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Towards a precision computation of $F_{\mathrm{B_s}}$ in quenched QCD * †

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We present a computation of the decay constant $F_{\mathrm{B_s}}$ in quenched QCD. Our strategy is to combine new precise data from the static approximation with an interpolation of the decay constant around the charm quark mass region. This computation is the first step in demonstrating the feasability of a strategy for F_{B} in full QCD. The continuum limits in the static theory and at finite mass are taken separately and will be further improved.

1. Introduction

A non-perturbative precise computation of F_{B} in full lattice QCD with reliable errorbars would be a major achievement with important phenomenological implications for CKM physics and the search for new fundamental processes. Such a computation has to meet three main obstacles: chiral extrapolation, the heavy b-quark and the large computational cost due to unquenching. We avoid chiral extrapolations for the time being by setting the light quark mass to the strange mass [1], thus addressing $F_{\mathrm{B_s}}$, which is of interest in itself, for example in $\mathrm{B_s} - \overline{\mathrm{B}}_{\mathrm{s}}$ mixing. A strategy to deal with the second problem has been suggested in [2,3]. The idea is to use HQET and the $1/m$ expansion after a non-perturbative matching to QCD in finite volume. No large lattices are required and the method can thus be applied to quenched as well as full QCD. This strategy seems to be viable now thanks to the considerable improvement of the statistical precision for computations in the static approximation to QCD that has been found in [4,5] The required order of the $1/m$ expansion is unknown. However, we hope that the size estimates of the terms in the HQET presented in [6] are correct and the linear term proves sufficient to keep the systematics under control. This can with the present computational power only be tested in the quenched approximation.

Here we compare the renormalization group invariant matrix element $\Phi^{\mathrm{stat}}_{\mathrm{RGI}}$ of the static axial current to relativistic data around the charm quark mass. Later we will also compute the $1/m$ term in the static approximation to complete the validation of our strategy for $F_{\mathrm{B_s}}$. We use an *interpolation* of our relativistic data and of $\Phi^{\mathrm{stat}}_{\mathrm{RGI}}$ to obtain a precise quenched value of $F_{\mathrm{B_s}}$ in the continuum limit. Previous results for $F_{\mathrm{B_s}}$ have been reviewd in [7] and a new computation has recently appeared [8].

2. Numerical Results

The renormalization group invariant matrix element $\Phi^{\mathrm{stat}}_{\mathrm{RGI}}$ at infinite mass is related to the pseu-

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doscalar decay constant $F_{\rm PS}$ at finite mass by a matching factor $C_{\rm PS}$,

$$F_{\rm PS}\sqrt{m_{\rm PS}} = C_{\rm PS}(\frac{M}{\Lambda_{\overline{\rm MS}}}) \times \Phi^{\rm stat}_{\rm RGI} + {\rm O}(\frac{1}{m}). \qquad (1)$$

$C_{\rm PS}$ can be expressed as a function of the renormalization group invariant heavy quark mass M,

$$\begin{aligned} x &= 1/\log(\frac{M}{\Lambda_{\overline{\rm MS}}}) \leq 0.62 \;\Rightarrow \\ C_{\rm PS}(x) &= x^{-\frac{2}{11}}(1 - 0.06814\,x - 0.08652\,x^2 \\ &\quad +0.07939\;x^3). \end{aligned} \qquad (2)$$

This expression is a simple and accurate parametrization of $C_{\rm PS}$ obtained by integration of the entering renormalization group functions as explained in [9], where the anomalous dimension of the static axial current is taken to 3 loops [10]. Its uncertainty is estimated to be smaller than two percent which is half of the difference between the 2-loop and the 3-loop result.

Our computational setup has been explained in [11,12]. In particular we use Schrödinger functional boundary conditions and employ non-perturbative O(a) improvement.

We obtain the decay constant at five meson masses from $\approx 1.7\,$GeV to $\approx 2.6\,$GeV at four lattice spacings a ranging from $0.1\,$fm to $0.05\,$fm. Since the data at the different lattice spacings are not at exactly the same pseudoscalar masses, we interpolate them linearly in $1/m_{\rm PS}$ and evaluate the interpolating functions at a few meson masses. These numbers can then be extrapolated linearly in a^2 to the continuum limit. This has to be done with care however, since the lattice artifacts depend on the mass. To avoid large discretization errors in the slope of the continuum extrapolation we thus follow the experience from perturbation theory [13] and leave out more and more coarse lattices at larger masses. At the largest masses only the finest lattice can be used and we take that point as the continuum limit. We add to its error the difference to the continuum value obtained from a linear fit from the two finest lattices. The continuum limits for two pseudoscalar masses are illustrated in figure 1 which shows the case with the two coarsest lattices left out in the upper part and the case where only one lattice contributes in the lower part.

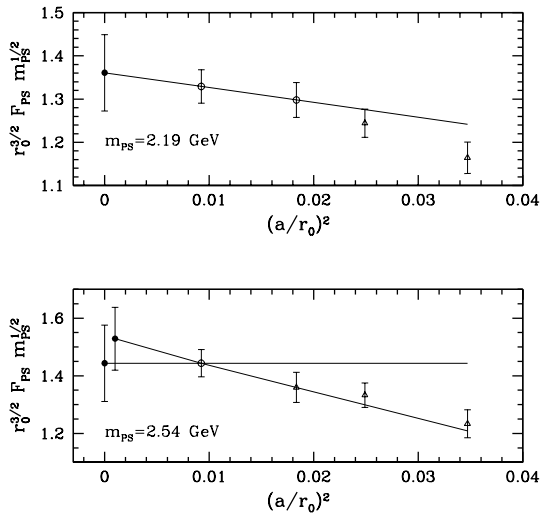

Figure 1. Continuum extrapolation of the decay constant $F_{\rm PS}\sqrt{m_{\rm PS}}$ at two meson masses from two resp. one lattice.

Our result is still preliminary since we will add an even finer lattice with $a \approx 0.03\,$fm (cmp. [12]) and since the details of the continuum extrapolations are still being discussed. Finally we choose five points close to our original data to be used in the interpolation between the static and the relativistic case.

To compare with $\Phi^{\rm stat}_{\rm RGI}$ we still have to compute the renormalization group invariant masses M that are needed for the evaluation of the matching factor $C_{\rm PS}$. Here we follow exactly [14] for meson masses in the range considered.

We take the static result from [4],

$$\Phi^{\rm stat}_{\rm RGI} = 1.74(13), \qquad (3)$$

which has been obtained in the continuum limit from three lattice resolutions with $a \approx 0.1\,{\rm fm}\ldots 0.07\,{\rm fm}$ with a new static action that uses HYP smeared links [15]. Furthermore, wave functions for the states at the Schrödinger functional boundary have been used together with an elaborate technique to reduce the contribution from the first excited state [4] and the renormalization factor has been computed non-perturbatively [9].

Our results are shown in figure 2 as a function of the inverse pseudoscalar mass. At those points where the number of lattices taken into account in the continuum extrapolation changes, there is a small systematic effect which explains

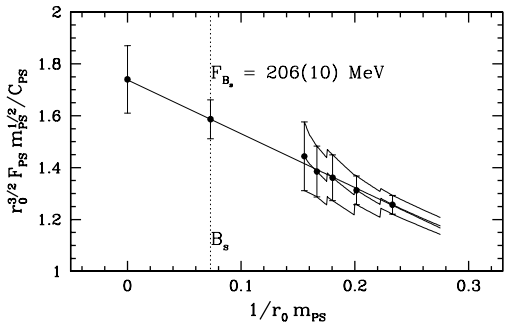

Figure 2. Comparison of the renormalization group invariant matrix element of the static axial current with relativistic data around the charm quark mass.

the zigzag behaviour of the 1-sigma band around the relativistic data. If we combine the five points selected above and the static point by an interpolation in $1/m_{\rm PS}$ we can obtain $F_{\rm B_s}$. To this end we use $M_{\rm b} = 6.96(18)\,\mathrm{GeV}$ [2] and $\Lambda_{\overline{\rm MS}}^{N_{\rm f}=0} = 238(19)\,\mathrm{MeV}$ [16] to evaluate the matching factor $C_{\rm PS}$ at the b-scale. Both, a linear and a quadratic interpolation lead to

$$F_{\rm B_s} = 206(10)\,\mathrm{MeV}. \qquad (4)$$

Here we have used $m_{\rm B_s} = 5.4\,\mathrm{GeV}$ and set the scale with $r_0 = 0.5\,\mathrm{fm}$.

Furthermore we notice, that the data are well described by the linear interpolation that is displayed in the plot. The slope of the fit is $-2.1(5)$ while without the constraint through the static approximation we would obtain $-2.1(1.1)$. To be able to compare this result with the slope computed from the HQET it is desirable to get this slope even more precise.

As in [11] we can use the slope of the interpolation to get an estimate of the quenched scale ambiguity of $F_{\rm B_s}$, i.e. of the effect of changing r_0 within 10 %. Under this change $F_{\rm B_s}$ changes by 12 %. Note that the true quenching error can of course only be estimated by an unquenched calculation.

3. Conclusion

Our comparison of the static and the relativistic results shows that at the current level of statistical precision nothing contradicts the hope that the $1/m$ expansion including only the linear term is enough to compute $F_{\rm B_s}$ from the HQET in full QCD at a sufficient precision. To further validate this strategy we will compute the $1/m$ term in the quenched approximation from HQET.

The combination of $\Phi_{\rm RGI}^{\rm stat}$ with our data at finite mass results in a new value for $F_{\rm B_s}$ in quenched QCD, $F_{\rm B_s} = 206(10)\,\mathrm{MeV}$, including all the systematic errors up to quenching. The quenched scale ambiguity is estimated by 12 %. Since we *inter*polate our data, this is a stand alone result that is almost independent of any effective theory, as is indicated by the fact that a linear and a quadratic function yield identical values.

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