqrt{2}\,\overline{Q}^2} \times\Big[c_0(\omega)+\sum_{n=2,4,\ldots}^{\infty} c_n(\omega,\log\overline{Q}^2)\,B_n\Big], \qquad (11)$$

where I have suppressed the dependence on the renormalization and factorization scale μ and chosen symmetric variables

$$\overline{Q}^2=\frac{1}{2}(Q^2+Q'^2), \qquad \omega=\frac{Q^2-Q'^2}{Q^2+Q'^2}. \qquad (12)$$

For symmetry reasons the coefficients c_n are even functions of ω. Evaluating them one finds the surprising behavior shown in Fig. 4.

We see that as soon as ω becomes different from one, the sensitivity to all but the lowest Gegenbauer moments B_n rapidly goes to zero. For our original goal of gaining information about the pion distribution amplitude one would thus focus on the region of $1-\omega$ shown in the second plot of the figure. As shown in [3], the transition form factor in this region has indeed some power to distinguish different scenarios for the Gegenbauer moments which are neither implausible nor ruled out by the existing CLEO data.

In a large region around $\omega=0$ the form factor is however hardly sensitive to the pion structure at all, except via the pion decay constant f_π. In fact, one finds that it can be expanded around

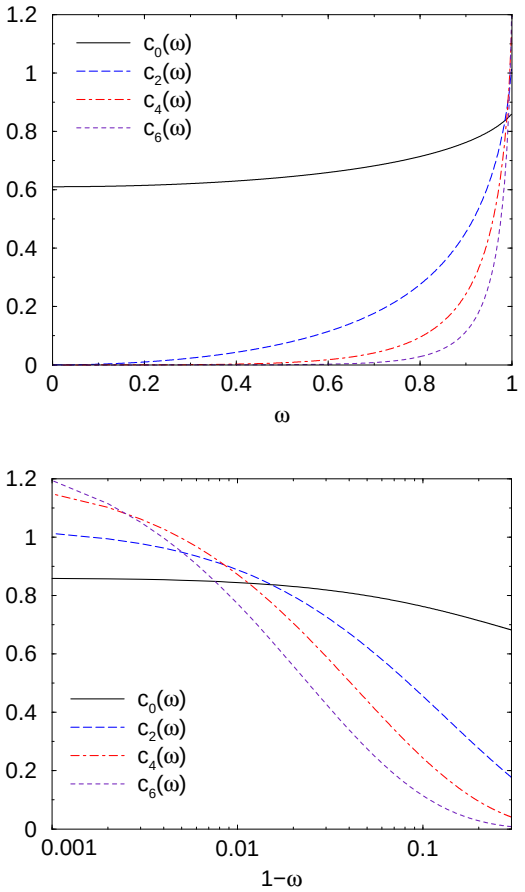

Figure 4. Coefficients c_n in the expansion (11) of $F_{\pi\gamma^*}$. Corrections of $O(\alpha_s)$ are included with the factorization and renormalization scale μ set to $\overline{Q}$ and taken as 2 GeV.

$\omega = 0$ as

$$F_{\pi\gamma^*}(\overline{Q}^2, \omega) = \frac{\sqrt{2} f_\pi}{3\overline{Q}^2} \left[1 - \frac{\alpha_s}{\pi} + \omega^2 \left(\frac{1}{5} - \frac{1}{3} \frac{\alpha_s}{\pi} \right) + \frac{12}{35} \omega^2 B_2 + O(\alpha_s \omega^2, \omega^4, \alpha_s^2) \right] \quad (13)$$

where higher Gegenbauer moments B_n always appear with at least a power ω^n. Given the results of [14] one can predict $F_{\pi\gamma^*}$ within QCD up to relative corrections of order

$$\omega^4, \ \alpha_s^3 \omega^2, \ \alpha_s^4, \ \Lambda^2/Q^2 \quad (14)$$

with a hadronic scale Λ, provided one takes the lowest coefficient B_2 as an input parameter. At small enough ω one may even neglect the terms with $\omega^2 B_2$ and then has a prediction for $F_{\pi\gamma^*}$ only in terms of α_s and f_π. The power corrections in Λ^2/Q^2 may at least be estimated using the results of [17], which would require knowledge of the matrix element $\langle \pi | \bar{d}\, g \tilde{G}^{\mu\nu} \gamma_\nu \, u | 0 \rangle$. Both this quantity and B_2 are in principle amenable to calculation from first principles in lattice QCD, and both can be constrained by phenomenological analysis, for instance of data for the transition form factor at ω close to 1.

The transition form factor $F_{\pi\gamma^*}$ can in this sense be regarded as a precision observable, whose measurement would allow a rather fundamental test of our understanding of QCD. It is very similar to the Bjorken sum rule for deep inelastic scattering, to which it is intimately related as explained in [14]. Note that compared with the efforts required to measure the Bjorken sum, measurement of $F_{\pi\gamma^*}$ can in principle be done in a single experiment for $e^+e^- \to e^+e^-\pi^0$ in suitable kinematics. The bad news is that the cross section is quite low: integrating over ω from -0.5 to $+0.5$ one obtains a differential e^+e^- cross section of $d\sigma/d\overline{Q}^2 \approx 0.5$ fb GeV^{-2} at $\overline{Q}^2 = 4$ GeV^2. Similarly low rates are obtained in the region $\omega \approx 1$ discussed above. Experimental investigation appears therefore difficult even at the high-luminosity machines BaBar and Belle in their present setups. The study of this fundamental process may however become feasible at possible luminosity upgrades of these facilities.

REFERENCES

1. M. Diehl, P. Kroll and C. Vogt, Phys. Lett. B **532**, 99 (2002) [hep-ph/0112274].
2. M. Diehl, P. Kroll and C. Vogt, Eur. Phys. J. C **26**, 567 (2003) [hep-ph/0206288].
3. M. Diehl, P. Kroll and C. Vogt, Eur. Phys. J. C **22**, 439 (2001) [hep-ph/0108220].
4. S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 1808 (1981).
5. A. Freund et al., Phys. Rev. Lett. **90**, 092001 (2003) [hep-ph/0208061].
6. D. Müller et al., Fortsch. Phys. **42**, 101 (1994) [hep-ph/9812448].
7. M. Diehl et al., Phys. Rev. Lett. **81**, 1782 (1998) [hep-ph/9805380].
8. A. Finch, in: Proceedings of PHOTON 2001, World Scientific, Singapore (2002).
9. K. Grzelak, in: Proceedings of PHOTON 2001, World Scientific, Singapore (2002).
10. P. Kroll, hep-ph/0302169.
11. M. Diehl et al., Phys. Lett. B **460**, 204 (1999) [hep-ph/9903268].
12. CLEO Collaboration, Phys. Rev. D **50**, 5484 (1994);
    VENUS Collaboration, Phys. Lett. B **407**, 185 (1997).
13. CLEO Collaboration, Phys. Rev. D **56**, 2485 (1997) [hep-ex/9701013];
    L3 Collaboration, Phys. Lett. B **536**, 24 (2002) [hep-ex/0204025].
14. B. Melić, D. Müller and K. Passek-Kumerički, hep-ph/0212346.
15. CLEO Collaboration, Phys. Rev. D **57**, 33 (1998) [hep-ex/9707031].
16. A. P. Bakulev et al., Phys. Rev. D **67**, 074012 (2003) [hep-ph/0212250].
17. A. Khodjamirian, Eur. Phys. J. C **6**, 477 (1999) [hep-ph/9712451].