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Towards High Precision Predictions for Top Quark Pair Production and Decay at a Linear Collider

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Abstract

We report on the progress in work on improving precision of the standard model theoretical predictions for the top quark pair production and decay into six fermions at a linear collider. Two programs have been combined into a single Monte Carlo program: `eett6f`, a MC program for $e^+e^- \to 6f$, and `topfit`, a program for electroweak radiative corrections to $e^+e^- \to t\bar{t}$. The MC program is described and preliminary numerical results are shown.

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I. INTRODUCTION

In order to disentangle possible new physics effects that may be revealed in the process of top quark pair production

$$e^+e^- \to t\bar{t} \tag{1}$$

at a linear collider from predictions of the standard model (SM), one needs to know the latter with high precision, possibly at the level of a few per mill [1]. That high precision level requires taking into account radiative corrections. Because the top (and antitop) quark of reaction (1) decays into a b (and $\bar{b}$) and fermion-antifermion pairs, we should study the 6 fermion reactions of the form

$$e^+e^- \to bf_1\bar{f}_1'\bar{b}f_2\bar{f}_2', \tag{2}$$

where $f_1, f_2' = \nu_e, \nu_\mu, \nu_\tau, u, c$ and $f_1', f_2 = e^-, \mu^-, \tau^-, d, s$.

Reactions (2) receive contributions from typically several hundred Feynman diagrams already at the tree level, e.g.the hadronic reaction $e^+e^- \to bu\overline{d}\overline{b}d\bar{u}$ in the unitary gauge, neglecting the Higgs boson couplings to fermions lighter than a b-quark, gets contributions from 1484 Feynman diagrams. This is the main reason why the calculation of the full $\mathcal{O}(\alpha)$ radiative corrections to any of reactions (2) seems not to be feasible at the moment.

Therefore, in order to improve the precision of the lowest order predictions one should try to include at least the leading radiative effects, such as initial state radiation (ISR) and factorizable radiative corrections to the process of the on-shell top quark pair production (1) and to the decay of the top and antitop quarks.

In the present lecture, we report on the progress of the SM theoretical predictions for the top quark pair production and decay into six fermions at a linear collider, paying special attention to the precision improvement.

We will show a sample of results on reactions (2) including some higher order effects including the ISR correction obtained with `eett6f` [2], a Monte Carlo program for reactions $e^+e^- \to 6$ fermions relevant for the top production. Further, we introduce the one–loop electroweak corrections to reaction (1) obtained with `topfit` [3], a numerical package for calculating electroweak radiative corrections to the on-shell pair production of a massive fermion–antifermion pair. The two programs have been combined into a single Monte Carlo program.

II. CALCULATING ISR WITH EETT6F

The ISR correction has been implemented recently into `eett6f` [4] in the leading logarithmic (LL) approximation utilizing the structure function approach. In this approach, the corrected differential cross section $\mathrm{d}\,\sigma^{LL}(p_1,p_2)$ of any reaction (2) reads

$$\mathrm{d}\,\sigma_{\mathrm{Born+ISR}}\,(p_1,p_2) = \int_0^1 \mathrm{d}\,x_1 \int_0^1 \mathrm{d}\,x_2\,\Gamma_{ee}^{LL}\left(x_1,Q^2\right)\Gamma_{ee}^{LL}\left(x_2,Q^2\right) \times \mathrm{d}\,\sigma_{\mathrm{Born}}\,(x_1p_1,x_2p_2)\,, \tag{3}$$

where p_1 (p_2) is the four momentum of a positron (electron), x_1 (x_2) is the fraction of the initial momentum of the positron (electron) that remains after emission of a collinear photon and $\mathrm{d}\,\sigma_{\mathrm{Born}}(x_1p_1,x_2p_2)$ is the lowest order cross section calculated at the reduced four momenta of the positron and electron. The structure function $\Gamma_{ee}^{LL}\,(x,Q^2)$ is given by Eq. (67) of [5], with `'BETA'` choice for non-leading terms. The splitting scale Q^2, which is not fixed in the LL approximation is chosen to be equal $s=(p_1+p_2)^2$.

The ISR corrected cross sections, $\sigma_{\mathrm{Born+ISR}}$, of different channels of (2) obtained with the current version of `eett6f` are compared with the results of LUSIFER [6] in Table 1 [4], where also the corresponding lowest order total cross sections σ_{Born} are compared. The reader is referred to [7] for a comparison of the lowest order predictions with other existing MC programs. The input parameters are the same as in [6] and cuts are given by

$$\begin{aligned} &\theta(l,e^{\pm})>\ 5^\circ,\ \theta(q,e^{\pm})>\ 5^\circ,\ \theta(l,l')>\ 5^\circ,\ \theta(l,q)>\ 5^\circ,\\ &E_l>\ 10\ \mathrm{GeV},\ E_q>\ 10\ \mathrm{GeV},\ m(q,q')>\ 10\ \mathrm{GeV}, \end{aligned} \tag{4}$$

where $\theta(i,j)$ is the angle between particles i and j in the laboratory system, q and l denote a quark and a final state charged lepton, respectively, and $m(q,q')$ is the invariant mass of a qq' quark pair.

The results of both programs agree nicely, basically within one standard deviation, except for $e^+e^-\ \to\ b\,\nu_e\,e^+d\,\bar{u}\,\bar{b}$, where the deviation is bigger, amounting to about 3 standard deviations. The discrepancy should be clarified by fine tuned comparisons. Differential cross sections obtained with `eett6f` that are plotted in Fig.1 agree with those presented in [6] within accuracy of the plots.

TABLE I: Comparison of the cross sections of different channels of (2) at $\sqrt{s} = 500$ GeV of LUSIFER [6] and `eett6f` [2, 4]. The input parameters and cuts, see Eqs. (4) are the same as in [6]. All cross sections are in fb. The number in parenthesis show the uncertainty of the last decimals.

	LUSIFER		eett6f	
$e^+e^- \to$	σ_{Born}	$\sigma_{\text{Born+ISR}}$	σ_{Born}	$\sigma_{\text{Born+ISR}}$
$b\,\nu_e\,e^+e^-\,\bar{\nu}_e\,\bar{b}$	5.8530(68)	5.6465(70)	5.8622(63)	5.6441(67)
$b\,\nu_e\,e^+\mu^-\,\bar{\nu}_\mu\,\bar{b}$	5.8188(45)	5.6042(38)	5.8189(37)	5.6075(59)
$b\,\nu_\mu\,\mu^+\mu^-\,\bar{\nu}_\mu\,\bar{b}$	5.8091(49)	5.5887(36)	5.8065(33)	5.5929(54)
$b\,\nu_\mu\,\mu^+\tau^-\,\bar{\nu}_\tau\,\bar{b}$	5.7998(36)	5.5840(40)	5.7992(32)	5.5844(33)
$b\,\nu_\mu\,\mu^+ d\,\bar{u}\,\bar{b}$	17.171(24)	16.561(24)	17.213(23)	16.569(17)
without QCD:	17.095(11)	16.454(10)	17.106(15)	16.457(16)
$b\,\nu_e\,e^+ d\,\bar{u}\,\bar{b}$	17.276(45)	16.577(21)	17.301(26)	16.741(43)
without QCD:	17.187(21)	16.511(12)	17.149(16)	16.522(17)

III. CALCULATING ONE-LOOP ELECTROWEAK CORRECTIONS WITH TOPFIT

`topfit` [3] is a numerical package for calculating the complete one–loop corrections to top-pair production (1) at a linear e^+e^- collider in the continuum energy region. The corrections are represented in terms of six independent form factors, which is suitable for implementation in the Monte Carlo generator. We recall the normalization of the differential cross section

$$\frac{\mathrm{d}\,\sigma(e^+e^- \to t\bar{t})}{\mathrm{d}\,\cos\vartheta} = \frac{\beta_t}{32\pi\,s} \sum_{\text{conf}} \left[|\mathcal{M}_B|^2 + 2\Re\mathrm{e}\,(\mathcal{M}_B^* \delta\mathcal{M})\right] \tag{5}$$

where ϑ is the antitop production angle with respect to the initial e^+ beam, $\beta_t = (1 - 4\,m_t^2/s)^{1/2}$ and the sum stands for the spin and color configuration of external fermions.

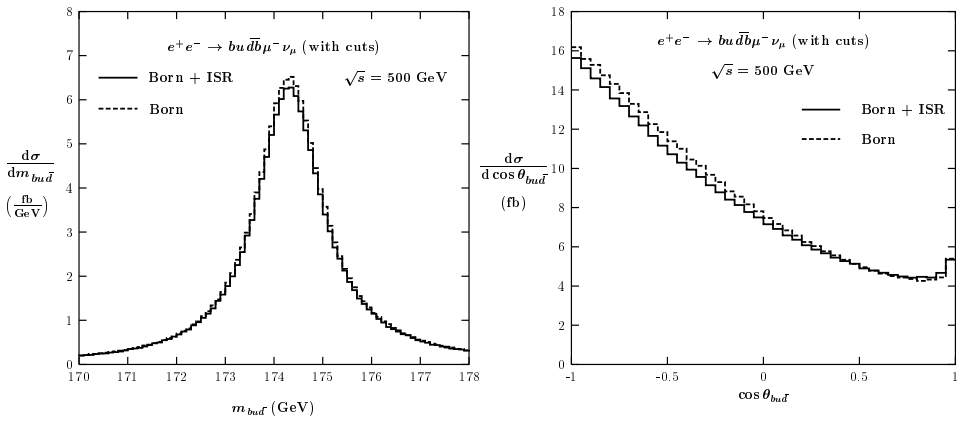

FIG. 1: Invariant mass (left) and angular (right) distributions of a $\bar{b}d\bar{u}$-quark triple at $\sqrt{s} = 500$ GeV with cuts.

There are four nonvanishing form factors F_{1B}^{ab} in Born approximation

$$\mathcal{M}_B = \sum_{a,b=\{1,5\}} \mathrm{F}_{1B}^{ab}\, \mathcal{M}_{1,ab}, \tag{6}$$

while the one–loop correction $\delta\mathcal{M}$ to the amplitude can be expressed in terms of six independent form factors: $\hat{F}_1^{ab},\ a,b = \{1,5\}$, plus additionally $\hat{F}_3^{11}$ and $\hat{F}_3^{51}$:

$$\delta\mathcal{M} = \sum_{a,b=\{1,5\}} \hat{F}_1^{ab}\, \mathcal{M}_{1,ab} + \hat{F}_3^{11}\, \mathcal{M}_{3,11} + \hat{F}_3^{51}\, \mathcal{M}_{3,51}. \tag{7}$$

The invariant amplitudes in Eqs. (6) and (7) are given by

$$\begin{aligned}
i\,\mathcal{M}_{1,\,ab} &= [\bar{v}(p_1)\ \gamma^\mu\, g_a\, u(p_2)\ ]\ [\bar{u}(p_3)\ \gamma_\mu\, g_b\, v(p_4)\ ], \quad a,b = \{1,5\}, \\
i\,\mathcal{M}_{3,11} &= -[\bar{v}(p_1)\ \not{p}_3\, \mathbb{1}\, u(p_2)\ ]\ [\bar{u}(p_3)\ \mathbb{1}\, v(p_4)\ ], \\
i\,\mathcal{M}_{3,51} &= -[\bar{v}(p_1)\ \not{p}_3\, \gamma_5\, u(p_2)\ ]\ [\bar{u}(p_3)\ \mathbb{1}\, v(p_4)\ ].
\end{aligned} \tag{8}$$

The normalization of the form factors is chosen such that the pure photonic Born amplitude becomes $F_{1B}^{11,\gamma} = e^2 Q_e Q_t / s$; (see [3] for the details on the analytic expressions).

The resulting corrected differential cross section formula can then be written in terms of the computed form factors, the top quark color factor (c_t) and the kinematical Mandelstam variables s,t and u ($s + t + u = 2m_t^2$):

$$\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,\cos\vartheta} = \frac{\beta_t c_t}{s}\frac{\alpha^2\pi}{s^2}\Re\mathrm{e}\Big\{ 4sm_t^2 \left(\bar{F}_1^{11}\bar{F}_{1B}^{11*} - \bar{F}_1^{15}\bar{F}_{1B}^{15*} + \bar{F}_1^{51}\bar{F}_{1B}^{51*} - \bar{F}_1^{55}\bar{F}_{1B}^{55*} \right)$$

TABLE II: Born and weak one–loop form factors for $e^+e^- \to t\bar{t}$ at $\sqrt{s} = 500$ GeV and $\cos\vartheta = 0.7$.

	Born F.F. (GeV^{-2})		Weak one–loop F.F. (GeV^{-2})	
	$\Re$e	$\Im$m	$\Re$e	$\Im$m
$\hat{F}_1^{11}$	$-2.50926472\cdot10^{-7}$	0	$1.08874859\cdot10^{-8}$	$-2.55015622\cdot10^{-9}$
$\hat{F}_1^{15}$	$1.56200966\cdot10^{-8}$	0	$-9.75979648\cdot10^{-9}$	$-9.02716758\cdot10^{-9}$
$\hat{F}_1^{51}$	$5.62400131\cdot10^{-8}$	0	$-7.00614109\cdot10^{-9}$	$-6.39336661\cdot10^{-9}$
$\hat{F}_1^{55}$	$-1.37479846\cdot10^{-7}$	0	$-1.11566191\cdot10^{-9}$	$7.86314816\cdot10^{-9}$
$m_t\hat{F}_3^{11}$	0	0	$8.14431322\cdot10^{-10}$	$-8.44590997\cdot10^{-10}$
$m_t\hat{F}_3^{51}$	0	0	$-9.09997715\cdot10^{-10}$	$4.88604910\cdot10^{-10}$

$$
\begin{aligned}
&+2(u^2 - t^2 - 2m_t^2(u-t))\left(\bar{F}_1^{11}\bar{F}_{1B}^{55^*} + \bar{F}_1^{55}\bar{F}_{1B}^{11^*} + \bar{F}_1^{51}\bar{F}_{1B}^{15^*} + \bar{F}_1^{15}\bar{F}_{1B}^{51^*}\right)\\
&+2(u^2 + t^2 + 2m_t^2(s-m_t^2))\sum_{a,b=\{1,5\}}\bar{F}_1^{ab}\bar{F}_{1B}^{ab^*}\\
&+4(ut - m_t^4)\left(\bar{F}_3^{11}\bar{F}_{1B}^{11^*} + \bar{F}_3^{51}\bar{F}_{1B}^{51^*}\right)\Big\}, \qquad (9)
\end{aligned}
$$

where the dimensionless *barred* form factors are defined for convenience as

$$
\begin{aligned}
\bar{F}_{1B}^{ab^*} &\equiv \frac{s}{e^2}F_{1B}^{ab^*},\\
\bar{F}_1^{ab} &\equiv \frac{s}{e^2}\left(\frac{1}{2}F_{1B}^{ab} + \hat{F}_1^{ab}\right), \qquad (10)\\
\bar{F}_3^{ab} &\equiv \frac{s}{e^2}m_t\hat{F}_3^{ab}.
\end{aligned}
$$

`topfit` is rather flexible and includes also real photonic bremsstrahlung. In order to fit the needs of the MC program described here, some specific values of flags have been chosen: `IWEAK` $= 1$ (inclusion of pure weak corrections), `IQED` $=$ `IQEDAA` $= 0$. The latter two settings switch off the calculation of photonic corrections due to bremsstrahlung (`IQED`, with the related virtual corrections) and of the fermionic vacuum polarization effects (`IQEDAA`). The sample corrections were computed using the same input parameters as in [8]. In Table II, form factors are given for $\sqrt{s} = 500$ GeV and $\cos\vartheta = 0.7$, and in Table III some differential cross section values for $\sqrt{s} = 500$ GeV.

TABLE III: Differential cross sections for $e^+e^- \to t\bar{t}$ at $\sqrt{s} = 500\text{GeV}$.

$\cos\vartheta$	$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\vartheta}\Big\vert_{\text{Born}}$ (pb)	$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\vartheta}\Big\vert_{\text{Born+Weak}}$ (pb)
-0.9	0.1088391941	0.1011751626
-0.7	0.1218770828	0.1136440560
-0.5	0.1422750694	0.1332668988
0.0	0.2254704640	0.2115640646
0.5	0.3546664703	0.3300633143
0.7	0.4192250441	0.3883965473
0.9	0.4911437158	0.4528185418

IV. NUMERICAL RESULTS

In the present section, we will show the results of implementation of the one–loop electroweak form factors for on-shell top quark pair production obtained with `topfit` in the MC program `eett6f`. Amplitudes (8) have been implemented in `eett6f` with the helicity amplitude method of [9, 10]. The actual numerical values of the form factors $F_{1B}^{ab}, \hat{F}_1^{ab}$, $a, b = \{1, 5\}$, $\hat{F}_3^{11}$ and $\hat{F}_3^{51}$ are computed by `topfit` and then transferred to `eett6f`. Unfortunately, the direct calculation of the one–loop corrections slows down the MC computation by more than a factor 1000. In practice, this problem has been solved applying the following strategy: the values of the one–loop form factors are computed for a given c.m.s. energy for a grid of several hundred, or a few thousand different values of cosine of the antitop scattering angle, $\cos\vartheta$. These fixed values of the form factors are stored in the computer memory and then used for computation of the form factors at any intermediate value of $\cos\vartheta$ by means of a linear interpolation between the neighbouring fixed values. The interpolation routine has been tested and shown to be very precise and efficient.

As a first step we calculate the differential cross section of reaction (2) in the narrow width approximation for the top and antitop, which is obtained by multiplying the one–loop corrected cross section (5) with the corresponding branching fractions leading to a specific

final state of (2)

$$\mathrm{d}\,\sigma\left(e^+e^- \to t\bar{t} \to bf_1\bar{f}_1'\bar{b}f_2\bar{f}_2'\right) = \mathrm{d}\,\sigma(e^+e^- \to t\bar{t}) \times \mathrm{d}\,\Gamma\left(t \to bf_1\bar{f}_1'\right)\ \mathrm{d}\,\Gamma\left(\bar{t} \to \bar{b}f_2\bar{f}_2'\right)/{\Gamma_t}^2. \tag{11}$$

This approach is gauge invariant and can be used as the reference for any other way of implementation of the one–loop electroweak corrections in the MC generator for a simulation of reactions (2) to $\mathcal{O}(\alpha)$.

Typical results for one specific channel of reaction (2) in the narrow top width approximation of Eq. (11) are shown in Figs. 2 and 3. The differential cross sections of reaction $e^+e^- \to b\nu_\mu\mu^+\bar{b}d\bar{u}$ at $\sqrt{s} = 500$ GeV without cuts are plotted on the left hand side of Fig. 2. Both the lowest order (dotted line) and one–loop corrected (solid line) differential cross sections in the b-quark energy are shown. The same differential cross sections with cuts of conditions (4) are plotted on the right hand side of Fig. 2. The corresponding differential cross sections in the energy of μ^+ are plotted in Fig. 3.

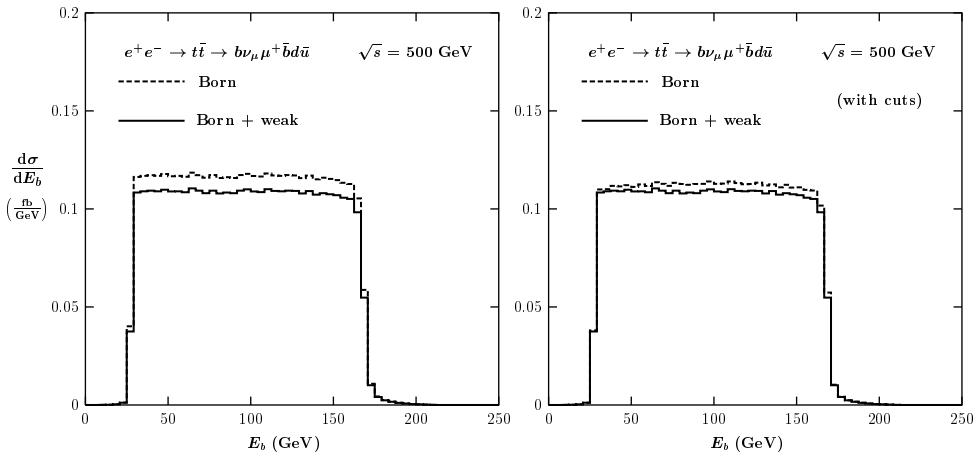

FIG. 2: Energy distributions of a b-quark at $\sqrt{s} = 500$ GeV without cuts and with cuts.

We see that the quite substantial effect of the one–loop electroweak correction on differential cross sections of $e^+e^- \to b\nu_\mu\mu^+\bar{b}d\bar{u}$ at $\sqrt{s} = 500$ GeV is reduced by the cuts (4). This kind of reduction is of course not desirable if one wants to test the consistency of the SM at the quantum level. However, it might be helpful in performing relatively fast numerical simulations of different channels of reaction (2) with the use of the MC generators working only with the lowest order amplitudes.

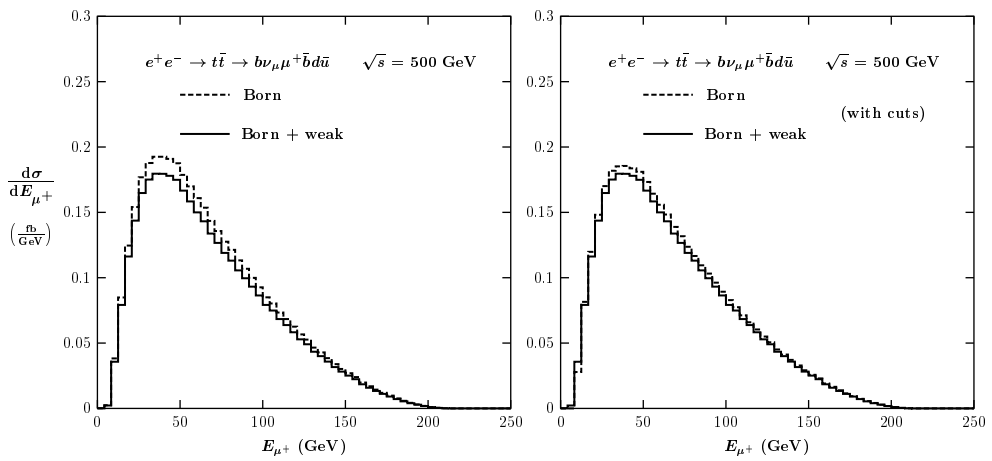

FIG. 3: Energy distributions of a μ^+ at $\sqrt{s} = 500$ GeV without cuts and with cuts.

V. SUMMARY AND OUTLOOK

We have reported on the progress in our work on improving precision of SM theoretical predictions for the top quark pair production and decay into six fermions at a linear collider. Apart from including the ISR effects in `eett6f` [4], we have created a MC program that combines two programs `eett6f` and `topfit`. The program allows for including one–loop electroweak corrections to the top quark pair production in reactions (2) in the narrow width approximation for the top and antitop. The description of these corrections to the on-shell top pair production has to be complemented by the corresponding ones to the decays of the top quarks [11], in order to complete the double pole approximation. This is in preparation [12].

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[1] R.-D. Heuer, D. Miller, F. Richard, P. M. Zerwas (eds.), *TESLA Technical Design Report, Part III: Physics at an e^+e^- Linear Collider*, DESY 2001-011, ECFA 2001-209, TESLA Report 2001-23, TESLA-FEL 2001-05, hep-ph/0106315;

T. Abe *et al.*, American Linear Collider Working Group Collaboration, *Resource book for Snowmass 2001*, SLAC-R-570, hep-ex/0106055-058;

K. Abe *et al.*, ACFA Linear Collider Working Group Collaboration, *Particle physics experi-*

ments at JLC, hep-ph/0109166.

[2] K. Kołodziej, Eur. Phys. J. C23 (2002) 471, hep-ph/0110063 and Comput. Phys. Commun. 151 (2003) 339, hep-ph/0210199;
A. Biernacik, K. Kołodziej, Nucl. Phys. Proc. Suppl. 116 (2003) 33, hep-ph/0210405.

[3] J. Fleischer, A. Leike, T. Riemann, A. Werthenbach, Eur. Phys. J. C31 (2003) 37, hep-ph/0302259;
J. Fleischer, A. Leike, A. Lorca, T. Riemann, A. Werthenbach, Fortran program `topfit` v.0.93 (10 Jun 2003); available at
`http://www-zeuthen.desy.de/~riemann/doc/topfit/topfit.html`.

[4] K. Kołodziej, in preparation.

[5] W. Beenakker *et al.*, in G. Altarelli, T. Sjöstrand, F. Zwirner (eds.), Physics at LEP2 (Report CERN 96-01, Geneva, 1996), Vol. 1, p. 79, hep-ph/9602351.

[6] S. Dittmaier, M. Roth, Nucl. Phys. B642 (2002) 307, hep-ph/0206070.

[7] S. Dittmaier, hep-ph/0308079.

[8] T. Hahn, W. Hollik, A. Lorca, T. Riemann, A. Werthenbach, hep-ph/0307132.

[9] K. Kołodziej, M. Zrałek, Phys. Rev. D43 (1991) 3619.

[10] F. Jegerlehner, K. Kołodziej, Eur. Phys. J. C12 (2000) 77.

[11] A. Denner and T. Sack, Nucl. Phys. B358 (1991) 46.

[12] K. Kołodziej et al., in preparation.