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Static quarks with improved statistical precision *

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We present a numerical study for different discretisations of the static action, concerning cut-off effects and the growth of statistical errors with Euclidean time. An error reduction by an order of magnitude can be obtained with respect to the Eichten-Hill action, for time separations beyond 1.3 fm, keeping discretization errors small. The best actions lead to a big improvement on the precision of the quark mass $M_{\rm b}$ and $F_{\rm B_s}$ in the static approximation.

1. INTRODUCTION

Euclidean correlation functions in the static approximation are known to be very noisy, the noise to signal ratio $R_{\rm NS}$ growing exponentially with the time separation. For the Eichten-Hill (EH) action [1], the law

$$R_{\rm NS} \equiv \frac{\rm noise}{\rm signal} \propto \exp\left(x_0\,\Delta\right)\ ,\ \Delta = E_{\rm stat} - m_\pi\,, \quad (1)$$

is roughly fulfilled [2], with the ground state energy of a heavy meson $E_{\rm stat}$ being linearly divergent while approaching the continuum limit.

Here we explore the possibility of reducing the exponent in eq. (1) by changing the discretisation of the static lattice action. On the other hand we want to retain some properties of the Eichten-Hill action in order to preserve the same level of O(a) improvement, in particular

- heavy quark spin symmetry,
- local conservation of heavy quark flavor number,

together with gauge, cubic and parity invariance and locality. Writing the static lattice action

$$S^{\rm W}_{\rm stat} = a^4 \sum_x \overline{\psi}_{\rm h}(x) D_0 \psi_{\rm h}(x), \quad (2)$$

$$D_0\psi_{\rm h}(x) = \frac{1}{a}[\psi_{\rm h}(x) - W^\dagger(x-a\hat{0},0)\psi_{\rm h}(x-a\hat{0})],$$

we studied the cases

$$W_{\rm S}(x,0) = V\left[\frac{g_0^2}{5} + \left(\frac{1}{3}{\rm tr}V^\dagger V\right)^{1/2}\right]^{-1}, \quad (3)$$

$$W_{\rm A}(x,0) = V\,, \quad (4)$$

$$W_{\rm HYP}(x,0) = V_{\rm HYP}\,, \quad (5)$$

where V is the average of the 6 staples around the link $U(x,0)$ and $V_{\rm HYP}$ is the HYP-link [3].[2]

2. NUMERICAL RESULTS

We mainly looked at the gain concerning noise reduction and at some scaling properties for the new actions. The setup is defined by the Schrödinger functional scheme implemented with non-perturbatively O(a) improved Wilson actions

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[2]For the coefficients α_1, α_2 and α_3 defining the HYP-smearing we used the values proposed in [3].

for the gauge and the light quark sectors. A more detailed discussion of the framework and definitions of the correlation functions can be found in [4]. Our aim is the computation of the B_s-meson decay constant as well as other b-physics matrix elements.

2.1. Noise reduction and cut-off effects

In Fig. 1 we show $R_{\rm NS}$ for the static-light axial correlator $f_{\rm A}^{\rm stat}$ as obtained from an ensemble of 2500 quenched configurations. This is one of the ensembles used for the computation of $F_{\rm B_s}\sqrt{m_{\rm B_s}}$ in the static limit. The Figure shows that more

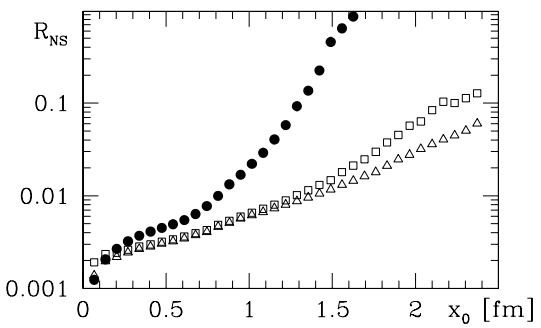

Figure 1. $R_{\rm NS}$ for $f_{\rm A}^{\rm stat}$ from a $24^3\times 36$, $\beta=6.2$ lattice. See text for the symbols.

than an order of magnitude can be gained in $R_{\rm NS}(x_0\simeq 1.5$ fm) with respect to the EH action (filled circles) by using $S^{\rm S}_{\rm stat}$ (empty squares), which behaves similarly to $S^{\rm A}_{\rm stat}$. The picture is even better for $S^{\rm HYP}_{\rm stat}$ (empty triangles).

We then studied the scaling behaviour for a set of observables. In [4] we reported about the step scaling function of the static-light axial current renormalisation constant $Z_{\rm A}^{\rm stat}$ (see [5]). Here we want to present a study of the quantity

$$h=\frac{f_{\rm A}^{\rm stat}(T/2)|_{\theta=0}}{f_{\rm A}^{\rm stat}(T/2)|_{\theta=1/2}}\quad\text{at}\quad m_{\rm q}=0, \tag{6}$$

where θ is the angle defining the periodicity of the fermions. Our results are shown in Fig. 2, they refer to four different lattice resolutions of an L^4 volume with $L=1.436r_0$ with $r_0=0.5$ fm.

Throughout all our computations we defined the O(a) improved correlator $f_{\rm A}^{\rm stat}$ by using the tree level value 1/2 for $b_{\rm A}^{\rm stat}$ [6] and the 1-loop

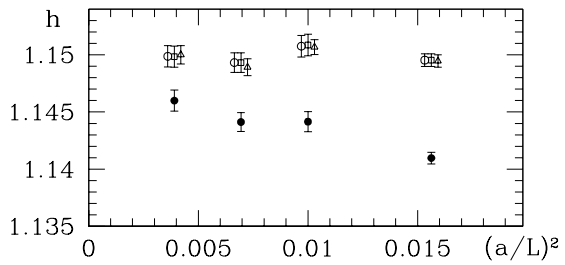

Figure 2. Scaling plot for h. Symbols as in Fig. 1 (empty circles refer to $S^{\rm A}_{\rm stat}$).

results for $c_{\rm A}^{\rm stat}$. In particular we set

$$\begin{aligned}
&c_{\rm A}^{\rm stat}=-0.08237g_0^2+{\rm O}(g_0^4),\quad S_{\rm stat}=S^{\rm EH}_{\rm stat},\\
&c_{\rm A}^{\rm stat}=-0.1164(10)g_0^2+{\rm O}(g_0^4),\ S_{\rm stat}=S^{\rm S}_{\rm stat},S^{\rm A}_{\rm stat},\\
&c_{\rm A}^{\rm stat}=-0.090(3)g_0^2+{\rm O}(g_0^4),\quad S_{\rm stat}=S^{\rm HYP}_{\rm stat}.
\end{aligned}$$

The first two numbers have been worked out in perturbation theory, while the last one has been numerically estimated by solving with respect to $c_{\rm A}^{\rm stat}(S^{\rm HYP}_{\rm stat})$ the implicit equation

$$\frac{f_{\rm A}^{\rm stat}(T/2)|^{\rm A}_{\theta}}{f_{\rm A}^{\rm stat}(T/2)|^{\rm A}_{\theta'}}\,\frac{f_{\rm A}^{\rm stat}(T/2)|^{\rm HYP}_{\theta'}}{f_{\rm A}^{\rm stat}(T/2)|^{\rm HYP}_{\theta}}=1. \tag{7}$$

This defines an improvement condition for a discretisation ($S^{\rm HYP}_{\rm stat}$), once the improvement coefficients for another discretisation ($S^{\rm A}_{\rm stat}$) are known. In Fig. 3 we show our numerical results for the coefficients $c_{\rm A}^{\rm stat,(1)}(S^{\rm S}_{\rm stat})$ and $c_{\rm A}^{\rm stat,(1)}(S^{\rm HYP}_{\rm stat})$, taking as input $c_{\rm A}^{\rm stat}(S^{\rm A}_{\rm stat})$. The same data sets used for computing h in eq. (6) have been used here ($\theta=0$, $\theta'=1/2$). The lower part of the plot can be regarded as a test of the method.

2.2. The B_s-meson decay constant

The setup has been used for computing

$$\Phi_{\rm RGI}\propto Z_{\rm RGI}\frac{f_{\rm A}^{\rm stat}(x_0)}{\sqrt{f_1}}e^{(x_0-T/2)E_{\rm stat}(x_0)}, \tag{8}$$

for which we have calculated the regularisation dependent part of the renormalisation constant $Z_{\rm RGI}$ exactly as done in [5] for the EH action. Here the B_s-meson boundary to boundary correlation function f_1 enters. The proportionality symbol in eq. (8) summarises volume factors coming from the normalisation of the correlation

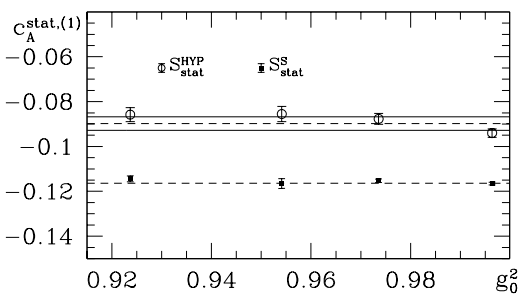

Figure 3. Numerical results for $c_{\rm A}^{\rm stat(1)}$. The dashed line in the lower part is the analytic perturbative result and the error band in the upper part is our estimate for $c_{\rm A}^{\rm stat}$ for $S_{\rm stat} = S_{\rm stat}^{\rm HYP}$.

functions, cmp. [4]. In addition we introduced hydrogen-like wavefunctions on the boundaries of the Schrödinger functional in order to minimise the overlap with the first excited state; we arrive at a plateau of length 0.8 fm in $\Phi_{\rm RGI}$ and the desired ground state matrix element can be extracted with confidence.

The quantity $F_{\rm B_s}\sqrt{m_{\rm B_s}}$ is related to $\Phi_{\rm RGI}$ via

$$\Phi_{\rm RGI} = \frac{F_{\rm B_s}\sqrt{m_{\rm B_s}}}{C_{\rm PS}(M_{\rm b}/\Lambda_{\overline{MS}})} + O(1/M_{\rm b}), \qquad (9)$$

the function $C_{\rm PS}(M_{\rm b}/\Lambda_{\overline{MS}})$ [5] being known in perturbation theory up to and including $\bar{g}^4(m_{\rm b})$ corrections to the leading order [7].

We computed $\Phi_{\rm RGI}$ for three different lattice resolutions ($\beta = 6, 6.1$ and 6.2) on a $L^3 \times T$ topology with $T/L = 3/2$ and $L = 1.5$ to 1.9 fm. The continuum limit extrapolation of $r_0^{3/2}\Phi_{\rm RGI}$, evaluated from $S_{\rm stat}^{\rm HYP}$ at $x_0 = T/2$, is shown in Fig. 4. The extrapolated value is

$$r_0^{3/2}\Phi_{\rm RGI} = 1.74(13). \qquad (10)$$

3. CONCLUSIONS

We have shown that the problem with the statistical precision of correlation functions computed with the Eichten-Hill action can be overcome by changing the lattice discretisation. In particular the largest gain is obtained by making use of the HYP-links in the static action. Cut-off effects for the proposed actions turned out to be of the same size as for the EH action, and are

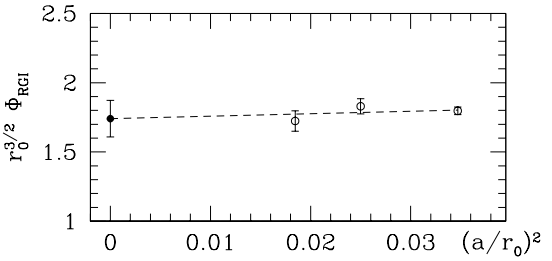

Figure 4. Continuum limit of $\Phi_{\rm RGI} r_0^{3/2}$.

in general rather small. The improvements presented led to a computation of the quantity $\Phi_{\rm RGI}$ in the continuum limit with an error of 7%. We are presently reducing this error further by adding one more lattice resolution ($\beta = 6.45$). The same data can be used for improving the results on the renormalisation group invariant b-quark mass along the lines described in [8]. Finally, a precise determination of $F_{\rm B_s}$ can be obtained by combining the result in eq. (10) with data around the charm quark mass [9,10].

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