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DIFFRACTION, EXCLUSIVE PROCESSES, AND PROTON STRUCTURE IN THREE DIMENSIONS*

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I briefly review how a three-dimensional picture of parton dynamics is rendered in the impact parameter representation, applied to generalized parton distributions and to the dipole picture of high-energy scattering.

1 The impact parameter representation

Processes like deeply virtual Compton scattering, elastic meson production, and hard diffraction tell us about the spatial distribution of quarks or gluons within a hadron. The impact parameter representation provides an adequate language to represent this information. It is a deep concept in field theory and has been used in various contexts of high-energy scattering.

To describe fast moving particles (hadrons or partons) it is useful to work with light-cone momenta $p^{\pm} = p^0 \pm p^z$. From the usual momentum eigenstates of a particle—labeled here by $|p^+, \mathbf{p}\rangle$ with boldface vectors referring to the x-y plane—one can form states

$$|p^+, \mathbf{b}\rangle = (16\pi^3)^{-1} \int d^2\mathbf{p}\, e^{-i\mathbf{p}\mathbf{b}}\, |p^+, \mathbf{p}\rangle \tag{1}$$

with definite plus-momentum and definite transverse position (called impact parameter). To avoid infinities one can use wave packets, where the transverse position is smeared out a bit. To retain the simple interpretation of p^+ as roughly twice p^z (for $p^z > 0$) one must restrict the range of $\mathbf{p}$ in the wave

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packet, giving a position uncertainty $\delta b \gg 1/p^+$ [1]. One can hence localize a particle in the x-y plane very precisely, provided it moves fast enough along z. This is different from trying to localize a particle in *all three* space dimensions, where one is limited by the Compton wavelength $1/m$. The original work of Burkardt [2] gives a hint at how this difference arises: it turns out that to achieve a simple interpretation one must approximate the energy p^0 with a constant for a given wave packet. To localize in three dimensions this restricts momenta to $(p^x)^2 + (p^y)^2 + (p^z)^2 \ll m^2$. For localizing in the transverse plane one needs $(p^x)^2 + (p^y)^2 \ll m^2 + (p^z)^2$, which is not restrictive for large p^z.

The states $|p^+, \mathbf{b}\rangle$ are eigenstates of a transverse position operator $\mathbf{R}$ [3]. It is associated with "transverse boosts", i.e., with Lorentz transformations

$$p^+ \to p^+, \qquad \mathbf{p} \to \mathbf{p} - p^+ \mathbf{v} \tag{2}$$

for some fixed vector $\mathbf{v}$. Notice the similarity with Galilean transformations in non-relativistic mechanics, where the mass of a particle replaces p^+. Noether's theorem relates Galilean invariance with the conservation of the center of mass of a many-body system. The conserved quantity for transverse boost invariance is hence the "center of (plus) momentum", $\sum p_i^+ \mathbf{b}_i \,/ \sum p_i^+$. The position $\mathbf{b}$ of a hadron state (1) can be understood as the center of momentum of its partons.

2 Parton distributions in impact parameter space

The impact parameter representation fits naturally with the concept of parton distributions, as realized long ago by Soper [4] and shown in detail by Burkardt [2, 5]. This can be used for "imaging" of hadrons [6]. The usual momentum space parton densities can be expressed through matrix elements $\langle p^+, \mathbf{p}| \, \mathcal{O} \, |p^+, \mathbf{p}\rangle$, where the quark or gluon operator $\mathcal{O}$ creates and annihilates a parton at impact parameter $\mathbf{0}$. I have omitted the dependence on polarization labels and on the usual plus momentum fraction x of the parton in the target. Inserting the same operator between the states (1) one finds

$$\langle p^+, \mathbf{b}' | \, \mathcal{O} \, |p^+, \mathbf{b}\rangle \propto \delta^{(2)}(\mathbf{b} - \mathbf{b}') \int d^2\mathbf{\Delta} \, e^{i\mathbf{b}\mathbf{\Delta}} \, \langle p^+, \mathbf{p}' | \, \mathcal{O} \, |p^+, \mathbf{p}\rangle, \tag{3}$$

where the matrix element on the right-hand side depends on $\mathbf{p}$ and $\mathbf{p}'$ only through $\mathbf{\Delta} = \mathbf{p}' - \mathbf{p}$ because of Lorentz invariance. The expression on the right

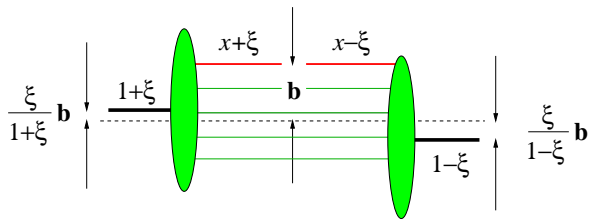

Figure 1: Impact parameter representation of a GPD in the region $x \geq \xi$. Plus momentum fractions $1 \pm \xi$ and $x \pm \xi$ refer to the average of initial and final proton momenta. The dashed line indicates the origin of transverse coordinates.

is the Fourier transform of a generalized parton distribution [7] at $t = -\mathbf{\Delta}^2$ and zero skewness ξ, whereas the left-hand side gives the joint density of partons with plus momentum fraction x and transverse position $\mathbf{b}$. It hence contains genuinely *three-dimensional* information about the target structure. Integrating (3) over $\mathbf{b}$ one recovers the usual parton densities with their one-dimensional information. If one integrates over x then $\mathcal{O}$ turns into a local operator, and the right-hand side becomes the Fourier transform of a form factor associated with this operator. This representation of a form factor in terms of a two-dimensional density is fully relativistic, unlike the usual representation through a three-dimensional density in a hadron at rest, which holds in the nonrelativistic limit and is limited to distance scales large compared with the Compton wavelength of the target. This is not a big restriction for a large and heavy nucleus, but it is quite restrictive for a proton. An interpretation of generalized parton distributions along these lines has been proposed in [8].

It is not known how to measure generalized parton distributions at $\xi = 0$ (except for certain moments in x), but the processes mentioned in the beginning involve these functions at nonzero ξ. A calculation analogous to (3) gives the picture shown in Fig. 1 for $x \geq \xi$, and a similar one for $x \leq \xi$ [1]. The center of momentum of the hadron is shifted by the interaction because the struck parton returns with a smaller momentum fraction. Note that this shift is independent of x, which is integrated over in the amplitudes of hard scattering processes. In a wide range of kinematics the shift is small compared with the distance of the struck parton from the "average" hadron position.

3 The dipole representation

A well-known formulation of BFKL dynamics at leading $\log \frac{1}{x}$ accuracy is the color dipole representation, shown in Fig. 2 for the virtual Compton amplitude. The sum of momentum space Feynman graphs can be rewritten in terms of photon wave functions $\Psi(z, \mathbf{r})$ and of the scattering amplitude $N(x_{bj}, \mathbf{r}, \mathbf{\Delta})$ for a $q\bar{q}$ dipole of size $\mathbf{r}$ on the target. The size $\mathbf{r}$, which is Fourier conjugate to the transverse quark momentum in the loop, is conserved in the scattering process as a nontrivial consequence of small-x kinematics, see e.g. [9]. After Fourier transform with respect to the transverse momentum transfer $\mathbf{\Delta}$ one obtains a representation fully in impact parameter space as shown in the figure. The conservation of the center of momentum in the transition between photon and $q\bar{q}$ pair has direct consequences for the dipole scattering amplitude, which is of the form [10]

$$\widetilde{N}(x_{bj}, \mathbf{r}, \mathbf{b}) = \int d^2\mathbf{\Delta}\, e^{i\mathbf{\Delta}\,[\mathbf{b}+(1-z)\mathbf{r}]} N(x_{bj}, \mathbf{r}, \mathbf{\Delta}). \tag{4}$$

We see that the relevant transverse distance in the Fourier transform is between the target and the quark in the dipole (not between the target and the photon). If one finally takes the leading double $\log \frac{1}{x} \log Q^2$ approximation, the amplitude $\widetilde{N}$ becomes proportional to the generalized gluon distribution at impact parameter $\mathbf{b}$, with the factorization scale $\mu \sim 1/|\mathbf{r}|$ set by the size of the dipole probing the gluons.

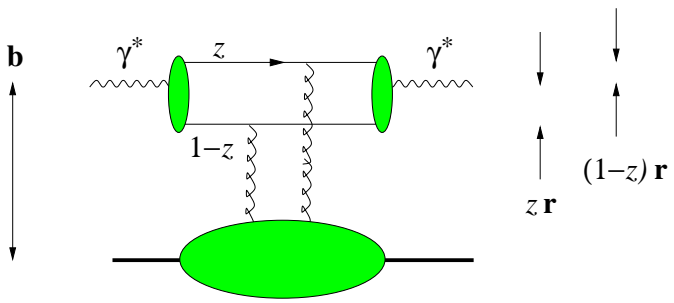

Figure 2: Impact parameter representation of the virtual Compton amplitude $\gamma^* p \to \gamma^* p$ in the dipole picture.

4 Conclusions

The impact parameter formalism gives a geometrical representation of hadron structure and dynamics, fully consistent with the principles of relativistic field theory. It naturally blends with the physical picture of the QCD parton model. Various aspects of physics become quite transparent in this representation, such as Gribov diffusion and diffractive shrinkage [11, 12], the pion cloud of the nucleon [13], or the limit $x \to 1$ of a fast parton in the target [5]. Much remains to be understood, as illustrated by the nontrivial interplay of $\mathbf{r}$ and $\mathbf{b}$ in saturation dynamics [14]. Measurement of the x_{bj} and t dependence in exclusive ep scattering processes can provide correlated information in longitudinal momentum and transverse position space, and thus in all three space dimensions.

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