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B_K from twisted mass QCD

ALPHA Collaboration

P. Dimopoulos[a] *, J. Heitger [b], C. Pena[c], S. Sint [d] and A. Vladikas[a]

[a]INFN, Sezione di Roma II, Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca scientifica 1, I-00133, Italy

[b]Westfälische Wilhelms-Universität Münster, Institut für Theoretische Physik, Wilhelm-Klemm-Strasse 9, D-48149 Münster, Germany

[c]DESY, Theory Group, Notkestrasse 85, D-22607 Hamburg, Germany

[d]Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI, Universidad Autónoma de Madrid, Cantoblanco E-28049 Madrid, Spain

We present some preliminary results for B_K at $\beta = 6.0$, using the twisted mass QCD formalism for the computation of bare matrix elements of the $\Delta S = 2$ operator. The main advantage of the method is that mixing with other $d = 6$ operators under renormalisation is avoided. Moreover the operator renormalisation is performed in the Schrödinger functional (SF) framework, using earlier results of our collaboration for the corresponding step scaling function.

1. B_K and tmQCD

Let us start by recalling in brief the tmQCD framework for the computation of B_K [1]. We start from a continuum fermion action of the form

$$S_{\rm F} = \int {\rm d}^4x \left\{ \bar\psi(x) \left[\not{D} + m_l + i\mu_l \gamma_5 \tau_3 \right] \psi(x) + \bar s(x) \left[\not{D} + m_s \right] s(x) \right\} , \qquad (1)$$

where $\psi = (u, d)^T$ is a doublet of degenerate light quarks, and τ_3 acts on isospin space. The axial transformation $\psi \to e^{i\alpha\gamma_5\tau_3/2}\psi$, $\bar\psi \to \bar\psi\, e^{i\alpha\gamma_5\tau_3/2}$ leaves the form of the action invariant, and induces a rotational transformation of the mass parameters. In particular, by choosing the rotation angle such that $\tan(\alpha) = \mu_l/m_l$ the standard QCD action is recovered. The quantity $M_l = \sqrt{m_l^2 + \mu_l^2}$ is left invariant, and can be identified with the physical light quark mass.

The above chiral rotation, on the other hand, can be seen as a change of fermion variables which induces a mapping between composite operators. If we denote with a prime the quantities in standard QCD variables then one has, in particular,

$$A'_\mu = \cos\left(\frac{\alpha}{2}\right) A_\mu - i \sin\left(\frac{\alpha}{2}\right) V_\mu \ , \qquad (2)$$

$$V'_\mu = \cos\left(\frac{\alpha}{2}\right) V_\mu - i \sin\left(\frac{\alpha}{2}\right) A_\mu \ , \qquad (3)$$

$$O'^{\Delta S=2}_{\rm VV+AA} = \cos(\alpha) O^{\Delta S=2}_{\rm VV+AA} - i \sin(\alpha) O^{\Delta S=2}_{\rm VA+AV} \ , \qquad (4)$$

where all the operators have an $s - d$ flavour structure. These tree-level properties are immediately extended to the renormalised quantum theory, provided renormalised quark masses and composite operators are used in the above relations. Then Eq. (4) implies that, for the special case $\alpha = \pi/2$, the physical matrix element of $O^{\Delta S=2}$ entering B_K is given in the twisted theory by a matrix element of the VA+AV part of the operator, which is protected from mixing under renormalisation by $\mathcal{CPS}$ symmetry and therefore renormalises multiplicatively [2].

To compute B_K we regularise the theory using a Schrödinger Functional (SF) action with Wilson fermions. The action is nonperturbatively O(a) improved in the bulk of the SF cylinder, and one-loop O(a) improved at the time boundaries. The

*Based on a poster presented by P. Dimopoulos at the LATTICE 2003 Conference (Tsukuba, Japan).

Set	$aM_{\rm PS}$	$aM'_{\rm PS}$	B_K
I	0.3901(15)	0.3892(16)	1.022(11)
II	0.3561(20)	0.3546(20)	0.999(24)
III	0.3298(19)	0.3284(19)	0.984(23)
IV	0.3175(23)	0.3040(24)	1.003(27)
		Extrap.	0.959(64)

Table 1
Values of the bare parameter B_K and pseudoscalar masses for the four sets of quark masses and extrapolation of B_K to the physical kaon mass. (Sets correspond to 245, 150, 150, 102 gauge configurations respectively.)

bare value of B_K is extracted from a fit to the central plateau of the ratio of correlation functions

$$R_{B_K}(x_0) = \frac{-i\langle \mathcal{O}' O^{\Delta S=2}_{\rm VA+AV} \mathcal{O}\rangle}{\frac{8}{3}\langle \mathcal{O}' \mathcal{K}(x)\rangle\langle \mathcal{K}(x)\mathcal{O}\rangle} \ , \qquad (5)$$

where $\mathcal{K} = \frac{1}{\sqrt{2}}[Z_A A_0 - iZ_V V_0]$ is the combination of renormalised operators which represents the physical axial current in twisted variables, and $\mathcal{O} = L^{-3}\sum_{\mathbf{x},\mathbf{y}} \bar{\zeta}_d(\mathbf{x})\gamma_5\zeta_s(\mathbf{y})$ is a pseudoscalar SF boundary source[1].

2. Results

Here we present results at $\beta = 6.0$, all of them on a $16^3 \times 48$ lattice, corresponding to an approximate physical volume of $(1.5 \text{ fm})^3 \times 4.5$ fm. All the simulations have been carried out on the APEmille machines at DESY-Zeuthen. Four sets of quark mass values have been considered. Three of them correspond to degenerate $s-d$ quark masses [2], and the fourth one introduces a small ($\approx 15\%$) $s-d$ isospin breaking.

The lowest pseudoscalar meson mass can be extracted either from an $s-d$ two-point correlation function with the appropriate quantum numbers or, alternatively, from an $s-d'$ correlation function, where d' is an untwisted down quark with exactly the same mass as s. In Table 1 (2nd and 3rd columns) we show the values $M_{\rm PS}$ and $M'_{\rm PS}$ for the corresponding pseudoscalar masses. If the twisted d quark is degenerate in mass with the s up to $\mathcal{O}(a^2)$ cutoff effects, then also the two meson masses should differ only by $\mathcal{O}(a^2)$ effects. This is clearly seen in the data, which show no noticeable difference between $M_{\rm PS}$ and $M'_{\rm PS}$ in the first three cases, whereas in the fourth case isospin breaking manifests itself in the non–equality of the masses.

[1] In Eq. (5), in standard SF fashion, the prime denotes the $x_0 = T$ boundary.
[2] The degeneracy holds for renormalised quark masses and up to $\mathcal{O}(a^2)$ cutoff effects.

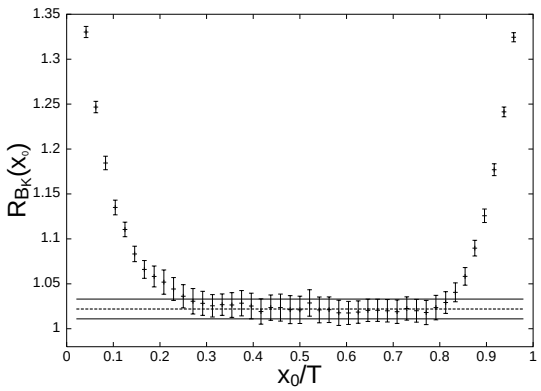

Figure 1. $R_{B_K}(x_0)$ and the fit to the plateau for B_K for the data set I of Table 1.

In Table 1 (4th column) we present our results for the bare value of B_K for each parameter set, as well as the result obtained by extrapolation to the physical kaon mass, which we take to be $aM_K = 0.2336$. In all cases the fitting interval is $x_0/a \in [16, 32]$. As an example in Fig. 1 we depict the ratio R_{B_K} and the fit to the plateau which gives the B_K for the data set I.

The data also allow to examine the chiral behaviour of the $\langle \bar{K}^0 | O^{\Delta S=2} | K^0 \rangle$ matrix element, which can be directly computed from the ratio

$$R_{O_{KK}}(x_0) = \frac{-i\langle \mathcal{O}' O^{\Delta S=2}_{\rm VA+AV} \mathcal{O}\rangle}{\langle \mathcal{O}'\mathcal{O}\rangle} \ , \qquad (6)$$

and then studied as a function of $(aM_K)^2$. The result is shown in Fig. 2. The operator $O^{\Delta S=2}_{\rm VA+AV}$ has been renormalised in the first of the SF schemes discussed in the next section, at the hadronic scale $\mu = 1/(2L_{\rm max})$. The intercept of the linear extrapolation is $C = -0.00038(27)$, i.e. compatible with zero within 1.5 standard deviations. We interpret this as a signal of correct

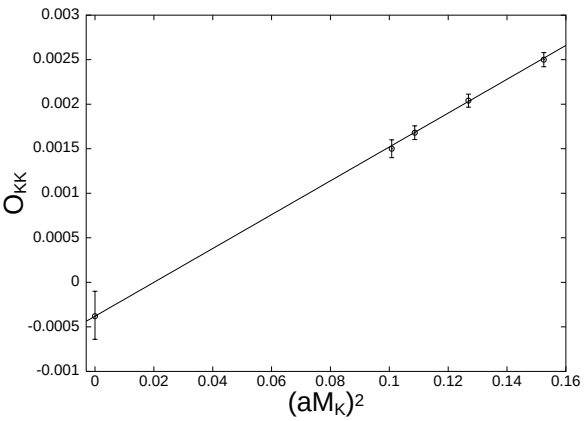

Figure 2. Extrapolation to the chiral limit of the $K^0 - \bar{K}^0$ matrix element.

chiral behaviour, in view of the fact that the prediction of a vanishing matrix element in the chiral limit is only true for this quantity in the continuum. Here we are working with an unimproved matrix element computed at fixed lattice spacing, and the behaviour in Fig. 2 is still affected by mild $\mathcal{O}(a)$ effects. Moreover, as in previous studies with Wilson fermions, the data lie in a region where LO chiral perturbation theory predictions have to be taken with care.

3. Renormalisation

For the nonperturbative renormalisation of $O^{\Delta S=2}_{\rm VA+AV}$ we use SF techniques, as described in [3]. Nine different SF renormalisation schemes are available, each one of them supplying a formally (up to statistical correlations) independent determination of the renormalisation group invariant parameter $B_K^{\rm RGI}$. Having only $\beta = 6.0$ results and thus no continuum extrapolation yet, an estimate of the latter is obtained by multiplying the bare value of B_K times the renormalisation constant at $\beta = 6.0$ and renormalisation scale $1/2L_{\rm max}$, after which the result is multiplied by the continuum quantity $B_K^{\rm RGI}/\bar{B}_K^{\rm SF}(1/(2L_{\rm max}))$, obtained using the corresponding nonperturbative step scaling function and NLO perturbation theory (see [3] for details). The renormalisation constant and the ratio are scheme-dependent quantities, whereas the final result is not. In Table 2 we supply the resulting estimate for $B_K^{\rm RGI}$ and $\bar{B}_K^{\overline{\rm MS}}(2\ {\rm GeV})$ in each of the nine schemes.

Scheme	$B_K^{\rm RGI}$	$\bar{B}_K^{\overline{\rm MS}}(2\ {\rm GeV})$
1	0.88(7)	0.64(5)
2a	0.94(8)	0.68(6)
2b	0.92(8)	0.67(6)
3a	0.89(7)	0.64(5)
3b	0.87(7)	0.63(5)
4a	0.94(7)	0.68(5)
4b	0.93(8)	0.67(6)
5a	0.94(8)	0.68(6)
5b	0.93(8)	0.67(6)

Table 2
Values for $B_K^{\rm RGI}$ and $\bar{B}_K^{\overline{\rm MS}}(2\ {\rm GeV})$ obtained after renormalisation at $\beta = 6.0$.

The combination of the nine estimates of $B_K^{\rm RGI}$ will ultimately yield a good control over systematic effects and a reduction of the uncertainty of the final result. It has to be stressed, though, that the results in Table 2, while being compatible within errors, involve different cutoff effects, and hence a reliable answer can be obtained only after extrapolation to the continuum limit (simulations at higher values of β will follow soon). Note, however, that the $\beta = 6.0$ result is already close to the continuum limit values found in the literature, which suggests that cutoff results in our framework might turn out to be remarkably small.

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