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Spinor Gravity

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A unified description of all interactions could be based on a higher-dimensional theory involving only spinor fields. The metric arises as a composite object and the gravitational field equations contain torsion-corrections as compared to Einstein gravity. Lorentz symmetry in spinor space is only global, implying new goldstone-boson-like gravitational particles beyond the graviton. However, the Schwarzschild and Friedman solutions are unaffected at one loop order. Our generalized gravity seems compatible with all present observations.

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The physical description of our world is based on fermionic and bosonic degrees of freedom, corresponding to particles with different statistical properties. In particular, bosons mediate the interactions between fermions as well as between themselves. One may consider two alternative approaches to a fundamental unified theory. Either one treats bosons and fermions on equal footing, attempting to link them by some symmetry principle - this is the road of supersymmetry. Alternatively, one considers the fermions as fundamental and explains the bosons as composite objects of an even number of fermions. The second approach is commonly followed in the treatment of strongly correlated electrons in statistical physics and is familiar from the Nambu-Jona-Lasinio [1] and Gross-Neveu [2] models[1].

In this letter we explore the hypothesis that a fundamental unified theory of the gravitational, electroweak and strong interactions can be formulated in terms of only fermionic degrees of freedom. Given that all bosons arise as composite fields, one immediately faces the fundamental challenge of describing the metric in terms of fermionic variables. In particular, one has to realize the symmetry of general coordinate transformations or diffeomorphisms by a purely fermionic functional integral *without* the use of a fundamental metric degree of freedom. The metric of space-time can then be associated with the expectation value of a suitable fermion bilinear. Our formulation uses the notion of d-dimensional coordinates x^μ, $\mu=0,\ldots,d-1$, only in order to describe spinor fields $\psi(x)$ in some local coordinate patch and for the definition of a derivative $\partial_\mu\psi$. The geometric and topological properties of space-time arise a posteriori as a consequence of the dynamics of the system, reflected in the fermion correlators. This realizes the idea of obtaining geometry from general statistics [4].

The idea of a purely fermionic fundamental theory has previously been explored on the basis of proposals for "pregeometry" [5,6] or "metric from matter" [7], where a diffeomorphism invariant spinor theory was formulated and the metric was obtained as a composite object. However, in [5] and most of the subsequent work on pregeometry, an (auxiliary) metric or vielbein field had to be introduced already at the level of the fundamental action. In the spirit of "induced gravity" [9] (see [10] for a review), fermionic loop corrections then generate the kinetic terms for these fields, transforming them into propagating degrees of freedom. By contrast, in [7] a purely spinorial action was proposed. However, it is of non-polynomial form (the covariant derivative involves terms $\sim(\psi\bar{\psi})^{-1}$). Since elements of the inverse matrix $(\psi\bar{\psi})^{-1}$ do not exist within the Grassmann algebra, it is not clear how a functional integral can be defined. A later supersymmetric proposal [8] involves again fundamental bosonic fields.

The proposal coming closest to our ideas about 'spinor gravity' is contained in Akama's pregeometry paper [6], where a purely spinorial, polynomial action with diffeomorphism invariance is given. However, after introducing bosonic auxiliary fields a singular limit is taken to realize local Lorentz symmetry. This limit is inconsistent with polynomiality of the original action. As will become clear below, our proposal is free of such problems because we do not insist on local Lorentz invariance at a fundamental level. Exploring the consequences of a spinor theory with only global Lorentz symmetry will lead us to a generalized gravity theory which is, in principle, distinguishable from Einstein gravity but appears to be consistent with all present experimental bounds.

Once an effective gravity theory is found, the extension to gauge interactions and scalar fields (needed for spontaneous symmetry breaking) is straightforward: one may go higher-dimensional [11]. Indeed, it is well known that non-abelian gauge interactions arise in the compactification of pure higher-dimensional gravity theories [12]. In contrast to supersymmetric unified models, the number of dimensions is not restricted here to $d\leq 11$. This opens the door for realizing the gauge symmetries of the standard model as isometries of "internal space", with light fermions protected by chirality. Rather realistic models of quarks and leptons with electroweak and strong interactions have already been proposed in the context of 18-dimensional gravity coupled to a Majorana Weyl spinor [13].

[1] There are also approaches of treating fermions as composite objects in a bosonic theory and one can imagine a dual description of a fundamental theory (see, e.g., [3]).

Spinors are Grassmann variables and our basic tool is the Grassmann functional integral for the partition function

$$Z[J] = \int \mathcal{D}\psi \exp\Big\{ -(S + S_J)\Big\}. \tag{1}$$

The dynamics is described by the "classical action" $S[\psi]$ which has to be a polynomial functional of the spinor fields. As a technical tool we have added to the action a source term S_J. It is linear in suitable sources J and facilitates the computation of expectation values of spinor bilinears. We will set $J = 0$ in the end to obtain expectation values in the vacuum. Our first task is to find a polynomial action S which is invariant under diffeomorphisms, so that no particular choice of coordinates is singled out. For this purpose we may use as a basic building block the "real" local fermion bilinears

$$\tilde{E}^m_\mu(x) = \frac{i}{2m}\Big\{\bar{\psi}(x)\gamma^m\partial_\mu\psi(x) - \partial_\mu\bar{\psi}(x)\gamma^m\psi(x)\Big\}. \tag{2}$$

Here $\psi(x)$ belongs to an irreducible spinor representation of the global d-dimensional "Lorentz group" SO(1,$d-1$) and γ^m are the associated Dirac matrices. The arbitrary mass scale m has only been introduced to make $\tilde{E}^m_\mu$ dimensionless. Under general coordinate transformations ψ transforms as a scalar and therefore $\tilde{E}^m_\mu$ as a vector. Interpreting $\tilde{E}^m_\mu$ as a $d \times d$ matrix and defining $\tilde{E} = \det(\tilde{E}^m_\mu)$, we propose the invariant action[2]

$$S = \alpha \int d^dx \tilde{E} = \frac{\alpha}{d!}\int d^dx \epsilon^{\mu_1\dots\mu_d}\epsilon_{m_1\dots m_d}\tilde{E}^{m_1}_{\mu_1}\dots\tilde{E}^{m_d}_{\mu_d}. \tag{3}$$

Clearly, no metric is needed in order to realize diffeomorphism invariance in a fundamental fermionic theory [6]. Since we demand that the action is polynomial in ψ, no inverse of $\tilde{E}^m_\mu$ can appear and all lower world indices from derivatives ∂_μ have to be contracted with the totally antisymmetric d-dimensional ϵ-tensor. This also ensures the cancellation of the Jacobian of reparameterizations. The action of Eq. (3) is invariant under global but not under local Lorentz rotations.

More general forms of invariant actions can be found in an accompanying paper [14]. As explained above, they have to contain precisely d derivatives by reparameterization invariance. The number of spinor fields contained in each invariant may vary, but it is bounded above due to the finite number of derivatives and because a sufficiently high power of a spinor field vanishes by anticommutativity. Thus, only a finite (though potentially large) number of independent invariants exists.

The basic composite bosonic field is the "global vielbein" E^m_μ associated with the expectation value of the fermion bilinear $\tilde{E}^m_\mu$

$$E^m_\mu(x) = \langle \tilde{E}^m_\mu(x)\rangle = \frac{\delta \ln Z}{\delta J^\mu_m(x)}. \tag{4}$$

Here we have specified the source term $S_J = -\int d^dx\, J^\mu_m(x)\tilde{E}^m_\mu(x)$. Using $\eta_{mn} = \mathrm{diag}(-1, 1\dots1)$, the metric can be constructed in the standard way

$$g_{\mu\nu}(x) = E^m_\mu(x)E_{\nu m}(x) = E^m_\mu(x)E^n_\nu(x)\eta_{mn}. \tag{5}$$

The inverse metric $g^{\mu\nu}$ exists whenever $E = \det(E^m_\mu) \neq 0$ and can be used to "raise indices". As required, the metric transforms as a symmetric tensor field. We will argue that the metric obeys field equations similar to the Einstein equations. It can be associated with a massless, composite graviton (see also [15] for some of the early as well as more recent related ideas). However, we expect the resulting effective gravity theory to be a generalization of Einstein gravity. Since the action of Eq. (3) is invariant under *global* but not under *local* Lorentz rotations, the vielbein describes additional massless degrees of freedom not associated with the metric. (This distinguishes our approach from earlier attempts to realize local Lorentz symmetry [5–7].) The resulting "generalized gravity" [16] will lead to a specific version of torsion. In suitable dimensions and for a suitable "ground state" E^m_μ, our fundamental spinor theory will describe both massless spinors and massless gravitational fields.

In order to get a first glance at the dynamics of the theory, we employ the method of partial bosonization [17]. Without a perturbative expansion in a small parameter we do not expect quantitatively accurate results, but the structural elements deriving from the effective degrees of freedom and symmetries will become visible. Up to an irrelevant normalization constant, we can write the partition function as an equivalent functional integral over fermions ψ and boson fields χ (cf. [2]),

$$Z[J] = \int \mathcal{D}\psi\mathcal{D}\chi^n_\nu \exp\Big\{ -S_B + \int d^dx\, J^\mu_m\chi^m_\mu\Big\},$$
$$S_B[\psi,\chi] = \alpha\int d^dx\Big[\det(\tilde{E}^m_\mu) - \det(\tilde{E}^m_\mu - \chi^m_\mu)\Big]. \tag{6}$$

The equivalent purely fermionic theory can be recovered by performing the Gaussian functional integral for the bosonic fields χ^n_ν. It differs from the theory defined by Eqs. (1) and (3) only in the connected correlation functions of at least order d in $\tilde{E}^m_\mu$ [14]. With $E^m_\mu = \langle\chi^m_\mu\rangle$, the effective action $\Gamma[E]$ is defined as

[2] In the conventions of Eq. (2) one has $\alpha \sim m^d$. There is, however, no need to introduce m. For dimensionless spinor fields $\psi(x)$ and omitting the factor m^{-1} in Eq. (2), the coefficient α becomes dimensionless. It can be scaled to arbitrary values by a rescaling of ψ. We employ here a "Euclidean notation" avoiding factors of i in the definition of the functional integral. For a Minkowski signature a factor i can be absorbed in α.

$$\Gamma[E] = -W + \int d^d x J^\mu_m(x) E^m_\mu(x), \tag{7}$$

$$\frac{\delta W[J]}{\delta J^\mu_m} = E^m_\mu\,, \qquad W[J] = \ln Z[J]\,. \tag{8}$$

By construction, $\Gamma[E]$ is invariant under diffeomorphisms and global Lorentz rotations if the functional measure is free of gravitational anomalies (which could, in principle, be present for $d = 2 \bmod 4$ [18]). The quantum field equations following from the variation of $\Gamma[E]$ are

$$\frac{\delta \Gamma}{\delta E^m_\mu} = J^\mu_m\,, \tag{9}$$

where $J^\mu_m = 0$ in the vacuum. Contributions from matter and radiation can be associated to an energy momentum tensor $T^{\mu\nu} = E^{-1} E^{m\mu} J^\nu_m$. The latter accounts for the incoherent fluctuations of ψ and of bosonic composite fields as well as for possible expectation values of bosonic composite fields other than E^m_μ.

At one loop order in the fermionic fluctuations, $\Gamma[E]$ is given by ($\tilde{\alpha} = (-1)^{d+1}\alpha$)

$$\Gamma[E] = \tilde{\alpha} \int d^d x E - \frac{1}{2} \,\mathrm{Tr}\, \ln(E\mathcal{D})\,, \tag{10}$$

where $E\mathcal{D}$ is the second functional derivative of S_B with respect to ψ. The generalized Dirac operator $\mathcal{D}$ involves the inverse vielbein E^μ_m and a "covariant derivative" $\hat{D}_\mu$,

$$\mathcal{D} = \gamma^\mu \hat{D}_\mu = E^\mu_m \gamma^m \hat{D}_\mu\,, \tag{11}$$

$$\hat{D}_\mu = \partial_\mu + \frac{1}{2E} E^m_\mu \partial_\nu (E E^\nu_m). \tag{12}$$

The evaluation of the one loop term can be performed by standard techniques (see, e.g., [19] and references therein). We expand in the number of derivatives and find (see below) for the term with two derivatives

$$\Gamma_{(2)} = \frac{\mu}{2} \int d^d x E \Big\{ -R + \tau [D^\mu E^\nu_m D_\mu E^m_\nu - 2 D^\mu E^\nu_m D_\nu E^m_\mu] \Big\}\,, \tag{13}$$

where $\tau = 3$ and

$$D_\mu E^m_\nu = \partial_\mu E^m_\nu - \Gamma_{\mu\nu}{}^\lambda E^m_\lambda. \tag{14}$$

Here $\Gamma_{\mu\nu}{}^\lambda$ and R are the usual Christoffel symbols and curvature scalar associated with the metric $g_{\mu\nu}$. Thus, the present rough approximation suggests that our spinor model leads to effective gravitational interactions.

Deviations from Einstein gravity result from the term $\sim \tau$. If E^m_μ were a *local* vielbein of conventional general relativity, this term would be forbidden by *local* Lorentz symmetry since D_μ contains no spin connection. In order to interpret the additional invariant, it is useful to study the linear expansion around flat space

$$E^m_\mu = \delta^m_\mu + \frac{1}{2}(h_{\mu\nu} + a_{\mu\nu}) \eta^{\nu m}\,, \tag{15}$$

where $h_{\mu\nu}$ and $a_{\mu\nu}$ are the symmetric and antisymmetric part. The former accounts for the metric fluctuations ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$) and can be decomposed in the usual manner,

$$h_{\mu\nu} = b_{\mu\nu} + \frac{1}{(d-1)} \left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \sigma + \frac{\partial_\mu \partial_\nu}{\partial^2} f + \partial_\mu v_\nu + \partial_\nu v_\mu\,, \tag{16}$$

with $b_{\mu\nu}\eta^{\mu\nu} = 0$, $\partial_\mu b^{\mu\nu} = 0$, and $\partial_\mu v^\mu = 0$ (v^μ and f being gauge degrees of freedom). The new degrees of freedom are associated with the antisymmetric part $a_{\mu\nu}$,

$$a_{\mu\nu} = c_{\mu\nu} + \partial_\mu (v_\nu + w_\nu) - \partial_\nu (v_\mu + w_\mu), \tag{17}$$

where $\partial_\mu c^{\mu\nu} = 0$ and $\partial_\mu w^\mu = 0$. Expanding the effective action of Eq. (13) to quadratic order yields

$$\Gamma_{(2)} = \frac{\mu}{8} \int d^d x \Big\{ \partial^\mu b^{\nu\rho} \partial_\mu b_{\nu\rho} - \frac{d-2}{d-1} \partial^\mu \sigma \partial_\mu \sigma + \tau \partial^\mu c^{\nu\rho} \partial_\mu c_{\nu\rho} \Big\}\,. \tag{18}$$

The terms involving $b_{\mu\nu}$ and σ are the same as in Einstein gravity. While v^μ and f are absent because of diffeomorphism invariance, the absence of w^μ may indicate a hidden nonlinear symmetry. The new massless particles described by $c_{\mu\nu}$ can be interpreted as the Goldstone bosons of the spontaneous breaking of the global Lorentz symmetry. Indeed, a ground state $E^m_\mu = \delta^m_\mu$ breaks the global SO(1,$d-1$) symmetry acting only on the indices m completely, while it is invariant under combined global Lorentz transformations acting simultaneously on the coordinates. This implies $d(d-1)/2$ Goldstone directions corresponding to $a_{\mu\nu}$. In Einstein gravity with local Lorentz symmetry $a_{\mu\nu}$ would be a gauge degree of freedom.

Furthermore, on flat space a spinor mass term is forbidden by global Lorentz rotations for irreducible spinors in $d = 2, 6, 8, 9 \bmod 8$ [21]. In these dimensions the spectrum contains both massless fermions and massless gravitons, where the latter can be interpreted as fermionic bound states. In a higher-dimensional theory ($d > 4$) this is a necessary [21] condition for obtaining a realistic low energy theory with quarks and leptons and the gauge symmetries of the standard model. Indeed, the most interesting solutions for the vielbein correspond to geometries where the characteristic size of $d-4$ dimensions is much smaller than the size of the 3+1 dimensional universe observable on large length scales. The effective four-dimensional theory is then given by "compactification". Part of the higher-dimensional symmetries (connected to isometries) appear in the effective theory as gauge symmetries. The chirality index [22] counting the number of chiral fermion generations with respect to these gauge symmetries depends on the particular geometry of "internal space". For gravity in $d = 18$ rather realistic settings for symmetries and charges have already been discussed

long ago [13] and it would be very interesting to see how such a construction could emerge from the generalization of gravity proposed in this letter.

We assume that, after compactification, the gravitational part of the effective four-dimensional action still has the form of Eq. (13), now with new effective coefficients μ_4, τ_4. The general form of the action is restricted by the symmetries, even if additional invariants are added to the classical action of Eq. (3). In this context we note that the two terms multiplying τ in Eq. (13) could, from the point of view of global Lorentz and diffeomorphism invariance, have coefficients with a ratio other than -2. In other words, there are really three independent invariants only two of which are actually generated by the loop calculation. If present, the third invariant would lead to more drastic deviations from Einstein gravity [14] than the invariant $\sim \tau$ generated in the one loop approximation. In this letter we assume that some deeper reason (symmetry?) protects the structure of the one loop expression Eq. (13) at higher-loop order. As a working hypothesis we take the coefficient of the other allowed invariant to be zero (or very small). As a test of the viability of a generalized gravity theory with only global Lorentz symmetry, we therefore compare the field equations obtained from the effection action Eq. (13) in $d = 4$ with observations. In this context we also assume that the cosmological constant in the effective 4-dimensional theory (almost) vanishes. Postponing a more detailed investigation of this interesting issue, we note that the classical cosmological term in Eq. (10) can be chosen to cancel the one-loop contribution, which amounts to the usual fine-tuning. This cancellation also depends on the internal compact space and extends to a more general form of the fundamental spinor action which could involve other invariants beyond Eq. (3).

For weak gravity the linear field equations follow from the variation of $\Gamma_{(2)}$ in Eq. (18) in presence of an energy momentum tensor $T_{\mu\nu}$. With energy density $T_{00} = \rho$ and Newtonian potential $\phi = -\frac{1}{2}h_{00}$, one finds for static solutions ($\Delta = \partial^i\partial_i$)

$$\Delta\phi = \frac{\rho}{2\mu}. \tag{19}$$

This associates μ with the reduced Planck mass $\bar{M}_p^2 = M_p^2/8\pi = (8\pi G_N)^{-1} = \mu$.

The new field $c_{\mu\nu}$ only couples to the antisymmetric part of the energy momentum tensor, $(T_A)_{\mu\nu}$, defined by the variation of the action with respect to the antisymmetric part of E_μ^m. As will become clear below, for spinors this corresponds to a coupling to the spin. Pointlike particles without internal structure (including massless particles) generate a symmetric energy momentum tensor. Therefore $(T_A)_{\mu\nu}$ can only be related to the internal structure – in our case it involves the spin vector. The "spin contribution" to the gravitational interactions mediated by the exchange of $c_{\nu\rho}$ does not contain a rotation invariant part since c_{0i} and $\epsilon_{ijk}c_{jk}$ transform as vectors. Only for macroscopic objects with a nonzero macroscopic spin vector the modifications of gravity would become observable.

It has been checked [14] that even beyond the linear approximation the modifications of gravity $\sim \tau$ neither affect the Schwarzschild solution nor the Friedmann cosmological solution. We conclude that our proposal of generalized gravity is compatible with all present tests of general relativity. Nevertheless, it is conceivable that the presence of new long range fields beyond the graviton may lead to new interesting cosmological solutions, possibly accounting for quintessence [23].

To make the relation of spinor gravity to conventional general relativity more explicit, it is useful to introduce nonlinear fields with local Lorentz invariance. We write the global vielbein as $E_\mu^m = e_\mu^n H_n{}^m$, where e_μ^n is a conventional "local vielbein" with local Lorentz index. The $\mathrm{SO}(1, d-1)$ matrix $H_n{}^m$ has one local and one global index. The new local Lorentz transformations present in the nonlinear formulation correspond to a reparameterization of the decomposition of E_μ^m in e_μ^n and $H_n{}^m$, leaving E_μ^m invariant. The nonlinear field $H_n{}^m$ characterizes the new degrees of freedom that are present in addition to those of conventional gravity. We define here the covariant derivative D_μ by the requirement of a covariantly constant local vielbein e_μ^m. This implies that the covariant derivative of E_ν^m is given by $D_\mu E_\nu^m = e_\nu^n D_\mu H_n{}^m$. Here the action of D_μ on the first index of H makes use of the spin connection defined by e_μ^m, while the second, global index is inert. Given that $E = e = \det(e_\mu^m)$, it is now obvious that Eq. (13) describes conventional gravity covariantly coupled to the non-linear field H.

Geometrically, the new degrees of freedom can be characterized by a further connection $\tilde{\Gamma}$, which is present in addition to the Riemannian connection Γ,

$$\tilde{\Gamma}_{\mu\nu}{}^\lambda = (\partial_\mu E_\nu^m) E_m^\lambda. \tag{20}$$

This connection is defined by demanding that E_μ^m be covariantly constant. The existence of a covariantly constant basis of vector fields implies that our new connection is curvature-free. It has, however, non-vanishing torsion

$$T_{\mu\nu\rho} = (\partial_\mu E_\nu^m - \partial_\nu E_\mu^m) E_{\rho m}. \tag{21}$$

Thus, in the bosonic formulation, we are dealing with general relativity with torsion, where the torsion tensor is restricted by the requirement of a curvature-free Riemann-Cartan connection. More precisely, given a metric and a torsion tensor such that the Riemann-Cartan connection has zero curvature, we can always define a global vielbein E_μ^m by parallel transporting a frame defined at an arbitrary but fixed point of the manifold. One can easily check that E_μ^m will then automatically satisfy Eq. (21). Note that such a theory is very different from general relativity with a generic (unconstrained) torsion tensor.

In the above geometrical formulation, the one-loop calculation leading to Eq. (13) is particularly easy to understand. The leading fermionic term in the lagrangian of Eq. (6) reads

$$\mathcal{L}_\psi \sim \bar{\psi} E_m^\mu \gamma^m \partial_\mu \psi + \text{h.c.} \tag{22}$$

Using the local Lorentz invariance introduced above, we choose $e_\mu^m = E_\mu^m$. In this local frame, the coefficients of the spin connection going with our Riemann-Cartan connection vanish, $\tilde{\omega}_\mu{}^{ab} = 0$. Thus, we have

$$\mathcal{L}_\psi \sim \bar{\psi} e_m^\mu \gamma^m \tilde{D}_\mu \psi + \text{h.c.}\,, \tag{23}$$

and the problem is reduced to evaluating the effective action induced by integrating out a fermion in a Riemann-Cartan background. In fact, we can introduce a new fermionic variable ψ' defined to coincide with ψ in the frame where $e_\mu^m = E_\mu^m$ and to transform like a usual general-relativity spinor under local Lorentz transformations of e_μ^m. (This corresponds to the field redefinition $\psi = S(H^{-1})\psi'$, where S denotes the spinor representation.) Now we have a formulation of partially bosonized spinor gravity with full local Lorentz invariance: The variables are ψ', e_μ^m and $T_{\mu\nu\rho}$ (constrained by the requirement of a curvature-free Riemann-Cartan connection). The quadratic part of the fermionic action is given by Eq. (23) with $\psi \to \psi'$, which has local Lorentz-invariance.

The fermionic one-loop contribution to the effective action for e_μ^m and $T_{\mu\nu\rho}$ is calculated from the square of the Dirac operator defined by Eq. (23). More specifically, the leading divergences are characterized by the DeWitt coefficients [12] of the relevant operator (see, e.g., [24]). While the highest divergence comes with the cosmological constant, the next contribution contains the leading terms with vielbein derivatives and torsion. Given that only the totally antisymmetric part of the torsion couples to fermions, it has to be of the form

$$\Gamma_{(2)} = \frac{\mu}{2} \int d^d x e \left\{ -R + \tau' T_{[\mu\nu\rho]} T^{[\mu\nu\rho]} \right\}, \tag{24}$$

The value $\tau' \equiv 3\tau/4 = 9/4$ is fixed, e.g., by the calculation of the relevant DeWitt coefficient in [19] (see also [20]). Although this formally agrees with the result of [6], the physical meaning is very different since our torsion variable is constrained. Thus, the second term in Eq. (24) is the kinetic term for our new light degrees of freedom rather than a torsion mass term. In the linear approximation one finds $T_{[\mu\nu\rho]} \sim \partial_{[\mu} c_{\nu\rho]}$. The source for $c_{\nu\rho}$ arises from the coupling $\sim c_{\nu\rho} \partial_\mu (\bar{\psi} \gamma^{[\mu} \gamma^\nu \gamma^{\rho]} \psi) \sim c_{\nu\rho} \epsilon^{\nu\rho\mu\sigma} \partial_\mu S_\sigma$, which follows from Eq. (23) with ψ replaced by ψ'. Because of the superposition of the microscopic contributions from individual fermions, a macroscopic source for $c_{\nu\rho}$ involves the macroscopic total spin vector S_σ. For known objects the total spin is small and the mass contribution to gravity dominates by far, rendering the macroscopic effects of the exchange of $c_{\nu\rho}$ unobservable. However, it is easy to check that the particles described by $c_{\nu\rho}$ are produced in electron-positron annihilation or similar processes and their t-channel exchange affects electron-positron scattering. Of course, due to the gravitational coupling strength the cross section is tiny and not observable, just as for the graviton.

The effective action of Eq. (24) depends on H only via the term quadratic in the totally antisymmetric part of the torsion $T_{[\mu\nu\rho]}$. Any solution $e_\mu^m(x)$ of Einstein gravity is therefore also a solution of the generalized field equations provided $T_{[\mu\nu\rho]} = 0$. In particular, the solution $E_\mu^m = e_\mu^m$ exists whenever the vielbein is diagonal in a suitable coordinate system and Lorentz frame. This holds for the Schwarzschild- and Robertson-Walker-metrics, explaining the result of the explicit computation of [14]. Similarly, $T_{[\mu\nu\rho]}$ vanishes in every rotation and parity invariant situation since no totally antisymmetric invariant tensor exists (ϵ_{ijk} being odd under parity). This concludes our argument that the generalized gravity of Eq. (13) cannot be distinguished from Einstein gravity by present observations. It may be a surprise that a principle as fundamental as local Lorentz symmetry is so little tested in practice. Global Lorentz symmetry seems to be a perfectly viable alternative, despite the presence of new massless fields in the gravitational sector.

Our ambitious proposal for a unified theory of all interactions stands, of course, only at the beginning of its development. Nevertheless, we believe that it offers a possible interesting alternative to superstring theories for a well-defined theory of quantum gravity. For this it is crucial that, unless affected by anomalies, spinor gravity is renormalizable: As explained above, the symmetries allow only a finite number of polynomial invariants. Thus, if the quantum theory respects the symmetries, all divergences can be absorbed in a finite number of tree-level coefficients (which is one of the definitions of a renormalizable field theory). Furthermore, even though global Lorentz and reparameterization symmetry allow for a large number of independent invariants, we expect that additional symmetry requirements can drastically reduce the number of fundamental parameters. One still needs to specify a regularized functional measure that preserves the symmetries. If this can be achieved, spinor gravity is well defined by an explicit functional integral. The expectation values, including the metric, become, in principle, calculable.

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